

Ragnar Frisch's "Circulation Planning": An Attempt at Modelling General Equilibrium

Ariane Dupont-Kieffer¹

We investigate how Ragnar Frisch's (1895-1973) came to use econometric tools and techniques for policy purposes and how this involved a general equilibrium approach.

Ragnar Frisch is well known for his contribution to the development of econometrics from the mid-1920s onwards, both on the scientific side, especially in the analysis of economic fluctuations, and on the institutional side. By the mid-1930s, however, he was taking an active part in the invention of new methodological and theoretical instruments based upon a specific shift from "political Economy" towards "rational Economy".

This article argues that "Circulation planning"—a controversial 1934 article inspired by a Walrasian organization of exchanges—is interesting on two accounts: firstly because Frisch elaborates his view of the economic crisis as a breakdown of the system of exchanges due to monetary causes, and secondly because he offers to solve the crisis by establishing a central agency in charge of the organisation of an optimal system of exchanges. The difficulty of such modelling and the conceptual dead-ends of the 1934 article led Frisch to abandon the General Equilibrium approach for a long time and, in the following years, to develop a specific methodology in order to be able to implement indirect planning.

Keywords : econometric modelling, economic policy and planning, general equilibrium

¹ Ariane Dupont-Kieffer, Université Paris-Est, IFSTTAR, DEST, F-93166 Noisy-le-Grand, France, Postal address: DEST, Ifsttar, 2 rue de la butte verte; 93166 Noisy le Grand Cedex, France, ariane.dupont@ifsttar.fr. I would like to thank the participants in the conference "General Equilibrium as Knowledge: From Walras Onwards" (Paris, September 2007) and with warm thoughts, I would like to thank John Aldrich and Isabelle Saint Saens for their help on editing and making ideas clearer and for their kind support. Special thanks to the two anonymous referees, Olav Bjerkholt, Jean-Sébastien Lenfant, and Julien Dupont for their comments, and fruitful discussions. All mistakes remain mine.

« Circulation Planning » : une tentative de Ragnar Frisch pour modéliser l'équilibre général

Cet article se penche sur les circonstances qui ont vu Frisch recourir à des techniques et des outils économétriques à des fins de politique économique, et ce à l'occasion de son approche de l'équilibre général.

Frisch est connu et reconnu pour sa contribution au développement de l'économétrie à partir du milieu des années 1920, contribution tant scientifique – notamment sur l'analyse des fluctuations de l'activité économique –, qu'institutionnelle. Toutefois, à partir du milieu des années trente, il prend une part active à l'invention de nouveaux instruments méthodologiques et théoriques dans le cadre du basculement de « l'économie politique » vers « l'économie rationnelle ».

Cette étape cruciale dans l'œuvre de Frisch, que représente son modèle d'équilibre général développé dans « Circulation Planning », article controversé de 1934 et inspiré par une organisation walrassienne des échanges, est intéressant à double titre : en premier lieu parce que Frisch y élabore sa compréhension des cycles économiques comme une défaillance d'origine monétaire du système des échanges et en second lieu, parce qu'il propose comme solution à la crise la création d'une agence centrale chargée d'organiser le système optimal des échanges.

Les difficultés inhérentes à une telle modélisation et les impasses théoriques rencontrées dans cet article de 1934 conduiront Frisch à abandonner pour un long moment l'approche de l'équilibre général, et à se concentrer sur l'élaboration, au cours des années suivantes, d'une méthodologie spécifique pour mettre en place une planification indirecte.

Mots clés : modélisation économétrique, politique et planification économique, équilibre général

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1. Introduction: Ragnar Frisch and his Seminal Heuristic Ambitions

Ragnar Frisch is well known for his contribution towards developing *econometrics*. Specifically, by the mid-1920s, Frisch devoted his efforts to the analysis of the fluctuations of economic activity, and this led him to the unification of mathematics, statistics and economic theory, which came to be the credo of econometrics bridging empirical quantitative economics and theoretical quantitative economics (Morgan, 1990, Dupont-Kieffer, 2003, chapter 5, Boumans, 2007, Bjerkholt and Dupont-Kieffer, 2009).

However, by the mid-thirties, his scientific work was also driven by political preoccupations. Frisch was very active in the invention of new methodological and theoretical instruments to promote economic policies and planning. In doing so, he took an active part in the contemporary specific shift from “political Economy” towards

"rational Economy" in the interwar years as described by Michel Beaud and Gilles Dostaler (1993, 69) and after the Second World War; this shift reflected the "triumph of interventionism" (*ibid.*). Indeed, the depth and extent of the economic crisis of the thirties and later the need for Europe to be rebuilt after World War II forced on politicians and citizens the need for an active, even radical, intervention of political authorities in economic affairs. From London to Moscow, new theoretical proposals and frameworks arose in order to define the nature of this intervention and to set its modalities. Econometricians, more specifically Jan Tinbergen and Ragnar Frisch, took an active part in this shift by developing new tools such as macrodynamic models and national accounts that can be used for economic policymaking. They gave to the shift a technical touch. They also helped to develop relations between the academic world, the political sphere and civil service, by training the future elites, politicians and civil servants, who would be in charge of economic policies and planning, especially in Norway and The Netherlands, after 1945. What was at stake was not only the teaching of econometric modelling and estimation techniques but also mutual help and cooperation in developing models and plans for economic development. Andvig (1985), Johansen (1974), Bjerkholt (1995b), Adrienne van den Bogaard (1998, 1999) showed that this intertwining in these two countries was based upon the definition and the use of econometric tools and techniques.

More specifically, the link established between modelling and economic planning in Norway has been tied for more than 60 years to the specific use of econometrics in that country (Bjerkholt, 1998), and would last until the seventies (Eriksen et al., 2008). This peculiar way of conducting economic policy can be partly explained by the historical and political context, but mainly by the conception of econometrics Frisch had embraced since the Great Depression: both a scientific analysis of socioeconomic phenomena and a practical handbook for economic policies.

Frisch devoted his career to developing a new methodology for economics in order "to subject abstract laws of theoretical political economy or 'pure' economics, to a numerical and experimental verification" (Frisch, 1926, 1). Structural modelling² can be seen as

2 Frisch was not talking *stricto sensu* about structural modelling. But, since 1928, he had been thinking out the idea that an econometric model has to be built up from autonomous and confluent equations. He had developed confluence analysis first by himself and then, in the second part of the 1930s, with Trygve Haavelmo. This eventually led to structural modelling such as it has been developed by the econometricians after WWII. Frisch's works on confluence analysis aimed to cope with the difficult task of connecting mathematical economics and empirical investigation, and identification issues (1928, 1929b, 1934b). In these papers, we see how Frisch developed the econometric methodology of structural modelling based

the outcome of Frisch's scientific ambitions (Morgan, 1990, Le Gall, 1994, Boumans, 1999, Dupont-Kieffer, 2003). The model appears in his works as the core of the scientific investigation on the one hand, and as the tool for connecting and "mediating" theoretical quantification ("mathematical economics") and empirical quantification ("statistical economics" after Frisch's terminology) on the other hand, the model being a "mediator" according to the analysis developed by Morgan and Morrison (1999, 10-11). The modelling methodology was elaborated in the course of investigating the causes and the shape of business cycles, bridging the current opposition of this field between mathematical economics and empirical and statistical economics.

The shift from intellectual ambitions to political preoccupations occurred in the context of his business cycle analysis. In the Propagation-Impulse model (1933a)³ Frisch advocated the use of systems of dynamic equations to account for the behaviour of the economy at the macro-level⁴. In this approach, which dominated the 1930s, the research focused on computing the parameters of the models, and raised such issues as the identification of the structural equations, estimations problems and calibration and data collection (Morgan, 1990). But Frisch seemed aware that this system-of-equations approach should be complemented by a general equilibrium approach, especially for policy purposes.

In this context, the decisive step in Frisch's shift from modelling for scientific purposes to modelling for planning purposes was the general equilibrium model developed in his long and controversial 1934 paper, "Circulation planning". It is noteworthy on two accounts: firstly because Frisch elaborated his interpretation of the economic crisis as a breakdown of the system of exchanges due to monetary causes, and secondly because he advocated solving the crisis by establishing a central agency in charge of the organisation of an optimal system of exchanges. Frisch conceived a clearing agency, alongside the market, that would allow a better circulation of information between producers and consumers, within particular sectors and among the different sectors of the national economy. This agency would be able to compute the optimal structure of exchanges using econometric modelling and heavy computational processes, and then distribute vouchers relative to this optimum.

upon the confluence analysis. Frisch already pitched into that idea in 1925 that "the statistical-econometric task is to specify the reference functions and to determine the value of the parameters and by the way to lead to numerical results" (manuscript found by Andvig quoted in his thesis in (Andvig, 1985, 24)).

3 The Propagation-Impulse model is also known as the Cassel Model since Frisch's article was published in a collective tribute to Gustav Cassel.

4 The economic variables (e.g. production of consumer goods, production of capital goods) in the Propagation-Impulse model (Frisch, 1933a) are aggregates.

In section 2 we will investigate how Frisch moved from the analysis of business cycles and the development of macrodynamics to the identification of the crisis as a circulation problem in the system of exchanges. Section 3 will deal with Frisch's proposals for coping with the crisis by means of a National Organisation of Exchange in charge of planning the exchanges aiming to compute a general equilibrium. In the last section we will show the conceptual and technical problems of the model which led him (i) to define a precise methodology for coordinating the work of academics, politicians and administrative authorities, and (ii) to develop specific tools for economic policies and indirect planning. Thus Frisch was promoting the figure of the social engineer.

2. The Analysis of Business Cycles: From the Propagation-Impulse Model (1933) to the "Incapsulating Phenomena" (1934)

For Frisch, in the context of the crisis of the interwar years, an understanding of the causes of business cycles was a prerequisite for taking action against the cycle. In the beginning of the 1930s, Frisch advanced different explanations for the persistence of disequilibria, complementary to the Propagation-Impulse model, and increasingly focused on monetary phenomena.

Since 1927, Frisch had been doing research on the *nature* of economic cycles (a question raised by his work on the decomposition of time series, which led him to suggest the distinction between free and forced oscillations) as well as on the *origin* of the cycles, the origin being found in the production of capital goods.

Considering together the proposals contained in the Propagation-Impulse model (Frisch, 1933a) and in "Circulation Planning" (Frisch 1934a), we can see that Frisch identified two types of explanations for the persistency of disequilibria: firstly, the special nature of capitalist production⁵ and secondly, the failures of the system of exchanges.

The explanation provided in the Propagation-Impulse model relied on the accelerator principle and the time lags occurring between the production of consumption goods and capital goods, the latter depending on the former and adjusting itself to the former with some delay. The relative weights of replacement investment and net investment (the latter is a function of the growth rate of the consumption of manufactured items) will determine the evolution of gross investment at the national level. Slack times for gross

5 Frisch's Propagation-Impulse model describes on the one hand what a macrodynamic modelling must be, and on the other hand it offers a whole theory of the economic cycles and crises, based upon the two major concepts of equilibrium and investment (Dupont-Kieffer, 2001, 2003).

investment and production of capital goods jeopardize the growth of consumption good production.

But these tensions due to time lags are strengthened by the reactions of the banking and monetary authorities which respond with some delay to variations in the demand for money. The third equation of the Propagation-Impulse model, named “the *encaisse désirée* equation” (Frisch’s wording for demand for cash), expresses the idea that the reaction delays of the banking system create tensions on the demand of money and in that way curtail the demand for consumption and capital goods. That is, restrictions on liquidity place restrictions on demand on the market of manufactured goods, and, as a consequence, on the market of capital goods.

In the 1934a paper, Frisch then focused on the fact that “violent depressions may be caused by the mere fact that the parties involved have a certain typical *behaviour* in regard to buying and loaning” (Frisch, 1934a, 271; Frisch’s emphasis). He then developed further the idea that the origin of the crises had to be sought in the behaviour of the banking and monetary authorities, leading to tensions on the money market—and stressing the cyclical fluctuations owed to the accelerator principle—and as a consequence on the organisation of exchanges. Moreover, the induced disturbances in the system of exchanges held up the free movement of the market mechanisms.

Frisch then recaps and makes his own the theory of the two interest rates developed by Wicksell.⁶ According to the theory of the cumulative process, the dynamics of the economic system is driven by a gap between two rates of interest. On the one hand, there is a banking rate of interest drawn up by the banks; on the other hand, there is a natural rate of interest, the one that would prevail under free competition by matching of global supply and global demand on the capital market. But the crucial role of the banking and monetary institutions, both of them ruled by routines, creates a gap between the equilibrium rate of interest and the actual one. This gap

6 The Swedish economist Knut Wicksell argued, in two major books, *Interest and Prices* (1898) and *Lectures II* (1906), that two concepts of interest rates were prevailing: the natural rate of interest—a rate of interest that equates aggregate demand and aggregate supply and depends on the marginal product of new capital—and the market rate of interest, quoted daily in financial markets, equivalent to the cost of borrowing money. When the market rate is left lower than the natural rate, the marginal product of capital is greater than its cost, and there is advantage for entrepreneurs to borrow funds from the banks and to invest in capital, stimulating excessively the aggregate demand which tends to exceed aggregate supply. Conversely, when the market rate remains above the natural rate, aggregate demand is choked off and tends to fall short of aggregate supply. A general rise, or fall, of prices is a disequilibrium that is not only permanent, as long as banks do not decide to equate interest rates, but also cumulative as long as there is a positive gap between aggregate demand and aggregate supply.

generates cumulative processes which deflect the market system from its general equilibrium position.

In "Circulation Planning", Frisch complements the Wicksellian idea of the disturbing role of the banking and monetary institutions on the exchange system with Fisher's idea that debt will distort the rules of exchanges by distorting the understanding of the true real volume of exchanges (Fisher, 1933). Frisch, without any rigorous argument, states that the anticipations made by the agents on the forthcoming growth influence directly the decisions of selling and purchasing. Note that Frisch refers to the Wicksellian cumulative processes and the Fisherian debt distortion without developing them.

On this basis, Frisch decides to focus on the analysis of the following paradox: "poverty is imposed on us in the midst of a world of plenty" (Frisch, 1934a, 259). Depressions are due not to shortages but to the "present form of organization of industry and trade" (*ibid.*, Frisch's emphasis), "and the disastrous effects produced... by the present organization of industry and trade is what might be called the *incapsulating phenomenon*"⁷ (*ibid.*). Considering that modern life is divided into a number of regions and groups, the phrase "incapsulating phenomenon" conveys the idea that where "the blind 'economic laws' will, under certain circumstances, create a situation where these groups [the modern economic life is divided into a number of regions and groups] are forced *mutually to undermine each other's position*" (*ibid.*, Frisch' emphasis). The idea is that certain agents following their pessimistic anticipations will deprive other agents of their outlets perspectives by cutting down their purchases and contracting the demand for manufactured and capital goods. The 'incapsulating phenomena' are conceived as a catch-22 where a set of contrary forces rules the market when the debt and the rate of interest disturb the agents' anticipations. Moreover Frisch explains in 1935 (Frisch, 1935, 16) that the initial problem in the circulation system is rooted in the fact that the majority of banks have an inflationist or deflationist psychosis, inducing pessimistic and optimistic anticipations.

With the lengthy "Circulation planning" paper published in *Econometrica* in 1934, it seems that Frisch was faced for the first time with the two possible sides of economic research, and consciously asserted the double role of the economist as theoretician and counsellor. In 1931, Frisch raised in a daily newspaper (*Tidens Tegn*) the possibility of using the budget deficit to support households and firms purchasing power and thus the global demand.⁸ Later on, in

7 The term is coined by Frisch (Frisch, 1934, 259)

8 Frisch linked the disturbances of the Norwegian monetary system and the monetary shocks to the survival of the Gold Exchange Standard (cf. *Troen på Nøkken*, 1933b in 1995, 90). The idea is that the unequal distribution of gold stocks in favor

1934, he would be personally involved in the definition of a draft for economic policies when collaborating with the Labour Party⁹

Norway was ruled at that time by non-socialist minority governments.¹⁰ Frisch's involvement can be partly explained by the emergency that he saw in finding a solution to the economic crisis and its social consequences (Andvig, 1985; Louça, 1999). This shift towards planning/engineering aimed at preserving the political and economic freedom of the individual. Frisch had been warning his compatriots since 1931 that if they were not ready to accept indirect planned economy and organised cooperation between the government and the various economic organisations, the only outcome of the crisis would be communism (Frisch, 1931a, 2). From Frisch's statements in newspapers and on the radio between 1931 and 1935, it appears that he was very confident that econometric computation and modelling could reconcile planning, individual freedom and market economy (pictured as "a big machinery" Frisch, 1931a, 2).

3. The Model of "Circulation Planning", or the Attempt at Modelling a General System of Exchanges

Frisch's paper "Circulation Planning" is quite lengthy and sometimes confused, but its main proposal is clear: to set up a National Exchange Organisation (NEO) the tasks of which is to collect information on expected demand and supply formulated by the agents, and then to compute the quantities to be exchanged in order to balance global demand and global supply. The computation of the better allocation, better in the sense of an equilibrium position or tending to the equilibrium position, relies on the definition and computation of the matrix¹¹ of the optimal rates of exchange.

of France and the United States would prevent the efficiency of gold as support for issuing paper money and allowing credits, since this non-optimal allocation of gold increases competition between nations for getting gold. The consequence of this gold rush is a fall of prices that throttles down the industrial and business activities.

9 For more details on the involvement of Frisch towards the Labor Party, see Andvig (1985, 374 and following).

10 Norwegian Minority Governments that ruled Norway in the 1930's were: 1928-1931 Liberal Government, 1931-1933 Agrarian Government, 1933-1935 Liberal Government and 1935-1940 Labor Government.

11 The inputs of the matrix are the aggregate supplies and demands for each market (here assimilated by Frisch to sub-sectors of the economy) and the core of the matrix shows the results of the computation of the optimal exchange rate for each market.

3.1. The National Clearing Agency: How to Deal with the Breakdown of the System of Exchanges

Frisch (1934a) underlined that the setting up of the NEO should be firstly complementary, or at least parallel, to the market system, and secondly temporary, depending on the time necessary to correct the failures of the exchange market system due to the crisis. Adhesion to the NEO should be free and voluntary. Frisch truly believed that the NEO could solve the dysfunctions of the market system (even if he ignored the monetary reforms), the disparate distribution of income among the industrial sectors and the fallacious relative prices due to the incapsulating phenomena.

The NEO would provide two kinds of solutions to the breakdown of the exchanges: firstly it would improve the collect and the circulation of the information on supplies and demands, and secondly it would build and compute the matrix of the optimal rates of exchanges.

Frisch presented the issue at stake as "an optimum problem" (Frisch, 1934a, 261), but without any reference to Walras or to the General Equilibrium framework. We will show that he described a process very similar to the Walrasian tâtonnement while never referring to it!¹² Tâtonnement was not his main interest, he saw it as a mere technical-computational problem. Moreover, Frisch tackled the issues of defining the equilibrium on all the markets only with the perspective of improving the organisation of the exchanges and the computation of the optimal exchange rates. He thus limited his investigation of the coordination of exchanges and of the optimum to computational and organisation matters. Indeed, the way Frisch sketched the issue is quite revealing: the optimum problem at the theoretical level is "quantitative" (Frisch, 1934a, 261). That is, Frisch understood the optimum problem as an econometric one (*ibid.*) in the sense that this problem was both mathematical and "actuarial".

More, the coordination of individuals' points of view is seen from a planning perspective (a top-down approach) and not from an individual choice perspective (a bottom-up approach) leading to questions such as: "how much should this activity be increased and that other curtailed?" (*ibid.*, Frisch's emphasis).

We will briefly focus on the first part of his article in order to show that he pictured the crisis as an organisational failure in the system of exchanges. This led him to offer a two-fold solution: an institutional one by the implementation of a National Exchange Organization

12 This is hardly surprising since Frisch seldom (and reluctantly) mentions his peers, with the exception of Irving Fisher when attempting to measure the marginal utility, or John Maurice Clark or Wesley Clair Mitchell when defining Business Cycles Analysis (see Dupont-Kieffer, 2003).

(NEO)¹³ and an econometric one by developing a solution to compute the optimal matrix of exchanges.

Frisch's presentation of the NEO is two-fold. Firstly he investigates the planning of the circulation of goods at the "theoretical" level by putting forward a simplified example, starting with two representative agents, then extending the analysis to a sector-based approach. In a second part,¹⁴ he deals with "practical aspects of the circulation scheme proposed" (Frisch, 1934a, 261), but the latter offers no clue for apprehending Frisch's understanding of optimum and coordination problems.

3.2. The Computation of the Optimal Structure of Exchange: Towards Rationing

Frisch begins the demonstration by a very simplified case with two representative agents, a shoemaker and a farmer. Thus, in the "theoretical" part Frisch assumes that each industrial sector is centrally conducted and behaves as one single supplier. He then assumes that each sector produces a single good and that all sectors are under the responsibility and the supervision of the NEO. The NEO will collect for each sector and within each region the expected demands and supplies and aggregate them. Frisch assumes that the suppliers on each market, i.e. on each sector in Frisch's framework, offered homogeneous or quasi-homogeneous items (or at least comparable). The NEO will then ask each supplier what quantity of the good he is ready to offer according to a fixed system of price.¹⁵

This system of collecting and matching individual demands and supplies is defined by a special framework that Frisch adopts, based upon the following principles:

13 After computing the optimal rate of exchanges, the NEO will issue vouchers. Each producer receives from the NEO a certain amount of vouchers that allows the producer to get the required inputs and the goods necessary to the proper functioning of the firm. The vouchers are supposed to be valid within a fixed period. Frisch thus hopes to prevent saving behavior from the producers—saving that maintains the incapsulating phenomena.

14 The second part aims at answering practical questions about the actual implementation of the NEO. Frisch then fell off the assumption of centralized sectors and inquired about the way the different partakers within a sector would react, but also among sectors, as it actually was in Norway in the 1930s. This second part is quite confused. We here focus on the first part where Frisch tried to compute the rates of exchange that allowed the optimal allocation of resources.

15 But the assumption was made that no producer can be asked to offer more than he has announced. Indeed, the production coefficients were supposed to be fixed in the short term. The NEO was supposed to guarantee outlets to each supplier.

First, a co-habitation between the NEO and the market with free participation in the NEO¹⁶;

Second, a region and sector based approach; this second principle relies on two sub-assumptions: (a) each sector is identified to one market on which one homogeneous type of goods is exchanged. In this sector, the supply- and respectively the demand, is presented by a single agent whose behaviour should describe the one of all the agents of the sector and, (b) for each sector, the production coefficients are fixed and the sector should send its inputs needs for any supply level to the NEO;

Third, for each price system, the NEO proposes a quantity adjustment of the supply and demand in order that, in the end, the global value of the aggregate supply should be equal to the global value of the aggregate demand of the sector, the market prices being the prices of reference to compute the value.

Frisch's interest in this process seems to focus on the practical details of collecting the data and not on the conceptual issues on the individuals' choices and maximisation routines. Indeed, the NEO is supposed to launch a stocktaking of the interested consumers for the relevant good, and set up a database of demands and supplies on each specific market, aiming to put in touch producers and consumers.

If the expectations of suppliers and consumers do not correspond at the aggregate level, the NEO would be in charge of adjusting aggregate demand and aggregate supply by issuing vouchers. He then turns to how to compute the optimal matrix of Exchange rates. The problem at stake is the following: Is it best to change the initial distribution, or to ration goods or to ration agents?

Frisch defines three coefficients for solving the regulation of the process: z_i represents the relative part of each sector in the total amount of goods bought; x_i represents the rationing for each good and y_i accounts for the part of the goods in the total amount of items purchased by each individual. Then the issue is to decide which coefficient has to be manipulated in order to settle the circulation crisis.

Frisch firstly accepts the idea of a system with an excess of demand, and then comes back to the equilibrium issue between the value of the aggregate demand and the aggregate supply by taxing the sector in excess demand. The "office" that should run the NEO would be in charge of setting up that tax ("the National Exchange Office", first quoted as so in Frisch, 1934a, 329).

16 Frisch is not concerned with defining competitive rules, but he is concerned by the co-existence of the NEO and the market, and by any risk of interference between the two systems. This is why he listed a number of conditions for issuing the NEO voucher: the vouchers could not be used as a mean of exchange and should be valid only within the NEO and for a certain period of time.

Frisch secondly contemplates relaxing the assumption of fixed production coefficients set and imagines that in this way, a substitution among production factors could occur and then lead to a decrease in production costs. He devotes pages to define the new supply and demand matrix which is meant to describe this substitution process. He then admits that this process should occur step by step, all the more so since a certain volume of transactions should be maintained in order to avoid a brutal collapse of the agents' purchasing power. Consequently, the steps of adjusting will be determined by the first sector to reach the amount of its initial offer. Then the "NEO Office" could fix the percentage reduction in demand of each sector.

Frisch is aware that the "free participation to the NEO"¹⁷ implies that if the initial bottleneck is not resolved then the suspension of the exchange system will be soon complete. But he maintains the "freedom axiom" even if it is risky, since he thinks that it guarantees the respect of individual freedom and free participation of the agents to the NEO. The latter is assumed to guarantee to the firms that the NEO will clear out their production if they participate in this system. The consequence of this commitment from the NEO to clear the firms' output will then be an adjustment on the demand side and a rationing of the demand. This organization of the exchanges could be compared to rationing techniques in war times (Frisch, 1934a, 323-324).

We see that Frisch is reluctant, or at least unable, to deal with the major issue of matching individuals' plans and the respect of individuals' choices: Frisch is broaching the optimum problem in a market economy as the definition of a calculus methodology and as the definition of the conditions for vouchers issuing in a parallel exchange system. But nonetheless, Frisch's interest for the general equilibrium is directly related with issues of computability and institutional settings. On this matter, it is noteworthy that the only reference in Frisch's text to the "general economic equilibrium theory" (Frisch, 1934a, 297) is done without any reference to any work or any economist, and is introduced when dealing determining the "minimum cost combination" (*id.*, 294) of production factors taking into account all commodities and not only one:

It is impossible here to proceed by the same methods which are—alas, so superficially—used in general economic equilibrium theory, namely to determine the minimum (or maximum) of an index function of several variables just by equating its partial derivatives to zero. (Frisch, 1934a, 297)

17 The participation to the NEO on both the supply and demand sides is voluntary based and any agent could quit the system when he wished.

He then concludes that he has to deal first with the "substitution problem... by small steps" keeping constant the volume of production, and then to turn to the "adaptation problem" while varying the volume of production (*ibid.*, 297-298) in a centralized economy and in a global perspective.

In a second part of his discussion, he abandons the assumption of centralized sectors. He then had to analyse the allocation of goods among sectors but also within each sector¹⁸. But there is no rigorous argument, no model, only the acknowledgement that the system would be very similar to the organisation of a rationed economy in wartimes (Frisch, 1934a, 323-324).

Solving the crisis by asking the NEO to organise the exchanges raises many problems: rationing, especially of the demand side, collecting data on demand and supply for each sector and each region, adjustment in time, the possibility to use vouchers as substitute to money, computation of the matrix of the optimal exchange rates, etc. The limitations are not only technical, and the NEO is not able to solve the breakdown in the market system of exchanges.

3.3. Econometric Modelling and Institutional Answers to the Technical and Conceptual Limitations?

Sometimes Frisch seems aware that his model of centralised exchanges is incomplete and imposes a demand (or supply) rationing. Indeed, the co-existence of the market and the NEO, implying a share of the economic activities between the two systems and, in addition, the guarantee offered by the NEO to the producers that their stocks would be cleared out by the NEO, means that the demand side will be constrained in the short term and that the adjustments will be made on the demand side. But he then expects that mathematics and new methods of computation will allow limiting this rationing (Frisch, 1934a, 282) especially on demand, which implies in return the rationing of supply. Frisch then suggests a reduction of the total volume of transactions in order to guarantee the relative distribution of purchasing power and a strict equality between the aggregate demand and the aggregate supply (Frisch, 1934a, 282). But this solution is still related to a constrained framework on agents'

18 He was very vague on the computation of the matrix that balanced demand and supply, and even on the initial distribution of the vouchers coined 'warrants', mentioning that he trusted the "natural mechanisms" of the exchange [throughout this second part, Frisch qualified the exchange mechanisms as "natural", and explained that supply and demand were "naturally" in business] in order to smooth the circulation of goods within the NEO. The latter should be in charge to distribute the warrants and to give the buyers a list of the possible suppliers for the considered item. The list should be sent to the regional agencies which would inform the agents. The problem was to match the sector-based approach of computing with the information collected on a geographical basis.

expectations; and Frisch appears more and more preoccupied by the fact that “the organized exchange should of course aim at producing the best possible harmony right from the beginning” (Frisch, 1934a, 284) and should also avoid any kind of discrimination between the partakers. We can see here that Frisch is concerned with the acceptability of the system by the different agents participating in the NEO, and the circulation of information.

Along the 176 pages of his demonstration, Frisch appears more and more concerned by the problems raised by any planning system i.e., acceptability, collection, compulsion and redistribution of information. But the interesting point is that he saw them as organisational (and partly computational) ones, with no concern with conceptual matters.

Indeed, according to Frisch the main problem the NEO would have to face is to collect very quickly the information, and to spot those sectors that are at an equilibrium position and those that are not; and to compute, for the latter, the percentage of the reduction for the demand side. Frisch has in mind an iterative process between the NEO office and the sectors, and assumes that 2 weeks will be necessary for a team of 150 econometricians to compute the matrix balancing demand and supply, in the case of a 50 sectors-based economy. Frisch is convinced that the process would be successful if the volume of information the office would have to deal with was not too huge, even if practical constraints and the co-existence with the free market are significant (Frisch, 1934a, 323). In order to simplify data and information collection, Frisch advocates creating local agencies,¹⁹ not sector-based but geographically based. These regional agencies would be in charge of centralising the expectations for each sector of demand and supply, and then to send the condensed information to the office.

Throughout the text, it appears that Frisch is not so much concerned by the issue of the optimum equilibrium and coordination of individuals' plans per se, and much more concerned with the technical and the computational aspects of his proposal. He is very confident in his proposal, and minimises the difficulties raised by the implementation of the NEO. Indeed, he mentions difficulties met when issuing the “warrants”, or when computing the matrix in a short time, or in developing quadratic programming. He is aware of the limitations to his solution raised by the co-existence of money and warrants, or by the challenge of defining prices in this two-faced exchange system. If he acknowledges that he had to face the co-existence of two systems, the one of the market and the one of

19 The collection of statistical data in Norway has been organized from the beginning on a regional basis.

the NEO, he minimises the implications of such a double system. The exchange rates prevailing in the NEO would be very different from those established on the market place. Frisch is not denying that the difference in the exchange rates might produce some disturbances in both systems. Because the participation of the agents to the NEO and to the market would be free and voluntary-based, it is then possible to consider the case where the agents would move from one system to another, according to the most interesting rate of exchange. Frisch, naïvely, considers that the risks of perturbation are limited, not only within each exchange system but also at the global level between the market and the NEO, and that the main task is to restore confidence and optimism, and then to facilitate the circulation of goods. His confidence in his own proposal seems unshakeable when concluding his very long article:

The organisation of a National Exchange service for a country seems both on theoretical and practical grounds, to be possible. There can scarcely be any doubt that such an arrangement would help greatly to be *break through* the obstacles of a purely circulation kind which the depression has created. Pessimism and lack of confidence would not be able to stop the exchange within such a system inasmuch as the participants would be placed face to face with the *fait accompli* that they *have* those things which will enable them to buy the goods they want. . . The vicious phase connection between sales and purchases would be broken. (Frisch, 1934a, 335-336; Frisch's emphasis)

Frisch's confidence seems to rely on the ability of the econometricians to develop quadratic programming and national accounting. The latter should provide an accurate picture of the national economy, sector by sector, but also by distinguishing two approaches, the "monetary" circulation and the "real" circulation.²⁰ The setting up of national accounts was the condition to improve the circulation of information, for analytical and political purposes. It was the subject to which Frisch would devote himself (and his assistants) in the following years. The description of the Norwegian economy in 1934 prefigured the development of a system of national accounts, sketched by Frisch as the Eco-Circ issued in 1935 and 1942.²¹ As in Eco-Circ, the division of the economy was sector-based, but in the "Circulation Planning" Frisch added to this a geographical division.

Combined with macrodynamics, national accounting must allow identifying the interconnection between the agents and the causes

20 Lindahl has also developed at the same time a two-fold national accounting, monetary-based and real-based. We can see there the influence of Wicksell on Scandinavian economics.

21 The Eco-Circ is the first sketch of National Accounts drawn by Frisch.

of the cycles and depressions. On the basis of this understanding, econometricians should be able to set up countercyclical policies.

The “Circulation Planning” model was met by indifference from the Norwegian politicians and by hostility from the economists (Bjerkholt, 1998) because of its focus on computational and organisational issues rather than on optimum issues, and of its inconsistencies.²² Frisch, disappointed by this negative reception, turns to national accounting, microeconomics and identification problems²³.

4. The Development of a Specific Methodology and of Tools for Indirect Planning

We will now shift our focus from Frisch’s design of the “Circulation Planning” model to its implementation, and then to Frisch’s will to develop a rigorous methodology for implementing and assessing economic policies.

Frisch’s work on business cycles lead him to accept the persistence of disequilibria. This recognition, along with the socioeconomic consequences of the 1929 crisis, explains his need to turn the econometric analysis into an instrument for implementing economic policies. As we have previously argued, his first outlines of solutions, in particular those offered in the 1934 article “Circulation Planning” (Frisch, 1934a), underline two significant elements of this theory: 1. if the crisis is a breakdown of the whole economic machine, then economic policies must aim to restart the machine and 2. economic policy should be restated as an optimization problem which can be solved through econometric calculation and modelling. But Frisch has failed to offer a theoretical solution to the question of the optimal coordination of individuals’ plans in a general equilibrium perspective, especially with the co-existence of two systems—the market and the NEO, and the rationing implied by the guarantee of clearing out proposed to the suppliers. These theoretical dead-ends and the computation problems enhanced by the 1934 paper drives Frisch to face the existence of a tension between an economic analysis based upon the concept of equilibrium and the economic policies aiming to face economic imbalances and to cope with the social and economic consequences of disequilibria.

The “Circulation Planning” paper clearly shows that the economic policy conceived by Frisch relies on the consideration of the economic

22 See Andvig (1995) and Bjerkholt (1995, 1998).

23 A comparison can be drawn with Jan Tinbergen’s own conception of economic policy and planning, as described and analyzed in details by Bogaard (1998, and 1999).

system as a whole and goes beyond the piecemeal measures that members of both the Labour Party and the non-socialist parties were thinking of. Andvig (1985) has underlined the ambivalence of Frisch in the 1930s, advocating sometimes comprehensive measures and sometimes partial and emergency ones. It seems that Frisch, during his collaboration with the Labour Party, increasingly faced the difficulty of defining and implementing large-scale Walrasian-based policies, and moved towards more pragmatic proposals. It is only after WWII that Frisch concretely develops econometrics as the essential support and guide to economic policy²⁴. This hope is voiced in the first issue of *Econometrica* after the worldwide conflict, where he calls for the social responsibility of the econometrician (Frisch, 1946).

The preface to the OSLO-Channel model²⁵ (Frisch, 1957) contains a classification of the different approaches to economic policies based upon econometric modelling. This classification reflects four stages in a process extensively based on the use of models: 1. the on-looker approach, 2. the *ad hoc* instrument approach, 3. the feasible instrument approach and 4. the optimization approach. The first approach aims to provide forecasts of economic activity, but also of what should be done in order to influence the future (Frisch, 1957, 250). In the second approach, the issue is to study the compatibility between targets and tools of economic policies, and the work of the econometrician should be understood as providing a macrodynamic model including a certain number of degrees of freedom. The third approach is to provide different alternative policies including various policy instruments, also taking into account the exogenous influences on the development of the national economy (or sectors). In the last approach, the time has come not only to grasp the possibilities but also to make choices on what will be done in terms of economic planning and policies. In that step, the close cooperation between political representatives, administrative authorities and econometricians is crucial (Frisch, 1957, 254).

After the publication of "Circulation Planning", Frisch would turn to a more pragmatic approach and he would develop a

24 As shown by Andvig (1985), Aukrust (1994), Bjerve (1950, 1959, 1989, 1998), Lie (1995) and Bjerkholt (1995, 1998), Norway succeed in developing close relationships between politicians and economists in order to define objectives of economic policies and to implement them. Moreover, as shown by Eriksen & al. (2007, 17-20), the Norwegian economy, with the full cooperation of industrialists and of the market sector, became a centrally planned economy.

25 The "Oslo Channel Model" aims to develop a tool to help defining and implementing an optimal planning of the investment at the national level in Norway. This model should account for the limited capacity of funding of the country. The model placed *pro facto* in a competitive perspective all the investment projects and should help to select those in the better position to increase the national production capacity.

methodology for the economic policy process in two stages: 1. selection of the policies and of the planning target and modalities, and 2. implementation of the selected policies. The first is based upon the definition of a *preference function* of the political agents, while the second relies upon the construction of *decision models*. The latter are sector and multisector structural econometric models. In that perspective, at a more technical level, Frisch emphasizes the necessity of investigating and improving the techniques of linear quadratic programming, which has been one of the stumbling blocks of "Circulation Planning".²⁶

Hence, Frisch makes a sharp distinction between the stage of *selection* and that of *implementation*. Selection consists in laying down specific economic objectives, while implementation consists in creating the organizations in charge of achieving these objectives. The phase of selection, i.e. of clarifying the normative aims, is more clearly specified by Frisch. He assumes that it is necessary to distinguish between what concerns the structure of the economy (which the structure of the model must account for) and what concerns national political aims. The moment of the selection must clarify the preference function. That way of doing implies that the political leaders start to formulate at this stage, through interviews by economists, their goals of social justice and economic development (GDP growth rate, environment goals, standards of education and of geographical balances). The task of the econometrician is then to lay down these various, even sometimes incompatible, goals. Selection consists in determining and ordering the preferences of the political leaders for the future of the Nation. The econometricians help them to formulate their preferences in an economic way, to indicate whether and to what extent their wishes are compatible, and up to what degree these aims can be accomplished, given the current state of the economy. Then the econometricians compute the optimal solution to reach their goals.

Frisch's ambition is to show that it is possible that the government could aim at an economic equilibrium, though less optimal from the point of view of the economic 'rational choice', has the advantage of being manageable and socially satisfactory.²⁷ Politicians and economists must then accept that a 'second best solution' in an

26 For more details, see Bjerkholt and Knell (2005).

27 After a strong opposition to the Soviet system of central planning, Frisch changed his mind and became a supporter of central planning systems in USSR and the East Europe countries. Eriksen *et al.* (2010, 13-14) very precisely show that this shift was publicized in newspapers, especially in the 1950s, and that Frisch was influenced not only by the Depression of the 1930s but also by his cooperation at that time with Ole Colbjørnsen, who had been working on the first Soviet 5 years plan.

economic perspective is more desirable and acceptable from a social and political point of view.

In order to develop relevant sector-based models,²⁸ Frisch has advocated since the 1930s the development of national accounts as the counterpart of modelling.²⁹ One of Frisch's ambitions for setting up national accounts was on the one hand taxonomic, in order to define precisely the key concepts of the economic system. On the other hand his aim is the quantification of the economic concepts using the modelling process. The Eco-Circ system issued in 1942 is the outcome of his ambition of classifying and harmonizing these concepts. It would provide a description of the economic system very much influenced by his analysis of the crisis as a failure of the exchange system due to monetary perturbations, as explained in "Circulation Planning". Frisch also favors a sector-based approach like the one Lindahl had developed in Sweden at that time, but is opposed to the approach developed by Richard Stone after WWII.

5. Conclusion

Frisch's attempt to develop the econometrics for solving "national circulation stagnation" (Frisch, 1934a, 260) and the disruptions in the exchanges system is based upon his belief that the optimum and general equilibrium issues can be reduced to computational challenges and organizational matters. But it can also be considered as a pioneer work compared to the further developments relative to business cycles in the 1950-1960s. Since the 1930s and during the 1940s, the econometric analysis of the business cycles is the system-of-equations approach deeply influenced by Frisch's. Since the system-of-equations focuses on invariant equations (and the correlated issues as identification problems and computation of the coefficients), it is unable to cope with economic policies assuming changing behaviour in a dynamic perspective, and to take into account the possible substitutions among goods, as well as production technologies. Frisch is facing these issues in "Circulation Planning", and in the end, Frisch was facing these issues in "Circulation

28 We give three examples of multi-sector-based decision models developed by Frisch later after WWII: 1/ the Median Model which attempts to analyze how the purchasing power is distributed from the production sectors to a certain number of households and how the income is influenced by taxation rules and grants. The MODIS models developed between 1960 and 1993 at the Norwegian Central Bureau of Statistics were sketched on the Median model; 2/ The REFI model was focusing on the discussion between 'real' circulation and 'financial circulation', especially concerning investments and 3/ the Oslo Channel Model was designed to plan the optimal structure of investment at the national level.

29 See Dupont-Kieffer (2012).

Planning” and we can identify the two following breaks in his research agenda following his investigation of the optimal matrix of exchanges: firstly, Frisch turns to a sector-based approach to business cycles and macroeconomics, and secondly he develops a very technical view of his research programme. This technical focus has to be understood in his view of a required if not necessary cooperation between econometricians and politicians to face economic crisis and economic growth path. Indeed, he thinks that the econometricians’ duties are to identify market failures and to propose ‘practical’ answers, while the politicians’ duties are to give the general guidelines for economic development and set priorities.

Indeed, it was one of the goals of the Institute of Economics of Oslo³⁰ to tackle these issues. Since 1936, the Institute had a new source of financial support from Norwegian industry.³¹ The collaboration with new partners reoriented the Institute activities, on the one hand, from macroeconomic models to sector econometric models treating (for example, supply and demand in the milk, chocolate or electricity markets), and, on the other hand, towards setting up the first framework of national accounts. The issue of computing the optimum equilibrium in the case of the interdependent markets was still alive after WWII. It would bloom again from within the econometric tradition approach to General Equilibrium only in the 1960s, with the works of Johansen in 1960 and Harberger in 1962, and later with the advent of large scale models after the work of Shoven and Whalley in 1972 and Scarf in 1973.

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30 The Institute was set up in December 1931 thanks to funds granted both by the Norwegian government and by the Laura Spelman Rockefeller Memorial—one branch of the Rockefeller Foundation (50,000 dollars for the 1932-1937 period, provided that the Institute was able to get an equal amount from Norwegian sources for these 5 years). Norwegian financial involvement grew as time went by, especially after 1936. The Institute at the University of Oslo was run by two directors of the new Institute of Economics. The other was Professor Ingvar Brynhjulf Wedervang (1891-1961), who in 1937 moved to Bergen and became rector of the newly founded Norwegian School of Economics and Business Administration.

31 On the initiative of Johan Throne Holst, the funding of the Institute relied also on the participation of the Norwegian Federation of Industries [*Norges Industriforbund*] and the Found for insurance of war goods [*Norsk Varekrigsforsikrings Fond*].

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