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A Discrete-Choice Model Approach to Optimal Congestion Charge

by

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April 10, 2008

Abstract:

We model the choice of transportation mode in a simplified Hotelling-like city, with a fixed number of total travellers, fixed road capacity and with no trade-off between when to travel and the time spent in a queue. A person that chooses to take her own car will inflict a congestion cost on all travellers. To get the travellers to internalise these external costs, a congestion charge has to be imposed. We derive an optimal congestion charge within in a discrete-choice framework, with a benevolent government maximising expected tax-adjusted social surplus.
The congestion charge to be imposed on private driving, beyond the opportunity cost – equal to the fare on public transportation – is shown to be a weighted average of a Ramsey-like term (capturing the goal to raise public revenue) and a Pigou-term capturing the environmental cost of a person's private driving. This property is similar to the optimal environmental tax derived by Sandmo (1975). However, the behavioural assumption underlying the present framework is quite different from the standard theory of consumer choice adopted by Sandmo.

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1. Introduction and main findings

Ever since Dupuit and Pigou, economists have been interested in pricing congested facilities, as the phenomenon is both socially important involving social gains if being implemented, while also providing a very nice example of a negative consumption externality. Sound economic principles are rarely implemented by the political system. However, congestion charges or road pricing, correcting for the negative externalities or time delays, are from time to time brought onto the political agenda. During the last few decades a number of cities have imposed such road charges or congestion tolls in order to reduce a costly over-utilization of restricted road or driving capacity during peak periods of the day. Despite very strong opposition ex ante, most hard-boiled opponents have admitted ex post that this measure has been a success, involving rather substantial economic benefits.

In this paper we characterise a second-best congestion toll within the context of a Hotelling-like linear city, with a fixed number of total travellers, living at the one extreme of the city while working at the other. Each traveller has a binary choice and inelastic demand – either taking her own car or taking public transportation. There are no substitution possibilities related to when to travel, because we do not model the off-peak periods. Choosing the private alternative (car) may give rise to congestion or queues, inflicting a time cost on any other traveller, including those taking public transportation (bus) as well. We employ a structural discrete choice model with random utility and extreme value distributed taste shifters. The model is simple but sufficiently rich so as to offer a set of interesting conclusions and testable hypothesis.

An optimal congestion charge is determined by a benevolent planner maximising expected tax-adjusted social surplus, within a partial equilibrium framework. The company offering transportation is public, and the revenue from charging drivers will be collected by the government, as well. Our model is formulated so as to allow us only to determine explicitly a congestion charge on a fixed road capacity, whereas the opportunity cost – the fare on public transportation – can be characterised by making use of the remaining equilibrium conditions of the model. The second-best optimal congestion charge imposed on private driving, beyond the opportunity cost equal to the fare on public transportation, is a weighted average of a Ramsey-like term (capturing the goal to raise revenue due to distortionary taxation elsewhere) and a Pigou-term capturing the congestion cost of a person's private driving. This property is similar to the optimal environmental tax derived by Sandmo (1975), but here we adopt a set-up based on different behavioural assumption rather than the one underlying standard theory of consumer choice.

The paper is organised as follows: In Section 2 we review some of relevant literature. In Section 3 the model, in particular the demand side, is outlined, whereas we in Section 4 derive the optimal congestion charge. Section 5 concludes.

2. A brief review of related literature

The issue of designing congestion charges (or more general road charges) has a long tradition in the economic literature. Even though the question has always been to correct for social costs or negative consumption externalities, the modelling approach differs substantially among the large group of contributors. Early contributors, in addition to Dupuit and Pigou, are Vickrey (1969) and Walters (1961), whereas Newbery (1988a, 1988b) has provided insight into various cost components, including the congestion cost, to be included in a general road charge.

Second-best optimal congestion tolls have been derived within a general equilibrium setting by Marchand (1968) and Sherman (1971), both influenced by a paper, published in French, by Levy-Lambert (1968), where each traveller can choose between different modes of transportation, as in the present paper, but their approach is based on standard theory of consumer choice.

Arnott, de Palma and Lindsay (1993) analyse a rich structural model of peak-period congestion, with special emphasis on the user's behavioural decision, but with only one mode of transportation. Their model is related to various pricing regimes with bottleneck congestion in the spirit of Vickrey (1969). Contrary to our approach, they offer a choice as to when to travel, whereas our approach considers only the congestion toll without introducing off-peak periods, and hence no trade-off between when to travel and the time stuck in a queue. Our approach to modelling the choice of transportation mode is quite different from those early papers. We adopt a discrete choice model; see Ben-Akiva and Lerman (1985). Each traveller has a choice between two modes of transportation. One of the modes (private use of cars) will inflict external congestion costs on all travellers, including those choosing public transportation as well. The cost due to congestion will be increasing in the number of cars on the road.

3. Transportation demand

For expository reasons we assume that the agents demanding transportation in an urban area have the same deterministic utility. A traveller can choose between driving her own car or use public transportation when going from home to work and back. Let U_{in} be the random utility of traveller n when choosing transportation mode or alternative i, where $i \in \{A, B\}$; A = automobile, B = bus or public transportation, whereas n denotes the individual $n = 1, 2, ..., N$, is supposed to be:

$$(1) \quad U_{An} = \alpha_A - \sigma P - g T_A + \sigma \varepsilon_{An}$$

$$(2) \quad U_{Bn} = \alpha_B - \sigma Q - g T_B + \sigma \varepsilon_{Bn}$$

Here α_i is a mode-specific constant, P is the cost of driving own car, while Q is the public transportation fare.[3] Travel time for alternative i is denoted T_i. Unobserved taste-shifters affecting utility are λ_{in}. These taste-shifters are assumed to be $i.i.d.$ extreme value distributed with standard deviation σ, and given by $\lambda_{in} = \sigma \varepsilon_{in}$, with ε_{in} being independently and identically extreme value distributed with zero expectation and unit variance. Dividing through with σ yields

$$(3) \quad \frac{U_{An}}{\sigma} = \frac{\alpha_A}{\sigma} - P - \frac{g}{\sigma} T_A + \varepsilon_{An} := a_A - P - \omega T_A + \varepsilon_{An}$$

$$(4) \quad \frac{U_{Bn}}{\sigma} = \frac{\alpha_B}{\sigma} - Q - \frac{g}{\sigma} T_B + \varepsilon_{Bn} := a_B - Q - \omega T_B + \varepsilon_{Bn}$$

[3] These prices are measured per unit distance, which for simplicity is assumed to be the same for all travellers in our linear city.

where $\omega := \frac{g}{b}$ shows the willingness to pay for one minute reduced travel time.

(Moreover $a_i := \frac{\alpha_i}{\sigma}$ for $i = A, B$.) Define also $U_i^* := \frac{U_{in}}{\sigma}$.

The cost of driving own car is $P = c_A + q$, where c_A is the private cost whereas q is a congestion charge or the toll. Q is the net price (net after marginal costs) of public transportation. For expository reasons we set the total number of daily travellers, N, equal to 1. We will assume that time spent on travelling is a given time t plus travel time due to congestions, which is being proportional to the expected number of travellers using own car. All travellers will be adversely affected by congestion. Expected travelling time is given by

(5) $T_A = t + h\varphi_A$

(6) $T_B = t + \beta\varphi_A$

where φ_A is the probability that car is the preferred mode of transportation. (The observed counterpart to this probability is the share of the population using own car when travelling.) By assuming $h \geq \beta$, we get that private transportation is more affected by congestion than public transportation.

The probability that agent n prefers to use her own car rather than taking the bus, φ_{An}, is given by

(7) $\varphi_{An} = \varphi_A = \Pr(U_{An}^* \geq U_{Bn}^*) = \frac{e^{v_A}}{e^{v_A} + e^{v_B}}$

where

(8) $v_A = a_A - P - \omega T_A$

(9) $v_B = a_B - Q - \omega T_B$

and v_i is the expected utility from choosing alternative i. The structure of the probability in (7) follows from the assumption that the taste shifters are i.i.d. extreme value across alternatives and individuals. The probability of using public transport is given by

(10) $\varphi_B = 1 - \varphi_A$

The model implies that traveller n's probability of travelling mode depends on the aggregate probability of travelling. Because we have assumed (for expository reasons only) that agents have the same deterministic utility function, all probabilities will be equal. The model suffers from the assumption of i.i.d., which can be avoided, for example, by assuming that the parameter ω is random across individuals. This will be important when taking the model to data, but here we will stick to the simplifying assumption that ω is the same for all travellers.

Define $r := h - \beta \geq 0$. Then we get the following derivatives and price elasticities of the demand probability φ_A, with respect to the prices q and Q, denoted e_q and e_Q, respectively:

(11) $$\frac{\partial \varphi_A}{\partial q} = \frac{-\varphi_A \varphi_B}{1 + \omega r \varphi_A \varphi_B} < 0$$

(12) $$\frac{\partial \varphi_A}{\partial Q} = \frac{\varphi_A \varphi_B}{1 + \omega r \varphi_A \varphi_B} > 0$$

(13) $$e_q := \frac{q}{\varphi_A} \frac{\partial \varphi_A}{\partial q} = -\frac{q \varphi_B}{1 + \omega r \varphi_A \varphi_B} < 0$$

(14) $$e_Q := \frac{Q}{\varphi_A} \frac{\partial \varphi_A}{\partial Q} = -\frac{Q \varphi_B}{1 + \omega r \varphi_A \varphi_B} > 0$$

We note from the structure of the model that $e_Q = -\frac{Q}{q} \cdot e_q > 0$. A higher toll rate will, as we should expect, reduce the use of cars and a higher fare rate on public transportation will increase the use of cars. The simplified demand structure is only for expository reason. Again, when the model is taken to data, one has to allow for a more flexible pattern of response to price changes.

4. Optimal congestion charge

To derive the optimal toll rate or congestion charge, we let the benevolent government maximise expected tax-adjusted social surplus, given by the sum of expected consumer surplus, expressed as $V(q,Q)$, and producer surplus, $\pi(q,Q)$, from both public transportation and tax revenues, with respect to the toll rate q, given the price of public transport Q. The objective function, with m as a positive (exogenous) marginal cost of public funds, is then:

$$(15) \quad W = V(q,Q) + (1+m)\cdot\pi(q,Q)$$

$$(16) \quad V(q,Q) = E[\max_{i=(A,B)} U_i^*] = \ln(e^{v_A} + e^{v_B})$$

$$(17) \quad \pi(q,Q) = q\varphi_A + Q\varphi_B - F$$

Prices (q and Q) are net of variable marginal costs, while F is a fixed cost related to toll stations and the public transport system. From (7) – (17) we then obtain

$$(18) \quad q = Q + \frac{m}{1+m}\frac{\varphi_A}{(-\partial\varphi_A/\partial q)} + \frac{\theta}{1+m}$$

or

$$(19) \quad q = Q + \frac{m}{1+m}\frac{q}{(-e_q)} + \frac{\theta}{1+m}$$

or

$$(20) \quad \frac{q-Q}{q} = \frac{m}{1+m}\cdot\frac{1}{(-e_q)} + \frac{1}{1+m}\cdot\frac{\theta}{q}$$

where

$$(21) \quad \theta := \omega\cdot(h\varphi_A + \beta\varphi_B) > 0$$

We can interpret θ as expected congestion cost. The first term on the right hand side of either (18) or (19) is the opportunity cost of car, which is here the public transportation fare. The second term is a Ramsey pricing term, which is due to the second best nature of the problem (distortionary taxes). The third term is one related to the marginal externality cost due to congestion. This last term may be denoted a Pigovian term.[4] In (20) we have the associated Lerner-index, showing how the

[4] See Sandmo (op.cit.).

optimal tax rate or the congestion charge rate should be stipulated. The optimal marginal cost of driving one's own car, $q-Q$, is a weighted average of the Ramsey-term and the Pigovian term, with weights $\frac{m}{1+m}$ and $\frac{1}{1+m}$.

We observe that

$$(22) \quad \lim_{m\to\infty}\frac{q-Q}{q}=\frac{1}{(-e_q)}$$

Thus when the marginal cost of public funds becomes extremely high, due to a costly tax system, the toll company should behave as a profit maximising monopolist, setting the toll rate so as to maximise public revenue from taxing private car use. The government will care only about tax revenue, in addition to the ticket revenues from public transportation. (Note that this rule corresponds to the "inverse-elasticity-rule" – the less elastic is demand, the higher can the charge be set.)

On the other hand, when the marginal cost of public funds becomes very low – in the extreme case the government can use lump-sum taxes, then

$$(23) \quad \lim_{m\to 0}\frac{q-Q}{q}=\frac{\theta}{q} \text{ or } \lim_{m\to 0} q = Q+\theta$$

In this case the toll rate q should be equal to the sum of the opportunity cost, Q, and the expected marginal cost of congestion, as given by the expected marginal congestion cost θ, evaluated for the social optimum. The amount paid by taxpayers depends on expected profits, evaluated at optimal prices. Whether one should have public transportation and toll system depends on whether ω, evaluated at optimal prices, will exceed zero. As an illustration of what empirics may yield, let $m=0,2$, $e_q=-2$ and $\theta=$ 5 € (per unit of trip per unit distance). Then we can calculate the optimal toll in € to be $q=1{,}091\cdot Q+4{,}55$.

Note that to find a closed form solution for the optimal toll rate, say from (19), one has to take into account that e_q and θ both depend on the congestion charge q. Because we have $\varphi_A + \varphi_B = 1$, we can also solve for the price, Q, of public transportation that is consistent with the optimal tolling. From (19) we observe that the optimal congestion charge or toll rate will always exceed the price of public transportation.

To allow for the willingness to pay for one minute shorter travel to vary among travellers is straightforward. Let ω_n be the willingness that individual n is willing to pay for one minute shorter travel. The expected number of travellers using own car, E[N_A] equals $\sum_n \varphi_{An}$. Then, $\frac{\partial E[N_A]}{\partial q} = \sum_n \frac{\partial \varphi_{An}}{\partial q}$. Let E_q denote the elasticity of E[N_A] with respect to the price q. Thus, $E_q = \frac{q}{E[N_A]} \frac{\partial E[N_A]}{\partial q} < 0$. Moreover the expected marginal congestion cost, Ω, is now given by

$$(24) \quad \Omega = E[N_A] \sum_n \omega_n h \frac{\partial \varphi_{An} / \partial q}{\partial E[N_A] / \partial q} + E[N_B] \sum_n \omega_n \beta \frac{\partial \varphi_{An} / \partial q}{\partial E[N_A] / \partial q} > 0$$

The equivalent to (19) is then

$$(25) \quad q = Q + \frac{m}{1+m} \frac{q}{(-E_q)} + \frac{\Omega}{1+m}$$

5. Conclusions

We have modelled the choice of transportation mode in a very simplified Hotelling-like city, with a fixed number of travellers, fixed road capacity and with no trade-off between when to travel and the time spent in a queue. A person that chooses to take her own car will inflict a congestion cost on all travellers. To get the travellers to internalise these external costs, a congestion charge has to be imposed.

We have derived an optimal congestion charge in a discrete-choice framework, when a benevolent government maximises expected tax-adjusted social surplus, in an imperfect environment with distortionary taxation.

The congestion charge to be imposed on private driving, beyond the opportunity cost – equal to the fare on public transportation – is shown to be a weighted average of a Ramsey-like term (capturing the goal to raise public revenue due to distortionary taxation elsewhere in the economy) and a Pigou-term capturing the environmental cost of a person's private driving. This property is similar to the optimal environmental tax derived by Sandmo (1975). We get a similar result as Sandmo did, but our set-up is based on a different behavioural assumption than standard theory of consumer choice.

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