What a Puzzle! Unravelling why UK Phillips Curves were Unstable

Jennifer L. Castle[a] and David F. Hendry[b*]
[a] Climate Econometrics and Calleva Project, Magdalen College, University of Oxford, UK;
[b] Climate Econometrics, Nuffield College, University of Oxford, New Road, OX11NF, UK

November, 2023

Abstract

Between 1860 and 2021, UK Phillips curves linking wage inflation (Δw) and unemployment (U_r) exhibit every slope in sub-period regressions from strongly negative, slightly negative, flat, slightly positive and strongly positive. These sub-period outcomes are predicted by an econometric model of real-wage growth expressed in terms of Δw. Correcting Δw for its regressors other than U_r, its sub-period regressions on U_r all have the same negative slope. However, 'shifts' in the real-wage model's variables do not explain the instabilities: surprisingly, the Phillips curves shift when some of the real-wage model's sub-period regressors are insignificant.

JEL classifications: C2, C5, J3.
Keywords: Phillips Curves; Wages; Unemployment; Inflation; Structural Breaks.

1 Introduction

The instability over time in Phillips curves is well known and well documented by both academics and policy-makers; see Del Negro et al. (2020) and Haldane and Quah (1999) for the former and Powell (2019) and Cunliffe (2017) for the latter, both from a US and UK perspective. This instability is demonstrated for the UK in Figure 2 which records five subsample estimates of the Phillips curve using annual data from 1860–2021.[1] Every slope in sub-period relationships between wage inflation (Δw, where lower case letters denote logs) and the unemployment rate (U_r) is observed, from strongly negative, slightly negative, flat, slightly positive and strongly positive. These outcomes are compared with those over the same sub-samples derived from the econometric model of real-wage growth in Castle and Hendry (2014) updated to 2021 in Castle et al. (2023), expressed in terms of Δw, revealing a close match in every sub-period. Thus, the constant real-wage growth model can account for the instabilities in the simple Phillips' curve, a successful mis-specification encompassing result (see Hendry, 1995, Ch. 14).

*We are pleased to acknowledge financial support from the Research Council of Norway, project 324472, on 'Model invariance and constancy in the face of large shocks to the Norwegian macroeconomic system', and Nuffield College, as well as helpful comments from Gunnar Bårdsen, Andrew B. Martinez, Ragnar Nymoen and participants at EcoMod2023. The data are fully documented and available. All calculations and graphs used *PcGive* (Doornik and Hendry, 2021). email: jennifer.castle@magd.ox.ac.uk and david.hendry@nuffield.ox.ac.uk

[1]Bill Phillips, whose amazing life is recounted by Alan Bollard (2016), is gratefully remembered by Hendry as his tutor at LSE in 1966–67 who successfully guided him through his initial struggles with econometrics.

We first compare Phillips Curves in price and wage inflation over the original sample in Phillips (1958), then extend the estimates to 2021 to establish its well-known instability. Next, we derive the nominal-wage inflation-unemployment relation from the real-wage model in Castle et al. (2023). We use that transform to demonstrate that its fitted values $\widehat{\Delta w_t}$ closely replicate the shifts for every sub-period. Thus the additional regressors must explain the shifts.[2] Confirmation that the underlying relationship between Δw_t and $U_{r,t}$ is constant conditional on the other regressors is shown by the relation between Δw_t and $U_{r,t}$ derived from the transformed real wage equation having essentially the same slope in every sub-period. However, it transpires that it is not 'shifts' in the real-wage model's regressors that explain the instabilities, which is a puzzle.

The structure of this paper is as follows. Section 2 compares Phillips Curves in price and wage inflation and replicates the original Phillips (1958) non-linear relation of wage inflation to the unemployment rate. Section 3 derives the nominal-wage inflation-unemployment relation from the real-wage model in CHM and Section 4 analyzes its sub-period implications. Section 5 tests the validity of conditioning on $U_{r,t}$. Section 6 examines subsample equations to ascertain what caused the shifts. Section 7 concludes and the Appendix 8 provides definitions and sources of the data series used.

2 Comparing Phillips Curves in price and wage inflation

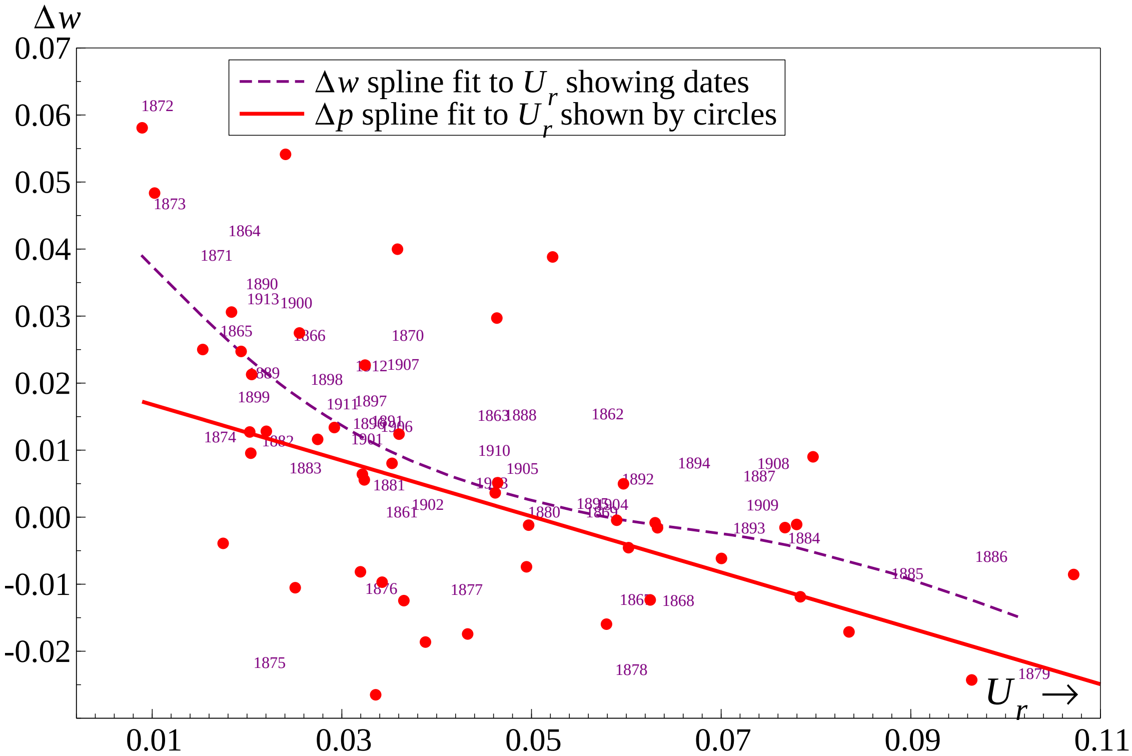

Figure 1: Comparing Phillips Curve in Δp with Δw.

Phillips (1958) related changes in nominal wages to the unemployment rate as both are labour market variables, but many recent variants use price inflation (see Forder, 2014 and Hoover, 2015,

[2] Similar non-invariance of new-Keynesian Phillips curves (NKPCs) is found by Castle et al. (2014).

for historical perspectives). Consequently, Figure 1 compares nominal wage and price inflation using annual observations on his data period 1860–1913. Phillips defined wage inflation, Dw, as $Dw = 0.5(W_{t+1} - W_{t-1})/W_t$, whereas we use the standard definition of the change in the log (there is a difference in timing of the peaks and troughs, but the two series are highly correlated: see Hendry, 2001). Phillips was aware that discrete approximations created moving-average errors (see Phillips, 2000), but in 1958 these were nearly impossible to estimate. He also knew that the 'loops' around his long-run relation represented dynamic adjustments, so calculated his equation from subsets of unemployment levels within which the average over a business cycle should be close to zero (see Desai, 1975).

Δp_t is price inflation measured by the GDP deflator. As Figure 1 shows, a cubic spline fitted to Δw matches Phillips' non-linear form, whereas price inflation results in a nearly straight line. Thus, all our results relate to wage inflation. Note that since Phillips conducted his study, pre-World War I (WWI) data on unemployment have been substantially revised by Boyer and Hatton (2002), but our pre-WWI results are close to those Phillips reported.

However, the simple bivariate relation between Δw and U_r was not to last as shown in Figure 2. The five sub-periods plots of Δw against U_r are chosen as the original Phillips' period 1860–1913 (before lags); WWI to the end of WWII; 1946–1980, namely post-war recovery till the end of the oil crisis; 1981–2011 which was the sample end in Castle and Hendry, 2014; and 2011–2021 which includes Brexit, the Covid-19 pandemic lockdowns and the UK government's furlough scheme to prevent excessive unemployment.

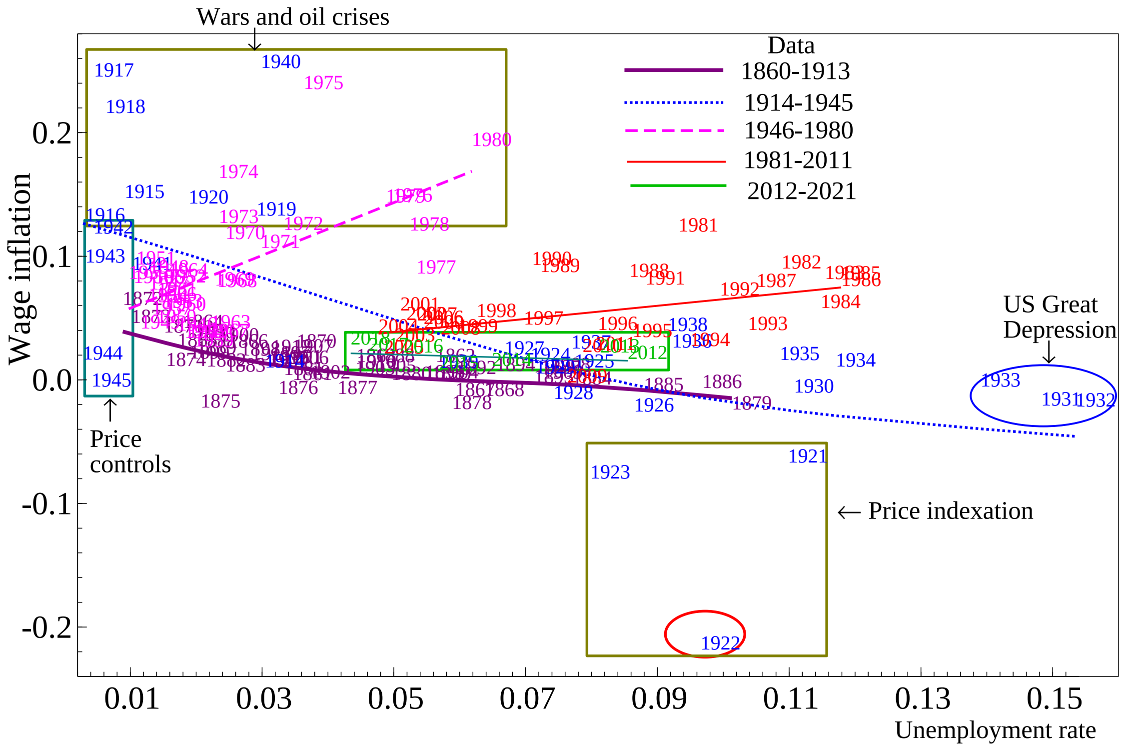

Figure 2: Shifts in wage inflation-unemployment relation.

That the Phillips Curve did not fail during the sub-sample which included WWI, the inter-war period with the post-war collapse and then the 'Great Depression', plus World War II is surprising (although the downward slope estimate doubled), but sloping up during the two less turbulent post-WWII periods, and flat since 2012 while U_r varied over 4%–8%, seem less excusable and confirms the unstable relation of Δw to U_r. The 'outliers' from wars, oil crises, price controls, indexation and the 'Great Depression' are highlighted in 'boxes' as they derive from different extraneous causes at different times. The resulting sub-sample coefficient estimates $\{\widehat{b}\}$ in the regression $\widehat{\Delta w_t} = \widehat{a} + \widehat{b}U_{r,t}$, with their heteroscedasticity and autocorrelation consistent (HAC) standard errors (see Andrews, 1991) are shown in Table 1.

	$\widehat{b}$	HAC standard errors
1861–1913	−0.67	0.10
1914–1945	−1.30	0.41
1946–1980	2.14	0.36
1981–2011	0.53	0.13
2012–2021	−0.27	0.15

Table 1: Estimates and HAC standard errors of $\widehat{b}$ in the regression $\widehat{\Delta w_t} = \widehat{a} + \widehat{b}U_{r,t}$

While Figure 1 shows cubic splines graphically fitted to the data given the non-linear relation in Phillips (1958), these are close to the linear sub-sample estimated regressions in Figure 2, confirming coefficients of U_r change substantially over time.

3 Deriving a wage inflation-unemployment relation from a real-wage model

The natural explanation of such unstable estimates is that relevant variables not included in the simple model experience shifts. If all excluded variables were stationary and maintained a constant correlation with U_r, its coefficient would be constant despite the omissions. Conversely, if the additional regressors included in the econometric model of real-wage growth in Castle et al. (2023) (reported for 1862–2021 in (1)) explained the shifts, its outcomes predicted for Δw_t in every sub-period should provide a close match. Figure 3 adds to Figure 2 the resulting sub-period plots of the full-sample $\{\widehat{\Delta w_t}\}$ against $U_{r,t}$ and the close match confirms that the regressors in (1) model account for the instabilities in the simple Phillips' curve. Since (1) is constant over the whole period T= 1862-2021 it successfully mis-specification encompasses the shifting Phillips curves.

$$
\begin{aligned}
\Delta\widehat{(w-p)}_t \;=\;& \underset{(0.04)}{0.40}\,\Delta(y-l)_t + \underset{(0.04)}{0.13}\,\Delta(y-l)_{t-1} - \underset{(0.03)}{0.14}\,\Delta^2 p_t - \underset{(0.03)}{0.18}\,(U_{r,t}-0.05) \\
& + \underset{(0.68)}{3.1}\,(U_{r,t}-0.05)^2 - \underset{(0.05)}{0.22}\,\Delta_2 U_{r,t} + \underset{(0.07)}{0.41}\,(\widetilde{f}_t \times \Delta p_t) - \underset{(0.01)}{0.13}\,S_{1939} \\
& + \underset{(0.02)}{0.18}\,S_{1940} - \underset{(0.01)}{0.07}\,S_{1941} - \underset{(0.01)}{0.05}\,I_{1916} - \underset{(0.01)}{0.05}\,I_{1977} + \underset{(0.01)}{0.03}\,I_{WWII} \\
& - \underset{(0.03)}{0.18}\,(w-p-y+l-\widehat{\mu})_{t-2} + \underset{(0.002)}{0.02}\,S_{2012} \qquad (1)
\end{aligned}
$$

$$\widehat{\sigma} = 1.1\% \quad \mathsf{R}^2 = 0.79 \quad \mathsf{F}_{\mathsf{ar}}(2,137) = 0.25 \quad \mathsf{F}_{\mathsf{arch}}(1,152) = 0.03$$
$$\chi^2_{nd}(2) = 0.62 \quad \mathsf{F}_{\mathsf{Het}}(19,130) = 2.5^{**} \quad \mathsf{F}_{\mathsf{reset}}(2,137) = 2.34 \quad \mathsf{F}_{\mathsf{nl}}(24,121) = 1.09$$

Coefficient standard errors are in parentheses (HAC in brackets), $\widehat{\sigma}$ is the residual standard deviation, F_{ar} tests residual autocorrelation (see Godfrey, 1978), $\mathsf{F}_{\mathsf{arch}}$ tests autoregressive conditional heteroscedasticity (see Engle, 1982), $\mathsf{F}_{\mathsf{het}}$ tests residual heteroskedasticity (see White, 1980), $\chi^2_{\mathsf{nd}}(2)$ tests non-Normality (see Doornik and Hansen, 2008), $\mathsf{F}_{\mathsf{reset}}$ tests non-linearity (see Ramsey, 1969), F_{nl} also tests non-linearity (see Castle and Hendry, 2010), and $\mathsf{F}_{\mathsf{chow}}$ tests parameter constancy (see Chow, 1960). One star indicates test significance at 5%, two at 1%. In (1), $\Delta\left(y-l\right)_t$ measures labour productivity and the labour share of income is given by $(w-p-y+l)_t$ where $\widehat{\mu}$ is its sample mean. S_{xxxx} is a step indicator taking the value 1 till the date $xxxx$ and 0 after, and I_{xxxx} is an indicator variable taking the value 1 for that observation only. Four selected consecutive impulse indicators during WWII are combined as their coefficients were equal and opposite signed ($I_{WWII} = I_{1942} + I_{1943} - I_{1944} - I_{1945}$) and:

$$\widetilde{f}_t = \frac{1}{0.88}\left(\left[1+\exp\left(-10\left(100\left(\Delta p_t\right)^2 - 0.2\right)\right)\right]^{-1} - 1\right). \tag{2}$$

is a logistic smooth transition function (see Luukkonen et al., 1988) where the scaling bounds the function between $[-1, 0]$).[3]

Most recently, (1) is constant over Brexit, the pandemic and the UK government's furlough scheme during lockdowns, and also passes a test for super-exogeneity of its contemporaneous regressors (see Engle et al., 1983 and Engle and Hendry, 1993).

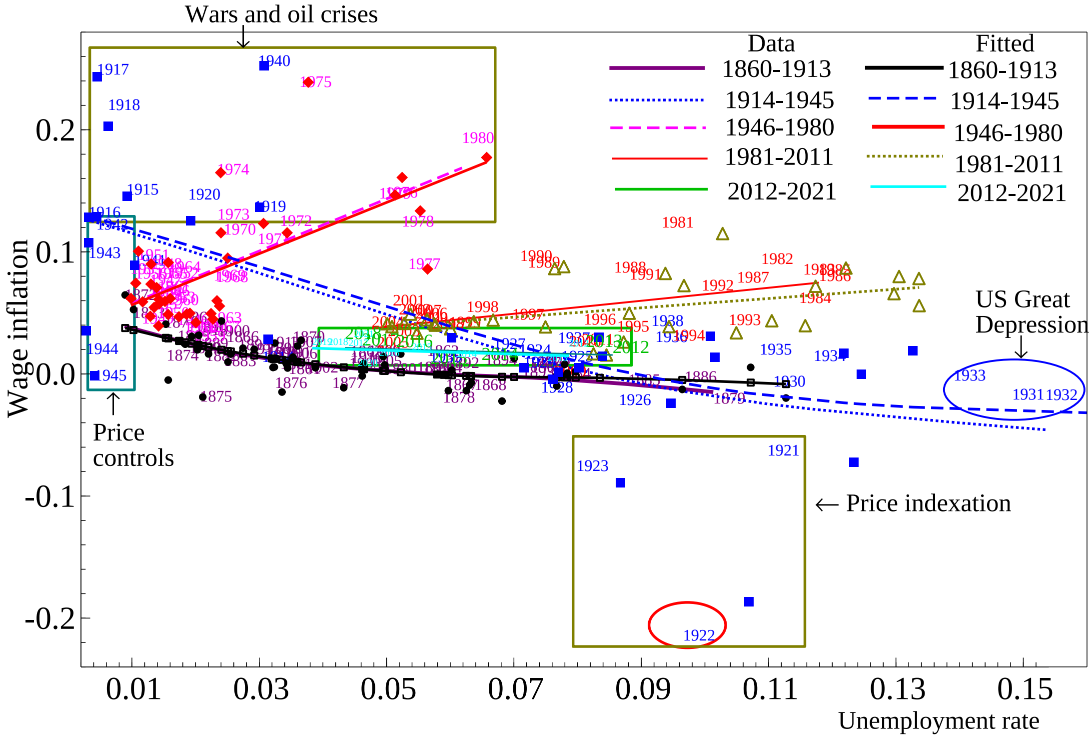

Figure 3: Comparing direct and derived Phillips Curves.

Expressing (1) in terms of Δw_t in relation to $U_{r,t}$ plus other drivers shown as $[\cdot]$ yields:

$$\begin{aligned}\widehat{\Delta w_t} \quad &= -0.71U_{r,t} + \big[3.1U^2_{r,t} + 0.22U_{r,t-2} + 0.025 + 0.86\Delta p_t + 0.14\Delta p_{t-1} \\ &+0.41(\widetilde{f}_t \times \Delta p_t) + 0.41\Delta(y-l)_t + 0.13\Delta(y-l)_{t-1} - 0.18(w-p-y+l-\widehat{\mu})_{t-2} \\ &-0.13S_{1939} + 0.18S_{1940} - 0.07S_{1941} + 0.02S_{2012} - 0.05I_{1916} - 0.05I_{1977} + 0.03I_{WWII}\big]\end{aligned} \tag{3}$$

[3] Castle and Hendry (2014) use a non-linear function of the form $f_t = -1/(1+1000(\Delta p_t)^2)$ which yields similar results.

The coefficient of Δp_t is carried over at unity from undoing $\Delta(w-p)_t$, so $0.86 = 1 - 0.14$ from 1), but is $0.96(0.08)$ if estimated unrestrictedly on the right-hand side of (1).

4 Sub-period implications

To evaluate if correcting for the additional drivers produced stable subsample estimates, we calculated $\widehat{x}_t$ as the sum of all the influences on Δw_t in $[\cdot]$ other than $U_{r,t}$ in (3) to derive $(\widehat{\Delta w_t | \widehat{x}_t})$ as the residuals from the full-sample regression of Δw_t on $\widehat{x}_t$, shown in (4) with HAC standard errors.

$$\widehat{\Delta w_t} = \underset{[0.007]}{-0.044} + \underset{[0.08]}{0.98}\ \widehat{x}_t \tag{4}$$

$$\widehat{\sigma} = 2.6\% \quad \mathsf{R}^2 = 0.80 \quad \mathsf{F_{ar}}(2,156) = 98^{**} \quad \mathsf{F_{arch}}(1,158) = 85^{**}$$
$$\chi^2_{nd}(2) = 3.3 \quad \mathsf{F_{Het}}(2,157) = 2.13 \quad \mathsf{F_{reset}}(2,156) = 7.3^{**}$$

Thus, wage inflation is only corrected by a scalar which uses the same coefficients in all sub-periods, leading to the full-sample regression recorded in (5). The resulting sub-period regressions are shown in Figure 4 and are nearly identical across all sub-periods, with the full-sample regression in (5).

$$(\widehat{\Delta w_t | \widehat{x}_t}) = \underset{(0.002)}{0.039} - \underset{(0.025)}{0.73}\ U_{r,t} \tag{5}$$

$$\widehat{\sigma} = 1.05\% \quad \mathsf{R}^2 = 0.84 \quad \mathsf{F_{ar}}(2,156) = 0.23 \quad \mathsf{F_{arch}}(1,158) = 0.03$$
$$\chi^2_{nd}(2) = 0.86 \quad \mathsf{F_{Het}}(2,157) = 0.47 \quad \mathsf{F_{reset}}(2,156) = 0.03$$

Table 2 reports the estimated coefficients of unemployment and their HAC standard errors, which stand in sharp contrast to the estimates in Table 1.

	coefficients	HAC standard errors
1860–1913	−0.76	0.07
1914–1945	−0.72	0.04
1946–1980	−0.85	0.15
1981–2011	−0.67	0.07
2012–2021	−0.84	0.11

Table 2: Coefficients of $U_{r,t}$ and HAC standard errors in the subsample $(\widehat{\Delta w_t | \widehat{x}_t})$ regressions

Although the Frisch and Waugh (1933) theorem suggests that $U_{r,t}$ should also be corrected for $\widehat{x}_t$, it was essentially orthogonal to $\widehat{x}_t$. This was a further surprise that the composite variable that explained most of the variance of real wages was not related to $U_{r,t}$. However, as shown in Table 3, while $U_{r,t}$ was uncorrelated with $\widehat{x}_t$ on the full sample, $U_{r,t}$ was significantly correlated with $\widehat{x}_t$ in those sub-samples where the Phillips curve shifted. The plot thickens...

In fact, the unconditional regression of Δw_t on $U_{r,t}$ delivers a similar coefficient of -0.72 to (5):

$$\widetilde{\Delta w_t} = \underset{[0.015]}{0.081} - \underset{[0.25]}{0.72}\ U_{r,t}$$

$$\widehat{\sigma} = 5.3\% \quad \mathsf{R}^2 = 0.17 \quad \mathsf{F_{ar}}(2,156) = 81^{**} \quad \mathsf{F_{arch}}(1,158) = 17.3^{**} \tag{6}$$
$$\chi^2_{nd}(2) = 34.3^{**} \quad \mathsf{F_{Het}}(2,157) = 0.28 \quad \mathsf{F_{reset}}(2,156) = 3.9^{*}$$

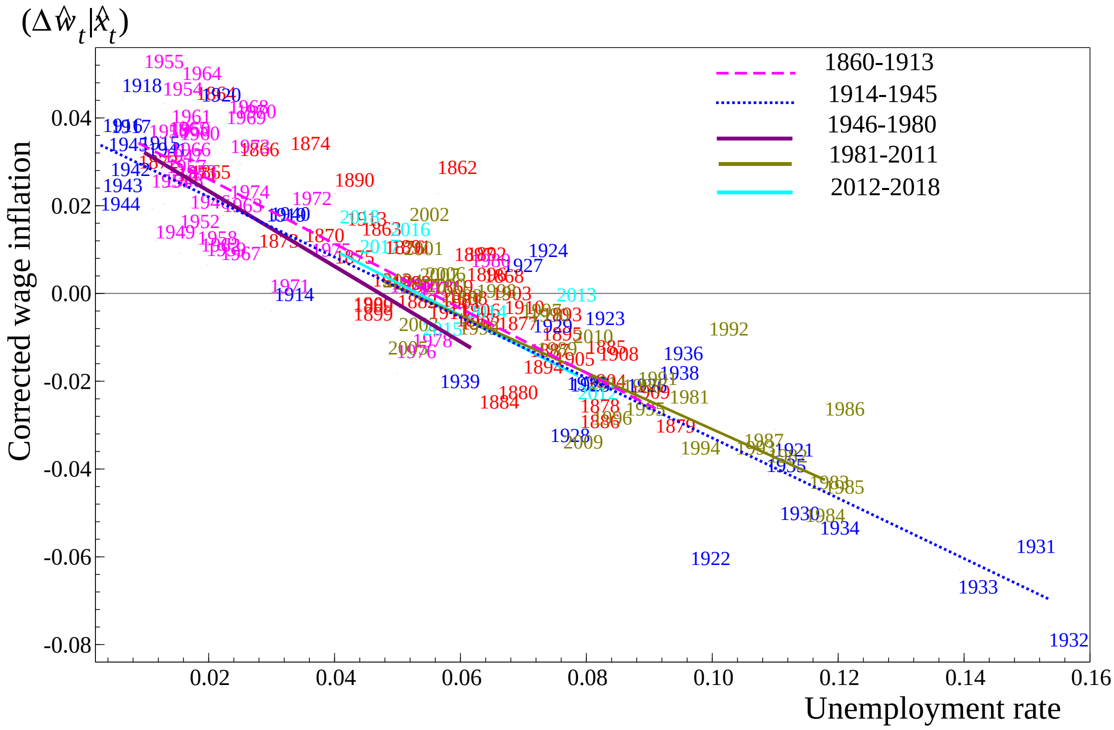

Figure 4: All sub-sample Phillips Curves for Δw_t corrected for $\widehat{x}_t$ on U_r.

	coefficients	standard errors	$\widehat{\sigma}$
1860–2021	0.002	0.05	0.033
1860–1913	0.11	0.18	0.019
1914–1945	−0.23	0.11	0.045
1946–1980	0.19	0.03	0.010
1981–2011	0.50	0.07	0.015
2012–2021	1.01	0.30	0.010

Table 3: Coefficients of $\widehat{x}_t$ and their standard errors with residual standard deviations $\widehat{\sigma}$ in the subsample $U_{r,t}$ regressions

Without accounting for price inflation, labour productivity, the wage share, non-linear wage-price spiral effects and exogenous shocks such as wars and oil crises, the Phillips curve is non-constant. These additional drivers obscure a constant nominal wage-unemployment rate relation over the last 160 years shown in Figure 5(b).

5 Testing the validity of conditioning on $U_{r,t}$

Valid conditioning in non-constant processes requires super exogeneity, see Engle et al. (1983) and Engle and Hendry (1993). Extending the automated test in Hendry and Santos (2010), we first apply impulse indicator saturation (IIS) and step indicator saturation (SIS) jointly at a significance level of 0.1% to the unconditional $U_{r,t}$ regression on a constant, shown in (7) to select step shifts over 1862–2021 to match the estimation samples above which were shorter from lagged variables.

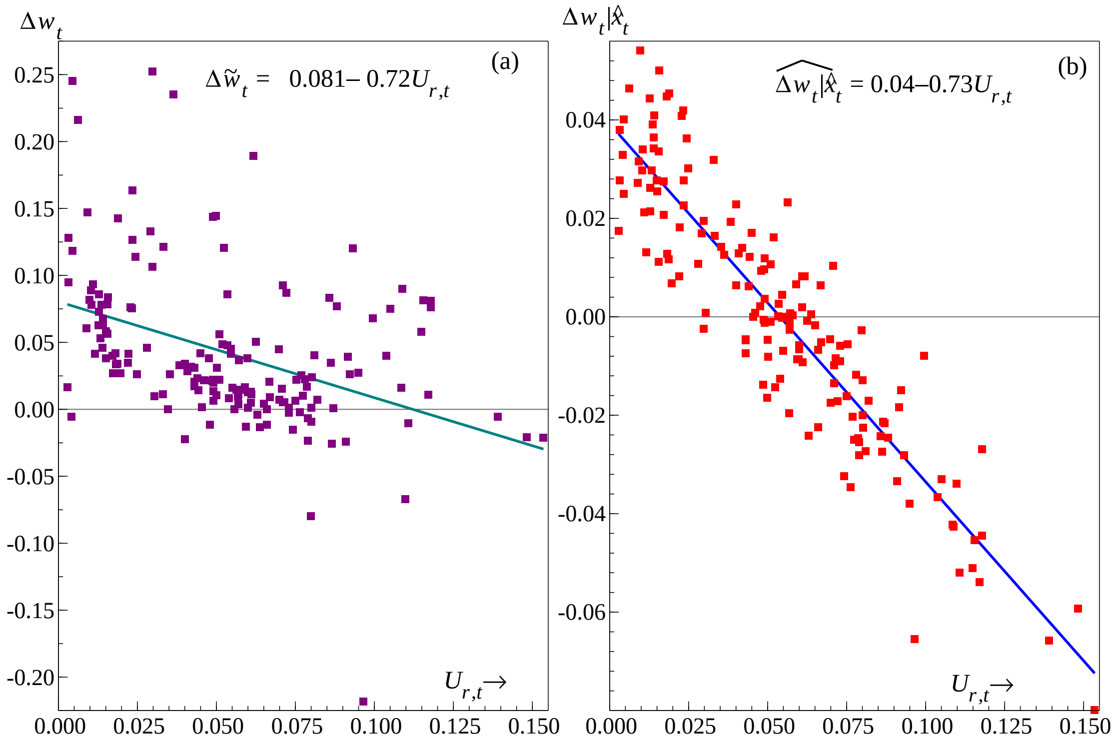

Figure 5: Full-sample Phillips Curves for (a) Δw_t on $U_{r,t}$; (b) $(\widehat{\Delta w_t | x_t})$ on $U_{r,t}$.

$$
\begin{aligned}
\widehat{U}_{r,t} = \; & \underset{[0.003]}{0.069} - \underset{[0.005]}{0.048}\, S_{1920} + \underset{[0.006]}{0.081}\, S_{1938} - \underset{[0.009]}{0.033}\, S_{1974} - \underset{[0.013]}{0.058}\, S_{1980} \\
& + \underset{[0.011]}{0.039}\, S_{1984} \qquad (7) \\
& \widehat{\sigma} = 0.02 \;\; \mathsf{R}^2 = 0.63 \;\; \mathsf{F}_{\mathsf{ar}}(2, 152) = 143^{**} \;\; \mathsf{F}_{\mathsf{arch}}(1, 158) = 91^{**} \\
& \chi^2_{nd}(2) = 1.7 \;\; \mathsf{F}_{\mathsf{reset}}(2, 151) = 0.00 \;\; \mathsf{F}_{\mathsf{exclude}}(5, 154) = 51.2^{**}
\end{aligned}
$$

In both (7) and (8), the standard errors reported are HAC, but the conventional standard errors also confirm significance as do the $\mathsf{F}_{\mathsf{exclude}}$ tests of excluding the indicators. Importantly, none of the step indicators in (7) also enter (3).

The five selected indicators are highly significant when added to (4) as reported in (8), which is expected as they are a 'big effects proxy' for the missing role of $U_{r,t}$.

$$
\begin{aligned}
\widehat{\Delta w_t} = \; & -\underset{[0.009]}{0.05} + \underset{[0.076]}{0.99}\, x_t - \underset{(0.01)}{0.04}\, S_{1920} - \underset{[0.01]}{0.06}\, S_{1938} \\
& + \underset{[0.009]}{0.03}\, S_{1974} + \underset{[0.004]}{0.04}\, S_{1980} - \underset{[0.008]}{0.03}\, S_{1984} \qquad (8) \\
& \widehat{\sigma} = 1.9\% \;\; \mathsf{R}^2 = 0.90 \;\; \mathsf{F}_{\mathsf{ar}}(2, 151) = 25.8^{**} \;\; \mathsf{F}_{\mathsf{arch}}(1, 158) = 7.3^{*} \;\; \chi^2_{nd}(2) = 0.13 \\
& \mathsf{F}_{\mathsf{Het}}(7, 152) = 3.57^{**} \;\; \mathsf{F}_{\mathsf{reset}}(2, 151) = 2.97 \;\; \mathsf{F}_{\mathsf{exclude}}(5, 153) = 30.7^{**}
\end{aligned}
$$

However, all the step indicators become insignificant when added to (5) as shown in (9) and no diagnostic tests reject. Thus the major shifts in $U_{r,t}$ do not enter the regression of $(\widehat{\Delta w_t|\widehat{x}_t})$ on $U_{r,t}$ confirming it is super exogenous in that model.

$$
\begin{aligned}
(\widehat{\Delta w_t|x_t}) \;=\; & \underset{(0.003)}{0.042} - \underset{(0.04)}{0.76}\ U_{r,t} - \underset{(0.004)}{0.001}\ S_{1920} + \underset{(0.005)}{0.004}\ S_{1938} + \underset{(0.005)}{0.002}\ S_{1974} \\
& - \underset{(0.007)}{0.004}\ S_{1980} - \underset{(0.006)}{0.002}\ S_{1984} \qquad (9)
\end{aligned}
$$

$$\widehat{\sigma} = 1.1\% \;\; \mathsf{R}^2 = 0.84 \;\; \mathsf{F_{ar}}(2,151) = 0.30 \;\; \mathsf{F_{arch}}(1,158) = 0.02 \;\; \chi^2_{nd}(2) = 1.3$$
$$\mathsf{F_{Het}}(7,152) = 1.74 \;\; \mathsf{F_{reset}}(2,151) = 0.09 \;\; \mathsf{F_{exclude}}(5,153) = 0.57$$

These results are consistent with the evidence in Castle and Hendry (2014) that most UK unemployment has been involuntary. Nevertheless, (5) and (9) are projections from a multivariate relation determining real wages, where the nominal level is determined by the price equation as in Hendry (2001).

6 Subsample equations: what caused the shifts?

Figure 6 plots the time series of Δw_t and $U_{r,t}$ with the sub-periods shown, where the bars mark the two world wars. Their patterns within each sub-period are very different, so it is unsurprising that the original Phillips curve would not be constant across the five subsamples, as shown in Figure 2.

Table 4 records the regression coefficient values with $|\mathsf{t}| \geq 2$ from fitting the general model to subsamples to examine what changes were due to which variables ($e_{t-2} = (w-p-y-l-\widehat{\mu})_{t-2}$). The first two sub-periods are similar to the full sample, but the next two differ in many respects (the final sample is too short to be reliable). In particular, the impacts of $(U_{r,t} - 0.05) = \bar{U}_{r,t}$ and $(\bar{U}_{r,t})^2$ are then insignificant, as are the non-linear inflation reactions. These *absent* impacts on wage inflation explain the upward slopes.

Variable	$\Delta(y-l)_t$	$\Delta(y-l)_{t-1}$	$\Delta^2 p_t$	$\bar{U}_{r,t}$	$(\bar{U}_{r,t})^2$	$\Delta_2 U_{r,t}$	$(\widetilde{f}_t \times \Delta p_t)$	e_{t-2}
1860–2021	0.41	0.13	−0.14	−0.18	3.1	−0.22	0.41	−0.18
1860–1913	0.18	0.17	−0.30	−0.20	3.3	−0.21	0.44	−0.10
1914–1945	0.34	0	−0.12	−0.22	3.4	−0.38	0.65	0
1946–1980	0.63	0.45	0	0	0	0	0	−0.41
1981–2011	0.49	0	−0.50	0	0	−0.47	0	0
2012–2021	0.23	0.42	−1.1	0.16	32	0	0.47	−0.61

Table 4: Coefficients with $|\mathsf{t}| \geq 2$ in the subsample regressions for the general model of $\Delta(w-p)_t$.

Imposing the whole sample coefficient estimates for the zero values over 1946–1980 in Table 4 yields an equation standard error of $\widetilde{\sigma} = 1.17\%$ as against the unrestricted fit of $\widehat{\sigma} = 1.11\%$. Similarly, for 1981–2011, $\widetilde{\sigma} = 0.91\%$ versus $\widehat{\sigma} = 0.98\%$ (the lower constrained value is an artefact of not counting restricted coefficients in the degrees of freedom). In neither case were any mis-specification tests significant for the constrained models, consistent with the overall constancy of the general model despite subsample estimate variations. Thus, the slope changes in the original Phillips curve are due to the lack of variability of the real-wage model regressors in the sub-samples: its stability needed hidden co-breaking (Hendry and Massmann, 2007) between the variables in the real-wage model, and the absence of some impacts stopped that occurring.

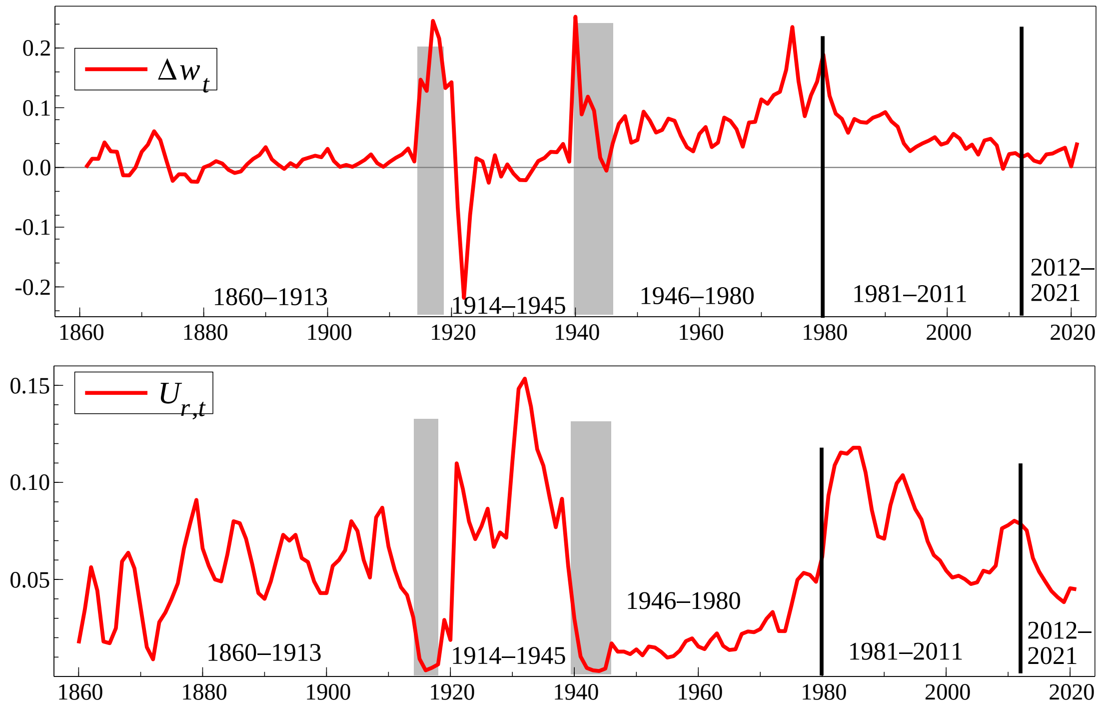

Figure 6: Time series of Δw_t and $U_{r,t}$ with the two World Wars shaded and the other subsamples shown by vertical lines.

7 Conclusion

The UK Phillips' curve relating changes in the log of nominal wages to unemployment is unstable. Sub-period relationships between wage inflation (Δw) and unemployment (U_r) can be strongly negative, slightly negative, flat, slightly positive and strongly positive in a time series from 1860 to 2021. Such behavior prompted five puzzles:

Puzzle 1: what caused the Phillips curve slopes to change so much?

Puzzle 2: why is the Phillips curve over the very turbulent period 1914–1945 similar to the original over the relatively stable 1860–1913?

Puzzle 3: can a constant-parameter real-wage model successfully encompass a shifting nominal wage equation?

Puzzle 4: does the lack of correlation of U_r with the full-sample estimated combination of the variables $\widehat{x}_t$ that explains changes in nominal wages, hold in subsamples?

Puzzle 5: why does correcting Δw_t by $\widehat{x}_t$ produce a near constant subsample set of equations for $\Delta w_t|\widehat{x}_t$ on $U_{r,t}$?

These puzzles concerning aspects of what caused the shifts in subsample Phillips curves can all be resolved as follows.

In Section 3, mis-specification encompassing (see Hendry and Nielsen, 2007, Ch. 13) revealed that the shifts in the subsample Phillips curves could be accounted for by a constant parameter real-wage equation;

In Section 4, partialling out from nominal wages the full-sample estimated coefficient linear combination of the regressors $\widehat{x}_t$, other than unemployment, showed that the resulting subsample equa-

tions had essentially the same downward slopes of between -0.67 to -0.85;
In Section 5 the validity of conditioning $(\widehat{\Delta w_t|\widehat{x}_t})$ on $U_{r,t}$ was confirmed;
In Section 6, the insignificance of estimated coefficients in subsample real-wage models in Table 4, matched when the Phillips curve shifted, as did the significance of the correlation of $U_{r,t}$ with $\widehat{x}_t$ in subsamples as in Table 3. Imposing the full-sample estimated values on insignificant subsample coefficients produced constant equations with no deterioration in fit, identifying the culprits behind the instability by their **absence**. Quite a surprise that the constancy of the nominal wage change-unemployment rate relationship depended on co-breaking of all the variables that it omitted from the constant real-wage model, then failed only when those lacked significance.

Although the whole sample regression of Δw_t on $U_{r,t}$ delivers the same coefficient as in a much more general constant parameter equation, it is not a useful way to model the inflation-unemployment relation important to economic policy. Instead, useful policy implications require taking account of the constant parameter, multivariate, non-linear, dynamic relationship for real wages that encompasses the original Phillips curve, interacted with a price inflation model to determine the overall level and persistence of inflation, as in Castle et al. (2023).

References

Andrews, D. W. K. (1991). Heteroskedasticity and autocorrelation consistent covariance matrix estimation. *Econometrica 59*, 817–858. https://doi.org/10.2307/2938229.

Bollard, A. (2016). *A Few Hares to Chase*. Oxford: Oxford University Press.

Boyer, G. R. and T. J. Hatton (2002). New estimates of British unemployment, 1870–1913. *Journal of Economic History 62*, 643–675. https://doi.org/10.1017/S0022050702001031.

Castle, J. L., J. A. Doornik, D. F. Hendry, and R. Nymoen (2014). Mis-specification testing: Non-invariance of expectations models of inflation. *Econometric Reviews 33*, 553–574. DOI:10.1080/07474938.2013.825137.

Castle, J. L. and D. F. Hendry (2010). A low-dimension portmanteau test for non-linearity. *Journal of Econometrics 158*, 231–245. https://doi.org/10.1016/j.jeconom.2010.01.006.

Castle, J. L. and D. F. Hendry (2014). Semi-automatic non-linear model selection. In N. Haldrup, M. Meitz, and P. Saikkonen (Eds.), *Essays in Nonlinear Time Series Econometrics*, pp. 163–197. Oxford: Oxford University Press. https://doi.org/10.1093/acprof:oso/9780199679959.003.0007.

Castle, J. L., D. F. Hendry, and A. B. Martinez (2023). The historical role of energy in UK inflation and productivity with implications for price inflation. *Energy Economics 126*. https://doi.org/10.1016/j.eneco.2023.106947.

Chow, G. C. (1960). Tests of equality between sets of coefficients in two linear regressions. *Econometrica 28*, 591–605. https://doi.org/10.2307/1910133.

Crafts, N. F. R. and T. C. Mills (1994). Trends in real wages in Britain, 1750-1913. *Explorations in Economic History 31*, 176–194. https://doi.org/10.1006/exeh.1994.1007.

Cunliffe, J. (2017). The Phillips curve: lower, flatter or in hiding? Speech, 14 November 2017, Bank of England. https://www.bankofengland.co.uk/-/media/boe/files/speech/2017/the-phillips-curve-lower-flatter-or-in-hiding-speech-by-jon-cunliffe.pdf.

Del Negro, M., M. Lenza, G. E. Primiceri, and A. Tambalotti (2020). What's up with the Phillips curve? *Brookings Papers on Economic Activity*, 301–357. https://www.jstor.org/stable/26996629.

Desai, M. J. (1975). The Phillips curve: A revisionist interpretation. *Economica 42*, 1–19. https://www.jstor.org/stable/2552982.

Doornik, J. A. and H. Hansen (2008). An omnibus test for univariate and multivariate normality. *Oxford Bulletin of Economics and Statistics 70*, 927–939. https://doi.org/10.1111/j.1468-0084.2008.00537.x.

Doornik, J. A. and D. F. Hendry (2021). *Empirical Econometric Modelling using PcGive: Volume I.* (9th ed.). London: Timberlake Consultants Press.

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity, with estimates of the variance of United Kingdom inflation. *Econometrica 50*, 987–1007. https://doi.org/10.2307/1912773.

Engle, R. F. and D. F. Hendry (1993). Testing super exogeneity and invariance in regression models. *Journal of Econometrics 56*, 119–139. https://doi.org/10.1016/0304-4076(93)90103-C.

Engle, R. F., D. F. Hendry, and J.-F. Richard (1983). Exogeneity. *Econometrica 51*, 277–304. https://doi.org/10.2307/1911990

Feinstein, C. H. (1972). *National Income, Expenditure and Output of the United Kingdom, 1855–1965*. Cambridge: Cambridge University Press.

Feinstein, C. H. (1990). New estimates of average earnings in the UK, 1880-1913. *Economic History Review 43*, 595–632. https://doi.org/10.2307/2596737.

Forder, J. (2014). *Macroeconomics and the Phillips Curve Myth*. Oxford: Oxford University Press.

Frisch, R. and F. V. Waugh (1933). Partial time regression as compared with individual trends. *Econometrica 1*, 221–223. https://doi.org/10.2307/1907330.

Godfrey, L. G. (1978). Testing for higher order serial correlation in regression equations when the regressors include lagged dependent variables. *Econometrica 46*, 1303–1313. https://doi.org/10.2307/1913830.

Haldane, A. and D. Quah (1999). UK Phillips curves and monetary policy. *Journal of Monetary Economics 44(2)*, 259–278. https://doi.org/10.1016/S0304-3932(99)00025-2.

Hendry, D. F. (1995). *Dynamic Econometrics*. Oxford: Oxford University Press.

Hendry, D. F. (2001). Modelling UK inflation, 1875–1991. *Journal of Applied Econometrics 16*, 255–275. https://doi.org/10.1002/jae.615.

Hendry, D. F. and N. R. Ericsson (1991). An econometric analysis of UK money demand in 'Monetary Trends in the United States and the United Kingdom' by Milton Friedman and Anna J. Schwartz. *American Economic Review 81*, 8–38. https://www.jstor.org/stable/2006786.

Hendry, D. F. and M. Massmann (2007). Co-breaking: Recent advances and a synopsis of the literature. *Journal of Business and Economic Statistics 25*, 33–51. https://doi.org/10.1198/073500106000000422

Hendry, D. F. and B. Nielsen (2007). *Econometric Modeling: A Likelihood Approach*. Princeton: Princeton University Press.

Hendry, D. F. and C. Santos (2010). An automatic test of super exogeneity. In M. W. Watson, T. Bollerslev, and J. Russell (Eds.), *Volatility and Time Series Econometrics*, pp. 164–193. Oxford: Oxford University Press. https://doi.org/10.1093/acprof:oso/9780199549498.003.0009.

Hoover, K. D. (2015). The genesis of Samuelson and Solow's price-inflation Phillips curve. *History of Economics Review 61(Winter)*, 1–16. https://doi.org/10.1080/18386318.2015.11681269.

Leeson, R. (Ed.) (2000). *A. W. H. Phillips: Collected Works in Contemporary Perspective*. Cambridge: Cambridge University Press.

Luukkonen, R., P. Saikkonen, and T. Teräsvirta (1988). Testing linearity against smooth transition autoregressive models. *Biometrika 75(3)*, 491–499. http://www.jstor.com/stable/2336599.

Mitchell, B. R. (1988). *British Historical Statistics*. Cambridge: Cambridge University Press.

Phillips, A. W. H. (1958). The relation between unemployment and the rate of change of money wage rates in the United Kingdom, 1861–1957. *Economica 25*, 283–299. https://www.jstor.org/stable/2550759. Reprinted in Leeson (2000), pp. 243–260.

Phillips, A. W. H. (2000). The estimation of systems of difference equations with moving average disturbances. See Leeson (2000), pp. 423–444. Walras-Bowley Lecture, 1966 Econometric Society Meeting, San Francisco.

Powell, J. (2019). December 2019 post-FOMC meeting press conference. Speech, 11 December 2019, Federal Reserve Board.
https://www.federalreserve.gov/mediacenter/files/fomcpresconf20191211.pdf.

Ramsey, J. B. (1969). Tests for specification errors in classical linear least squares regression analysis. *Journal of the Royal Statistical Society B, 31*, 350–371. https://www.jstor.org/stable/2984219.

Shadman-Mehta, F. (1995). An empirical study of the determinants of real wages and employment: The Phillips curve revisited. Unpublished thesis, Université Catholique de Louvain, Belgium.

Sleeman, A. (1981). The relation between unemployment and the rate of change of money wage rates in the United Kingdom, 1851-1979. Paper presented to the Atlantic Economic Society, LSE, London.

Thomas, J. J. (1984). Wages and prices in the United Kingdom, 1862-1913: A re-examination of the Phillips curve. Presentation, ESRC Quantitative Economic History Study Group, Oxford.

White, H. (1980). A heteroskedastic-consistent covariance matrix estimator and a direct test for heteroskedasticity. *Econometrica 48*, 817–838. https://doi.org/10.2307/1912934.

8 Appendix: data definitions and sources

Y_t	$=$	real GDP, £million, 1985 prices	[6], p.836, [5]a (1993), ONS code:YBHH at 2005 prices, [9].
P_t	$=$	implicit deflator of GDP, (1860=1)	[3], p.836, [5]a (1993), ONS code: ABML, [9].
U_t	$=$	unemployment	[4], [5]c (1993), ONS code: MGSC.
$Wpop_t$	$=$	working population	[4], [5]c (1993), ONS code: MGSF.
$U_{r,t}$	$=$	$U_t/Wpop_t$ (unemployment rate, fraction)	
L_t	$=$	employment (= $Wpop_t - U_t$)	[1], [5]
W_t	$=$	average weekly wage earnings	[7], [8], ONS code: LNMM
$W_{r,t}$	$=$	nominal wage rates	[2], [6], [8]
Δz_t	$=$	$(z_t - z_{t-1})$ for any variable z_t	
$\Delta^2 z_t$	$=$	$\Delta z_t - \Delta z_{t-1}$	

Sources:
[1] Shadman-Mehta (1995) (who cites Sleeman (1981) and Thomas (1984) as sources);
[2] Phillips (1958);
[3] Mitchell (1988);
[4] Feinstein (1972) and Boyer and Hatton (2002);
[5] Bean ((a) *Economic Trends Annual Supplements*, (b) *Annual Abstract of Statistics*, (c) *Department of Employment Gazette* and (d) *National Income and Expenditure*, as well as other sources cited here);
[6] Office for National Statistics, Blue Book;
[7] Crafts and Mills (1994);
[8] Feinstein (1990);
[9] ONS.
Hendry and Ericsson (1991) and Hendry (2001) provide detailed information about many of these series.