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Procrastination, partial naivete, and behavioral welfare analysis.

Geir B. Asheim*

February 1, 2007

Abstract

This paper has a dual purpose. *First*, I present a new modeling of partial naivete, and apply this to the analysis of procrastination. The decision maker is assumed to have stationary behavior and to be partially naive in the sense of perceiving that his current preferences may persist in the future. The behavioral implications of such partial naivete differ from those of related literature. *Second*, I suggest a general principle for welfare analysis in multi-self settings through a new application of Pareto-dominance, which is motivated by the existence of time-inconsistency and coincides with dominance relations used elsewhere in game theory. In the case of procrastination, it leads to a clear welfare conclusion: Being partially naive reduces welfare.

Keywords and Phrases: Procrastination, partial naivete, time-inconsistency, game theory, behavioral welfare economics.

JEL Classification Numbers: C70, D11, D60, D74, D91, E21

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1 Introduction

People procrastinate tasks with immediate cost and future benefits, even when they prefer to do the task now compared to the much delayed execution that the procrastination leads to. Why does this happen and what are the welfare consequences?

Why do people procrastinate? Contributions to behavioral economics (in particular, O'Donoghue and Rabin, 2001; see also, e.g., Brocas and Carrillo, 2001, and Fischer, 1999) explain procrastination by endowing the decision maker with the following two properties:

(1) The decision maker has "present-biased preferences". This phenomenon seems to be supported by experimental evidence (as reported by Loewenstein and Prelec, 1992, and Frederick, Loewenstein and O'Donoghue, 2003, although contested by Rubinstein, 2003). A common formulation (originating from Phelps and Pollak, 1968, and Elster, 1979, and employed by Laibson, 1994, 1997, and others) is to assume that the decision maker has preferences that, in addition to time-consistent discounting, give extra weight to current well-being over future well-being (so-called (β, δ)–preferences).

(2) The decision maker is not fully aware of the self-control problems that such present-biased preferences lead to; instead he is *partially naive* (in the terminology of O'Donoghue and Rabin, 2001) about future preferences.

The present-biased preferences provide an incentive for the decision maker to postpone the task to the next period, while the deficient awareness of the self-control problems leads him to believe falsely that the task will actually be performed then. These two elements are sufficient to model behavior whereby the decision maker postpones the task period by period.

While obviously being a very important and influential contribution, the analysis presented by O'Donoghue and Rabin (2001) has features that might make it

Table 1: Payoffs of the decision maker as a function of postponement

Postponement	**Payoff**
0	$-1 + \frac{1}{2}\frac{4}{5}5 = 1$
1	$\frac{1}{2}\frac{4}{5}\big(-1 + \frac{4}{5}5\big) = \frac{6}{5}$
2	$\frac{1}{2}\big(\frac{4}{5}\big)^2\big(-1 + \frac{4}{5}5\big) = \frac{24}{25}$
...	...
n	$\frac{1}{2}\big(\frac{4}{5}\big)^n\big(-1 + \frac{4}{5}5\big) = \frac{3}{2}\big(\frac{4}{5}\big)^n$
...	...
∞	0

worthwhile to consider alternative ways to model procrastination, while keeping the key ingredients (1) and (2): (β, δ)-preferences and partial naivete.

The alternative modeling choices that I propose in the present paper can be illustrated through the following example. Consider a decision maker who performs a task at one of the stages 0, 1, 2, ..., or not at all. Performing the task leads to an immediate cost equal to 1, and enables the decision maker to reap benefits equal to 5 at the next stage. Assume a constant per-period discount factor, $\delta = 4/5$. Moreover, assume that future costs and benefits are discounted by an additional factor, $\beta = 1/2$, leading to present-biased preferences. The payoffs of the decision maker as a function of the number of stages he postpones the task are given in Table 1. Hence, the decision maker prefers to postpone the task one and only one period.

O'Donoghue and Rabin (2001) study pure strategies, leading to perhaps unconvincing patterns of behavior obtained through backward induction. E.g., in the example above, a sophisticated decision maker (i.e., who is fully aware of his future self-control problems) might decide to do the task now, because he believes that otherwise he will postpone the task in exactly 2 periods. Instead, I assume that the decision maker uses a *stationary* (i.e., Markovian and time-invariant) strategy, where the same mixed action is chosen if the task has not yet been performed. E.g., a sophisticated decision maker will do the task with probability 1/2, since then the payoff (= 1) he can ensure himself by doing the task now is equal to the expected payoff of doing the task at stages 1, 2, 3, ... with probability 1/2, 1/4, 1/8, ...

O'Donoghue and Rabin (2001) assume that the partially naive decision maker is consistently mistaken about future preferences, still quite sophisticated when constructing his decision rule. E.g., if the decision maker believes that his β at future stages will not be 1/2, but rather exceed 5/8 so that

$$-1 + \beta \tfrac{4}{5} 5 \; > \; \beta \tfrac{4}{5}\big(-1 + \tfrac{4}{5} 5 \big) \, ,$$

then he believes that he will do the task for sure at the next stage, entailing that he strictly prefers *not* to do the task now. However, at each new stage he discovers that his preferences are as present-biased as before, and that the task remains undone.

Here, I model partial naivete as follows: The decision maker *perceives that his present preferences will persist with positive probability* at the next stage, entailing that his future preferences will *not* be present-biased and effectively enabling him to commit to his present decision rule. E.g., if the decision maker believes with at least probability 1/2 that his present preferences will persist at the next stage, then it follows from the results of the present paper that in the unique stationary equilibrium the decision maker chooses to postpone the task for sure. Still, his understanding of the decision problem is not contradicted by the flow of events, since—as long as he is not fully naive—he perceives that there is at each stage a positive probability that the present preferences will *not* persist.

My definition of partial naivete is defined for general intertemporal preferences and is not limited to the special case of (β, δ)–preferences. Moreover, as I will show, it leads to different behavioral implications.

What are the welfare consequences? O'Donoghue and Rabin (2001) along with most of the behavioral economics literature on time-inconsistency employ two different welfare criteria: (i) "Long-run utility" meaning that one sets $\beta = 1$ for welfare analysis. (ii) The Pareto-criterion is used to evaluate two alternatives by comparing payoffs along the implemented paths at *all times*. These welfare criteria are not uncontroversial—see, e.g., Bernheim and Rangel (2005, Section 2C), Gul and Pesendorfer (2005, Section 6.4) and O'Donoghue and Rabin (2001, footnote 21)—and may not be considered fully compelling. In particular, "long-run utility" does not respect the preferences of the decision maker, and by applying the Pareto-criterion to contrast payoffs along the implemented paths one is lead to make comparisons across different histories.

Here, I suggest to avoid these issues by using the Pareto-criterion to evaluate two alternatives by comparing payoffs at all *decision nodes*. This new application of Pareto-dominance in multi-self settings is motivated by the existence of time-inconsistency, and it coincides with dominance relations used elsewhere in game theory (e.g., in the literature on renegotiation-proofness in repeated games). In the case of procrastination, the strategies for different levels of partial naivete turn out to be Pareto-ranked and, hence, the analysis leads to a clear welfare conclusion: Being partially naive decreases welfare. The general principles involved in the welfare analysis have significance beyond the problem of procrastination.

The paper is organized as follows. In Section 2 I present a general model of partial naivete, where the decision maker is partially naive by perceiving that his present preferences may persist. In Section 3 I present results on procrastination as equilibrium behavior in a situation where the person has one task to perform, with immediate cost and future benefits, and show how the degree of procrastination varies with the perceived preference persistence. In Section 4 I turn to a general discussion of welfare analysis in multi-self settings, based on which, in the following Section 5, I evaluate the welfare effects of procrastination when the decision maker is partially naive in the sense of perceiving that his present preferences may persist. I summarize my results in the concluding Section 6.

2 A general model of partial naivete

Consider a T-stage decision problem where T can be finite or infinite. The set of *histories* is defined inductively as follows: The set of histories at the beginning of the first stage 0 is $H_0 = \{h_0\}$. Denote by H_t the set of histories at the beginning of stage t. At $h \in H_t$, the decision maker's action set is A^h. Define the set of histories at the beginning of stage $t+1$ as follows:

$$H_{t+1} = \{(h, a) \mid h \in H_t \text{ and } a \in A^h\}.$$

This concludes the induction. Let H denote the set of *decision nodes.* If $T < \infty$, then $H = \bigcup_{t=0}^{T-1} H_t$; otherwise, $H = \bigcup_{t=0}^{\infty} H_t$. Assume that, for all $h \in H$, A^h is non-empty and finite. A trivial decision is made at h if A^h is a singleton. Refer to $Z := H_T$ as the set of *outcomes.* Denote by $t(h)$ the stage which starts at $h \in H$. Denote by H^h the set of decision nodes equal to or following $h \in H$:

$$H^h := \{h' = (h_0, a_0, \dots, a_{t(h')-1}) \in H \mid t(h') \geq t(h) \text{ and } h = (h_0, a_0, \dots, a_{t(h)-1})\},$$

and denote by Z^h the set of outcomes following $h \in H$:

$$Z^h := \{z = (h_0, a_0, a_1, \dots) \in Z \mid h = (h_0, a_0, \dots, a_{t(h)-1})\},$$

The preferences of the decision maker at $h \in H$ are represented by a Bernoulli utility function $u^h : Z^h \to \mathbb{R}$ which assigns *payoff* to every outcome following h. The preferences are *time-consistent* if, for all $h \in H$ and $a \in A^h$, $u^{(h,a)}$ is a positive affine transformation of u^h restricted to $Z^{(h,a)}$.

For every $h \in H$ and $h' \in H^h$, the decision maker perceives at h that his preferences at h' will equal $\tilde{u}^{h|h'}$. His system of perceived preferences over H^h are defined inductively as follows: (1) $\tilde{u}^{h|h} = u^h$, and (2) for every $h' \in H^h$ and $a \in A^{h'}$, $\tilde{u}^{h|(h',a)} = \tilde{u}^{h|h'}|_{Z^{(h',a)}}$ with probability p and $\tilde{u}^{h|(h',a)} = u^{(h',a)}$ with probability $1-p$. Refer to $p \in [0, 1]$ to as the *perceived preference persistence*, reflecting the probability with which the decision maker perceives at h that his preferences at stage t ($\geq t(h)$)

will persist at $t+1$. Note that since $\tilde{u}^{h|h} = u^h$ for every $h \in H$, there is no *actual* preference persistence, except if preferences are time-consistent.

A *decision rule* at $h \in H$, r^h, assigns to every $h' \in H^h$ a mixed action $r^h(h') \in \Delta(A^{h'})$. Denote by R^h the set of decision rules at $h \in H$. A *decision rule* r^h is *optimal* at $h \in H$ if the probability measure over Z^h generated by r^h is weakly preferred to the probability measure over Z^h generated by any alternative decision rule $\tilde{r}^h$, when evaluating the probability measures by means of expected payoff. The decision rule at h specifies the behavior of the decision maker as long as the preferences at h, u^h, persist.

A *strategy* s assigns to every $h \in H$ a decision rule $s(h) \in R^h$. Denote by S the set of strategies. At h_0, the strategy s prescribes that the decision rule $s(h_0)$ be selected. At any later decision node $h \in H \backslash \{h_0\}$, the strategy s prescribes that $s(h)$ be selected if and only if prior preferences do not persist. Note that, with a positive perceived preference persistence p, it is essential that a strategy assigns to each $h \in H$ a decision rule rather than just a mixed action.

For every $h \in H$ and $h' \in H^h$, the strategy $s \in S$ and perceived preference persistence $p \in [0,1]$ generate a probability measure $\tilde{P}^{h|h'}(p,s)$ over $Z^{h'}$ as perceived at h, given that preferences persist from h to h'. Let $\tilde{v}^{h|h'} : [0,1] \times S \to \mathbb{R}$ assign to each $p \in [0,1]$ and $s \in S$ the perceived expected payoff $\tilde{v}^{h|h'}(p,s) = E_{\tilde{P}^{h|h'}(p,s)} u^h(z)$ that this combination of strategy and perceived preference persistence leads to at $h' \in H^h$ as perceived at $h \in H$, given that preferences persist from h to h'.

Denote by (r^h, s^{-h}) the strategy that is obtained from s by substituting r^h for $s(h)$ at h. We can now state our main definition.

Definition 1 A strategy $s \in S$ is a *multi-self subgame-perfect equilibrium (*MSSPE*) with perceived preference persistence* p if, for all $h \in H$, $h' \in H^h$ and $r^h \in R^h$,

$$\tilde{v}^{h|h'}(p,s) \geq \tilde{v}^{h|h'}(p,(r^h, s^{-h}))\,.$$

Hence, an MSSPE is a subgame-perfect equilibrium of the game induced by treating each self of the decision maker as a separate player having decision making authority as long as his preferences persist, with the decision rules of the selves corresponding to the strategies of the players, and the strategy of the decision maker corresponding to the strategy profile of the induced game. In particular, existence results for the concept of subgame-perfect equilibrium can be applied.

In words, a strategy $s \in S$ is an MSSPE with perceived preference persistence p if there exists no decision nodes $h \in H$ and $h' \in H^h$ such that the decision maker at h gains at h' by selecting an alternative decision rule to the one assigned by s, given that preferences has persisted from h to h', s will be followed at all later decision nodes where prior preferences do not persist, and the decision maker perceives that the preference persistence probability is p.

If the decision maker has time-consistent preferences, then a strategy assigning an optimal decision rule at all decision nodes is an MSSPE, independent of the perceived preference persistence. On the other hand, if the decision maker has time-inconsistent preferences, then a strategy assigning an optimal decision rule at all decision nodes is an MSSPE when the perceived preference persistence is 1; not necessarily otherwise.

The above model nests both sophisticated behavior and fully naive behavior.

- By setting $p = 0$ we get ordinary sophistication: The decision maker acknowledges how his preferences will change, and selects at each decision node an optimal action, given the actions that the strategy prescribes at future decision nodes.

- By setting $p = 1$ we get full naivete: The decision maker perceives incorrectly at every decision node that his current preferences will persist, implying that the selected decision rule will be followed throughout the T-stage decision problem. However, when reaching the next stage, the decision maker discovers that his preferences have changed and he selects a new decision rule.

With $0 < p < 1$ we obtain a new theory of partial naivete: The decision maker perceives incorrectly that there are positive probabilities p, p^2, p^3, ... that his current preferences will persist and a selected decision rule will be followed at subsequent stages. The fact that his preferences do not persist does not contradict p, since it can be interpreted by the decision maker as a bad draw. In a Bayesian framework, the partially naive decision maker is rational if his prior distribution over values of p has all measure concentrated on one point so that updating does not change his perceived preference persistence. Although extreme, it is not irrational.

My new definition of partial naivete is general, in the sense that it can be applied to any kind of system of conditional preferences. In contrast, O'Donoghue and Rabin's (2001) definition of partial naivete is tailored for (β, δ)–preferences, since it relies on the decision maker's point estimation of his future β. In the next section I will show how my alternative definition has different behavioral implications than the definition of partial naivete proposed by O'Donoghue and Rabin.

3 Procrastination as equilibrium behavior

In the present section I consider a special case of the general model presented in Section 2. I model a decision problem with an infinite number of stages, which concerns the timing of a task with immediate cost and future benefits. The set of decision nodes H is partitioned into two states ω^0 and ω^1: $H = \omega^0 \cup \omega^1$, with $\omega^0 \cap \omega^1 = \emptyset$. Here ω^0 denotes the state in which the decision maker has not performed the task, while ω^1 denotes the state in which the decision maker has performed the task. If the task has not been performed, i.e., $h \in \omega^0$, then the action set A^h equals $\{a^0, a^1\}$, where a^0 means to do nothing, while a^1 means to do the task. On the other hand, if the task has already been performed, i.e., $h \in \omega^1$, then the action set A^h equals $\{a^0\}$. Naturally, $(h, a^0) \in \omega^0$ and $(h, a^1) \in \omega^1$ if $h \in \omega^0$, while $(h, a^0) \in \omega^1$ if $h \in \omega^1$. With the assumption that the history at stage 0, h_0, is contained in ω^0—entailing that the task has not been performed at the root of the decision problem—the determination of the set of histories is complete. An outcome specifies in which stage the task is done; at stage 0, 1, 2, ..., or not at all.

To specify the decision maker's preferences, let $c > 0$ be the cost accruing at the stage in which the task is performed, and let $v > 0$ be the benefits accruing at the next stage. Assume that the decision maker has (β, δ)–preferences,

$$u^h(z) = v_{t(h)} + \beta \sum\nolimits_{t=t(h)+1}^{\infty} \delta^{t-t(h)} v_t$$

where $0 < \beta \leq 1$ and $0 < \delta < 1$, and where v_t denotes the periodic payoff at stage t, with $v_t = -c$ if $a_t = a^1$, $v_t = v$ if $a_{t-1} = a^1$, and $v_t = 0$ otherwise. Hence, the payoff at decision node h of doing the task now is

$$-c \ + \ \beta\delta v \, ,$$

the payoff at decision node h of doing the task $t - t(h) \geq 1$ stages from now is

$$\beta\delta^{t-t(h)}\big(-c + \delta v\big) \, ,$$

while the payoff at decision node h of not doing the task at all is 0.

Assumption 1 It is better to perform the task now than not doing it at all, but even better to postpone the task to the next stage:

$$0 < -c + \beta\delta v < \beta\delta\big(-c + \delta v\big) \, .$$

The example of the introduction has parameter values ($\beta = 1/2$, $\delta = 4/5$, $c = 1$, and $v = 5$) that satisfy this assumption. Assumption 1 is satisfied if and only if

$$\beta\delta v \ \ > \ \ c \ \ > \ \ \beta\delta v \frac{1-\delta}{1-\beta\delta} \, .$$

Since $\beta < 1$ is necessary for these inequalities to hold, Assumption 1 implies that the decison maker has "present-biased preferences". With (β, δ)–preferences, it is w.l.o.g. to assume that the benefits accrue at the stage following the stage in which the task was performed, since v may represent the present value of future benefits discounted back to this stage by discount factor δ. For later use, denote by A the payoff ratio of (i) doing the task now and (ii) postponing it to the next period.

$$A := \frac{-c + \beta\delta v}{\beta\delta\big(- c + \delta v\big)} \, .$$

Assumption 1 implies that $A \in (0, 1)$.

Note that a decision rule specifies a non-trivial action choice only at decision nodes corresponding to the state in which the decision maker has not performed the task. Under the above assumptions the optimal decision rule is to do nothing *now*, and instead perform the task *at the next stage*. Of course, this is not time-consistent. For a large perceived preference persistence p, a strategy that always assigns the optimal decision rule will turn out to constitute an MSSPE. For a small p this will not be the case, as the decision maker will perceive that the selected decision rule will not be followed at the next stage with a sufficiently high probability $1 - p$.

I will consider an MSSPE where the decision maker uses a stationary (i.e., Markovian and time-invariant) strategy, entailing that the same decision rule is assigned whenever prior preferences do not persist and the task has still not been performed. Hence, the decision maker is assumed *not* to let his selection depend on time *nor* to use self-enforcing schemes of self-reward and self-punishement to overcome his commitment problems.

When a stationary strategy is used, the equilibrium is characterized in terms of single decision rule. Note that a decision rule assigned by a stationary strategy constituting an MSSPE has the following two properties:

- Any such decision rule specifies that the task be performed at all future stages with probability 1, provided that prior preferences persist.
- Any such decision rule does not specify that the task be performed now with probability 1. The reason is that then the decision maker would believe that

the task would be done with probability 1 at the next stage, independently of whether prior preferences persist. Hence, he would want to delay performing the task until the next stage, contradicting that the stationary strategy assigning this decision rule is an equilibrium.

For any decision rule r^h, let $r^h(h')$ denote the probability with which r^h specifies the choice of a^1 at $h' \in H^h$. Hence, if $h' \in H^h \cap \omega^0$, $r^h(h') \in [0,1]$, while if $h' \in H^h \cap \omega^1$, $r^h(h') = 0$. The following result characterizes the unique stationary MSSPE.

Proposition 1 *For given perceived preference persistence $p \in [0,1]$, there exists a unique stationary* MSSPE *s_p. The* MSSPE *s_p has the following properties:*

(1) If $h \in \omega^0$, then $s_p(h) = r_p^h$ where

$$r_p^h(h') = \begin{cases} q & \text{if} \quad h' = h, \\ 1 & \text{if} \quad h' \in H^h \backslash \{h\} \quad \text{and} \quad h' \in \omega^0 \\ 0 & \text{if} \quad h' \in H^h \backslash \{h\} \quad \text{and} \quad h' \in \omega^1 , \end{cases}$$

and $q \in (0,1)$ is determined by

$$A = \frac{p + q - pq}{1 - (1-p)(1-q)\delta} \tag{1}$$

if $A > p/\big(1-(1-p)\delta\big)$, while $q = 0$ if $A \leq p/\big(1-(1-p)\delta\big)$.

(2) If $h \in \omega^1$, then $s_p(h) = r_p^h$ where

$$r_p^h(h') = 0 \quad \text{for all} \quad h' \in H^h .$$

Proof. The discussion preceding the proposition entails that only q is to be determined. Denote by $V(p,q)$ the expected present value of future payoffs conditional on the task not having been performed yet and not being performed now, discounted back to the next stage using $(1,\delta)$–discounting, under the assumptions that (i) preferences persist at the next stage with probability p, and (ii) a stationary strategy is applied, prescribing a decision rule which specifies that the task be performed with probability q now and with probability 1 at the next stage. Since

$$V(p,q) = (p+q-pq)\big(-c+\delta v\big) + (1-p)(1-q)\delta V(p,q)\,,$$

we obtain that

$$V(p,q) = \frac{p+q-pq}{1-(1-p)(1-q)\delta}\big(-c+\delta v\big)\,.$$

It follows that s_p is an MSSPE if and only if one of the three cases holds:

(a) $q = 0$ and $-c + \beta\delta v \leq \beta\delta V(p,q)$

(b) $q \in (0,1)$ and $-c + \beta\delta v = \beta\delta V(p,q)$

(c) $q = 1$ and $-c + \beta\delta v \geq \beta\delta V(p,q)$

Since by Assumption 1 case (c) can never hold, we are left with the two remaining cases. If $p = 1$, then by Assumption 1 we must be in case (a), with $q = 0$ and $A < 1 = p/\big(1-(1-p)\delta\big)$. If $p \in [0,1)$, then

$$\frac{p+q-pq}{1-(1-p)(1-q)\delta}$$

is an increasing function of q, and it follows that q is uniquely determined as specified in the proposition. This establishes the proposition. ■

One may interpret q as the decision maker's belief about his future actions; it does not necessarily entail that the decision maker randomizes. This is consistent with the usual interpretation of Nash equilibrium as an equilibrium in beliefs.

Turn next to the comparative statics: how does the unique stationary MSSPE s_p vary with the perceived preference persistence? For the statement of this result, denote by $q(p)$ the probability determining the decision rule r_p^h assigned to $h \in \omega^0$ by s_p.

Proposition 2 *There exists* $\bar{p} \in (0,1)$ *such that*

$$q(p) \begin{cases} \in \ (0,1) & \textit{if} \quad p \ \in \ [0,\bar{p}) \\ = \ 0 & \textit{if} \quad p \ \in \ [\bar{p},1]\,. \end{cases}$$

The probability $q(p)$ *is a continuous function for* $p \in [0,1]$*, continuously differentiable for* $p \in (0,\bar{p})$*, strictly decreasing function in* p *on* $[0,\bar{p}]$*, and constant in* p *on* $[\bar{p},1]$*. The critical perceived preference persistence* $\bar{p}$ *is determined by*

$$A = \frac{\bar{p}}{1-(1-\bar{p})\delta}\,.$$

Proof. Write $f(p) := p/\big(1-(1-p)\delta\big)$. Since $A \in (0,1)$ (by Assumption 1), $f(0) = 0$, $f(1) = 1$, and $f(\cdot)$ is continuous and strictly increasing, we obtain that $\bar{p} \in (0,1)$ is uniquely determined. It now follows from Proposition 1 that $q(p)$ is a continuous function for $p \in [0,1]$, $q(p) = (0,1)$ if $p \in [0,\bar{p})$ and $q(p) = 0$ if $p \in [\bar{p},1]$. Finally, for $p \in (0,\bar{p})$, $q(p)$ is continuously differentiable and

$$q'(p) = -\frac{1-q(p)}{1-p} < 0\,,$$

since $A = \big(p+q-pq\big)/\big(1-(1-p)(1-q)\delta\big)$ in this range of p values. ■

Proposition 2 entails that an increased perceived preference persistence p decreases the probability of doing task now, up to a critical level $\bar{p} \in (0,1)$, above which the task is postponed for sure.

Hence, the behavioral implications of the present modeling of procrastination (in the setting considered) can be described as follows:

(a) A sophisticated decision maker, having a perceived preference persistence equal to zero, will do the task in any stage with a probability between 0 and 1. This probability reflects the decision maker's own uncertainty concerning whether the task will be performed at any future stage.

(b) A higher level of partial naivete, in the form of increased perceived preference persistence, lowers this probability *continuously* up to a critical level, above which the task is postponed for sure.

It is of interest to point out how these behavioral implications differ from those of O'Donoghue and Rabin (2001), on the one hand, and Gul and Pesendorfer (2004), on the other hand, in the same setting.

With full sophistication, the modeling of O'Donoghue and Rabin (2001) is the same as mine, and behavioral implication (a) can be obtained within their framework by allowing for a stationary strategy implementing a mixed action as long as the task has not been performed. However, their modeling cannot replicate behavioral implication (b). To see this, assume that the decision maker is partially naive in the sense of O'Donoghue and Rabin (2001). E.g., with the parameter values used in the example of the introduction, suppose that the decision maker believes his future β will be 3/5, entailing partial naivete as his true β equals 1/2. Assuming that he believes that his future selves will apply a stationary strategy, it then follows (by setting $\beta = 3/5$ and $p = 0$ and applying Proposition 1) that the decision maker believes his future selves will perform the task in any stage with probability 7/8. This means that he now (and in the future) strictly prefers to postpone the task to the next stage, implying that the task will never be performed. This generalizes to any combination of parameter values satisfying Assumption 1: Any degree of partial naivete leads to the task being postponed forever, provided that the decision maker believes a stationary strategy will be used by his future selves. Hence, going from full sophistication to a tiny degree of partial naivete in the sense of O'Donoghue and Rabin (2001) leads to a discontinuous change in behavior.

When modeling the present setting by means of Gul and Pesendorfer's (2004) 'dynamic self-control' preferences, the decision maker has recursive preferences which take into account the temptation of delaying the task to the next stage. It holds generically that the decision maker, when faced with the binary choice of performing the task now or delaying the task to the next stage, will either do the task now, or not do the task at all. One will not observe that the task is postponed for a finite number of periods before, finally, being performed. Note also that partial naivete, as such, cannot be explicitly modeled in the framework of Gul and Pesendorfer (2004).

4 Welfare analysis in multi-self settings

To motivate the the general discussion of welfare analysis with time-inconsistent preferences, consider the model of Section 3 with $c = 1$ and $v = 5$. If the task is performed at stage 1, then the stream of periodic payoffs is

$$0, \;\; -1, \;\; 5, \;\; 0, \;\; 0, \;\; \dots \, ,$$

while if the task is performed at stage 2, then the stream of periodic payoffs is

$$0, \;\; 0, \;\; -1, \;\; 5, \;\; 0, \;\; \dots \, .$$

With time-consistent preferences, say $\beta = 1$ and $\delta = 4/5$, the decision-maker at both stage 0 (i.e., at decision node h_0) and stage 1 (i.e., at decision node (h_0, a^0)) agree that performing the task at stage 1 (outcome z_1) is better than performing the task at stage 2 (outcome z_2):

$$u^{h_0}(z_1) = \tfrac{12}{5} > \tfrac{48}{25} = u^{h_0}(z_2) \quad \text{and} \quad u^{(h_0,a^0)}(z_1) = 3 > \tfrac{12}{5} = u^{(h_0,a^0)}(z_2)\, .$$

The situation is different with time-inconsistent preferences, say $\beta = 1/2$ and $\delta = 4/5$. Then the decision-maker at stage 0 (i.e., at decision node h_0) prefers to perform the task at stage 1, while the decision maker at stage 1 (i.e., at decision node (h_0, a^0)) prefers to perform the task at stage 2:

$$u^{h_0}(z_1) = \tfrac{6}{5} > \tfrac{24}{25} = u^{h_0}(z_2) \quad \text{and} \quad u^{(h_0,a^0)}(z_1) = 1 < \tfrac{6}{5} = u^{(h_0,a^0)}(z_2)\, . \tag{2}$$

This conflict of interests in the case of time-inconsistent preferences is the motivation for using the Pareto-criterion in such cases. While with time-consistent preferences, all selves on the path leading up to a decision (in this case, whether to do the task at stage 1 or 2) agree on what to do, this is the not the case with time-inconsistent preferences. So with time-inconsistent preferences, we cannot in this case rank the alternatives due to the conflict of interests between the selves at h_0 and (h_0, a^0).

Note that all other decision nodes are irrelevant for deciding between z_1 and z_2. At decision nodes where the task has been performed (e.g., after (h_0, a^1)), there is no task to perform, while when stage 2 or later has been reached, it is no longer possible to perform the task at stage 1. The general principle is to compare decision rules at each and every decision node. A decision rule at h_0 implementing z_1 is

$$r_1^{h_0}(h) = \begin{cases} 1 & \text{if} \quad h \in H^{(h_0,a^0)} \cap \omega^0 \\ 0 & \text{otherwise,} \end{cases}$$

while a decision rule at h_0 implementing z_2 is

$$r_2^{h_0}(h) = \begin{cases} 1 & \text{if} \quad h \in H^{(h_0,a^0,a^0)} \cap \omega^0 \\ 0 & \text{otherwise.} \end{cases}$$

Hence, $r_1^{h_0}$ ($r_2^{h_0}$ respectively) specifies to perform the task at stage 1 (2 respectively) and, if it is not done by then, to do it as soon as possible. These two decision nodes implement the same outcome at all decision nodes except for $h = h_0$ and $h = (h_0, a^0)$. It follows from (2) that the decision maker at $h = h_0$ prefers $r_1^{h_0}$ to $r_2^{h_0}$, while the opposite is the case for the decision maker at $h = (h_0, a^0)$.

Turning now to the general model of Section 2 and taking explicitly into account that the decision maker is not able to commit to one decision rule throughout the decision problem, one must compare alternative strategies (not decision rules) *at each and every decision node.* I will take the position that the comparisons should be based on the actions that will actually be taken throughout the decision problem, not the actions that the decision maker naively thinks he will take.

Since there is no actual preference persistence, it follows that for every $h \in H$, a strategy $s \in S$ generates an actual probability measure $P^h(s)$ over the set of outcomes Z^h, independently of the perceived preference persistence. Let $v^h : S \to \mathbb{R}$ assign to each $s \in S$ the actual expected payoff $v^h(s) = E_{P^h(s)} u^h(z)$ that this strategy leads to. A formal definition of Pareto-dominance in the context of our general model of Section 2 can now be stated.

Definition 2 A strategy $s' \in S$ *Pareto-dominates* another strategy $s'' \in S$ if

$$v^h(s') \geq v^h(s'') ,$$

for all $h \in H$, with strict inequality for some $h' \in H$.

I use this criterion to evaluate the welfare effects of partial naivete in Section 5. It is of interest to note that Pareto-dominance as defined in Definition 2 is closely related to the dominance relation used to define concepts of renegotiation-proofness in repeated games (see, e.g., Farrell and Maskin, 1989, and their concept of weak renegotiation-proofness). I have in Asheim (1997) used the same dominance relation to refine of the concept of MSSPE in the context of individual planning with time-inconsistent preferences. In that paper I explain and exploit the analogy between, on the one hand, a single decision maker with inconsistent preferences revising his decision rule and, on the other hand, the grand coalition in a repeated game renegotiating away from a continuation equilibrium that punishes all players.

In the behavioral economics literature on time-inconsistency and procrastination, other welfare criteria have been applied. In the case of (β, δ)–preferences, the most common practice seems to be to set $\beta = 1$ and treat $\sum_{t=0}^{\infty} \delta^t \upsilon_t$ as the welfare criterion, where υ_t denotes the periodic payoff at stage t. O'Donoghue and Rabin (1999, 2001) use this welfare criterion, which they refer to as "a person's long-run utility", while Gul and Pesendorfer (2005, Section 6.4) question its justification.

Measuring welfare by means of "long-run utility" can be motivated by a paternalistic concern for the well-being of the decision maker, where $\beta < 1$ is interpreted as a defect of his decision making capabilities. This entails the normative position that β *should* equal 1.

"Long-run utility" utility can also be provided with a non-paternalistic justification, if we are willing to assert the following:

- Assume that the decision maker has no decision to make at stage 0 (i.e., at the root of the decision tree h_0, the action set A^{h_0} is a singleton), entailing that "life" starts at stage 1.
- Adopt the normative position that *only* the self of the decision maker at the root of the decision tree h_0 has normative significance.

The combination of these two points means that β plays no rule and the ranking of strategies are made in accordance with "long-run utility". Hence, they imply in the setting of the general model of Section 2 that $v^{h_0}(s)$ measures the welfare generated by strategy s, provided that A^{h_0} is a singleton.

The present paper is based on the presumption that the problem of the decision maker is not his preferences, but his naivete, and his lack of access to a commitment mechanism. Hence, for the purpose of this paper, I do not adopt a paternalistic position. Furthermore, even if A^{h_0} is a singleton, so that there is no decision to make at stage 0, it might be difficult to argue that the self at the root of the decision tree should be treated as a dictator, trumping the interests of all future selves.

Some researchers (e.g., Goldman, 1979; O'Donoghue and Rabin, 2001) have used the Pareto criterion differently from Definition 2. In their use of the concept, one stream of periodic payoffs is considered unambiguously as good as another if it is weakly preferred by the decision maker *at all stages.* Returning to the two streams of periodic payoffs given at the beginning of this section, they compare not only $u^{h_0}(z_1)$ to $u^{h_0}(z_2)$ and $u^{(h_0,a^0)}(z_1)$ to $u^{(h_0,a^0)}(z_2)$, but also $u^{(h_0,a^0,a^1)}(z_1) = 5$ to $u^{(h_0,a^0,a^0)}(z_2) = 1$ and $u^{(h_0,a^0,a^1,a^0)}(z_1) = 0$ to $u^{(h_0,a^0,a^0,a^1)}(z_2) = 5$. Hence, at stage 0 the decision maker prefers z_1 to z_2, at stage 1 he switches to preferring z_2 to z_1, at stage 2 he wishes the task had already been done and reswitches back to preferring z_1 to z_2, while finally, at stage 3 he thinks the later execution date is better and prefers once more z_2 to z_1.

However, the comparisons at stages 2 and 3 are made across different histories. That such comparisons lead to nonsensical results is illustrated by O'Donoghue and Rabin (2001) in their footnote 21. And as O'Donoghue and Rabin (2001, footnote 21) observe, this use of the Pareto-criterion leads to a conflict of interest between different selves even if the decision maker has time-consistent preferences. In contrast, the concept of Pareto-dominance proposed in Definition 2 reduces to the usual criterion for intertemporal choice with time-consistent preferences.

5 Welfare effects of procrastination

In this section I use Pareto-dominance as proposed in Definition 2 for the welfare analysis of procrastination. In particular, I do comparative statics of the equilibrium strategies established in Proposition 1 w.r.t. perceived preference persistence p.

Following Definition 2, one strategy s' Pareto-dominance another s'' if $v^h(s') \geq v^h(s'')$ for all $h \in H$, with strict inequality for some $h' \in H$. When applied to the equilibria of the special model of Section 3, Definition 2 is simple to apply. The reason is that, for each perceived preference persistence p, the MSSPE s_p of Proposition 1 is a stationary strategy, entailing that the same decision rule is assigned whenever prior preferences do not persist and the task has still not been performed. Hence, the welfare indicator $v^h(s_p)$ is the same at all decision nodes h at which the task has not yet been performed (i.e., $h \in \omega^0$). Since there is only one feasible decision rule at decision nodes h at which the task has already been performed (i.e., $h \in \omega^1$), and hence, the problem of selecting a decision rule is trivial at such nodes, we need only be concerned with the common value of $v^h(s_p)$ for all $h \in \omega^0$.

The main welfare result is stated by the following proposition.

Proposition 3 *The welfare indicator $v^h(s_p)$ is strictly decreasing in p on $[0, \bar{p}]$ and constant in p on $[\bar{p}, 1]$ for all $h \in \omega^0$, and constant in p on $[0, 1]$ for all $h \in \omega^1$.*

Proof. The statements for $h \in \omega^1$ are trivially true. Hence, consider $h \in \omega^0$. Applying the notation of Section 3, we have that

$$v^h(s_p) = q(p)\big(-c + \beta\delta v\big) + (1 - q(p))\beta\delta V(0, q(p)) \tag{3}$$

if $h \in \omega^0$. Since, by Proposition 2, $q(p)$ is a continuous function for $p \in [0,1]$ and continuously differentiable for $p \in (0, \bar{p})$, it follows that $v^h(s_p)$ is a continuous function of p for $p \in [0,1]$ and continuously differentiable for $p \in (0, \bar{p})$.

Propositions 1 and 2 imply that, for $p \in [0, \bar{p})$,

$$-c + \beta\delta v \;=\; \beta\delta V(p, q(p)) \;\geq\; \beta\delta V(0, q(p))$$

since

$$\frac{p + q - pq}{1 - (1-p)(1-q)\delta}$$

is a nondecreasing function of p. Hence, it follows from (3) that, for $p \in (0, \bar{p})$,

$$\frac{dv^h(s_p)}{dp} \;<\; q'(p)\Big(\big(-c + \beta\delta v\big) - \beta\delta V(0, q(p))\Big) \;\leq\; 0$$

since $q'(p) < 0$ by Proposition 2 and

$$\frac{q}{1 - (1-q)\delta}$$

is an increasing function of q. Hence, $v^h(s_p)$ is decreasing in p on $[0, \bar{p}]$. ■

Proposition 3 yields an unambiguous welfare conclusion: *Having a positive perceived preference persistence reduces welfare, since such partial naivete leads to welfare reducing procrastination.*

If I instead had followed O'Donoghue and Rabin (2001) and ranked the strategies in accordance with "long-run utility", the conclusions of Proposition 3 would still hold, since "long-run utility" gives less relative weight to the immediate cost associated with performing the task. However, Proposition 3 has the interesting feature that the negative welfare effects of partial naivete can be established even if we evaluate the strategies of the decision maker by his actual preferences.

The present analysis can also be used to evaluate the welfare effects of introducing a commitment device. E.g., suppose that a mechanism was offered, enabling the decision maker at stage t (where $t = 0, 1, \dots$) to make a costless commitment to performing the task at stage $t + 1$. If the perceived preference persistence is smaller than 1, then a stationary MSSPE leads to the choice of the commitment mechanism at any decision node where the task has not already been performed. Furthermore, Definition 2 entails that this equilibrium welfare dominates any other stationary strategy, since comparisons need only be made at decision nodes where the task has still not been performed, and the decision maker prefers to make the commitment at all such nodes. This means that policy offering the decision maker an opportunity to choose such a commitment device is welfare enhancing. In particular, according to Definition 2 it is irrelevant that the decision maker will regret his commitment when being forced to perform the task, as long as his selves up to the point at which the commitment was made unanimously agreed.

6 Concluding remarks

In this paper I have presented a new modeling of partial naivete, and applied this to the case of procrastination. By combining my modeling with an application of Pareto-dominance which is motivated by the existence of time-inconsistency and which coincides with dominance relations used elsewhere in game theory, I have been able to provide a clear welfare conclusion: The decision maker's welfare is reduced if he is partially naive in the sense of perceiving that his present preferences may persist.

Furthermore, I have presented a critical discussion of how to do welfare analysis in models with time-inconsistent preferences. The general principles I have suggested for such welfare analysis have significance beyond the problem of procrastination.

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