Time series estimation and forecasting of Covid in Norway

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Preliminary.

Introduction

- Covid time series are typical examples of changing data-generating processes,
  - because of mutations and policy responses
  - over time and across economies
- Likely that constant-parameter models will fail.
- An approach with exogenous shocks/breaks and effects of corresponding policy responses might be a useful complementary addition to the toolbox.

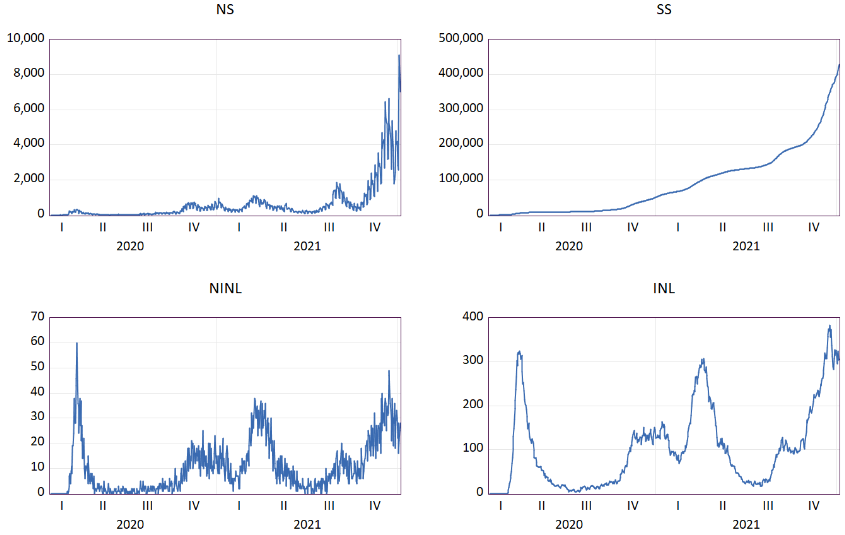
The model

- Builds on Nymoen (2022).
- The model contains four endogenous variables:
  - NS_t, number of new infected with Covid-19, day t.
  - SS_t, accumulated number of new infected with Covid-19, day t.
  - $NINL_t$, number of new hospitalisations wih Covid-19, day t.[1]
  - INL_t, number of hospitalisations with Covid-19, day t.[2]
- The model belongs within the class of autoregressive systems. It has a recursive solution: First NS_t and SS_t are decided. Conditional upon these outcomes, $NINL_t$ is determinded and finally INL_t is solved for.

[1] https://www.fhi.no/sv/smittsomme-sykdommer/corona/dags--og-ukerapporter

[2] https://www.helsedirektoratet.no/statistikk/antall-innlagte-med-pavist-covid-19-for-nedlasting

The data


NS
10,000
8,000
6,000
4,000
2,000
0
I
II
III
IV
I
II
III
IV
2020
2021
SS
500,000
400,000
300,000
200,000
100,000
0
I
II
III
IV
I
II
III
IV
2020
2021
NINL
70
60
50
40
30
20
10
0
I
II
III
IV
I
II
III
IV
2020
2021
INL
400
300
200
100
0
I
II
III
IV
I
II
III
IV
2020
2021

Use two standard models of breaks

Dummy model and smooth transition model

- Model:

$$y_t = \beta_0 + \beta_1 x_t + \beta_2 x_t D_t + \beta_3 x_t G_t + u_t$$

- Exogenous event makes binary variable D_t change effect of x_t on y_t.
- Smooth transition model[3] part $G(s_{t-k}; \gamma, r)$ that changes smoothly from 0 to 1 with increasing policy target variable s_{t-k}.
- Hospitalisations INL used as policy target variable.
- The transition function $G(INL_{t-k}; \gamma, r)$ is the logistic specification

$$G(INL_{t-k}; \gamma, r) = \frac{1}{1 + \exp\left[-\gamma\left(INL_{t-k} - r\right)\right]}.$$

- the threshold r is estimated.
- The parameter γ decides the steepness of the transition function.

Use two standard models of breaks (cont.)

Dummy model and smooth transition model

- If $INL_{t-k} = r$, then

$$G\left(INL_{t-k}; \gamma, r\right) = 0.5.$$

- with $\left(INL_{t-k} - r\right) \rightarrow -\infty$

$$G\left(INL_{t-k}; \gamma, r\right) \rightarrow 0$$

- with $\left(INL_{t-k} - r\right) \rightarrow \infty$

$$G\left(INL_{t-k}; \gamma, r\right) \rightarrow 1.$$

[3] See f. ex. van Dijk, Teräsvirta, and Franses (2002).

Breaks and regime shifts with policy response

- Model abrupt exogenous events with D.
- Model *effects* of policy response with G.
- Covid:
  - Abrupt exogenous events, like mutations (D).
  - *Effects of* policy response as target variable increases (G).
  - Development of pandemic modeled and forecasted as
    1. Breaks in dynamic time series model.
    2. Use INL_{t-5} as target variable.
    3. Effects of policy response when hospitalisations approach threshold level r.

The *NS* equation

The equation for NS_t is estimated as:

$$\begin{aligned} NS_t &= \underset{(0.003)}{0.060}\,(SS_{t-1} - SS_{t-14}) \\ &\quad + \underset{(0.0009)}{0.007}\,(SS_{t-1} - SS_{t-14})\,D_t \\ &\quad - \underset{(0.003)}{0.007}\,(SS_{t-1} - SS_{t-14})\,G_t \\ &\quad + \text{lagged}(\Delta\, NS_{t-j}) + \text{ residual} \\ T &= 15.2.2020 - 5.1.2022,\; 691 \text{ obs.} \end{aligned} \tag{1}$$

where

$$D_t = f(\mathit{Tyrol}, \mathit{Alpha}, \mathit{Delta}, \mathit{Omicron})$$

$$G_t = \frac{1}{1 + \exp\left[\underset{(0.028)}{-0.038}\left(INL_{t-5} - \underset{(22.58)}{294}\right)\right]}.$$

The NS equation (cont.)

By definition the equation for SS_t is:

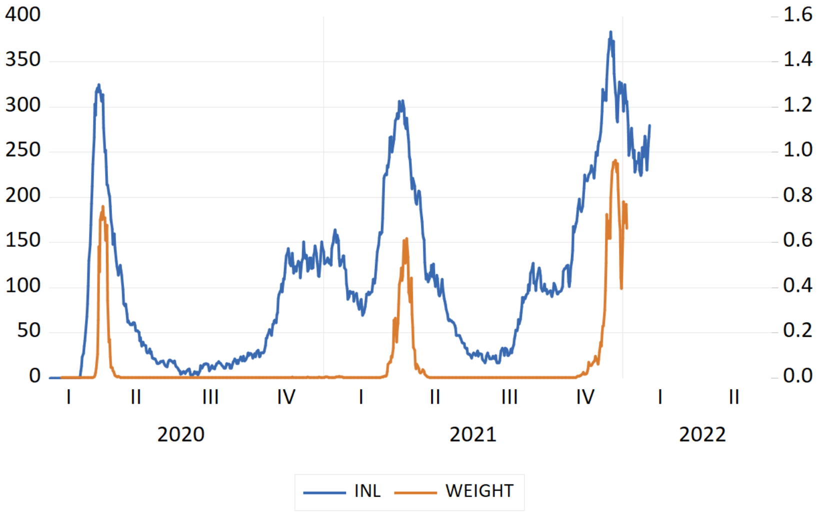
$$SS_t = NS_t + SS_{t-1} \, ,$$

The scale of incidence will be a function of the infection level in the population, which is unobservable. In equation (1) the change in the accumulated level of cases over a two-week period, $(SS_{t-1} - SS_{t-14})$ is used as a an indicator of the the infection level. Note that

$$(SS_{t-1} - SS_{t-14}) = \sum_{j=1}^{14} NS_{t-j} \, , \tag{2}$$

so (1) is an autoregressive model.

Hospitalisations INL_t and smooth transition function



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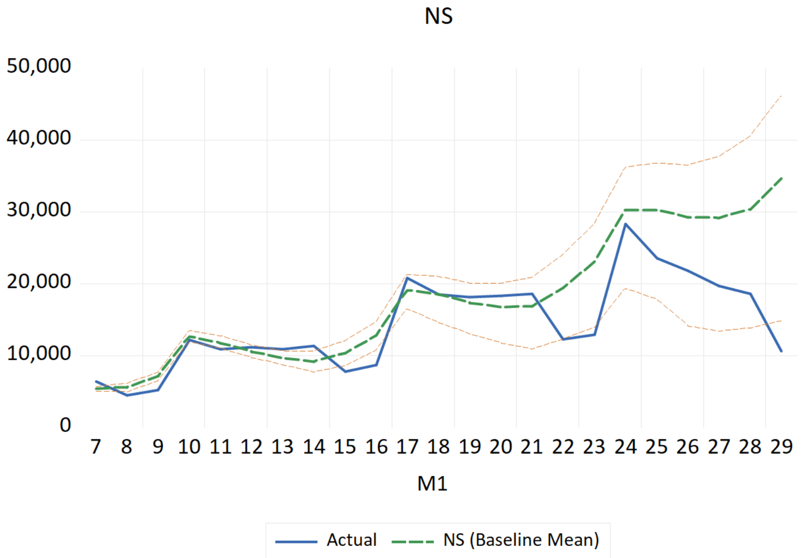
The forecasts and realizations of *NS*

Forecast intervals including parameter uncertainty

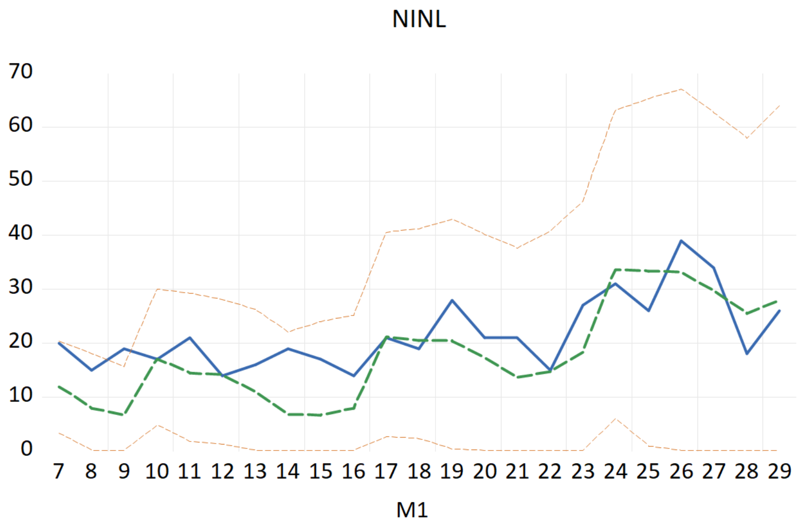


The *NINL* and *INL* equations

$$
\begin{aligned}
NINL_t =& -\underset{(0.0001)}{0.0006}\,(SS_{t-3} - SS_{t-9})\, Omicron_{t-6} \\
& + \underset{(0.001)}{0.012} NS_t \\
& - \underset{(0.0009)}{0.007}\, NS_t \times NINLDN_t \qquad (3) \\
& - \underset{(0.0004)}{0.002}\, NS_t \times NINLDNnov21_t \\
& + \text{lagged}(NINL_{t-j}) + \text{ residual} \\
INL_t =& \underset{(0.002}{0.901} INL_{t-1} + INL_t \qquad (4) \\
T =& 14.7.2020 - 5.1.2022,\; 541 \text{ obs.}
\end{aligned}
$$

The forecasts and realizations of *NINL*

Forecast intervals including parameter uncertainty

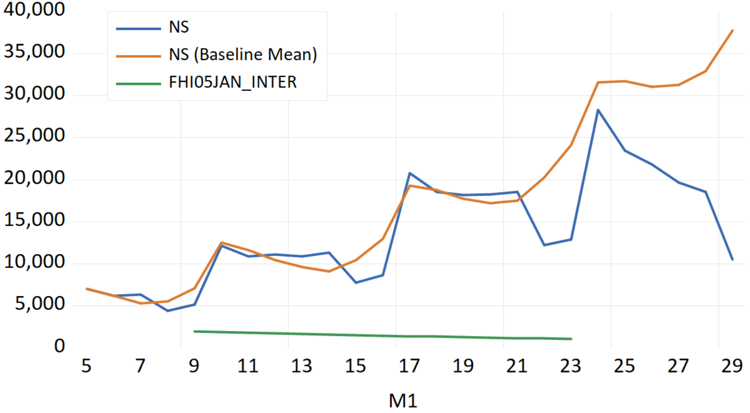


The forecasts and realizations with parameter uncertainty

The forecasts and realizations without parameter uncertainty

Comparisons with *NIPH (FHI)*

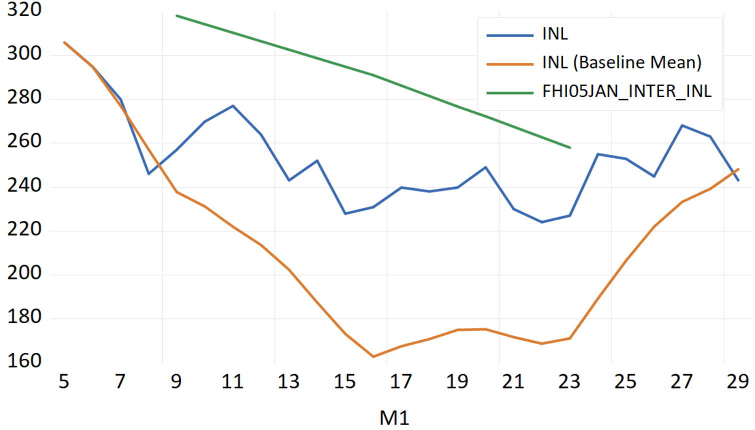
Incidents[4]



[4] "Situational awareness and forecasting for Norway".FHI COVID-19 modelling team Week 1, 5 January 2022. Table 2. Linear interpolation between 7, 14 and 21 days ahead forecasts from forecast origin date.

Comparisons with *NIPH (FHI)*

Hospitalisations[5]



[5] "Situational awareness and forecasting for Norway".FHI COVID-19 modelling team Week 1, 5 January 2022. Table 2 and Figure 5. Linear interpolation between 7, 14 and 21 days ahead forecasts from forecast origin date.

Conclusions

- Covid time series are typical examples of changing data-generating processes,
  - both because of mutations and policy responses
  - both over time and across economies
- Likely that constant-parameter models will fail.
- Time series models with exogenous shifts/breaks and corresponding effects of policy response might be a useful complementary addition to the toolbox.

References

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