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Economic Covid-19 effects analysed by macro econometric models.

The case of Norway

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13 October, 2022

Contents

If we denote the n endogenous variables in period t by y_t and let x_t represent m exogenous variables, an explanatory macro model can be expressed compactly as:

$$y_t = f_y(y_{t-1}, ..., y_{t-p}, x_t, ..., x_{t-p}, D_{yt}, \varepsilon_{yt}), \tag{1}$$

where $f_y(\cdot)$ denotes a function.

D_{yt} represents deterministic terms (constants, trends, seasonals and dummy variables for interventions or shocks).

ε_{yt} represents random error-terms that are unpredictable by conditioning on the other arguments in the function.

A possible definition of a large shock is that it can be found as a significant impulse indicator variable, by the use of statistical tests and conventional significance levels.

This means that the shock can be "picked up" by a (zero-one) indicator variable which is an element in D_{yt}.

I suggest therefore that a shock can be large in this meaning of the word without necessarily leading to further structural changes in the equations of the model.

Conversely, estimation of coefficients of economic variables in model equations that include a significant set of indicator variables can have an interpretation of robust estimators, statistically speaking (Johansen and Nielsen (2009).

Hence, inclusion of indicators variables can be seen as feasible route to specification of models that represent "normal economic behaviour".

The extended model

A large shock can affect the data generation of all the economic variables of the model, not just the endogenous ones.

The analysis becomes more relevant and valid if the model is completed by adding a module that endogenizes the variables in the x_t vector:

$$x_t = f_x(x_{t-1}, ..., x_{t-p}, D_{xt}, \varepsilon_{xt}) \tag{2}$$

I will refer to (1)-(2) as the extended model.

The extended model can be written compactly by stacking y_t and x_t in the $m + n$ vector $\boldsymbol{y}_t$, the two error-terms in ε_t and the deterministic terms in $\boldsymbol{D}_t$:

$$\boldsymbol{y}_t = f(\boldsymbol{y}_{t-1}, ..., \boldsymbol{y}_{t-p}, \boldsymbol{D}_t, \varepsilon_t). \tag{3}$$

Baseline and counterfactual solutions

Let $\mathcal{I}_D$ denote the information set that the solution is based on. Two main cases:

1. All the intervention dummies in $\mathcal{I}$ are zero in all time periods, denoted by $\mathcal{I}_{D=0}$.
2. At least one intervention dummy is set to 1 in at least one period, denoted by $\mathcal{I}_{D=1}$.

I will refer to a solution based on $\mathcal{I}_{D=1}$ as a baseline solution and denote it by $\boldsymbol{y}_t^b; t =, 1, 2, ..., T$. A solution based on $\mathcal{I}_{D=0}$ is a counterfactual solution, $\boldsymbol{y}_t^c; t =, 1, 2, ..., T$.

The consequence of a large shock on the system can be defined as the difference between two conditional expectations:

$$\text{Diff}_I \boldsymbol{y}_t = E(\boldsymbol{y}_t^c \mid \mathcal{I}_{D=0}) - E(\boldsymbol{y}_t^b \mid \mathcal{I}_{D=1}); t =, 1, 2, ..., T. \qquad (4)$$

$\text{Diff}_I \boldsymbol{y}_t$ is an *ex post* measure as it in practice will be based on the empirical identification of the shock periods, eg., $\mathcal{I}_{D=1}$.

Another measure which has been used in appreciations of the economic effects of Covid-19, is the difference between an economic forecast produced before the outbreak of the pandemic and the actuals during the pandemic, see eg., Bougroug et al. (2021), Andersen et al. (2022).

With the notation above it can be formalized as:

$$\text{Diff}_{II} \boldsymbol{y}_t = E(\boldsymbol{y}_t^c \mid \mathcal{I}_{D=0,\text{other}}) - \boldsymbol{y}_t ; t =, 1, 2, ..., T. \tag{5}$$

where the counterfactual $E(\boldsymbol{y}_t^c \mid \mathcal{I}_{D=0,\text{other}})$ denotes a forecast with date of origin prior to the shock.

The two differences estimate the same phenomenon, but

$$\text{Diff}_I \boldsymbol{y}_t \neq \text{Diff}_{II} \boldsymbol{y}_t,$$

in general.

$\text{Diff}_l \boldsymbol{y}_t$ is a function of time. To illustrate how the function is determined, consider a cointegrated system with two variables Y_t and X_t:

$$\Delta Y_t = \tilde{c}_{10} + \tilde{c}_{11}\Delta X_t + \tilde{c}_{1d} D_t + \tilde{\alpha}_{11}(Y_{t-1} + \beta_{12} X_{t-1}) + \tilde{\varepsilon}_{1t} \quad (6)$$
$$\Delta X_t = c_{20} + c_{2d} D_t + \alpha_{21}(Y_{t-1} + \beta_{12} X_{t-1}) + \varepsilon_{2t} \quad (7)$$

where β_{12} is the cointegration parameter. (6) is a conditional model equation and (7) is a marginal model equation, hence the two error-terms are uncorrelated.
The reduced form (or VAR) is:

$$Y_t = Y_{t-1} + c_{10} + c_{1d} D_t + \alpha_{11}(Y_{t-1} + \beta_{12} X_{t-1}) + \varepsilon_{1t} \quad (8)$$
$$X_t = X_{t-1} + c_{20} + c_{2d} D_t + \alpha_{21}(Y_{t-1} + \beta_{12} X_{t-1}) + \varepsilon_{2t} \quad (9)$$

where it is understood that $c_{10} = \tilde{c}_{10} + \tilde{c}_{11} c_{20}$, and similarly for c_{1d}, α_{11} and ε_{1t} as a result of the substitution.

The closed system

Let D_t denote an impulse indicator that represent a *singl*e shock, that occurs in period t_0. The difference between the counterfactual and the baseline is a function of h, the number of periods after the shock:

$$\text{Diff}_I Y_{t_0+h} = \delta_{y0} + \delta_{y1}\frac{1-\lambda_2^h}{1-\lambda_2},\ h = 0, 1, 2, ...,\ \ \lambda_2^0 \underset{\text{def}}{=} 1, \tag{10}$$

λ_2 is the stable root, the other parameters of the function are:

$$\delta_{y0} = -c_{1d}, \tag{11}$$

$$\delta_{y1} = \lambda_2(-c_{1d}) + (\alpha_{21}\beta_{12} + 1)c_{1d} - \alpha_{11}\beta_{12}c_{2d}. \tag{12}$$

If h becomes infinitely large we obtain:

$$\text{Diff}_I Y_{t_0+h\to\infty} = \frac{\alpha_{21}\beta_{12}c_{1d} - \alpha_{11}\beta_{12}c_{2d}}{1-\lambda_2} \tag{13}$$

which is also different from zero in general.

For X_t the corresponding expressions become:

$$\begin{aligned} \text{Diff}_I X_{t_0+h} &= \delta_{x0} + \delta_{x1}\frac{1-\lambda_2^h}{1-\lambda_2},\ h = 0, 1, 2, ...,\ \lambda_2^0 \underset{\text{def}}{=} 1, & (14) \\ \delta_{x0} &= -c_{2d}, & (15) \\ \delta_{x1} &= \lambda_2(-c_{2d}) + (1+\alpha_{11})c_{2d} - \alpha_{21}c_{1d}, & (16) \\ \text{Diff}_I X_{t_0+h\to\infty} &= = \frac{\alpha_{11}c_{2d} - \alpha_{21}c_{1d}}{1-\lambda_2}. & (17) \end{aligned}$$

$$\text{Diff}_I Y_{t_0+h\to\infty} = \text{Diff}_I X_{t_0+h\to\infty}$$

in the special case of long-run homogeneity, $\beta_{12} = -1$.

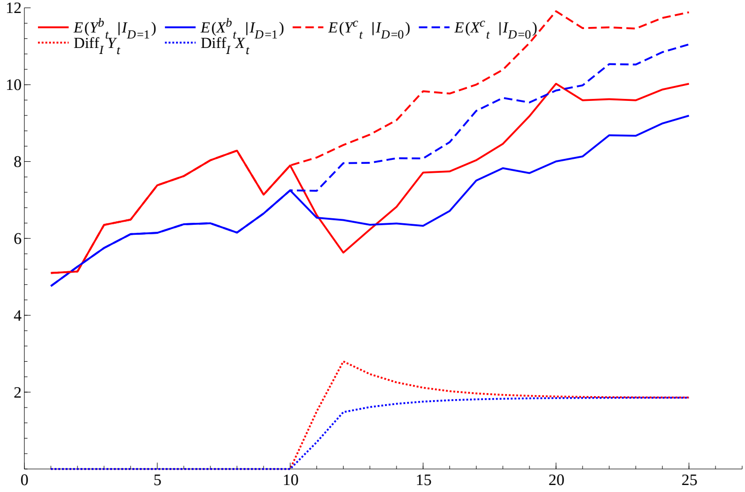


Figure 1: Simulation of a cointegrated two-variable system subject to impulse indicators in period 11.

- Figure 1 illustrates that in general for cointegrated systems, temporary shocks have permanent effects on the solution paths of the endogenous variables.
- The effects of a negative shock do not in general “go away” unless there are counteracting shocks, simultaneously or later in the solution period.
- If there are structural breaks in β_{12},α_{11} or α_{21} after the period of the shock, the solutions for ΔY_t and ΔX_t will be affected.
- Would be a deep change in behaviour that can be confirmed empirically as post crisis data come in.

Open system

The qualitative effects of single period shocks are the same as for the closed system.

However if we more generally include separate impulse indicators in say D_{Yt} and D_{Xt}, it is only D_{Xt} that affects the level of both X_t and Y_t.

If the shock is “limited to” Y_t so that it is captured by D_{Yt}, the solution for the level of Y_t will not be permanently affected, because the level of Y_t is determined by the level of X_t in the open system case.

NAM

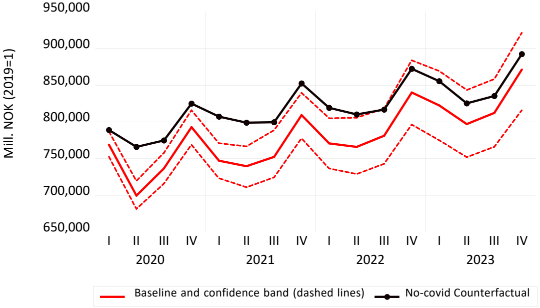
- Norwegian Aggregate Model (NAM) is an quarterly empirical econometric model.
- Indicator variables for each quarters from 2020q1 to 2022q1 were included in all the empirical equations. Retained if the t-values were significant at the 5 % level.

Table 1: Number of equations where Covid-19 impulse indicators are included

Quarter	Impulse Indicator	Model version	
		Standard (120 eqs)	Extended (133 eqs)
2020(1)	$D_{\text{Covid},t}$	12	23
2020(2)	$D_{\text{Covid},t-1}$	26	38
2020(3)	$D_{\text{Covid},t-2}$	15	26
2020(4)	$D_{\text{Covid},t-3}$	13	23
2021(1)	$D_{\text{Covid},t-4}$	11	21
2021(2)	$D_{\text{Covid},t-5}$	12	20
2021(3)	$D_{\text{Covid},t-6}$	11	28
2021(4)	$D_{\text{Covid},t-7}$	7	9
2022(1)	$D_{\text{Covid},t-8}$	9	12

Examples of NAM model equations with or without Covid-19 impulse indicators

With	**Without**
Value added, service production	Value added, Manufacturing
	Value added, Other products
Private consumption	Capital formation private business
Imports	
Export of services	Exports of products (non-oil)
Foreign export markets	
	Wage formation ("almost")
Foreign producer prices	Value added deflators
Foreign short interest rate	
Policy interest rate	


950,000
900,000
850,000
800,000
750,000
700,000
650,000
Mill. NOK (2019=1)
I II III IV I II III IV I II III IV I II III IV
2020 2021 2022 2023
Baseline and confidence band (dashed lines)
No-covid Counterfactual

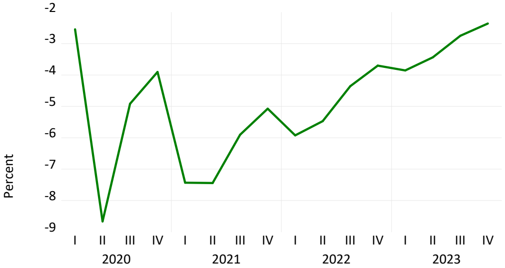

-2
-3
-4
-5
-6
-7
-8
-9
Percent
I II III IV I II III IV I II III IV I II III IV
2020 2021 2022 2023

Table 2: GDP Mainland-Norway. Difference between baseline (impulse indicators included) and no-covid counterfactual. NAM simulation.

	2020	2021	2022	2023
Mill NOK (2019=1)	-156710	-209920	-160688	-105296
Percent	-5.2	-6.9	-5.1	-3.2
Memo:				
Bjertnæs et al. (2021)	-4.7	-3.8	-2.2	-0.5
von Brasch et al. (2022)	-4.6	-2.4	-2.1	+0.9

The numbers in the memo section are, broadly speaking, interpretable as values of functions of type $\text{Diff}_{II}\boldsymbol{y}_t$.

The NAM results are values of the $\text{Diff}_{I}\boldsymbol{y}_t$ function defined above.

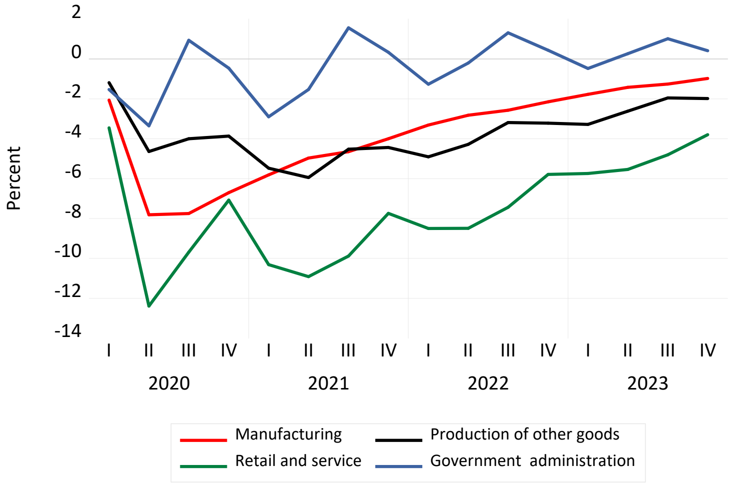


Figure 3: Value added in Mainland-Norway sectors. Deviation between baseline and no-covid solutions, in percent of no-covid solution.

Empirical final form equations

In principle, each final form equation implied by a model like NAM is an ARMA(p,q) model augmented by impulse indicators.

ARMA(p,q) can be approximated by an AR(p') with $p' > p$

In order to decide the lag order p' and which indicator variables to include, have used the machine learning algorithm Autometrics with Impulse Indicator Saturation in PcGive, Doornik (2009), Hendry and Doornik (2014), Doornik and Hendry (2018).

The estimation for GDP Mainland-Norway (OLS), with Target size $= 1\%$:

$$\begin{aligned}\Delta log(YF)_t \;=\;& -\underset{(0.07)}{0.56}\;\Delta log(YF)_{t-1} - \underset{(0.07)}{0.23}\;\Delta log(YF)_{t-2} + \underset{(0.06)}{0.18}\;\Delta log(YF)_{t-12}\\ &+\underset{(0.002)}{0.009} - \underset{(0.008)}{0.04}\;S_t - \underset{(0.007)}{0.060}\;S_{t-1} - \underset{(0.006)}{0.064}\;S_{t-2}\\ &+\underset{(0.017)}{0.065}\;D_{1985(1)} + \underset{(0.017)}{0.057}\;D_{1997(2)} - \underset{(0.017)}{0.080}\;D_{\text{Covid},t-2}\end{aligned} \qquad (18)$$

OLS	Sample: 1981(2) - 2022(1)	Number of obs.: $= 164$
$\hat{\sigma}100 = 1.67$	$R^2 = 0.85$	
AR_{1-5} :	$F(5,149) = 2.48[0.04]$	
$ARCH_{1-4}$:	$F(4,156) = 0.69[0.60]$	

An overall target significance level of 1 % is relatively strict. It may explain the meagre catch of indicator variables.

When the target size was set to the more liberal 4.0 %, there were 16 additional indicators variables that were retained by Autometrics.

The larger indicator set is a concise summary of events which represents the main large shocks to the macro economy over the last 40 years.

The rest of the specification is unchanged.

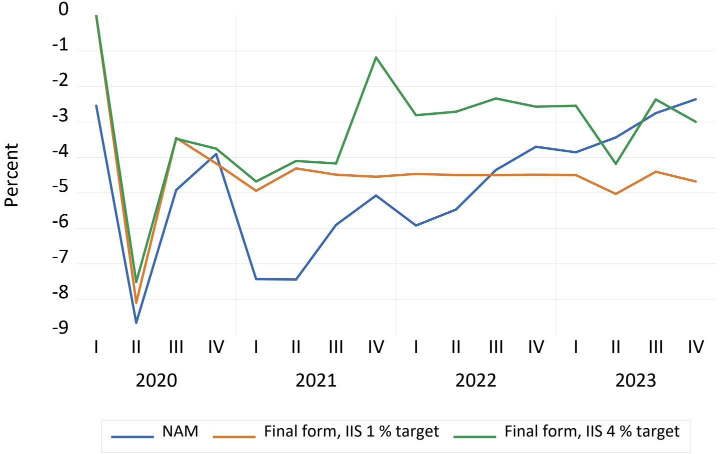


Figure 4: GDP for Mainland-Norway. Simulated effects of the Covid-19 impulse indicators in NAM, and in two empirical final form models. Percentage deviation between baseline and the No-Covid scenario.

Summary

- In the co-integrated model, large shocks can have persistent effects (also) when we assume that the deep parameters like cointegration coefficients and the stable characteristic roots are invariant to the shock.
- Empirically, the GDP response in the operative model NAM to Covid-19 impulses was quite persistent.
- The magnitude of the response was similar, though somewhat larger, than in existing studies based on forecasts with date or origin before the Covid-19 shocks.
- A separate assessment using automatic modelling of GDP growth with IIS corroborated the NAM results (in terms of magnitude).
- In further work:
  - Non-linear co-integration allows us to relax the assumption of invariant adjustment coefficient (stable roots).
  - In practical modelling that implies non-linear functional forms that incorporate the potential for change in behaviour also during normal times as well as of policy responses.
  - Breaks in cointegration parameters, would be an even deeper structural change. Will be relatively easy to detect with the the tools and tests that already are available for model maintenance.

References I

Andersen, T., S. Holden and S. Honkapoja (2022). Economic consequences of the pandemic-The Nordic countries. Underlagsrapport till SOU 2022:10 Sverige under pandemin, Socialdepartementet, Stockholm.

Bjertnæs, G. H. M., T. von Brasch, Å. Cappelen, S. Holden, H. E, O. Slettebø and J. Zulavona (2021). COVID-19, tapt verdiskaping og finanspolitikkens rolle. Utredning for Koronakommisjonen. RAPPORTER/REPORTS 2021/13, Statistisk sentralbyrå, Statistics Norway, Oslo-Kongsvinger.

Blytt, J. P., A. Bougroug and P. Sletten (2022). Økonomisk utvikling gjennom Covid19. En oppdatert sammenligning av Norge, Sverige og Danmark. RAPPORTER/REPORTS 2022/14, Statistisk sentralbyrå, Statistics Norway, Oslo-Kongsvinger.

References II

Bougroug, A., O. Krag Kjos and P. Sletten (2021). Økonomisk utvikling gjennom Covid-19. En sammenligning av utviklingen i Norge, Sverige og Danmark. RAPPORTER/REPORTS 2021/14, Statistisk sentralbyrå, Statistics Norway, Oslo-Kongsvinger.

Doornik, J. A. (2009). Autometrics. In Castle, J. and N. Shephard (eds.), *The Methodology and Practice of Econometrics*, chap. 8, 88—121. Oxford University Press, Oxford.

Doornik, J. A. and D. F. Hendry (2018). *Empirical Econometric Modelling PcGive 15. Volume 1*. Timberlake Consultants, London.

Hendry, D. F. and J. A. Doornik (2014). *Empirical Model Discovery and Theory Evaluation. Automatic Selection Methods in Econometrics*. Arne Ryde Memorial Lectures. MIT Press, Cambridge,Mass.

References III

Johansen, S. and B. Nielsen (2009). Analysis of the Indicator Saturation Estimator as a Robust Regression Estimator. In Castle, J. L. and N. Shephard (eds.), *The Methodology and Practise of Econometrics*. Oxford University Press, Oxford.

Rungcharoenkitkul, P. (2021). Macroeconomic effects of Covid-19: a mid-term review. *Pacific Economic Review*, *20*(4), 439–458.

von Brasch, T., Å. Cappelen, S. Holden, E. L. Lindstrøm and J. Skretting (2022). COVID-19, tapt verdiskaping og finanspolitikkens rolle. Utredning for Koronakommisjonen. RAPPORTER/REPORTS 2022/15, Statistisk sentralbyrå, Statistics Norway, Oslo-Kongsvinger.