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Simulating labor supply behavior when workers have preferences for job opportunities and face nonlinear budget constraints

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Simulating labor supply behavior when workers have preferences for job opportunities and face nonlinear budget constraints

Abstract:
This paper analyzes the properties of a particular sectoral labor supply model developed and estimated in Dagsvik and Strøm (2006). In this model, agents have preferences over sectors and latent job attributes. Moreover, the model allows for a representation of the individual choice sets of feasible jobs in the economy. The properties of the model are explored by calculating elasticities and through simulations of the effects of particular tax reforms. The overall wage elasticities are rather small, but these small elasticities shadow for much stronger sectoral responses. An overall wage increase and, of course, a wage increase in the private sector only, gives women an incentive to shift their labor supply from the public to the private sector. Marginal tax rates were cut considerably in the 1992 tax reform. We find that the impact on overall labor supply is rather modest, but again these modest changes shadow for stronger sectoral changes. The tax reform stimulated the women to shift their labor from the public to the private sector and to work longer hours. A calculation of mean compensated variation shows that the richest households benefited far more from the 1992 tax reform than did the poorest households.

Keywords: Labor supply, married females, structural model, sectoral choice, wage elasticities, evaluation of tax reforms

JEL classification: J22, C51

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1. Introduction

To sort out and assess effects of policy interventions beyond popular beliefs, it is necessary to apply models that can simulate behavioral responses. Although nonbehavioral models are useful for studying direct effects on income distributions resulting from policy changes (conditional on household behavior), their disadvantage is that they are unable to provide answers to most of the questions raised in the political debate, such as how a suggested tax reform will change labor supply, and who loses and who gains from the reform (Creedy et al., 2002).

Since Ashenfelter and Heckman (1974) and Heckman (1974a and 1974b), empirical labor supply models have been embedded in a behavioral framework. One important extension of this line of research was made by Burtless and Hausman (1978), who proposed a method which explicitly accounted for piecewise linear and nonconvex budget sets. That paper and others by Hausman led to an outpouring of empirical research based on his approach; see Blundell and MaCurdy (1999) for a survey of these studies, and also MaCurdy et al. (1990) and Heim and Meyer (2003). In this research and also in more recent work on labor supply, such as van Soest et al. (2002), it is assumed that the choice variable is hours of work, which can be chosen freely in the market. Wage rates are determined by human capital characteristics and, together with the chosen hours of work and the tax structure facing the agents, the disposable household incomes follow. In our opinion, an important weakness with this standard approach is the neglect of the possibility that nonpecuniary job attributes may matter a great deal to the agents' labor market choices. In the conventional approach, it is simply assumed with no further discussion that disposable income and leisure are the only decision variables that affect preferences.

In contrast, the point of departure in this paper is that in addition to leisure and disposable income, "job type" is an important decision variable. Type of job and other nonpecuniary job attributes may matter a great deal for the chosen labor market affiliation of the individuals. Some jobs may be more interesting and challenging than other jobs and to explain the long working hours among scientists and government bureaucrats, for instance, solely with reference to after-tax wage rates may be quite misleading. In Dagsvik and Strøm (2006), as well as in previous contributions, cf. Aaberge, Dagsvik and Strøm (1995), Dagsvik and Strøm (1997), Aaberge, Colombino and Strøm (1999), it is assumed that the agent faces a choice set of feasible jobs with job-specific (given) hours of work and job-specific wages. Thus, in this setup, realized hours of work are equal to job-specific hours of the chosen job. This seems to be consistent with labor markets throughout the industrialized world where it is typically found that hours of work are fixed for many types of jobs. This is due to firm technology, government regulations and/or the outcome of negotiations between unions and employers associations in unionized economies. Thus, to change working load within this setup, one has to change jobs; see Altonji and Paxson (1988) for findings that support this view. Although most job attributes are unobserved (apart from the sector aspect), in our application this alternative point of departure has important implication for the econometric modeling framework, which differs radically from the conventional approach. Specifically, it implies a new way of interpreting and dealing with quantity constraints in the labor market. Typically, data on hours of work show at peaks full-time and part-time hours of work. Within our alternative approach, this is interpreted as resulting from institutional constraints in the labor market, implying that most jobs offer typical full-time or part-time hours of work. For a review of discrete choice approaches, see, for example, Creedy and Kalb (2005).

The purpose of this paper is to explore in detail some features of the alternative approach developed and estimated by Dagsvik and Strøm (2006). Dagsvik and Strøm (2006) extend Aaberge, Dagsvik and Strøm (1995) and Aaberge, Colombino and Strøm (1999) in that one important job attribute is observed, namely which sector the chosen job belongs to, either “public” or “private”. To this end, in this paper we have carried out a wide range of simulation experiments that demonstrate the degree of magnitude and heterogeneity in responses, and the results of these experiments are summarized below. Dagsvik and Strøm (2006) extend the previous work of this type in another aspect, namely by providing a theoretical justification of the functional form of the utility function.

To account for differences in socioeconomic characteristics is of great importance in the assessment of how changes in tax rules affect labor supply and household welfare; see Atkinson (1995). In our framework, as well as in other discrete choice approaches, observed as well as unobserved heterogeneity plays a more important role than in the standard approach owing to the fact that we allow for more general functional form specifications and more variety in the distribution of taste shifters because of the assumption that agents have preferences over nonpecuniary job attributes. However, in the two-sector model developed by Dagsvik and Strøm (2006), additional observed heterogeneity is accounted for through the choice of sector. Type of job and sector affiliations may matter for labor supply responses when tax systems are changed, as the more interesting and challenging a job is, the less important may be the net wage (above a certain level). Those who have these types of jobs are not randomly chosen in the population; they tend to be well educated, with high wage incomes, and their spouse may also fit the same characteristics. This kind of behavior may have strong implications for how tax rules should be changed to stimulate labor supply. Improved economic incentives should be targeted towards those who respond, not necessarily towards those with the highest education and income levels, who face the highest marginal tax rates.

The sector dimension of the model allows us to go beyond overall labor supply responses to changes in wages and tax rates. Our hypothesis is that although overall labor supply may be rather inelastic, these modest labor supply responses may shadow for stronger responses with respect to sectoral choice. Highly educated women are often found working in the public sector in a welfare state such as Norway. Job security is higher than in the private sector, human capital seems to be priced slightly higher and the public sector may offer better opportunities to find subsidized childcare facilities. On the other hand, in the private sector, wages are more dispersed and hours are less regulated. We should thus expect that stronger incentives to work, like higher wages or lower marginal tax rates, may have an impact on the sectoral choice of working women. Higher wages, in particular in the private sector, or lower marginal tax rates, may give women an incentive to shift labor supply away from the public towards the private sector. A typical example is a part-time nurse or a medical doctor in a public hospital who shifts her labor supply to a private clinic with longer working hours. However, the income of the spouse may affect the choice of the wife and we should remember that matching in the marriage market is not random. Typically, a woman with a high potential wage in the market is married to a man with similar opportunities.

The results of the policy experiments are reported through different measures. First, we report labor supply responses, overall and across sectors. We also report labor supply responses according to deciles in the distribution of disposable household income. To compute, say, wage elasticities, conditional on disposable household income, we derive from the model the joint probability of sector choice, hours of work and disposable household income. From this joint probability, we derive the conditional probability of sector choice and hours of work, conditional on household income. The computation of wage elasticities, conditional on disposable income, is then straightforward. Second, to evaluate a policy intervention that consists in implementing a particular tax reform, we report compensating variation (CV) figures using the methodology of Dagsvik and Karlström (2005). In the 1980s and early 1990s, many OECD countries reformed their tax systems. Progressive tax schedules were changed towards proportional taxation. Marginal tax rates were cut, in particular for high incomes. The Norwegian tax reform we analyze here was similar and took place in 1992. The top marginal tax rate on wage income was reduced from 0.654 to 0.495. Our analysis shows that such a sharp reduction in marginal tax rates stimulates overall labor supply to some extent, but it gives married women an incentive to move from the public sector to the private sector where hours are less constrained and wage dispersion is higher. Despite the fact that labor supply increases, which enlarges the tax base, tax revenues are reduced. Thus, the view held by some politicians that government revenue can be increased by cutting tax rates does not to seem to hold even in the highly tax-progressive Scandinavian welfare states. An important aspect of the paper is the calculation of the mean value of the change in household welfare (CV) that follows from the tax reform. We find that the rich gained far more than did the poor from the 1992 tax reform.

The paper is organized as follows. In the next section, the model is explained briefly. Data are described in Section 5 and the empirical specification and estimation results follow in the next section. Labor supply elasticities are reported and interpreted in Section 7. In Section 8, we estimate the model under the restriction that the distribution of hours of work among the feasible jobs is uniform. In Sections 9 and 10, the implications of two tax reforms are analyzed.

2. The one-sector model

In what follows, we give a simplified description of the model. For more details and for justifications of the assumption made, we refer the reader to Dagsvik and Strøm (2004, 2006). The present paper is about the labor supply of married and cohabiting women. The labor supply and hence the wage income of the husband is exogenously given. For expository reasons, we begin by assuming that the agent can choose between jobs in one sector versus not working. Later, we will show how the model can be extended to deal with choice of jobs in different sectors of the economy.

The household is assumed to derive utility from household consumption, here set equal to household disposable income, leisure and nonpecuniary attributes of jobs. Let $U(C,h,z)$ be the utility function of a household, where *C* is disposable household income, *h* is hours of work of the married woman, *z* indexes jobs ($z=1,2,3,\ldots$) and $z=0$ represents not working. The reason why the index z enters the utility function is that job-specific attributes beyond wage and hours of work may affect the utility of the agents. Examples of such attributes are type of work, stimulating colleagues, location of the work place and subsidized kindergarten. In principle, some of these job-specific attributes can be observed, but it is obvious that the researcher cannot observe many of them, and that many cannot be represented quantitatively. The assessments of these unobserved job-specific attributes may vary across agents. Thus, as an outside observer, the researcher has to assume that the utility function is random.

For given hours of work *h* and wage rate *w*, disposable household income is given by

$$C=f\left(hw,I\right), \tag{1}$$

where $f(\cdot)$ is a function that transforms pretax incomes into after-tax incomes. The pretax incomes are the wage income of the married female (*hw*) and three nonlabor income components included in the vector *I*. These three incomes are the wage income of the husband, the capital income of the household and child allowances, which vary with the number of children up to the age of 18. Child allowances are not taxed. All details of the tax structure are taken into account in the estimation of the model. The tax functions of wage income in 1994, as well as child allowances, are given in Appendix B. From there, we note that the tax functions differ depending on whether both spouses are working. Capital income is taxed at a flat rate of 0.28. The utility function has the structure

$$U\left(C,h,z\right)=v\left(C,h\right)\varepsilon(z), \tag{2}$$

for $z=0,1,2,3,\ldots$, where $v(\cdot)$ is a positive deterministic function and $\varepsilon(z)$ is a positive random taste shifter. The taste shifter accounts for unobserved individual characteristics and unobserved job-specific attributes. These taste shifters $\{\varepsilon(z)\}$, are assumed to be i.i.d. across jobs and agents, with c.d.f. $\exp\left(x^{-1}\right)$, $x>0$. To the outside observer, the agent's choice set of feasible jobs is not known and it may differ from the choice sets of other agents. To represent such unobserved heterogeneity in opportunities, it is desirable to apply a framework that allows for a convenient representation of stochastic choice sets. Such a representation is discussed in Dagsvik and Strøm (2004, 2006). Here, in this section, and for expository reasons only, we describe the model when choice sets vary solely by observable characteristics.

Assume that the agent faces a fixed (individual-specific) wage rate in the labor market and let $B(h)$ denote the agent's sets of available jobs with offered hours of work h. Let $m(h)$ be the number of jobs in the choice set $B(h)$. Although $m(h)$ may also depend on the wage rate, we suppress this in the notation here. The nonmarket choice consists of one alternative so that $B(0)=\{0\}$ and $m(0)=1$. Let D be the set of feasible annual hours. To simplify exposition let

$$\psi\left(h,W,I\right)=v\left(f\left(hW,I\right),h\right). \tag{3}$$

For the sake of interpretation and empirical specification, it will be convenient to express $m(h)$ as follows. Let $\theta=\sum_{h>0}m(h)$ and $g(h)=m(h)/\theta$, which yields $m(h)=\theta g(h)$. Note that θ is the total number of jobs available to the woman in the market, whereas $g(h)$ is the fraction of feasible jobs (for the woman) with h hours of work.

The assumptions made above allow us to derive the probability that an agent will choose a job with hours of work h within the choice set $B(h)$. By well-known results, it follows that if the agent is a utility maximizer, then the probability that job z within the choice set $B(h)$ is chosen, is given by

$$P\left[\psi\left(h,W,I\right)\varepsilon(z)=\max_x \max_{k\in D(x)}\left(\psi\left(x,W,I\right)\varepsilon(k)\right)\right]=\frac{\psi\left(h,W,I\right)}{\sum_{x\in D}\sum_{k\in B(x)}\psi\left(x,W,I\right)}=\frac{\psi\left(h,W,I\right)}{\sum_{x\in D}m(x)\psi\left(x,W,I\right)}. \tag{4}$$

Let $\varphi\left(h;W,I\right)$ denote the probability of choosing *any* job within the choice set $B(h)$, and let H denote the hours of work of the chosen job. Obviously, $\varphi\left(h;W,I\right)$ must be equal to the sum of the probabilities of choosing a specific job in the choice set $B(h)$, summed over all jobs in this choice set. Then, from (3) and (4), we get

$$
\begin{aligned}
\varphi\left(h;W,I\right) &= P\left(H=h \mid W,I\right) = \sum_{z\in B(h)} P\left[\psi\left(h,W,I\right)\varepsilon(z) = \max_x \max_{k\in B(x)} \psi\left(x,W,I\right)\varepsilon(k)\right] \\
&= \sum_{z\in B(h)} \frac{\psi\left(h,W,I\right)}{\sum_{x\in D} m\left(x,W,I\right)\psi\left(x;W,I\right)} = \frac{m(h)\psi\left(h,W,I\right)}{\psi\left(0,0,I\right)+\sum_{x>0,x\in D} m(x)\psi\left(x,W,I\right)} \\
&= \frac{\theta g(h)\psi\left(h,W,I\right)}{\psi\left(0,0,I\right)+\theta\sum_{x>0,x\in D} g(x)\psi\left(x,W,I\right)}.
\end{aligned} \tag{5}
$$

Note that the choice probability of not working, $\varphi\left(0,W;I\right)$, follows from (5) by replacing the numerator with $\psi(0,0,I)$. Note also that we have used that $\psi\left(0,W,I\right)=\psi\left(0,0,I\right)$. We note that the numerator in the choice probability in (5) can be interpreted as the representative value of jobs with hours of work h (the $\psi(\cdot)$ term), weighted by the number of feasible jobs in the market with the respective characteristics (the $m(\cdot)$ term). The denominator represents the corresponding sum of weighted values of the representative indirect utility, including the option of not working.

It should be emphasized that the weighting of utilities in the choice probabilities by the respective number of feasible jobs is justified with a reference to how the labor market is organized and regulated. A change in these institutional constraints in the labor market will change the labor supply probabilities. These eventual changes are driven by changes in demand-side factors (firm technology) and/or labor market institutions (government regulation or outcomes about working hours in the negotiations between unions and employers' associations) and not by changes in preferences. This setup should be contrasted with the *ad hoc* approach of van Soest (1994) and Callan and van Soest (1996), who include a penalty rate for particular hours in the utility function so that the probabilities at these points are reduced; see Creedy and Kalb (2005) for a review of these and related approaches. In the approach of van Soest (1994), a change in a labor market institution or firm technology that changes the constraints on hours offered in the market will falsely be attributed to changes in preferences.

In the modeling framework discussed above, it is assumed that the wage rates are specific to each individual. This differs from the setup in Aaberge, Dagsvik and Strøm (1995) and Aaberge, Colombino and Strøm (1999) where the wage rates are allowed to vary across jobs. The problem with the approach with job-specific wage rates is that it is difficult to separate variations in the wage rates across jobs for a given individual from variation in the wage rates across individuals. For example, in the works referred to above, identification is facilitated by the restriction that random effects are ruled out. This is unreasonable as it is reasonable to believe that, within a group of individuals of a given age and educational level, the variation in wage rate levels across individuals is substantially greater than variation in wage rate opportunities across jobs, for a given person. This motivates the approach taken in Dagsvik and Strøm (2006), where the emphasis is on variation in wage rates across individuals due to unobservables (random effects).

3. Unobserved heterogeneity in choice sets

In the modeling setup above, the choice sets of feasible jobs were assumed to be equal across observationally identical individuals. This is clearly unsatisfactory as it is highly likely that choice sets vary according to unobserved individual characteristics. Thus, it would be desirable to extend the modeling framework to allow for stochastic choice sets. This issue has been discussed in Dagsvik (1994) and in the context of labor supply modeling in Dagsvik and Strøm (2006). Dagsvik and Strøm (2006) discuss how a continuous "weighted multinomial logit" version of (5) is consistent with an interpretation with random choice sets. However, the discrete version discussed above is also consistent with stochastic choice sets. (By letting the c.d.f. *G* be a discrete distribution, the desired result follows from Theorem 7 in Dagsvik (1994).) To clarify and interpret the formalism of Dagsvik (1994), we outline the key concepts below.

To this end, let $H(z)$ denote the (fixed) hours of work associated with job z with taste shifters $\varepsilon(z)$. Recall that the taste shifters' $\varepsilon(z)$ represents the utility value of the nonpecuniary unobservable attributes of job z, that is $\varepsilon(z)=\varepsilon^{*}\left(T(z)\right)$, where $\varepsilon^{*}(\cdot)$ is a suitable deterministic function and $T(z)$ is a vector of unobserved qualitative attributes of job z. Let $\left\{\left(H(z),T(z)\right), z=1,2,...\right\}$ be the set of feasible offered hours and qualitative attributes. A key assumption is that $\left(H(z),T(z)\right)$, $z=1,2,...$, are independently and randomly scattered in some suitable set Ω. In addition, $H(z)$ and $T(z)$ are independent. A formal representation of such "spatial" stochastic processes is the bivariate Poisson process, with components $H(z)$ and $T(z)$ that are independent. If the Poisson process is homogeneous, the points are randomly but evenly distributed on Ω. If the Poisson process is nonhomogeneous, the points are unevenly distributed in the sense that it is likely that the concentration of points in some parts of Ω is higher than in other parts. It follows that the transformed process with points $\left\{\left(H(z),\varepsilon(z)\right), z=1,2,...\right\}$ is also a Poisson process. The concentration of points in a Poisson process can be represented by the so-called *intensity measure*, which in this case equals $\theta dG(h)\cdot d\varepsilon/\varepsilon^{2}$, for

$\varepsilon > 0$, $h > 0$, where $G(\cdot)$ is a c.d.f. function and $\theta > 0$ is a constant. The interpretation of the intensity measure is that $\theta dG(h) \cdot d\varepsilon / \varepsilon^2$ is the probability that a job with $H(z) \in (h, h + dh)$, $\varepsilon(z) \in (\varepsilon, \varepsilon + d\varepsilon)$ is feasible to the agent. If $G(h)$ is a step function, it follows that $dG(h)$ is zero, except at points where $G(h)$ "jumps". The number of points in the Poisson process with this particular intensity measure, with $G(h)$ being a step function with a finite number of steps, can be shown to be infinite (but countable). However, there will only be a finite number of points (jobs) with different hours of work, corresponding to the hours in *D*. Let $g(h) = dG(h)$. Then, $g(h)$ will be zero if $h \notin D$ and positive otherwise. As demonstrated in Dagsvik (1994), the choice probability density of *H* is given by (5). Note that the c.d.f. *G* in the intensity measure can be a completely general step function, whereas the particular functional form of the other factor, $d\varepsilon / \varepsilon^2$, is necessary for obtaining the weighted multinomial logit structure. In the case where *G* is absolutely continuous and thus differentiable, the corresponding choice model will be a continuous probability density, as discussed in Dagsvik and Strøm (2006).

4. The two-sector model

The two-sector model is similar to the model for one sector. The utility function is assumed to have the form $U(C, h, j, z) = \mu_j v(C, h) \varepsilon_j(z)$, where $j = 1, 2$ indexes the sectors and μ_j represents the average taste for working in sector j. The agent is assumed to face two wage rates, W_1 and W_2, specific to each sector. Not working is indexed by $j = 0$, in which case $z = 0$. Let $W = (W_1, W_2)$. Availability of jobs in the two sectors is allowed to vary across the two sectors and also across the human capital characteristics of the agents. For many reasons, most women are working in the service branch of the economy. In Norway, most of the services are provided by the public sector (health services, education etc) and many of these jobs require higher education, whereas the services provided in the private sector, say in retail sales, are typically based on low-skilled labor. Hence, it makes sense to assume that the availability of jobs in the two sectors depends on education level.

Let $\varphi_j(h; W, I)$ be the probability of choosing sector *j* and hours of work *h*. Similarly to (5), it follows that

(6)
$$\varphi_j(h; W, I) = \frac{\psi(h, W_j, I) g_j(h) \theta_j \mu_j}{\psi(0, 0, I) + \sum_{k=1}^{2} \sum_{x>0, x \in D} \psi(x, W_k, I) g_k(x) \theta_k \mu_k},$$

for $h > 0, \quad j = 1, 2$, and

(7) $$\varphi_0(0;W,I)=\frac{\psi(0,0,I)}{\psi(0,0,I)+\sum_{k=1}^{2}\sum_{x>0,x\in D}\psi(x,W_k,I)g_k(x)\theta_k\mu_k},$$

for $h=0$. Here $g_j(h),\ j=1,2,$ denotes the opportunity density of offered hours in sector *j*. The densities of offered hours are assumed to be uniform, apart from peaks at typical full-time and part-time hours. This accounts for the fact that there are more jobs available in the labor market with part-time hours and full-time hours. This assumption means that when we observe many people working, say full time, rushing to their job at 7 a.m. and rushing back home at 5 p.m., this may reflect constraints on offered hours in the market rather than individuals preferring to have the exact same working load. In a perfect competitive economy with no constraints on offered hours, offered hours are uniformly distributed and hence $g_j(h)$ is a constant. Similarly to the one-sector model, the terms $\theta_j,\ j=1,2,$ represent the availability of jobs in sector *j*. Note that the preference parameter μ_j cannot be separated from θ_j without additional assumptions. To achieve identification, we have therefore set $\mu_j=1$. Below, we will assume that there are more jobs available for the higher educated in the public sector than in the private.

5. Data

Data on the labor supply of married women in Norway used in this study consist of a merged sample of the "Survey of Income and Wealth, 1994" and the "Level of living conditions, 1995" (Statistics Norway, 1994 and 1995, respectively). Data cover married couples as well as cohabiting couples with common children. The ages of the spouses range from 25 to 64. None of the spouses is self-employed and none of them is on disability or other type of benefits. A person is classified as a wageworker if their income from wage work is higher than their income from self-employment. All taxes paid are observed and in the assessment of disposable income, at hours not observed, all details of the tax system are accounted for. Hours of work are calculated as the sum of hours of the main job as well as those of any side jobs. In 1994, the unemployment rate in Norway was rather low by international standards. For that reason, the fact that we employ actual hours worked instead of desired hours (which were unavailable) is unlikely to be of significance.

Wage rates above NOK 350 or below NOK 40[1] are not utilized when estimating the wage equations. The wage rates are computed as the ratio of annual wage income to hours worked. When computing annual wage income, we take into account the fact that some women have multiple jobs.

[1] In June 2006, 1 USD≈NOK 6.20

The size of the sample used in estimating the labor supply model is 810. Descriptions of variables and summary statistics are given in Appendix A and the tax function and child allowances are given in Appendix B.

6. Empirical specification and estimation results

The choice set of offered hours is assumed to be represented by seven intervals. The medians of the intervals range from 315 annual hours to 2600 annual hours and are given by $D = \{0, 315, 780, 1040, 1560, 1976, 2340, 2600\}$. The midpoints in the intervals for part-time and full-time jobs are 1040 and 1976 annual hours, respectively.

Wage rates are assumed to depend on human capital characteristics such as potential work experience and education. Both the levels of and the returns to human capital are allowed to vary between the public and the private sectors. When estimating the model given in (6) and (7), we face two problems. First, sector j wage rates are observed only for those who work in sector j. Second, wage rates may be endogenous in the sense that they may be correlated with the taste shifters. To deal with these issues, sector-specific wage equations are estimated and used as instrument variables. In the wage equations, log wage rates are specified as a linear function of experience (defined as age minus years of education and minus six), experience squared and education level. The random error terms are assumed to be independent across sectors and normally distributed. To control for selectivity, wage equations are estimated separately in a two-step procedure; see Dagsvik and Strøm (2004). Subsequently, the sector-specific wage rates in the model are replaced by the respective estimated wage equations, including the error term. As the wage equations contain these random error terms, we must take the expectation of the choice probabilities (6) and (7) with respect to these error terms. We refer the reader to Dagsvik and Strøm (2004) for further details.

The estimates of the parameters of the wage equations obtained by Dagsvik and Strøm (2006) are given in Appendix C. From the results given there, we observe that the wage level, given the observed covariates, is predicted to be slightly higher in the private sector than in the public sector. On the other hand, human capital variables like experience and education are priced marginally higher in the public sector compared to the private sector. From the estimates given in Appendix C, we observe that the standard deviation of the error term in the public sector, σ_1, is estimated to be 0.243, whereas in the private sector the corresponding standard deviation σ_2 is estimated to be 0.274. Thus, the wage level, as well as the dispersion in wages, is slightly higher in the private sector than in the public sector, whereas observed human capital is priced higher out on the margin in the public sector.

Next, consider the structure of θ_j. As discussed in Section 2, in general this term will depend on the distribution of preferences of the workers and profit (cost) function of the firms through equilibrium conditions. To this end, Dagsvik (2000) discusses equilibrium issues in matching markets with heterogeneous suppliers and demanders. In fact, he demonstrates that labor supply models of the type considered in this paper are special cases within the framework in Dagsvik (2000, see section 6.2). There, he shows that one can express the opportunity density $\theta_j g_j(h)$ as the product of the number of vacancies in sector j times the conditional profit (cost) function, conditional on hours of work. This implies that the sector-specific vacancy levels are sufficient statistics for the equilibrium conditions in the sense that if the vacancy levels are observed, then $\theta_j g_j(h)$ divided by the number of vacancies in sector j will depend only on the systematic part of the conditional profit functions of the firms. This property of the model could be utilized to identify and estimate structural specifications of the conditional profit functions, which would, apart from the vacancy levels, imply a structural specification of the opportunity densities. It is, however, far beyond the scope of our paper to identify and estimate a structural specification of the conditional profit functions of the firms. Therefore, we choose a reduced form specification, namely

(8) $$\log \theta_j = f_{j1} + f_{j2} S,$$

for $j = 1, 2$, where S is the length of education.

The functional form of the deterministic part of the utility function is a critical issue in structural empirical analysis. In most studies, a class of functional form is selected in a purely ad hoc manner. Dagsvik and Strøm (2006) postulated a particular invariance axiom that the model should satisfy and demonstrated that this implies that the deterministic term of the utility function has a Box–Cox type of functional, as given in Appendix C. For the reader's convenience, we also report the estimates obtained by Dagsvik and Strøm (2006) in Appendix C. The estimated coefficients imply that the deterministic part of the utility function is quasi-concave. The interaction term between consumption and leisure is negative and significantly different from zero, which means that separability between consumption and leisure is rejected. Marginal utilities with respect to consumption and leisure are positive. The marginal utility of leisure declines with age to around 32 years of age and thereafter it increases with age. The number of young and "old" children has a similar and positive effect on the marginal utility of leisure. Thus, when the woman is young and has children, she has a reduced incentive to take part in work outside the home and when the children have grown up, her incentive to participate in the labor market again weakens because she is becoming older.

We would expect that offered hours in the public sector are more concentrated at full-time hours than in the private sector. The unions are stronger with a much higher coverage in the public than in the private sector. We would also expect that there are more jobs available for the higher educated woman in the public sector than in the private sector. These expectations are confirmed by the estimates given in Appendix C. The estimated model fits the data quite well; see Dagsvik and Strøm (2006). The model predicts that 8.3 per cent of married women are not working. The predicted split between the public and the private sector is almost equal, with slightly more married women working in the public sector than in the private sector. Hours worked, conditional on working, are predicted to be almost the same across sectors, with a few more hours predicted for the private sector than for the public. Moreover, we show that in our empirical application, this class of functional form is fairly flexible, as compared to the class of third- to fifth-order polynomials in consumption and leisure; see van Soest et al. (2002). For more details regarding estimates and predictions, and comparisons with other model specifications, we refer the reader to Dagsvik and Strøm (2004, 2006).

Table 1 reports how the predicted choice probabilities vary with socioeconomic characteristics. The probability of *not working* decreases with age and education, and sharply increases with the number of children. The older the woman and the lower is her level of education, the more likely it is that she works in the private sector.

The probability of working in the public sector is remarkably similar across varying numbers of children. In contrast, the probability of working in the private sector declines rather strongly with the number of children. These findings accord well with widely held conjectures that childcare facilities and leave with pay at the time of giving birth are more easily available in the public sector than in the private. The predictions in Table 1 also accord with the "observation" that for highly educated women, who tend to be married to educated and well-paid men, there are more interesting and challenging jobs in the public than in the private sector. We observe that participation in the public sector increases rather sharply with the years of education of the woman.

Table 2 provides predictions of the conditional expectations of hours and their variation with socioeconomic characteristics. Expected hours, given working, are predicted to vary little across ages. They drop sharply in both sectors when the household has two or more children. Of particular interest is the prediction of how hours vary with education in the two sectors. In the public sector, hours increase slightly with years of education, whereas in the private sector, the highly educated woman is predicted to work rather long hours. As mentioned above, highly educated women tend to prefer the public rather than the private sector, but those who do work in the private sector work long hours. Although our estimates indicate that human capital is priced higher out at the margin in the public sector, we should keep in mind that hours are less regulated in the private sector and wage dispersion is higher. Examples of well-paid women working long hours in the private sector are women in leading management positions and female doctors working in private clinics rather than in public hospitals. The question is whether improvements in job opportunities like higher wages, lower taxes and less regulated hours will move more women with high education from the public sector to the private sector. These are some of the issues that we discuss in the next sections.

Table 1. Choice probabilities and their variation with socioeconomic variables for married women, Norway, 1994. Per cent

Variables	Not working	Public sector	Private sector
Age range:			
25–34	10.45	47.32	42.33
35–44	7.75	49.05	43.20
43–64	6.80	44.71	48.49
Number of children:			
0	4.89	46.02	49.09
1	6.18	48.88	44.94
2	10.09	46.76	43.15
More than 2	16.79	47.03	36.18
Education:			
Less than 9 years	9.71	27.54	62.74
Intermediate	9.05	43.42	47.52
High, 15–17 years	4.42	73.27	22.31

Table 2. Conditional expectations of annual hours and their variation with socioeconomic variables for married women, Norway, 1994

Variables	Public sector	Private sector
Age range:		
25–34	1530	1576
35–44	1571	1631
43–64	1598	1608
Number of children:		
0	1689	1694
1	1627	1662
2	1490	1530
More than 2	1310	1363
Education:		
Less than 9 years	1535	1531
Intermediate	1552	1604
High, 15–17 years	1607	1768

7. Elasticities

This section contains simulation results that throw light on the properties of the model. All simulations are based on analytic expressions of the respective choice variables such as "hours of work", "disposable income", "choice of sector", and "working versus not working". Above we obtained an expression for the probability for the choice of sector, hours of work and not working. We shall now consider the derivation of the joint probability of sector choice, hours of work and disposable income (consumption). This simultaneous choice probability is needed for computing the conditional elasticities such as wage elasticity of labor supply, given the level of consumption.

Let

$$\tilde{\varphi}_j\left(h,y;I\right)=P\left(H=h,J=j,C\le y\,|\,I\right).$$

It is implicit in the above definition that we condition on the explanatory variable in the wage equations. The error terms in the wage equations will be integrated out. From (1), it then follows that

$$\begin{aligned}\tilde{\varphi}_j\left(h,y;I\right)&=P\left(f\left(hW_j,I\right)\le y,H=h,J=j\,|\,I\right)\\&=E_W\left\{P\left(f\left(hW_j,I\right)\le y\,|\,H=h,J=j,W\,|\,I\right)\varphi_j\left(h;W,I\right)\right\},\end{aligned}\tag{9}$$

where E_W denotes the expectation with respect to W, where W is the vector of sector wage rates. Note that when $\left(H,J,W\right)$ are given, then $f\left(hW_j,I\right)$ is nonstochastic and the probability

$$P\left(f\left(hW_j,I\right)\le y\,|\,H=h,J=j,W,I\right)$$

is equal to zero or one. Let

$$\kappa\left(x,I,y\right)=\begin{cases}1 & \text{if } f\left(x,I\right)\le y\\0 & \text{otherwise}.\end{cases}$$

Then, we can express (9) as

$$\tilde{\varphi}_j\left(h,y,I\right)=E_W\left\{\kappa\left(hW_j,I,y\right)\varphi_j\left(h;W,I\right)\right\}\tag{10}$$

for $j=1,2$. Similarly,

$$\tilde{\varphi}_0\left(0,y,I\right)=\kappa\left(0,I,y\right)E_W\varphi_0\left(0;W,I\right).\tag{11}$$

In practice, the probability in (10) is computed by stochastic simulation as follows. Let W_j^r be given by the wage equation of sector *j* as

$$\log W_j^r = X\beta_j + \sigma_j \eta_j^r \tag{12}$$

where $\eta_j^r, r = 1,2,...,M,$ are independent draws from $N(0,1)$. If *M* is large

$$\tilde{\varphi}_j(h, y, I) \cong \frac{1}{M}\sum_{r=1}^{M} \kappa\left(hW_j^r, I, y\right)\varphi_j\left(h; W^r, I\right) \tag{13}$$

where $W^r = \left(W_1^r, W_2^r\right)$. Once we have obtained $\tilde{\varphi}_j(h, y, I)$ it follows immediately that

$$P\left(H = h, J = j \,|\, y_1 < C \le y_2, I\right) = \frac{\tilde{\varphi}_j(h, y_2, I) - \tilde{\varphi}_j(h, y_1, I)}{\sum_{k>0}\sum_{x>0, x\in D}\left(\tilde{\varphi}_k(x, y_2, I) - \tilde{\varphi}_k(x, y_1, I)\right) + \tilde{\varphi}_0(0, y_2, I) - \tilde{\varphi}_0(0, y_1, I)}. \tag{14}$$

Eq. (14) expresses the conditional density of chosen hours and sector, given that disposable income lies within the interval (y_1, y_2). From this expression, we can compute the corresponding conditional mean hours of work and different types of conditional elasticities, such as the conditional elasticity of mean hours, given that consumption lies within some interval. On the basis of (13), we can also simulate the marginal distribution of consumption, as this distribution is given by

$$\tilde{\varphi}(y, I) \equiv \sum_{k=1}^{2}\sum_{x>0, x\in D}\tilde{\varphi}_k(x, y, I) + \varphi_0(0, y, I). \tag{15}$$

From (15), we can derive the decile limits. To this end, let *N* be the number of households and let

$$\Phi(y) = \frac{1}{N}\sum_{i=1}^{N}\tilde{\varphi}^i(y, I_i). \tag{16}$$

Define y_n, by $\Phi(y_n) = 0.n$ for $n = 1$ and 9. Thus, if disposable income of household *i*, C_i, is equal to or below y_1, it belongs to the first decile, whereas if $y_1 < C_1 \le y_9$, the disposable income belongs to the second to ninth deciles in the distribution of disposable income. If $C_1 > y_9$, the disposable income belongs to the tenth decile. Below, we use these decile limits when we report the wage elasticities for households with income in the first, second to ninth and the tenth deciles. From (14) to (16), we observe that the denominator in (14) equals 0.1N, 0.8N and 0.1N for the first, second to ninth and the tenth deciles, respectively.

In Tables 3–5, we report wage elasticities in labor supply among married women when the hourly wage rates are increased. The choice probabilities are used to calculate these elasticities. We have used stochastic simulation to calculate the expectation of the choice probabilities with respect to the error terms in the wage equations. The marginal effects are calculated for each individual and thereafter aggregated, and subsequently the corresponding elasticities are calculated. We term them aggregate elasticities. They measure the elasticities of aggregate labor supply (participation, expected hours worked) with respect to the wage rates.

An overall wage increase and overall labor supply

The first column of Table 3 defines the categories for which the elasticities are calculated. We have calculated the aggregated elasticities for the whole sample and according to deciles in the distribution of disposable income. The second column gives the elasticities of the probabilities of working, working in the public sector and working in the private sector. For simplicity, we term these elasticities the working sector elasticities. The next column gives the elasticities of hours of work, given that the individual works either in the public sector or in the private sector. The last column gives the elasticities of the unconditional expectation of labor supply with respect to wage rate changes.[2]

The sector dimension introduced here plays a novel role in how increased wage rates may affect behavior. In the public sector, human capital variables are priced marginally higher than in the private sector which makes the public sector more attractive for women with a higher education. On the other hand, hours are more regulated and the level of wage rates is lower and wage dispersion is less than in the private sector. The prospect of a wage increase may thus give the woman working in the public sector an incentive to move to the private sector. A typical example is a nurse or a medical doctor working in a public hospital who starts working in a private clinic because it offers higher wages.

From Table 3, we first observe that the probability of participation in the labor market is low in the first decile in the household income distribution, whereas in the upper deciles, participation is rather high, most likely the highest in the world. We also observe that hours supplied increase with the deciles in the household income distribution.

When turning to the simulated elasticities in Table 3, we note that an overall wage increase implies an elasticity with respect to working (in any sector) of 0.14. The elasticity of hours supplied, conditional on working, is slightly higher, 0.19, which means that the aggregate elasticity of labor supply in the population of married females in Norway in 1994 sums up to around 0.34.

[2] The last column is approximately equal to the sum of the preceding columns. The equality is not exact due to aggregation.

The next lines in Table 3 give the same elasticities according to the deciles in the distribution of disposable household income. Women in the lower deciles in the distribution of disposable household income tend to be less responsive to wage increases than women belonging to higher deciles. Elasticities of participation are nearly the same across the deciles, whereas the elasticities of expected hours, conditional on working, increase with the deciles in the household income distribution.

An overall wage increase and sectoral responses

Turning now to the choice of sector and the sector-specific supply of hours, we observe that, in the public sector, the wage elasticities related to participation are declining with the deciles in the distribution of disposable household income. The negative sign of the elasticities of the public sector choice probability with respect to an overall wage increase for women in the upper deciles in the wage rate distribution (–0.18) indicates that women earning the highest wage rate may have some incentive to shift from working in the public sector to working in the private sector. The "high" elasticity of private sector choice probability (0.40) in the upper deciles supports this hypothesis.

In the private sector, we observe that the elasticity of the choice probability is increasing with the deciles in the disposable household income distribution. As mentioned above, education and experience is priced marginally higher in the public than in the private sector, which means that women with higher education and more experience should be expected to favor the public sector over the private. However, as demonstrated above, wage rate levels and the dispersion of wage rates, captured by the variance in the distribution of the random variables that affect wage rates, are higher in the private than in the public sector. Moreover, offered hours tend to be less spiked at full-time hours in the private sector than in the public. All of these factors are accounted for in the model. A higher chance of finding jobs with longer working hours and higher wage levels may be the reason why women with the highest wage rate would like to shift their labor supply from the public to the private sector, and to find jobs with longer working hours, when there is an overall increase in wage rates. The elasticity of expected hours, conditional on working, is slightly higher in the public than in the private sector.

We also report the elasticity of tax revenue with respect to an overall increase in wage rates. Tax revenues are increased for two reasons. A higher wage rate yields higher earnings, given labor supply. A higher wage rate stimulates labor supply. We observe that the elasticity is estimated to be 0.69, which is clearly less than 1.

A wage increase in the public sector only

In Table 4, we report the wage elasticities when only the wage rate in the public sector is increased. Comparing Tables 3 and 4, we observe that the effects on overall labor supply are considerably weaker when the wage rates in the public sector only are increased. The most important result is that the modest wage elasticities related to work in any sector (overall labor supply) shadow for much higher intersector wage elasticities. An increase in wage rates in the public sector gives women an incentive to move from the private to the public sector. Hours of work, given the sector, are only modestly affected.

A wage increase in the private sector only

The same pattern emerges when the wage rates in the private sector only are increased, as shown in Table 5. The wage elasticities related to participation in the public sector decline with the deciles in the distribution of disposable household income, whereas the opposite occurs in the private sector. We note that women in the higher deciles in the household income distribution have a strong incentive to shift jobs from the public to the private sector as a response to a wage increase in the private sector.

Table 3. Aggregate elasticities of labor supply with respect to an overall wage increase in the public and the private sector. Married Norwegian females, 1994

	Mean working sector		Mean conditional expected hours		Mean unconditional expected hours	
	Probability per cent	Elasticities	Hours	Elasticities	Hours	Elasticities
All (Private and Public)		0.14	1559	0.19	1455	0.34
Deciles of Disp. Income:						
1	56.8	0.16	911	0.07	517	0.23
2–9	95.5	0.14	1600	0.19	1527	0.32
10	96.5	0.16	1877	0.28	1812	0.44
Public	91.7	0.06	1551	0.23	737	0.28
Deciles of Disp. Income:						
1	33.7	0.25	931	0.14	314	0.39
2–9	49.5	0.07	1596	0.22	790	0.29
10	40.4	–0.18	1812	0.30	733	0.11
Private	44.7	0.22	1563	0.15	718	0.39
Deciles of Disp. Income:						
1	23.1	0.03	882	–0.04	203	–0.01
2–9	46.0	0.21	1604	0.15	737	0.36
10	56.1	0.40	1923	0.26	1079	0.67
Total tax revenue						0.69

Table 4. Aggregate elasticities of labor supply with respect to a wage increase in the public sector. Married Norwegian females, 1994

	Mean working sector		Mean conditional expected hours		Mean unconditional expected hours	
	Probability per cent	Elasticities	Hours	Elasticities	Hours	Elasticities
All (Private and Public)	91.7	0.08	1559	0.12	1455	0.20
Deciles of Disp. Income:						
1	56.8	0.11	911	0.04	517	0.15
2–9	95.5	0.08	1600	0.13	1527	0.21
10	96.5	0.08	1877	0.08	1812	0.16
Public	47.0	1.46	1551	0.22	737	1.76
Deciles of Disp. Income:						
1	33.7	0.31	931	0.11	314	0.42
2–9	49.5	1.41	1596	0.22	790	1.65
10	40.4	2.93	1812	0.27	733	3.28
Private	44.7	–1.37	1563	0.00	718	–1.40
Deciles of Disp. Income:						
1	23.1	–0.18	882	–0.01	203	–0.27
2–9	46.0	–1.35	1604	0.01	737	–1.34
10	56.1	–1.98	1923	0.01	1079	–1.97
Total tax revenue						0.28

Table 5. Aggregate elasticities of labor supply with respect to a wage increase in the private sector. Married Norwegian females, 1994

	Mean working sector		Mean conditional expected hours		Mean unconditional expected hours	
	Probability per cent	Elasticities	Hours	Elasticities	Hours	Elasticities
All (Private and Public)	91.7	0.07	1559	0.10	1455	0.18
Deciles of Disp. Income:						
1	56.8	0.04	911	0.02	517	0.06
2–9	95.5	0.07	1600	0.08	1527	0.15
10	96.5	0.11	1877	0.25	1812	0.37
Public	47.0	–1.36	1551	0.01	737	–1.42
Deciles of Disp. Income:						
1	33.7	–0.08	931	–0.00	314	–0.08
2–9	49.5	–1.32	1596	0.00	790	–1.32
10	40.4	–2.82	1812	0.03	733	–2.80
Private	44.7	1.58	1563	0.14	718	1.80
Deciles of Disp. Income:						
1	23.1	0.21	882	0.06	203	0.26
2–9	46.0	1.57	1604	0.14	737	1.73
10	56.1	2.22	1923	0.24	1079	2.52
Total tax revenue						0.49

8. The impact on labor supply of constraints on offered hours

In the model, $\{\theta_j g_j(h)\}$ represents the choice restrictions of available jobs at different hours. As discussed above, the density of offered hours have two peaks, relating to part-time hours of work and full-time hours of work. The alternative, which is that offered hours are assumed to be uniformly distributed, corresponds to the standard assumption in neoclassical labor supply models. To illustrate the role of this latter assumption, we have used the model to simulate the impact on labor supply of replacing our opportunity density, with spikes at part-time and full-time hours, by uniformly distributed offered hours. To do so, we have adjusted the coefficients in the indicator for job availability in the two sectors so that the total number of available jobs remains constant. How this is done is set out in Appendix C.3.

The impact on the choice probabilities is not shown here, but it is negligible. The probability of not working is 0.083 when hours are constrained and 0.086 when offered hours are uniform. The impact on the choice probabilities of working in the public and private sectors, respectively, is of the same negligible order of magnitude.

Of greater interest is the impact on hours worked (Table 6) and wage elasticities (Table 7). Table 6 shows that replacing the current constraints on offered hours with uniformly distributed hours has a *negative* effect on annual hours supplied, conditional on working, and more so in the public sector than in the private. The reduction in overall labor supply, conditional on working, amounts to 5.5 per cent.

Because the spikes play a more important role in the public than in the private sector, we would expect that labor supply is more affected in the public sector than in the private sector when the constraint on hours is removed. Removing the constraint on offered hours makes it more likely that a woman will find jobs with fewer than full-time hours, in particular when working in the public sector. It depends on preferences and on the nonlabor income, whether labor supply will increase or decrease. However, for many women, say those with children and with a husband in a full-time job, fewer hours than implied by a full-time job may be preferred. Our simulation results confirm this conjecture. The expected labor supply declines by 8.8 per cent in the public sector and by 2 per cent in the private sector. Thus, it seems that the constraints on offered hours in the Norwegian welfare state force the married women to work longer hours, in particular in the public sector and in the lowest deciles in the distribution of household incomes.

Table 6. Mean expected annual hours, conditional on working. Married women, Norway, 1994. Constrained hours versus uniform hours when the amount of available jobs is kept the same

Categories	Constrained offered hours	Uniform offered hours
	Working in any sector	
All	1559	1474
1 decile*	911	808
2–9 deciles	1600	1506
10 deciles	1877	1888
	Working in the public sector	
All	1551	1415
1 decile	931	813
2–9 deciles	1596	1447
10 deciles	1812	1757
	Working in the private sector	
All	1563	1532
1 decile	882	799
2–9 deciles	1604	1569
10 deciles	1923	1923

*Decile(s) refers to the deciles in the distribution of disposable household income, 1994.

Table 7 gives the wage elasticities, conditional on working, and we see that with uniformly distributed offered hours, labor supply becomes more responsive, in particular among those working in the public sector and in the lowest deciles. The reason is that with fewer constraints on choices, labor supply becomes more responsive.

Table 7. Aggregate elasticities of labor supply with respect to an overall wage

Categories	Constrained offered hours	Uniform offered hours
	Working in any sector	
All	0.19	0.44
1 decile*	0.07	0.44
2–9 deciles	0.19	0.44
10 deciles	0.28	0.44
	Working in the public sector	
All	0.23	0.53
1 decile	0.14	0.54
2–9 deciles	0.22	0.53
10 deciles	0.30	0.54
	Working in the private sector	
All	0.15	0.42
1 decile	–0.04	0.42
2–9 deciles	0.15	0.42
10 deciles	0.26	0.45

*Decile(s) refers to the deciles in the distribution of disposable household income, 1994.

9. Labor-supply effects of tax reforms

In 1992, the Norwegian tax system was reformed, with a move towards lower and less progressive tax rates. In subsequent years, the tax structure remained virtually unchanged. Therefore, to assess the effects on labor supply we have chosen to focus on 1991, the year prior to the tax reform, and a postreform year, 1994.

The tax rates on labor incomes in these years are set out in Appendix B, and we observe that the 1992 reform considerably reduced the top marginal tax rate from 0.654 to 0.495. To assess the labor supply responses to this reform, we have employed our model to simulate the labor supply among married women. Because the 1992 reform was a move towards less progressive taxes, we have also used the model to simulate the impact on labor supply of replacing the 1994 tax system with a flat and revenue-neutral tax system. The results are reported in Table 8. Note that when taxes are changed, this also implies a change in the taxation of the wage income of the spouse.

In our model, when the 1991 tax regime is replaced by the 1994 tax regime, we get an increase in labor market participation from 88.4 per cent to 91.7 per cent. The increase in participation is highest in the lowest deciles in the distribution of disposable household income (in 1994). There is a slight reduction in public sector participation, but there is a considerable increase in participation in the private sector. Thus, the labor supply effects of the tax reform of 1991 imply that married women are given a stronger motive to find work outside home and to work in the private sector. The expected hours of work, given participation in any sector, increase by around 250 hours per year, mostly in the upper deciles in the distribution of disposable household income; the increase in expected working hours is somewhat higher for women working in the private sector. Despite the fact that labor supply is stimulated by the reform, tax revenue goes down. The reason is that lower tax rates have a negative effect on tax revenue, which outweighs the positive effect on tax revenue from the increase in labor supply.

Accounting for labor supply responses, a flat tax of 29 per cent on all incomes is found to yield the same tax revenue as the 1994 tax system. By introducing a flat tax system, the labor supply responses to the 1992 tax reform are reinforced. There is no change in overall participation, but there is a further shift in participation away from the public sector towards the private sector. Working hours are predicted to increase further, in particular in the upper deciles and in the private sector. However, it should be noted that there is also a rather strong increase in supply of labor in the lowest deciles.

In Table 9, we report how choice probabilities vary with socioeconomic characteristics. We note that the flat tax system reinforces the labor supply effects of the 1992 tax reform.

Table 10 reports the mean of expected hours, conditional on working in the public or the private sector, and grouped according to socioeconomic characteristics. The most notable result is the large increase in hours worked in the private sector by women with the highest education level in response to the tax reforms.

Table 8. Labor supply responses to the tax reform of 1992 and to a flat tax of 0.29

	Mean working probabilities, per cent			Mean conditional expected annual hours			Mean unconditional expected annual hours		
	1991	1994	Flat tax	1991	1994	Flat tax	1991	1994	Flat tax
All sectors (Public and private)	88.4	91.7	91.9	1427	1559	1679	1291	1455	1566
In deciles:									
1	44.0	56.8	61.1	833	911	981	366	517	600
2–9	93.1	95.5	95.6	1462	1600	1718	1361	1527	1643
10	95.2	96.5	92.9	1742	1877	2062	1657	1812	1916
Public sector	47.5	47.0	44.4	1472	1551	1647	700	737	731
In deciles:									
1	27.1	33.7	35.3	860	931	1000	233	314	353
2–9	50.2	49.5	48.3	1484	1596	1686	745	790	797
10	47.2	40.4	30.7	1733	1812	1936	819	733	594
Private sector	40.7	44.7	47.4	1444	1563	1754	588	718	831
In deciles:									
1	16.9	23.1	25.9	789	882	955	133	203	247
2–9	42.9	46.0	48.3	1436	1604	1750	616	737	846
10	47.9	56.1	62.2	1750	1923	2125	839	1079	1322
Tax revenue, Mill 1994 NOK							130	113	113

Table 9. Choice probabilities and their variation with socioeconomic variables. Per cent

Variable	1991 tax system			1994 tax system			Flat tax of 29%		
	Not working	Public sector	Private sector	Not working	Public sector	Private sector	Not working	Public sector	Private sector
Age range									
25–34	14.43	47.56	38.01	10.45	47.32	42.23	10.12	45.06	44.82
35–44	11.19	49.71	39.10	7.75	49.05	43.20	7.24	46.32	46.43
43–64	9.90	45.27	44.83	6.80	44.71	48.49	7.44	41.79	50.77
Number of children									
0	7.31	47.27	45.42	4.89	46.02	49.09	5.55	42.92	51.53
1	9.11	50.03	40.86	6.18	48.88	44.94	5.39	46.42	48.20
2	14.39	46.62	38.99	10.09	46.76	43.15	9.71	44.39	45.90
more than 2	22.37	45.71	31.92	16.79	47.03	36.18	16.75	44.67	38.57
Woman's education									
low (≤9 years)	13.37	26.54	60.09	9.71	27.54	62.74	8.90	27.48	63.62
Intermediate (10–13 years)	12.82	43.46	43.72	9.05	43.42	47.52	9.24	41.19	49.57
High (15–17 years)	6.84	76.41	16.75	4.42	73.27	22.31	3.98	67.37	28.65

Table 10. Conditional expected annual hours under different tax rate systems by several variables and ranges

Variable	1991 tax system		1994 tax system		Flat tax of 29%	
	Public sector	Private sector	Public sector	Private sector	Public sector	Private sector
Age range						
25–34	1465	1434	1530	1576	1589	1706
35–44	1470	1462	1571	1631	1656	1785
43–64	1481	1438	1598	1608	1695	1764
Number of children						
0	1587	1528	1689	1694	1775	1843
1	1533	1496	1627	1662	1699	1806
2	1393	1369	1490	1530	1569	1677
more than 2	1215	1215	1310	1363	1399	1523
Woman's education						
low (≤9 years)	1455	1406	1535	1531	1605	1642
Intermediate (10–13 years)	1464	1446	1552	1604	1628	1747
High (15–17 years)	1494	1531	1607	1768	1702	1968

10. Compensating variation

To further evaluate the 1992 tax reform, we calculate the change in household welfare. To do so, we employ a recent method developed by Dagsvik and Karlström (2005) to calculate Compensating Variation (CV). The calculation of CV is not straightforward in a random utility model when utility is not linear in household income. A random utility function implies that CV is also random. What we do is to calculate the expected value of CV for each individual and thereafter we calculate the population density of the individual mean CV and the mean CV within selected deciles.

We define

$$\tilde{V}_j\left(h,W_j,I,f\right)=\max_{z\in B_j(h)} U\left(f\left(hW_j,I\right),h,z\right). \tag{17}$$

$\tilde{V}_j\left(h,W_j,I,f\right)$ denotes the conditional indirect utility, given hours of work h in sector j, with wage rate W_j, nonlabor income I and tax system f. From (2) and (3), we have that

$$\tilde{V}_j\left(h,W_j,I,f\right)=\psi\left(h,W_j,I\right)\max_{z\in B_j(h)} \varepsilon_j(z). \tag{18}$$

Owing to the fact that the random taste shifters are extreme value distributed, it follows that we can write

(19)
$$\max_{z \in B_j(h)} \varepsilon_j(z) \overset{d}{=} \theta_j g_j(h) \tilde{\varepsilon}_j(h),$$

where $\overset{d}{=}$ denotes equality in distribution and $\tilde{\varepsilon}_j(h)$ has c.d.f. $\exp(-1/x), x > 0$. Moreover, $\tilde{\varepsilon}_j(h)$, $j = 0,1,2$, $h = 0,1,\ldots$, are independent. (Recall that we use the convention that $h = 0$ implies $j = 0$.) For the reader's convenience, we provide a proof of this in Appendix C. As a result, we can express the conditional indirect utility as

(20)
$$\tilde{V}_j(h, W_j, I, f) = \psi(h, W_j, I) \theta_j g_j(h) \varepsilon_j(h)$$

for $h > 0$, $j = 1,2$, and

(21)
$$\tilde{V}_0(0, W_j, I, f) \equiv \tilde{V}_0(0,0,I,f) = \psi(0,0,I) \varepsilon_0(0)$$

for $h = 0$. For notational simplicity, let $V_j(h, W_j, I) = \psi(h, W_j, I) \theta_j g_j(h)$ for $h > 0$ and $V_0(0, W_j, I, f) \equiv V_0(0,0,I,f) = \psi(0,0,I)$.

Let $\tilde{V}(W, I, f)$ be the unconditional indirect utility, defined as

(22)
$$\tilde{V}(W, I, f) = \max\left[\tilde{V}_0(0,0,I,f), \max_{j=1,2} \max_{h>0, h \in D} \tilde{V}_j(h, W_j, I, f)\right].$$

The compensating variation CV (for an individual), is defined implicitly through

(23)
$$\tilde{V}(W, I, f_0) = \tilde{V}(W, I - CV, f_1),$$

where f_0 denotes the initial budget constraint and f_1 denotes the budget constraint after the tax reform. In Dagsvik and Karlström (2005), it is demonstrated that the distribution of $Y \equiv I - CV$ is given by

(24)
$$P(Y > y) = \frac{\sum_j \sum_{h \in D} R_j(h, y) V_j(h, W_j, I, f_0)}{K(y)},$$

where

$$R_j\left(h,y\right)=\begin{cases}1 & \text{if} \quad V_j\left(h,W_j,y,f_1\right)<V_j\left(h,W_j,I,f_0\right)\\ 0 & \text{otherwise},\end{cases}$$

and

$$(25) \qquad K(y)=\max\left(\psi\left(0,0,I,f_0\right),\psi\left(0,0,y,f_1\right)\right)+\sum_{j=1}^{2}\sum_{h>0}\max\left(V_j\left(h,W_j,I,f_0\right),V_j\left(h,W_j,y,f_1\right)\right).$$

The difference between the case considered here and the treatment in Dagsvik and Karlström (2005) is that, in their case, *Y* is positive whereas in the present case, *Y* can attain negative values. As a result, we cannot use Lemma 1 in Dagsvik and Karlström (2005) to compute the mean. Instead, we use the following.

Suppose *Y* is distributed on $\left[-a,\infty\right)$, $a>0$, with c.d.f. $F(y)$. Then

$$(26) \qquad EY=\int_{-a}^{\infty}\left(1-F(y)\right)dy-a\,.$$

The proof of (26) is straightforward. From (24), (25) and (26) it follows that the *individual* mean *CV*, conditional on wage rates, nonlabor income and other characteristics (suppressed in the notation below) is given by

$$(27) \qquad E\left[CV\,|\,W,I\right]=I-EY=I+a-\sum_{j=1}^{2}\sum_{h>0}V_j\left(h,W_j,I,f_0\right)\int_{-a}^{y_j(h)}\frac{dy}{K(y)}-\psi\left(0,0,I,f_1\right)\int_{-a}^{y(0)}\frac{dy}{K(y)}$$

where $y_j(h)$ and $y(0)$ are defined by

$$(28) \qquad V_j\left(h,W_j,I,f_0\right)=V_j\left(h,W_j,y_j(h),f_1\right),$$

$$(29) \qquad V\left(0,0,I,f_0\right)=V\left(0,0,y(0),f_1\right).$$

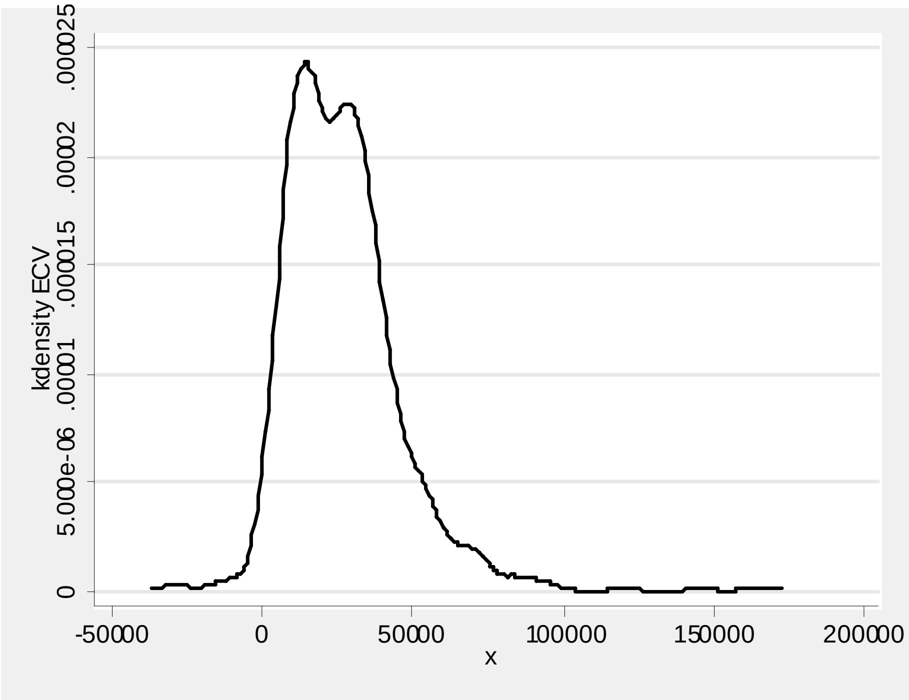
It is important to emphasize that the formula in (27) gives the mean *CV* conditional on wage rates, nonlabor income and other individual (observed) characteristics. The next step is to compute the conditional mean *CV* given nonlabor income, education and the demographic variables that enter the model, i.e., the mean is taken with respect to the wage rates. This is done by drawing independent error terms from the standard normal distribution and thereafter inserting these error terms into the wage equations. This yields a set of random wage rates for each woman. From these simulated wage rates, one can compute (simulate) the conditional mean, $E\left(CV\,|\,I\right)$ given nonlabor income and other individual characteristics, by taking the expectation with respect to the wage rates distribution for each woman. Below, we report the mean and spread in the population.

Table 11. Expected value of compensating variation, an estimate of the welfare changes for households from the 1992 tax reform. NOK 1994, with the 1991 tax system used as a reference against the 1994 tax system

	E(CV)
All	27078
Deciles in the distribution of household disposable income, 1991:	
1 (poor)	3338
2–9 (middle)	23727
10 (rich)	68509

From Table 11, we observe that the mean household in the sample gained NOK 27078 from the 1992 tax reform. The richest household gained almost 20 times more than the poorest. The distribution of expected gain across households is given in Figure 1, and we observe that most of the households will benefit from the 1992 tax reform. Thus, such a reform would have attained support from a clear majority at an election, as happened in reality in Norway.

Figure 1. Population density of expected Compensating Variation. Distribution of E(CV), comparing the 1991 tax regime against the 1994 tax regime

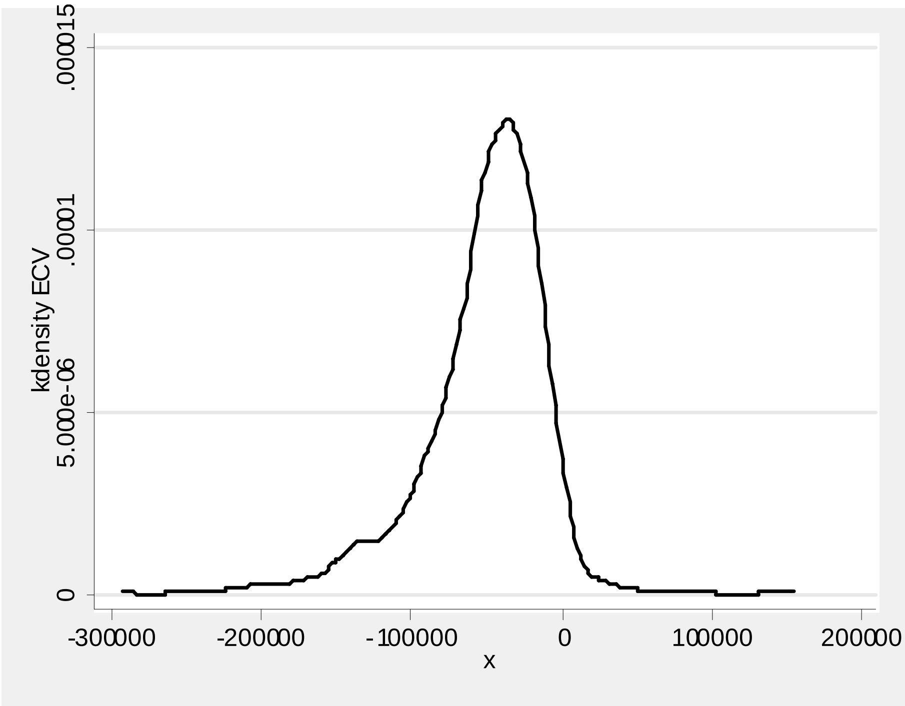


We have also calculated the expected value of compensating variation of a flat tax reform. In the calculations, the tax-revenue-neutral flat tax reform of 29% is used as a reference. Negative values mean that these values have to be subtracted from household incomes under the flat tax regime in order to make the households indifferent in welfare terms between the 1994 regime and the flat tax regime. Table 12 then says that, on average, the households will gain NOK 51528 if there is a shift from the 1994 tax regime to a flat tax regime. The richest households gain around 11 times more than the poorest. Thus, in a distributional sense, the richest household benefited more from having the 1991 regime replaced with the 1994 tax regime than they would have in the case of a shift from the 1994 tax regime to a flat tax regime. In Figure 2, we show the population density of the individual mean CV. We observe that a vast majority will benefit from the replacement of the 1994 tax regime with a flat tax regime.

Table 12. Expected value of compensating variation, an estimate of the welfare changes for households from a flat tax reform. NOK 1994, with a flat tax regime used as a reference against the 1994 tax regime

	E(CV)
All	–51437
Deciles in the distribution of household disposable income, flat tax:	
1 (poor)	–17155
2–9 (middle)	–53093
10 (rich)	–146966

Figure 2. Population density of expected Compensating Variation. Distribution of E(CV), with the flat tax system of 29% used as a reference against the 1994 tax regime



11. Conclusions

A female labor supply model, estimated on Norwegian data from 1994, has been used in selected simulation experiments. Some of these experiments illustrate the effect of changes in wage rates, whereas others illustrate the effect of a tax reform. The overall elasticities are rather small, but these small elasticities shadow for much stronger sectoral responses.

We find that a wage increase, overall and by sectors, has a strong positive impact on sector choice probabilities. A wage increase gives women an incentive to shift their labor supply from the public to the private sector. This is particularly true for women belonging to the highest deciles in the distribution of disposable household income. This pattern occurs despite the fact that education and experience yield slightly higher returns in the public than in the private sector. The reasons for our results are that the wage rate levels and dispersion are higher in the private than in the public sector. Moreover, offered hours of work are less regulated in the private sector compared to in the public sector.

The Norwegian tax reform of 1992 implied a considerable reduction in the top marginal tax rate, but the tax rates in lower brackets were also reduced. We find that the impact on overall labor supply is rather modest, but again these modest changes shadow for stronger sectoral changes. The tax reform stimulated the women to shift their labor from the public to the private sector and to work longer hours. The latter was particularly true for women belonging to the upper deciles in the income distribution and working in the private sector. Despite the fact that labor supply was stimulated, the tax reform implied a reduction in tax revenue. Thus, the lower tax rates implied lower tax revenues when labor supply responses were taken into account. A calculation of the expected value of changes in household welfare demonstrated that the richest households benefited far more from the tax reform than the poorest household.

A flat tax reform, with the same tax revenue as in 1994, would reinforce the labor supply responses of the 1992 reform. In relative terms, the richest households benefit more from the 1992 tax reform than from a having a further reform towards a flat tax regime.

References

Aaberge, R., J. K. Dagsvik and S. Strøm (1995): Labor Supply and Welfare Effects of Tax Reforms. *Scandinavian Journal of Economics*, **97**, 635–659.

Aaberge, R., U. Colombino and S. Strøm (1999): Labor Supply in Italy: An Empirical Analysis of Joint Household Decisions with Taxes and Quantity Constraints. *Journal of Applied Econometrics*, **14**, 403–422.

Altonji, J.G. and C.H. Paxson (1988): Labor Supply Preferences, Hours Constraints and Hours–Wage Tradeoffs. *Journal of Labor Economics*, **6**, 254–276.

Ashenfelter O. and J. J. Heckman (1974): The Estimation of Income and Substitution Effects in a Model of Family Labor Supply. *Econometrica*, **42**, 73–86.

Atkinson, A.B. (1995): The Welfare State and Economic Performance. *National Tax Journal*, XLVIII, **2**, 171–198.

Blundell, R. and T. MaCurdy (1999): Labor Supply: A Review of Alternative Approaches. In O. Ashenfelter and D. Card (eds): *Handbook of Labor Economics*, New York, North–Holland.

Burtless, G. and J. A. Hausman (1978): The Effect of Taxation on Labor Supply: Evaluating the Gary Negative Income Experiment. *Journal of Political Economy*, **86**, 1103–1130.

Callan, T. and A. van Soest (1996): Family Labor Supply and Taxes in Ireland. Mimeo, Tilburg University.

Creedy, J., A. S. Duncan, M. Harris and R. Scutella (2002): *Microsimulation modeling of taxation and the labor market*. Edward Elgar, Northampton.

Creedy, J. and G. Kalb (2005): Discrete Hours Labor Supply Modeling: Specification, Estimation and Simulation. *Journal of Economic Surveys*, **19**, 697–734.

Dagsvik, J. K. (1994): Discrete and Continuous Choice, Max-stable Processes and Independence from Irrelevant Attributes. *Econometrica*, **62**, 1179–1205.

Dagsvik, J. K. and A. Karlström (2005): Compensating Variation and Hicksian Choice Probabilities in Random Utility Models that are Nonlinear in Income. *Review of Economic Studies,* **72**, 57–76.

Dagsvik, J. K. and S. Strøm (1997): A Framework for Labor Supply Analysis in the Presence of Complicated Budget Restrictions and Qualitative Opportunity Aspects. Memorandum from Department of Economics, University of Oslo, No. 22.

Dagsvik, J. K. and S. Strøm (2004): Sectoral Labor Supply, Choice Restrictions and Functional Form. Discussion Papers no. 388, Statistics Norway.

Dagsvik, J. K and S. Strøm (2006): Sectoral Labor Supply, Choice Restrictions and Functional Form. Forthcoming in *Journal of Applied Econometrics* (September 2006).

Heckman, J. J. (1974a): Shadow Prices, Market Wages and Labor Supply. *Econometrica*, **42**, 679–694.

Heckman, J. J. (1974b): The Effect of Day Care Programs on Women's Work Effort. *Journal of Political Economy*, **82**, S136–S163.

Heim, B.T. and B. D. Meyer (2003): Structural Labor Supply Models when Budget Constraints are Nonlinear. Manuscript, Duke University.

MaCurdy, T., D. Green and H. Paarsch (1990): Assessing Empirical Approaches for Analyzing Taxes and Labor Supply. *Journal of Human Resources,* **25**, 415–490.

Statistics Norway (1994): Survey of Income and Wealth. www.ssb.no/emner/05/01/.

Statistics Norway (1995): Level of Living Conditions. www.ssb.no/emner/00/02/.

Van Soest, A. (1994): Structural Models of Family Labor Supply: A Discrete Choice Approach. *Journal of Human Resources*, **30**, 63–88.

Van Soest, A., M. Das and X. Gong (2002): A Structural Labor Supply with Flexible Preferences, *Journal of Econometrics*, **107**, 345–374.

Description of variables and summary statistics

Table A.1. Description of the variables used in the analysis (values in NOK, 1994)

Symbols	Description	
FNR	Identification number	
FAR	Woman's year of birth	
B02	Number of children, 0–2 years	
B36	Number of children, 3–6 years	
B717	Number of children, 7–17 years	
B06	Number of children, 0–6 years	
MALDER	Age in years (man)	
MUTD	Education in years (man)	
KALDER	Age in years (woman)	
KUTD	Education in years (woman)	
INR	Choice variable of working hours: 1–15	
ARBTID	Annual hours of work as follows:	
	INR = 1; ARBTID = 0	
	Public sector	**Private sector**
KAPINNT	Household capital income	
MANNLONN	Men's wage income per year	
Variable generated:		
KUTD_100	Woman's education in years (KUTD) /100	
SKILL	Work experience = woman's age – woman's education in years (KUTD) – six (starting school age)	
SK_100	SKILL/100	
SK2_100	$(SKILL/100)^2$	
CAPINC	Net capital income (CAPINC) = KAPINNT–CHALL as KAPINNT includes CHALL. CHALL refers to child allowances; see Appendix B	
W_PU	Women's hourly wage in the public sector	
W_PR	Women's hourly wage in the private sector	

Table A.2. Descriptive statistics, number of observations = 810

Variable	Mean	Std. Dev.	Min	Max
FAR	53.92	9.04	30.00	69.00
B02	0.23	0.45	0.00	2.00
B36	0.30	0.56	0.00	3.00
B717	0.66	0.85	0.00	4.00
MALDER	42.80	9.17	25.00	66.00
MUTD	12.05	2.49	9.00	19.00
KALDER	40.07	9.04	25.00	64.00
LNKALDER	3.66	0.22	3.22	4.16
KUTD	11.61	2.15	9.00	17.00
INR	7.83	4.01	1.00	15.00
B06	0.54	0.77	0.00	3.00
ARBTID	1482.89	664.97	0.00	2600.00
SEKTOR	1.34	0.61	0.00	2.00
KUTD_100	0.11	0.02	0.09	0.17
SKILL	22.45	9.63	2.00	49.00
SK_100	0.22	0.09	0.02	0.49
SK2_100	0.05	0.04	0.0004	0.24
KAPINNT	32306.71	42378.48	0.00	568403.00
CHALL	13094.37	12154.01	0.00	60084.00
KVLONN	149751.97	83060.53	0.00	581693.00
MANNLONN	274372.89	106239.67	17312.00	1184861.00
W_PU	89.36	12.09	64.88	132.34
W_PR	109.77	13.68	80.14	156.44

Tax functions and child allowances

Table B.1. Tax function in 1994 for a married nonworking woman whose husband is working, OK 1994

Mannlonn, Y_{male}	Tax T
0–41907	0
41907–140500	$0.302Y_{male}$–12656
140500–252000	$0.358Y_{male}$–20524
252000–263000	$0.453Y_{male}$–44464
263000–	$0.495Y_{male}$–55510

Table B.2. Tax function in 1994 for a married working woman or man, NOK 1994

Wage income, Y	Tax T
0–20954	0
20954–140500	0.302Y–6328
140500–208000	0.358Y–14196
208000–236500	0.453Y–33956
236500–	0.495Y–43889

In 1994, the child allowances were:

- One child between 0 and 17 years: NOK 10416
- Two children between 0 and 17 years: NOK 21336
- Three children between 0 and 17 years: NOK 33696
- Four children between 0 and 17 years: NOK 46692
- Five children or more between 0 and 17 years: NOK 60084

Table B.3. Tax function in 1991 for a married nonworking woman, whose husband is working, NOK 1994

Mannlonn, Y_{male}	Tax T
0–38392	0
38392–70746	$0.303Y_{male}$–11642
70746–171915	$0.343Y_{male}$–14455
171915–200567	$0.418Y_{male}$–27348
200567–264239	$0.558Y_{male}$–55428
264239–	$0.654Y_{male}$–80509

Table B.4. Tax function in 1991 for a married working woman, or working man. NOK 1994

Wage income Y	Tax T
0–19596	0
19596–22639	0.343Y–6722
22639–70746	0.303Y–5832
70746–137956	0.343Y–8634
137956–174037	0.418Y–18981
174037–219669	0.558Y–42964
219669–	0.654Y–64214

C.1 The wage equations

Wage equations accounting for heterogeneity, with η_j denoting the random effect are

$$W_{ji} = w_{ji}^{*}\eta_{1i} = \exp\left(\alpha_{j0} + \alpha_{j1}Z_{1i} + \alpha_{j2}Z_{2i} + \alpha_{j3}Z_{3i} + \sigma_{j1}\log\eta_{ji}\right),$$

for sector $j = 1,2$, where $Z_{ki}, k = 1,2,3,$ are respectively, the woman's experience (divided by 100), the square of the previous variable, and woman's education in years (divided by 100), and η_j the random effect on wages.

Table C.1 Estimates of wage equations. Norwegian women, 1994

Variables	Public sector		Private sector	
	Estimates	t-values	Estimates	t-values
Constant	3.37	13.5	3.70	25.2
Experience in years/100	3.21	6.0	2.55	5.1
(Experience in years/100)^2	–4.75	–5.3	–3.80	–4.2
Education in years/100	5.57	4.9	5.26	4.2
Log (Probability of working in the chosen sector)	–0.12	–2.0	0.06	0.9
Variances	0.059	18.6	0.075	17.0
No of observations	691		580	
R^2	0.14		0.08	

The Box–Cox type of specification for the deterministic part of the utility function is given by

$$\log v(C,h) = \alpha_2\left(\frac{[10^{-4}(C-C_0)]^{\alpha_1}-1}{\alpha_1}\right) + \left(\frac{(L-L_0)^{\alpha_3}-1}{\alpha_3}\right)\left(\alpha_4 + \alpha_5\log A + \alpha_6(\log A)^2 + \alpha_7 CU6 + \alpha_8 CO6\right)$$
$$+\alpha_9\left(\frac{[10^{-4}(C-C_0)]^{\alpha_1}-1}{\alpha_1}\right)\left(\frac{(L-L_0)^{\alpha_3}-1}{\alpha_3}\right)$$

where *A* is the age of the married woman, *CU*6 and *CO*6 are the number of children less than six years and above six years, *L* is leisure, defined as

$$L - L_0 = 1 - h/3640,$$

and α_j, $j = 1, 2, ..., 9$, are unknown parameters. Observe that from total annual hours we have subtracted a "subsistence" level, L_0, amounting to 5120 hours, which corresponds to about 14 hours per day reserved for sleep and rest, and similarly a subsistence level, C_0, for consumption chosen to be close to the official estimate of a subsistence level in Norway (NOK 60 000). Total consumption C is measured as the sum of the annual wage income of the woman and her husband after tax, household capital income after tax and child allowances. If $\alpha_1 < 1$, $\alpha_3 < 1$, $\alpha_2 > 0$,

$$\alpha_4 + \alpha_5 \log A + \alpha_6 (\log A)^2 + \alpha_7 CU6 + \alpha_8 CO6 > 0,$$

and α_9 is positive, or if negative, sufficiently small numerically, then $\log v(C, h)$ is increasing in C, decreasing in (h) for fixed C and strictly concave in (C, h).

C.2. Estimates of the labor supply model.

Table C.2. Estimation results for the parameters of the labor supply probabilities

Variables	Parameters	Estimates	t-values
	Uniformly distributed offered hours, but with part-time and full-time peaks		
Preferences:			
Consumption:			
Exponent	α_1	0.64	7.6
Scale 10^{-4}	α_2	1.77	4.2
Subsistence level C_0 in NOK per year		60 000	
Leisure:			
Exponent	α_3	–0.53	–2.1
Constant	α_4	115.02	3.2
Log age	α_5	–63.61	–3.2
$(\log \text{age})^2$	α_6	9.20	3.3
# children 0–6 years	α_7	1.27	4.0
# children 7–17 years	α_8	0.97	4.1
Consumption and Leisure, interaction	α_9	–0.12	–2.7
Subsistence level of leisure in hours per year		5120	
The parameters θ_1 and θ_2; $\log\theta_j = f_{j1} + f_{j2}S$ *			
Constant, public sector (sector 1)	f_{11}	–4.20	–4.7
Constant, private sector (sector 2)	f_{21}	1.14	1.0
Education, public sector (sector 1)	f_{12}	0.22	2.9
Education, private sector (sector 2)	f_{22}	–0.34	–3.3
Opportunity density of offered hours, $g_j(h)$, j = 1, 2			
Full-time peak, public sector (sector 1)**	$\log\left(g_1(h_{Full})/g_1(h_0)\right)$	1.58	11.8
Full-time peak, private sector (sector 2)	$\log\left(g_2(h_{Full})/g_2(h_0)\right)$	1.06	7.4
Part-time peak, public sector	$\log\left(g_1(h_{Part})/g_1(h_0)\right)$	0.68	4.4
Part-time peak, private sector	$\log\left(g_2(h_{Part})/g_2(h_0)\right)$	0.80	5.2
# observations			810
Log likelihood			–1760.9

* The estimates of f_{j1} are not "correct" as we have not normalized the $g_j(h)$ functions to be probability density functions. However, in the simulation experiments, we have used a normalized version.

** The notation h_0 refers to an arbitrary level of hours of work different from typical full-time and part-time hours.

C.3. Adjustment of coefficients, constraint on hours versus no constraints

Let f_{j1}^* denote the adjusted constant for sector *j* in equation (8) above and let θ_j^* denote the corresponding new value for θ_j. Let γ_{jF} and γ_{jP} denote the parameters associated with full-time and part-time hours in sector *j*. It can be shown that the new values of b_j and f_{j1} are given by

$$\log\theta_j^* = f_{j1}^* + f_{j2}S$$

and

$$f_{j1}^* = f_{j1} + \log\left[\frac{5+\exp(\gamma_{jF})+\exp(\gamma_{jP})}{7}\right]$$

for $j = 1,2$. Note that we have seven categories of hours, of which two are part-time and full-time workloads. With the estimates in Table 1 ($\gamma_{1F} = 1.58$, $\gamma_{1P} = 0.68$, $\gamma_{2F} = 1.06$, $\gamma_{2P} = 0.8$, $f_{11} = -4.20$ and $f_{21} = 1.14$) we find that the new constants have to increase to –3.68 ($= f_{11}^*$, sector 1, the public sector) and to 1.51 ($= f_{21}^*$, sector 2, the private sector). Because more jobs are concentrated around full-time hours in the public sector than in the private sector, the increase in the θ_j for the public sector has to be the larger.

A proof of eq. (19)

Consider the distribution of the second factor on the right-hand side of (21). As $\varepsilon_j(z), z = 0,1,2,...,$ are i.i.d. with c.d.f. $\exp(-1/x)$, it follows that

$$P\left(\max_{z\in B_j(h)}\varepsilon_j(z)\le x\right) = P\left(\bigcap_{z\in B_j(h)}\left(\varepsilon_j(z)\le x\right)\right) = \prod_{z\in B_j(h)}\exp\left(-\frac{1}{x}\right) = \exp\left(-\sum_{z\in B_j(h)}\frac{1}{x}\right).$$

As $1/x$ does not depend on *z,* and the number of jobs in $B_j(h)$ equals $\theta_j g_j(h)$, we obtain that

$$P\left(\max_{z\in B_j(h)}\varepsilon_j(z)\le x\right) = \exp\left(-\sum_{z\in B_j(h)}\frac{1}{x}\right) = \exp\left(-\frac{\theta_j g_j(h)}{x}\right).$$