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Michael Hoel

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Department of Economics
University of Oslo

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P. O.Box 1095 Blindern
N-0317 OSLO Norway
Telephone: + 47 22855127
Fax: + 47 22855035
Internet: http://www.oekonomi.uio.no
e-mail: econdep@econ.uio.no

In co-operation with
The Frisch Centre for Economic Research

Gaustadalleén 21
N-0371 OSLO Norway
Telephone: +47 22 95 88 20
Fax: +47 22 95 88 25
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Environmental taxes in an economy with distorting taxes and a heterogeneous population*

Michael Hoel[†]

15 January 2008

Abstract

During the last couple of decades, there has been a large literature discussing how the properties of emission taxes are affected by the existence of distortionary taxes. Most of this literature ignores distributional aspects of environmental taxes and other types of environmental policy instruments.The present paper considers a very simple model with heterogeneous households, differing in income earning ability. The tax system is not necessarily fully optimal. Instead, a tax function is assumed to be exogenously given, but the parameters of this tax function are optimally chosen. The rule for the second-best optimal environmental tax is derived and compared with the Pigovian rule. The results derived in the present paper are related to the results from the literature on public goods provision under distortionary taxes.

Keywords: environmental taxes, public goods, distortionay taxation

JEL classification: H23, H41, Q58

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[†]Department of Economics, University of Oslo. P.O Box 1095 Blindern, N-0317 Oslo. Email: mihoel@econ.uio.no

1 Introduction

The idea that there is a so-called "double dividend" of environmental taxes goes back to the late 1980s.[1] The background for the double dividend hypothesis is the observation that there are distorting taxes in the economy. By introducing an environmental tax one therefore not only increases the environmental quality (the first "dividend") but the environmental tax also raises revenue that can be used to reduce other distorting taxes so that the performance of the economy is improved (the second "dividend"). While this might seem plausible at first sight, it is obviously not generally correct. If the tax system before any considerations of the environment is optimally designed, social welfare (ignoring environmental aspects) is by definition maximized subject to whatever constraints there are on what taxes and tax rates one may use. Changing the initial tax system by introducing a new tax and adjusting other taxes will in such a situation necessarily reduce social welfare (ignoring environmental aspects). The second dividend is thus not present. Introducing an environmental tax may of course nevertheless be welfare improving; this will be the case if the improvement in environmental quality is considered larger than the loss in "non-environmental social welfare" (i.e. social welfare ignoring environmental aspects).

If the initial tax system is *not* optimally designed, there will by definition exist changes in the tax system that improve social welfare. In particular, among the set of such possible welfare improving tax changes there may be changes that involve introducing a tax that may be labeled "environmental" and adjusting some other taxes. If this is the case, non-environmental social welfare will increase, in addition to the environmental benefit the environmental tax gives. In this case we thus have a double dividend.

There was a considerable literature in the early 1990s identifying cases were the initial tax system was non-optimal and where the introduction of an environmental tax in combination with a reduction of other tax rates gave a double dividend. However, one can question how much insight this literature gave, and in particular what implications it had for environmental policy. The "second dividend" (i.e. the increase in non-environmental social welfare) in all these cases had nothing to do with the environmental tax as such, it was simply a "dividend" from changing a non-optimal tax system

[1] The first use of the concept "double dividend" seems to have been by Pearce (1991), but the general idea goes back to at least Terkla (1984).

to some tax system that was better (measured by non-environmental social welfare). Such a tax reform therefore made economic sense even if there was no concern for the environment. The fact that such a tax reform also improved the environment was of course nice, but irrelevant.

A related, and in my opinion more interesting, issue that has been discussed during the last couple of decades, is how the properties of emission taxes and other environmental policy instruments are affected by the existence of distortionary taxes. An important insight from this literature is that policy instruments that raise revenue (such as an emission tax or auctioned quotas) become more favorable compared with non-revenue raising instruments (such as direct regulation or non-auctioned quotas) than in a situation without distorting taxes, see e.g. Parry (1997) and Goulder et al. (1999). A second important question that has been raised is how the optimal rate of an emission tax will be affected by the existence of distorting taxes. From the earlier double dividend literature discussed above it might be argued that even if there is no double dividend, at least some of the costs of introducing an environmental tax are offset by the reduction in other distorting taxes. Since this reduces the cost of environmental policy, one could furthermore argue that the optimal environmental tax should be higher than the Pigovian level (i.e. the sum of everyone's willingness to pay for improving the environment). Lee and Misiolek (1986) give a formal analysis with this conclusion: The rate of the environmental tax should be higher than if the tax was chosen only to control the amount of pollution, provided the revenue from the environmental tax is increasing in the tax rate at the optimum. However, studies by e.g. Bovenberg and van der Ploeg (1994), Bovenberg and de Mooij (1994), Goulder (1995), Parry (1995), Bovenberg and Goulder (1996), Bovenberg (1999), and Metcalf (2003) have shown that it is not so obvious whether or not the optimal environmental tax should exceed the Pigovian level. Most of these studies consider the case of a proportional income tax. An important result derived in several of these papers is that if preferences are separable between pollution and other goods, and the uncompensated labour supply elasticity is positive, the second-best optimal emission tax is *lower* than the Pigovian level.

Somewhat surprisingly, there is relatively little in this literature concerning distributional aspects of environmental taxes and other types of environmental policy instruments. The reason why this is surprising is that distributional considerations are the reason one has distorting taxes. Without any concern for income/consumption distribution, the necessary tax revenue could simply be raised by an equal tax per person, and there would be no interaction (except income effects) between such taxation and the use of environmental taxes. There is a small literature that discusses environmental taxation in models that explicitly allow for various types of heterogeneity in the population. Perhaps the earliest contribution was Sandmo (1975), who adressed this issue with a relatively restricted set of possible taxes. More recently, Cremer et al. (1998), Cremer and Gahvari (2001), and Pirttila and Tuomala (1997) consider a heterogeneous population, and income taxation is in these studies assumed to be optimally designed.

The present paper starts by introducing a simple one person economy that is used to explain the result that the second-best optimal emission tax is *lower* than the Pigovian level (section 2). In section 3 the model is extended to a heterogeneous population and the rules for an optimal emission tax are derived in a setting with a simple two-parameter linear income tax. The special case of a homogeneous population is briefly discussed again in section 3.1, while section 3.2 gives results for the case of a heterogeneous population.

In Section 4 it is shown that there is a strong similarity between the issue of optimal emission taxation and the optimal provision of public goods. While this similarity is particularly transparent for the model used in the present analysis, the similarity is a general feature: Optimal emission taxation is simply a question of the optimal amount of a public bad, which obviously is almost equivalent to the question of the optimal amount of a public good. There is a large literature on the optimal provision of public goods in the presence of distorting taxation, going back to Pigou (1947), and with important early contributions including Diamond and Mirrlees (1971), Stiglitz and Dasgupta (1971) and Atkinson and Stern (1974). Contributions such as Feldstein (1997) have argued that the presence of distorting taxation implies that public goods should be supplied by a lower amount than what is implied by the Samuelson rule (i.e. marginal costs of producing a public good should be equal to the sum of willingness to pay for the good). The formal analyses of e.g. Stiglitz and Dasgupta (1971) and Atkinson and Stern (1974) imply that when a public good is optimally supplied, the marginal costs of producing it should be lower than the sum of willingness to pay for the good, provided the public good is separable from consumption and leisure in preferences and that the labour supply curve is upward sloping. This corresponds completely to the claim that the optimal emission tax should be lower than the Pigovian level. Unlike the environmental literature however, the literature on public goods has for a long time explicitly taken population heterogeneity into consideration. In addition to Diamond and Mirrlees (1971), early contributions include Hylland and Zeckhauser (1979) and Christiansen (1981). The latter contribution shows that if (a) income taxes are optimal and (b) labour is separable from other goods in preferences, the optimal provision of public goods is given by the Samuelson Rule. This result is discussed further by Boadway and Keen (1993), and it is extended by Kaplow (1996). Kaplow shows that even if the tax system is not initially optimal, it is possible to achieve a Pareto improvement if the Samuelson Rule does not hold, assuming that labour is separable from other goods and that a particular type of tax adjustment is possible simultaneously with a change in the supply of the public good.

In the literature on environmental taxes, there has been remarkably little reference to the large literature on the optimal supply of public goods. None of the contributions on environmental taxes referred to above refer to the important results of Christiansen, Boadway and Keen, and Kaplow. It is particularly surprising that the most recent of the above contributions do not refer to Kaplow's result, since Kaplow explicitly shows that his result is of direct relevance to the question of how large environmental taxes should be.[2] In Section 4, the results derived in Section 3 are related to the results from the literature on public goods provision. Section 5 concludes.

2 Environmental taxes in the presence of a tax wedge in the labour market

Consider a consumer with a utility function $u(C, L, E)$ where C is consumption, L is labour supply and E is aggregate emissions of some pollutant. This utility function is increasing in C and decreasing in L and E. The economy's resource constraint is $C = F(L, E) - G$, where G is an exogenously given public expenditure and $F(L, E)$ is a reduced form (net) production function. If e.g. the pollutant is greenhouse gases, the interpretation of E could be the use of fossil fuels in gross production $\Phi(L, E)$. If these fuels have an

[2] Pirttila and Tuomala refer briefly to Boadway and Keen (1993) and Cremer and Gahvari refer briefly to Kaplov (1996), but neither refer explicitly to the results in these papers and their relevance for optimal environmental taxation.

international price equal to p, the country's net production (i.e. GDP) is $F(L,E) = \Phi(L,E) - pE$. Without any concern for the environment, the country would maximize its welfare by choosing E so that $\Phi_E = p$, giving $F_E = 0$. If the use of fossil fuels is reduced below this level, we get $F_E = \Phi_E - p > 0$, and F_E may be interpreted as a tax on the use of fossil fuels in production.

The optimal choice of E is the choice that maximizes $u(C,L,E) = u(F(L,E) - G, L, E)$, and the first-order condition of this maximization problem is

$$u_C F_E + u_E + (u_C F_L + u_L) \frac{dL}{dE} = 0$$

or

$$F_E = \frac{-u_E}{u_C} - \left(F_L - \frac{-u_L}{u_C} \right) \frac{dL}{dE} \tag{1}$$

As explained above, F_E is equal to the emission tax. Moreover, the gross wage is equal to F_L. Without any tax wedge in the labour market, the consumer chooses his labour supply so that $\frac{-u_L}{u_C}$ is equal to the gross wage rate. In this case the second term on the r.h.s. of (1) is therefore zero, and we get the standard expression for the Pigou tax: The tax should be equal to the marginal willingness to pay for reduced emissions. However, with a positive tax wedge in the labour market, the term in brackets on the r.h.s. of (1) is positive. The optimal environmental tax is therefore lower or higher than the Pigovian level depending on whether $\frac{dL}{dE}$ is positive or negative. The sign of $\frac{dL}{dE}$ generally ambiguous. An important special case is characterized by preferences being weakly separable in emissions and the two other goods (consumption and leisure), i.e. that $u(C,L,E) = \tilde{u}(w(C,L),E)$. In this case the size of E will have no *direct* effect on L. In this case the sign of $\frac{dL}{dE}$ will depend only on the sign of the uncompensated supply elasticity for labour: As E is increased, i.e. environmental policy is relaxed, the after-tax wage rate goes up (otherwise there would have been no cost of reducing emissions). If the labour supply responds positively to this increase in the after-tax wage, it follows that $\frac{dL}{dE}$ is positive, implying that the optimal emission tax is below the Pigovian level $\frac{-u_E}{u_C}$.

3 Environmental taxes with an optimal linear income tax.

We now turn to the more interesting case of a heterogeneous population. There are I persons in the economy, and persons i's labour supply in efficiency units is $\alpha^i \ell^i$ when his or her labour supply measured in hours is ℓ^i. The parameters α^i reflect differences in productivity across the population. The production function is as before $F(L, E)$, where aggregate labour input in efficiency units now is equal to $L = \sum_i \alpha^i \ell^i$. The gross income of person i is $F_L \alpha^i \ell^i$, and the tax is $tF_L \alpha^i \ell^i - s$, where t is positive and s may be positive or negative. The net (after tax) marginal wage rate (in efficiency units) is denoted n, and is

$$n = (1 - t) F_L \tag{2}$$

since the marginal productivity of labour F_L is equal to the gross wage rate. The disposable income of person i is thus $n\alpha^i \ell^i + s$.

It is useful to derive the indirect utility function of each person. This is defined by

$$v^i = v(n\alpha^i, s, E) = \max \left\{ u(C^i, \ell^i, E) \text{ s.t. } C^i = n\alpha^i \ell^i + s \right\} \tag{3}$$

The solution of this maximization problem gives demand and supply functions $C(n\alpha^i, s, E)$ and $\ell(n\alpha^i, s, E)$, and the aggregate labour supply is

$$L(n, s, E) = \sum_i \alpha^i \ell(n\alpha^i, s, E) \tag{4}$$

The government's optimization problem can be formulated as

$$\max \sum_i v(n\alpha^i, s, E) \tag{5}$$

$$\text{s.t. } F\left(L(n, s, E), E\right) - \sum_i C(n\alpha^i, s, E) - G \geq 0 \tag{6}$$

The Lagrangian to this problem is

$$\Psi = \sum_i v(n\alpha^i, s, E) + \mu \left[F\left(L(n, s, E), E\right) - \sum_i C(n\alpha^i, s, E) - G \right] \quad (7)$$

and the first order conditions determining E, s and n are

$$\sum_i v_E^i + \mu \left[F_E + F_L L_E - \sum_i C_E^i \right] = 0 \quad (8)$$

$$\sum_i v_s^i + \mu \left[F_L L_s - \sum_i C_s^i \right] = 0 \quad (9)$$

$$\sum_i \alpha^i v_{n\alpha}^i + \mu \left[F_L L_n - \sum_i \alpha^i C_{n\alpha}^i \right] = 0 \quad (10)$$

Our concern here is the optimal emission tax, i.e. F_E. In Appendix A it is shown that (8) implies (using well-known properties of the indirect utility function given by (3))

$$F_E = \frac{\sum_i (-u_E^i)}{\mu} - tF_L L_E \quad (11)$$

Before discussing this expression further for the general case, it is useful to briefly consider the special case of a homogeneous population, which is identical to the one person case.

3.1 A homogeneous population

In the Appendix it is shown that when all I persons have the same value of α, (10) gives

$$\frac{1}{\mu} = \frac{1}{u_C} \left(1 - \frac{t}{1-t} \varepsilon \right) \quad (12)$$

where ε is the average labour supply elasticity ($= L_n n / L$)

Inserting (12) into (11) gives

$$F_E = \left[1 - \frac{t}{1-t}\varepsilon\right] I \left(\frac{-u_E}{u_c}\right) - tF_L L_E \tag{13}$$

If preferences are weakly separable in emissions and the two other goods as explained in Section 2, $L_E = 0$. If the term in square brackets was 1, we would have the Pigou rule for the I-person case, i.e. $F_E = I\left(\frac{-u_E}{u_c}\right)$. When the tax rate t is positive, the term in square brackets will be smaller than or larger than one depending on whether the labour supply elasticity is positive or negative. This confirms the result we derived somewhat less formally in Section 2 (and which is well known from previous literature, see e.g. Bovenberg and de Mooij, 1994). Even if the labour supply is independent of the net wage rate ($\varepsilon = 0$), the second-best-optimal emission tax will differ from the Pigovian level if preferences are not separable. The tax is higher than the Pigovian level if the labour supply is increasing in emissions (e.g. if higher emissions make time intensive leisure activities less attractive) while the opposite is true if the labour supply is decreasing in emissions (e.g. if higher emissions have a negative effect on health and thus on labour supply).

Equation (13) is however not particularly interesting. If the population is homogeneous, it well known from elementary economic theory that there is no reason to have a distortionary tax if a uniform head tax is feasible. And if $t = 0$, (13) simply gives us the standard Pigou tax. The reason we in practice see distortionary taxes used, is that the population is not homogeneous, and that the government has some distributional preferences. We therefore now return to the case of a heterogeneous population.

3.2 Environmental taxes with a heterogeneous population.

Consider again equation (11). When there is separability in preferences between emissions and the other two goods, the second term in (11) is zero. In this case (11) is very similar to the standard Pigou tax. The difference is that the Pigou tax in the $I-$person case is $\sum_i(\frac{-u_E^i}{u_C^i})$, while it in the present case is $\frac{\sum_i(-u_E^i)}{\mu}$. Cremer and Gahvari (2001) conclude from this that in the case of separability between emissions and other goods, the optimal emission tax is equal to the Pigovian level. As we shall see below, this seems to be a rather misleading use of the term "Pigovian level".

It is shown in Appendix A that (9) may be rewritten as

$$\frac{1}{\mu} = \frac{1}{\bar{u}_C}\left(1 - tF_L\bar{L}_s\right) \tag{14}$$

where the two averages $\bar{u}_C$ and $\bar{L}_s$ are defined by

$$\bar{u}_C = \frac{\sum_i u_C^i}{I} \tag{15}$$

and

$$\bar{L}_s = \frac{\sum_i \alpha^i \ell_s^i}{I} \tag{16}$$

Inserting (14) into (11) gives

$$F_E = \left[1 - tF_L\bar{L}_s\right]\frac{\sum_i(-u_E^i)}{\bar{u}_c} - tF_L L_E$$

or

$$F_E = \left[1 - tF_L\bar{L}_s\right]\sum_i \frac{u_c^i}{\bar{u}_c}\left(\frac{-u_E^i}{u_c^i}\right) - tF_L L_E \tag{17}$$

As before, the last term in (17) is zero if preferences are weakly separable in emissions and the two other goods. The term $\sum_i \frac{u_c^i}{\bar{u}_c}\left(\frac{-u_E^i}{u_c^i}\right)$ is a weighted sum of the willingness to pay for reduced emissions for all persons. The weights are marginal utilities of consumption. With realistic assumptions about the utility function, consumption will be higher and marginal utility of consumption lower the higher is the productivity α (and thus also wage per hour). In other words, high income persons will have a lower weight in the term $\sum_i \frac{u_c^i}{\bar{u}_c}\left(\frac{-u_E^i}{u_c^i}\right)$ than low income persons. A natural phrase for this term is therefore the equity weighted Pigovian level of the emission tax. Notice that if lump-sum taxation was possible, i.e. different values of s for different persons, u_c^i would be identical for all persons, so there would be no difference between a weighted and unweighted sum of the willingness to pay.

The term $\left[1 - tF_L\bar{L}_s\right]$ is larger than one provided that $\bar{L}_s < 0$, i.e. provided that leisure is a normal good (on average). For the reasonable case that leisure is a normal good, the optimal emission tax is thus higher than the equity weighted Pigovian level (in the case of weakly separable preferences).

Returning to the expression (17), an obvious question is why the optimal emission tax should be higher than the equity weighted Pigovian level for the case when preferences are separable from the two other goods. The interpretation is as follows. Starting with a tax at the weighted Pigovian level, a small increase in the emission tax will have a small cost for households, through a lower transfer s to all persons. Disregarding induced effects on labour supply, this change in the emission tax has an impact of measure zero on welfare. However, since s goes down, labour supply increases (when leisure is a normal good). Due to the positive marginal income tax, labour supply is lower than what is optimal. The increase in labour supply thus gives an increase in the welfare level of measure one, thus dominating the direct welfare loss of deviating from the weighted Pigovian level.

An obvious question is how the expression for the second-best tax given by (17) differs from the Pigovian level (i.e. the unweighted sum of the willingness to pay for reduced emissions for all persons). It is generally not possible to say in what direction this difference goes. The fact that the term in square brackets is larger than one (when leisure is a normal good) draws in the direction of the second-best tax being higher than the Pigovian level. On the other hand, if the willingness to pay for reduced emissions is higher for high-income than for low-income persons, the weighing of the willingness to pay terms in (17) draws in the direction of the second-best tax being lower than the Pigovian level.

Although we have restricted the analysis in this section to the case of a linear income tax, we have assumed that the two parameters in this tax function (t and s) are optimally chosen. To understand the importance of the tax rate being optimally chosen, consider a starting point with given values of E, t and s. Assume that for the given value of E, both t and s are higher than their optimal values. Moreover, assume that a small reduction in E will lead to a surplus in the government's budget, which can be used to either reduce t or increase s. Since t and s by assumption are both higher than their optimal values, it is clear that a small reduction in E is more favorable if it is accompanied by a reduction in t than if it is accompanied by an increase in s. It may even be the case that if the tax rate t is simultaneously adjusted, welfare increases if E is reduced, while if s is simultaneously adjusted welfare increases if E is increased. The optimal environmental tax will thus in this case depend on which tax rate is assumed to be adjusted if the environmental tax is changed.

4 Environmental taxes and the optimal provision of public goods

The model we have used may easily be re-interpreted as that of a model of public goods provision. Redefine E to be the provision of a particular public good. The signs of u_E and F_E must be reversed compared with our analysis so far, so that $u_E > 0$ and $F_E < 0$ ($-F_E$ is thus the marginal cost of producing the public good). The first-best optimum for the supply of this good is given by the Samuelson Rule:

$$-F_E = \sum_i \left(\frac{u_E^i}{u_c^i} \right) \tag{18}$$

The interpretation of this equation is that the marginal cost of producing the public good should be equal the sum of the willingness to pay for good. In the early literature on the optimal provision of public goods it has often been argued that one should provide less of the good than (18) suggests, due to the distortion caused be raising taxes to finance the good.[3] This argument is very much in the spirit of the argument leading to (13), suggesting that the optimal emission tax should be lower than the Pigovian level (in the case of weakly separable preferences).

The analysis in Section 3 remains valid with the reinterpretation of E, so that the results (13) and (17) remains valid. Rewriting these we get

$$-F_E = \left[1 - \frac{t}{1-t} \varepsilon \right] I \left(\frac{u_E}{u_c} \right) + tF_L L_E \tag{19}$$

and

[3]For instance, Wilson (1991) cites Stiglitz (1988, p. 180): "Since it becomes more costly to obtain public goods when taxation imposes distortions, normally this will imply that the efficient level of public goods is smaller than it would have been with non-distortionary taxation".

$$-F_E = \left[1 - tF_L\bar{L}_s\right] \sum_i \frac{u_c^i}{\bar{u}_c} \left(\frac{u_E^i}{u_c^i}\right) + tF_L L_E \tag{20}$$

Expressions similar to (19) for the case of a homogeneous population - but with somewhat different notation - were derived by e.g. Stiglitz and Dasgupta (1971) and Atkinson and Stern (1974). In the absence of lump-sum taxes the public good should be supplied at a level making the marginal cost of producing this good lower than the marginal willingness to pay, provided labour supply is increasing in the net wage rate and preferences are weakly separable in the public good and the two private goods (consumption and leisure).

For the case of a heterogeneous population and the possibility of a uniform lump-sum tax it is from (20) not obvious whether the supply of the public good should be lower or higher than the level implied by the Samuelson Rule. The fact that we in (20) have an equity weighted sum of the willingness to pay for all persons will tend to make the r.h.s. of (20) lower than the r.h.s. of (18), as long as the public good is a normal good. On the other hand, if leisure is a normal good the term in square brackets in (20) tends to make the r.h.s. of (20) higher than the r.h.s. of (18). Finally, if the supply of the public good has a direct effect on labour supply, the second term in (20) will be a third source of difference between (20) and (18).

Unlike the literature on emission taxes, there is a large literature on the optimal provision of public goods where distributional considerations are explicitly taken into account. An expression similar to (20) - but with somewhat different notation and assumptions - can for instance be found in Atkinson and Stiglitz (1980, Sec. 16.3). An important early contribution to this literature is Christiansen (1981), who shows that if the income tax is optimally designed, and if consumers' preferences have separability between labour and other goods, then the optimal supply of the public good is characterized by the Samuelson Rule (18). Notice that the separability assumption used is different from the separability assumption required for the second tern in (20) to become zero. The latter condition was that $u(C, L, E) = \tilde{u}(w(C, L), E)$, while the assumption used by Christiansen is that $u(C, L, E) = \tilde{u}(\phi(C, E), L)$.

Kaplow (1996) has generalized the result of Christiansen to cases where the initial taxes are not necessarily second best optimal.[4] With the same sep-

[4]A related result was derived already in 1979 by Hylland and Zeckhauser.

arability assumption as Christiansen, Kaplow shows that unless (18) holds, it is possible to change the provision of public goods and simultaneously adjust taxes so that a Pareto improvement is possible. A Pareto optimal outcome must thus be characterized by the Samuelson Rule (18). Kaplow also shows that his result applies to the issue of an optimal environmental tax, thus arguing that the Pigovian tax is optimal also in a second-best setting.

It is important to note that Kaplow's result assumes the possibility of adjusting taxes so that a Pareto improvement is possible for any change in the supply of the public good if (18) does not hold initially. This requires quite a rich tax system (see also Christiansen, 2007). With the simple linear tax function used in section 3 this will generally not be possible, so that we with this tax function cannot expect the Samuelson Rule for public goods to hold. Similarly, we cannot expect the optimal emission tax to be equal to the Pigovian level for the simple linear tax function used in Section 3.[5]

An obvious question is whether the strong similarity between the optimal emission tax and the optimal provision of a public good is simply a particular property of the specific model used, or whether it holds more generally. To see that it is a quite general property, consider the following generalization of our model: Let the economy's resource constraint be $H(\mathbf{C},G,L,E) \geq 0$ in stead of our previous resource constraint $C = F(L,E) - G$. H is increasing in L and E (which as before represent labour input and emissions) and decreasing in the other variables: $\mathbf{C} = (C_1, ..., C_K)$ is a vector of private consumption goods, and G is now the supply of public goods (generalization to several public goods is trivial). Let C_1 be the numeraire good. The marginal production cost of private good k is in terms of the numeraire good given by (in obvious notation) $\frac{H_k}{H_1}$, and the marginal production cost of the public good is (in terms of the numeraire good) $\frac{H_G}{H_1}$. Notice that both of these marginal production costs include environmental costs through a shadow price of E, since E is held constant. The marginal abatement cost in terms of the numeraire good (i.e. the emission tax) is equal to $\frac{H_E}{-H_1}$, and the marginal productivity of labour (i.e. the gross wage rate) is $\frac{H_L}{-H_1}$.

In much of the literature on environmental taxes it is assumed that emissions are linked to the use of "dirty" consumer goods, i.e. as $E = \sum_k \delta_k C_k$ where δ_k is positive for all "dirty" goods. This case can be approximated by the general description of the production possibilities given above. A simple

[5] An example of a utility function implying that tha Kaplov result holds with the linear tax function assumed in Section 3 is given in Appendix B

case is where producer prices (excluding environmental costs) p_k and gross wages w are exogenous. Let

$$H(\mathbf{C},G,L,E) = -\sum_k p_k C_k - p_k C_k + wL - \left(\sum_k \delta_k C_k - E\right)^{2\sigma}$$

where σ is "large". Moreover, let $H_1 = -p_1 = 1$, and let the numeraire good be "clean", so that $\alpha_1 = 0$. Then the marginal shadow price of emissions is $H_E = 2\sigma \left(\sum_k \delta_k C_k - E\right)^{2\sigma-1}$ and the marginal cost of consumer good k, including environmental costs, is $-H_k = p_k + \delta_k H_E$. Finally, the optimum $\sum_k \delta_k C_k$ can be made arbitrarily close to E by choosing a sufficiently large value of σ. The case often considered in the environmental literature is thus a special case of the model used in the present section.

When there are several consumer goods, C^i must be replaced by $\mathbf{C}^i$ in the utility function, and C^i in the budget equation (3) must be replaced by $\sum_k q_k C_k^i$, where q_k is the consumer price of good k. The commodity tax of good k is thus $\tau_k = q_k - \frac{H_k}{H_1}$. For the numeraire good we have $q_1 = 1$ and $\tau_1 = 0$. Without loss of generality the numeraire good can be chosen so that all commodity taxes are non-negative.

Proceeding as we did in Section 3, it is straightforward to derive the following formulas for the optimal emission tax and the optimal provision of the public good (for the case of a linear income tax):

$$\frac{H_E}{-H_1} = \left[1 - t\frac{H_L}{-H_1}\bar{L}_s + \sum_k \tau_k \bar{C}_{ks}\right] \sum_i \frac{u_1^i}{\bar{u}_1} \left(\frac{-u_E^i}{u_1^i}\right) - t\frac{H_L}{-H_1}L_E + \sum_k \tau_k C_{kE} \tag{21}$$

$$\frac{H_G}{H_1} = \left[1 - t\frac{H_L}{-H_1}\bar{L}_s + \sum_k \tau_k \bar{C}_{ks}\right] \sum_i \frac{u_1^i}{\bar{u}_1} \left(\frac{u_G^i}{u_1^i}\right) + t\frac{H_L}{-H_1}L_G - \sum_k \tau_k C_{kG} \tag{22}$$

where the marginal utility of income is now equal to the marginal utility of the numeraire good, and where

$$C_{kE} = \sum_i \frac{\partial C_k^i}{\partial E}$$

$$C_{kG} = \sum_i \frac{\partial C_k^i}{\partial G}$$

and

$$\bar{C}_{ks} = \frac{1}{I} \sum_i \frac{\partial C_k^i}{\partial s}$$

Equations (21) and (22) are practically identical, and correspond completely to the previous expressions (17) and (20) except for the additional terms involving how the non-numeraire consumption goods are effected by changes in E and s. Notice that if all goods are normal and all commodity taxes are non-negative, the terms containing $\bar{C}_{ks}$ in the square brackets are positive, so that the terms in square brackets as before are larger than one. Also, if E and G are separable form the private goods in preferences, the second and third terms in (21) and (22) will be zero.

The results above hold for any arbitrarily given commodity taxes. Without any restrictions on preferences, the optimal commodity taxes will typically be positive for some goods. However, although the choice both of E and G typically will affect producer prices (i.e. marginal production costs) of the private consumption goods, these choices will have no consequence for the rule for determining optimal commodity taxes. This point was first made by Sandmo (1975).

5 Concluding remarks

The previous sections have demonstrated that there is no simple answer to the question of how large the second-best optimal emission tax should be compared to the Pigovian level. In particular, the answer to this question does not depend on simple properties such as whether the tax revenue form the environmental tax is (locally) increasing in the tax rate, or the sign of the labour supply elasticity. Even with specific assumptions about separability in preferences we do not know the answer to this question. If e.g. preferences are weakly separable in emissions and the two other goods (consumption and leisure) the last term in (17) is zero. However, as argued after (17), even if this case we do not generally know whether the optimal emission tax is higher or lower than the Pigovian level.[6].

From the literature on the provision of public goods we know that if consumers' preferences have separability between labour and other goods, and the income tax system is sufficiently "rich" so that Kaplow's result holds, then the optimal environmental tax rate is equal to the Pigovian tax rate. In practice, these two conditions will seldom be fulfilled. However, an interesting question would be to analyze what the welfare loss would be from (wrongly) setting the tax rate equal to the Pigovian rate for realistic deviations from the two assumptions above. Obviously, this is a very difficult question to answer. If I were to guess, my guess would be that this welfare loss would be much lower than the loss we may get from an inaccurate measurement of peoples' willingness to pay for reduced emissions, given the large and well-known difficulties in making such measurements.

Appendix A: Derivations of the second-best optimal emission tax

The demand and supply functions corresponding to (3) satisfy

$$\begin{aligned} C_{n\alpha}(n\alpha^i, s, E) &= \ell + n\alpha^i \ell_{n\alpha}(n\alpha^i, s, E) \\ C_s(n\alpha^i, s, E) &= 1 + n\alpha^i \ell_s(n\alpha^i, s, E) \\ C_E(n\alpha^i, s, E) &= n\alpha^i \ell_E(n\alpha^i, s, E) \end{aligned} \tag{23}$$

Moreover, the envelope theorem applied to (3) gives

[6] In an earlier version of this paper I have shown that a similar conclusion is true for a more general tax function than the simple tax function considerred in Sections 3-5. This issue is also discussed by Wilson (1991) for quite general assumptions about what taxes are feasible.

$$
\begin{aligned}
v_s(n\alpha^i, s, E) &= u_C(C^i, \ell^i, E) \\
v_{n\alpha}(n\alpha^i, s, E) &= u_C(C^i, \ell^i, E)\ell(n\alpha^i, s, E) \\
v_E(n\alpha^i, s, E) &= u_E(C^i, \ell^i, E)
\end{aligned}
\tag{24}
$$

Using (2), (23) and (24) we can rewrite (8) as

$$
\sum_i u_E^i + \mu \left[F_E + (n + tF_L)L_E - nL_E\right] = 0
$$

Rearranging gives (11).

Using (2), (23) and (24) we can rewrite (9) as

$$
\sum_i u_C^i + \mu \left[(n + tF_L)L_s - I - nL_s\right] = 0 \tag{25}
$$

Together with (15) and (16) this gives (14).

The optimal tax can alternatively be derived by using (10) together with (2), (4), (23) and (24). This gives

$$
\sum_i \alpha^i \ell^i u_C^i + \mu \left[(n + tF_L)\sum_i (\alpha^i)^2 \ell_{n\alpha}^i - \sum_i \alpha^i(\ell^i + n\alpha^i \ell_{n\alpha}^i)\right] = 0
$$

or

$$
\sum_i \alpha^i \ell^i u_C^i + \mu \left[tF_L L_n - L\right] = 0
$$

This can be rewritten as

$$
\frac{1}{\mu} = \frac{1}{\tilde{u}_C}\left(1 - \frac{t}{1-t}\bar{\varepsilon}\right) \tag{26}
$$

where $\bar{\varepsilon}$ is the average labour supply elasticity ($= L_n n/L$) and $\tilde{u}_C$ is the following weighted average of u_C^i:

$$\tilde{u}_C = \frac{\sum_i \alpha^i \ell^i u_C^i}{\sum_i \alpha^i \ell^i} \tag{27}$$

This weighted average of marginal utilities differs from the unweighted average $\bar{u}_C$ defined in (15). Since $\alpha\ell$ is higher the higher is α (at least if consumption is a normal good), low values of the marginal utility of consumption are given more weight in $\tilde{u}_C$ than in $\bar{u}_C$. We therefore have $\tilde{u}_C < \bar{u}_C$.

Inserting (27) into (11) gives

$$F_E = \left[1 - \frac{t}{1-t}\bar{\varepsilon}\right] \frac{\bar{u}_C}{\tilde{u}_C} \sum_i \frac{u_c^i}{\bar{u}_c} \left(\frac{-u_E^i}{u_c^i}\right) - tF_L L_E \tag{28}$$

For the special case of a homogeneous population, $\bar{u}_C = \tilde{u}_C = u_C$, so that this equation may be rewritten as (13). For this case it also follows from from (13) and (17) that $t = 0$. This confirms the well-known result that distortionary taxes are not needed in an economy with a completely homogeneous population.

Appendix B: The Kaplow result

Assume that preferences are given by the separable utility function

$$u(C^i, \ell^i, E) = \tilde{u}(\phi(C^i, E), \ell^i) \tag{29}$$

Consider a change in E combined with a change in taxes so that for each person the utility level is unchanged for any potential choice of labour supply. (We shall return to the question of whether such a tax change is feasible.) Given the utility function (29), no one will change his or her choice of labour supply, since the effect of changing ℓ^i on $\phi(C^i, E)$ will be the same as before the change in E and taxes. For person i the change in consumption that compensates the change in E, denoted ΔE, is

$$\Delta C^i = \frac{-u_E^i}{u_C^i} \Delta E$$

so that the change in total consumption must be

$$\sum_i \Delta C^i = \sum_i \left(\frac{-u_E^i}{u_C^i} \right) \Delta E$$

Since labour supply is unchanged, the change in total output ($F(L, E)$), denoted ΔF, is

$$\Delta F = F_E \Delta E$$

The surplus (production minus consumption) from the combined change in E and taxes, which will be a surplus in the government budget since households consume all of their income, is thus

$$\Delta F - \sum_i \Delta C^i = \left[F_E - \sum_i \left(\frac{-u_E^i}{u_C^i} \right) \right] \Delta E \tag{30}$$

If the emission tax ($= F_E$) is not at the pigovian level, i.e. if the term in square brackets in (30) is not zero, the government will get a surplus by a suitable choice of ΔE. (ΔE will be positive (negative) if the initial tax is above (below) the Pigovian level.) This surplus can be used to make everyone better off by an appropriate change in tax rates, for instance by giving a uniform tax credit. Since a Pareto improvement is possible if the Pigou condition

$$F_E = \sum_i \left(\frac{-u_E^i}{u_C^i} \right) \tag{31}$$

does *not* hold, this condition must hold at any Pareto optimum.

The reasoning above assumed that it was possible to supplement a change in emissions with tax changes so that for each person the utility level was unchanged for any potential choice of labour supply. Gahvari (2006) has shown that if the only restriction on the tax function is the information constraint (i.e. that the government knows the distribution of productivities, but not each individual's productivity), then such a compensating tax change is possible. In this case we can therefore conclude that the optimal emission tax rate is the Pigovian rate (provided the assumptions about preferences given initially hold).

For a simple linear tax as given in section 3 it is generally not possible to adjust taxes in response to a change in E as described above. Such a tax adjustment is however possible for a particular specification of the utility function: In the utility function (29), let ϕ be a Cobb Douglas function in consumption and "environmental quality" $Q-E$,where Q is a positive parameter (representing environmental quality in the absence of any emissions):

$$\phi(C^i, E) = (C^i)^a (Q - E)^b \tag{32}$$

where a and b are positive parameters. With this specification

$$\frac{-u_E^i}{u_C^i} = \frac{-\phi_E^i}{\phi_C^i} = \frac{C^i}{Q - E}$$

i.e. the marginal willingness to pay for improved environmental quality is for any given E proportional to the person's consumption of private goods.

Inserting the households' budget equations into (32) gives

$$\phi(C^i, E) = \left[(1-t)F_L \alpha^i \ell^i + s\right]^a (Q - E)^b \tag{33}$$

The tax adjustment described in the beginning of this Appendix required that as E is changed, $\phi(C^i, E)$ must be left unchanged for all α^i and all ℓ^i. From (33) it is clear that a change in E must be accompanied by a change in the term in square brackets that is proportional to the initial value of this term. This can be achieved by adjusting s and t so s and $(1-t)F_L$ are changed in the same proportion, and with a change just sufficient to make $\phi(C^i, E)$ unchanged in spite of E being changed. After such a combined change in E and the tax rates t and s it follows from the reasoning above that if (31) does not hold it is possible to change E, t and s such that all utility levels remain unchanged while (6) will hold with a strict inequality. From this position it is possible to achieve a Pareto improvement by increasing s and adjusting t so $(1-t)F_L$ is left unchanged, see (2) and (3). For the preference function given by (29) and (32) we can thus conclude that the optimal emission tax is equal to the Pigovian level.

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