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Rotten Parents and Disciplined Children: A Politico-Economic Theory of Public Expenditure and Debt

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Rotten Parents and Disciplined Children: A Politico-Economic Theory of Public Expenditure and Debt*

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Abstract

This paper proposes a dynamic politico-economic theory of fiscal policy in a world comprising a set of small open economies, whose driving force is the intergenerational conflict over debt, taxes, and public goods. Subsequent generations of voters choose fiscal policy through repeated elections. The presence of young voters induces fiscal discipline, i.e., low taxes and low debt accumulation. The paper characterizes the Markov-perfect equilibrium of the voting game in each economy, as well as the stationary equilibrium debt distribution and interest rate of the world economy. The equilibrium can reproduce some salient features of fiscal policy in modern economies.

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There are large differences in fiscal policies and government debt across countries and over time. In spite of this, there is still a limited theoretical understanding of the politico-economic determinants of debt dynamics. Debt breaks the link between taxation and expenditure, allowing governments to shift the fiscal burden to future generations. In a non-Ricardian world, this raises an intergenerational conflict. Since only current generations vote, there is a politico-economic force for debt accumulation. What, then, prevents the current generations from passing the entire bill for current spending to future generations?

To address this question, we construct a dynamic general equilibrium model of small open economies where voters each period choose domestic public good provision and its financing through debt and taxes. Debt and capital are traded on worldwide markets. We abstract from sovereign debt issues by assuming that governments are committed to debt repayment. Within each country, old agents support high spending on public goods, high labor taxes, and large debt. The young dislike debt, since it crowds out public good provision within their lifetime.

We characterize the equilibrium in a world where countries care to a different extent for public goods relative to private consumption. Such heterogeneity can reflect either preferences or differences in the quality of public good provision. A strong preference for public goods strengthens the fiscal discipline and keeps the government debt low. This can explain why economies with large governments such as the Scandinavian countries run tighter fiscal policies than countries such as Greece and Italy which have large debt and, arguably, provide public goods less efficiently. The theory predicts a stark divergence when governments can use lump-sum taxes: All countries, except those which are most concerned with public goods, accumulate large debts and fall into immiseration in the long run, in the sense that they provide no public goods and use all tax revenue to service their foreign debt. Such a dichotomy is averted when taxation is distortionary. Then, the level of debt still depends on the country's preference for public goods, but the country need not fall into public poverty. A calibrated version of the model delivers an empirically plausible cross-country distribution of debt and fiscal policy.

The theory is also consistent with a number of empirical observations. First, the response of debt to fiscal shocks is persistent, but mean reverting, in both the theory and the data. Second, the steady-state debt-GDP ratio is positively correlated across countries with (inverse) measures of government efficiency proxying for the quality of public good provision. Finally, the theory predicts a response to a demographic transition consisting of a "baby boom" followed by a "baby bust," which resembles the post-war pattern for debt in many OECD countries.

We contrast the results with an environment in which fiscal policy is delegated to a Ramsey planner who attaches independent decaying weight to all future generations, as in Farhi and

Werning (2007). We emphasize cases in which the planner would plunge the economy into public poverty, while a sequence of selfish short-lived agents would not do so. The outcome hinges on the lack of commitment in the political process. If an elected government could commit fiscal policy over two periods, no disciplined fiscal policy could be sustained. Thus, the lack of commitment may *benefit* future generations more than would a paternalistic planner.

Our paper contributes to the politico-economic literature of government debt. Existing papers have analyzed the determinants of debt policy in closed economies. These include Cukierman and Meltzer (1989), Persson and Svensson (1989), Alesina and Tabellini (1990), and, more recently, Battaglini and Coate (2008), Azzimonti Renzo (2009) and Yared (2010).[1] Our paper is also related to the recent literature studying the intergenerational conflict on taxes and transfers abstracting from explicit debt (Bassetto 2008, Gonzalez-Eiras and Niepelt 2008, Lancia and Russo 2011 and Mateos-Planas 2010). These papers impose a government balanced-budget constraint, whereas our focus is on debt dynamics and on the forces that might induce public poverty. Methodologically, our paper is related to Klein *et al.* (2008), who also characterize the Markov-perfect equilibrium (MPE) of a dynamic game in terms of a generalized Euler equation (GEE) for government expenditures. However, they focus on a balanced budget and have no intergenerational conflict. None of the existing papers characterize the general equilibrium of a world comprising integrated small open economies with independent political processes.

1 Environment and Political Equilibrium

The model economy consists of a set of small open economies of a total unit measure populated by overlapping generations of two-period lived agents who work in the first period and live off savings in the second period. The total population is constant. Agents consume two goods: a private good (c) and a domestic public good (g) provided by each economy's government.

We assume additively separable logarithmic preferences over private and public goods.[2] The utility of a young agent in country $j \in [0,1]$, born in period t, can be written as $U_{Y,j,t} = \log(c_{Y,j,t}) + \theta_j \log(g_{j,t}) + \beta(\log(c_{O,j,t+1}) + \lambda\theta_j \log(g_{j,t+1}))$, where β is the discount factor, and θ_j and $\lambda\theta_j$ capture the preference weight on public goods for young and old, respectively. Cross-country differences in θ's may reflect cultural diversity or differences in the efficiency and quality of public good provision, related to the technology and organization of the government sector. We let θ_j be drawn from a finite-valued set, $\theta_j \in \{\underline{\theta}, \theta^1, \theta^2, .., \theta^M, \bar{\theta}\} \equiv \Theta$, and denote

[1] Battaglini and Coate (2008) analyze a model where legislators can extract pork barrel transfers. They focus on a different political mechanism (legislative bargaining) and abstract from intergenerational conflict.

[2] Log utility is for tractability. In Appendix B, available online, we generalize the analysis to CRRA utility.

by $\upsilon > 0$ the measure of countries with $\theta_j = \bar{\theta}$. Hereafter, we omit t indexes and use recursive notation with x' denoting next-period x.

The private good is produced with capital and labor as inputs in the production function $Y_j = QK_j^{\alpha} H_j^{1-\alpha}$. Capital is perfectly mobile and depreciates fully after one period. We denote by R the (endogenous) world interest rate and by w_j the workers' pre-tax wage. In a competitive equilibrium, $K_j = K(R, H_j) = (\alpha Q/R)^{1/(1-\alpha)} H_j$, and $w_j = w(R) = (1-\alpha) Q^{1/(1-\alpha)} (\alpha/R)^{\alpha/(1-\alpha)}$. Since the focus of our analysis will be on stationary equilibria, we characterize the allocations of individual countries as functions of a constant R.

Domestic fiscal policy is determined through repeated elections. Government debt is traded at worldwide asset markets. Given an inherited debt b, the elected government chooses the labor tax rate ($\tau \leq 1$), public good expenditure ($g \geq 0$), and the debt accumulation (b'), subject to a dynamic budget constraint:[3]

$$b'_j = g_j + Rb_j - \tau_j w(R) H(\tau_j), \tag{1}$$

where aggregate labor supply $H(\tau)$ captures that τ may distort labor supply. Governments are committed to not repudiating the debt. Then, sovereign debt cannot exceed the present value of the maximum feasible tax revenue (the natural debt limit). In an environment with a constant interest rate, the constraint is $b_j \leq \bar{\tau} w(R) H(\bar{\tau}) / (R-1) \equiv \bar{b}(R)$, where $\bar{\tau} \equiv \arg\max_{\tau} \tau \cdot H(\tau)$ denotes the top of the Laffer curve.

Logarithmic utility implies that $c_Y = C_Y(\tau, R) \equiv (1+\beta)^{-1} A(\tau; R)$ and $c_O = (1+\beta)^{-1} \beta R A(\tau; R)$, where $A(\tau; R)$ is the present value of after-tax lifetime income. Thus, ignoring irrelevant constants, and denoting by τ_{-1} the tax rate in the previous period, we can express the indirect utility of young and old voters, respectively, as

$$U_Y(\tau_j, g_j, g'_j; \theta_j, R) = (1+\beta) \log(A(\tau_j; R)) + \theta_j \log(g_j) + \beta\lambda\theta_j \log(g'_j), \tag{2}$$

$$U_O(g_j; \theta_j, R) = \log(A(\tau_{-1,j}; R)) + \lambda\theta_j \log(g_j). \tag{3}$$

We model the political mechanism as a probabilistic voting model à la Persson and Tabellini (2000, pp. 54-58). An explicit microfoundation is provided in Appendix B. In this model, the equilibrium fiscal policy maximizes a weighted sum of young and old voters' utilities. The weights capture the relative political clout of each group, reflecting on the one hand its relative size and on the other hand exogenous group-specific characteristics, such as the voting turnout or the relative salience of the fiscal policy for that group relative to other issues. Formally,

[3] We abstract from capital income taxation. Note that capital mobility would curtail the government's ability to tax assets. For instance, if capital tax were source-based and assets could move after the tax announcement, the tax rate in the political equilibrium outlined below would be zero. Details are available upon request.

the *political objective function* is given by $U\left(\tau_j, g_j, g_j'; \theta_j, R\right) = (1-\omega)\, U_Y\left(\tau_j, g_j, g_j'; \theta_j, R\right) + \omega U_O\left(g_j; \theta_j, R\right)$, where ω is the relative weight on old agents.[4]

Political Equilibrium. The world equilibrium is a set of (country-specific) policy functions and laws of motion for government debt, private wealth, and the world capital stock. In each country, fiscal policy is determined by the dynamic games between successive generations of voters. The world interest rate is pinned down by an international asset market clearing condition. We restrict attention to Markov-perfect equilibria (MPE) where voters condition their strategies only on payoff-relevant state variables. Since private wealth does not affect the political preference of old voters, b_j is the only domestic payoff-relevant state variable. The policy of an individual country does not affect the interest rate, so voters take the equilibrium interest rate sequence as given.

In general, the distributions of debt and wealth across countries would be state variables, of which the policy rules and the world interest rate are functions. Following the literature initiated by Aiyagari (1994), we focus on stationary equilibria featuring a constant interest rate.[5] Consistent with this approach, when we consider fiscal policy transitions for individual countries, we maintain that a unit measure of them is in a steady state. In a companion paper (Song et al. 2011), we provide a definition of MPE in non-stationary environments and characterize non-stationary equilibria in the case of inelastic labor supply.

Definition 1 *A stationary Markov-perfect political equilibrium (SMPPE) is an interest rate R, a stationary debt distribution $\{b_j\}_{j\in[0,1]}$, and a 3-tuple $\langle B, G, T\rangle$, where $B : \left[\underline{b}, \bar{b}\right] \times \Theta \times \mathbb{R}^+ \to \left[\underline{b}, \bar{b}\right]$ is a debt rule, $b' = B\left(b; \theta, R\right)$, $G : \left[\underline{b}, \bar{b}\right] \times \Theta \times \mathbb{R}^+ \to [0, \bar{g}]$ is a government expenditure rule, $g = G\left(b; \theta, R\right)$, and $T : \left[\underline{b}, \bar{b}\right] \times \Theta \times \mathbb{R}^+ \to [0,1]$ is a tax rule, $\tau = T\left(b; \theta, R\right)$, such that:*[6]

1. *$\langle B\left(b; \theta, R\right), G\left(b; \theta, R\right), T\left(b; \theta, R\right)\rangle = \arg\max_{\left\{b' \in \left[\underline{b}, \bar{b}\right], g \geq 0, \tau \leq 1\right\}} U\left(\tau, g, g'; \theta, R\right)$, subject to (1), where $g' = G\left(b'; \theta, R\right)$, and the government's budget constraint is satisfied:*

$$B\left(b; \theta, R\right) = G\left(b; \theta, R\right) + Rb - T\left(b; \theta, R\right) w\left(R\right) H\left(T\left(b; \theta, R\right)\right). \tag{4}$$

2. *The world asset market clears:*

$$\int_{j\in[0,1]} \frac{\beta}{1+\beta} A\left(T\left(b_j; \theta_j, R\right); R\right) = \int_{j\in[0,1]} b_j' + \int_{j\in[0,1]} K_j', \tag{5}$$

[4] Several papers use probabilistic voting models in dynamic games. Hassler *et al.* (2005) and Gonzalez-Eiras and Niepelt (2008) focus, as we do, on Markov-perfect equilibria. Sleet and Yeltekin (2008) and Farhi et al. (forthcoming) analyze environments with public insurance/taxation and private information.

[5] Aiyagari (1994) analyzes individual consumption-saving decisions in an economy with a stationary distribution of households and a constant (endogenous) interest rate. Here, we model a continuum of countries issuing debt in an integrated world market, where aggregation generally fails.

[6] For standard technical reasons, we impose a lower bound on debt, $\underline{b}$. Such lower bound must be chosen sufficiently low so as not to be binding in equilibrium.

where $b'_j = B\left(b_j; \theta_j, R\right)$ *and* $K'_j = K\left(R, H\left(T\left(b'_j; \theta_j, R\right)\right)\right)$.

3. *The debt distribution is stationary and consistent with the policy rule, i.e., a unit measure of economics keeps debt constant:* $b_j = B\left(b_j; \theta_j, R\right)$ *for almost all* $j \in [0,1]$.

We impose a natural stability condition, requiring that, given R, a perturbation of the steady-state debt level of an individual country does not trigger diverging debt dynamics. For instance, if an exogenous shock increases a country's debt, debt tends to revert to its steady-state level, or at least does not move further away.

Definition 2 *A SMPPE is said to be "stable" (SSMPPE) if, for all* $\theta_j \in \Theta$, *the fixed point of the difference equation* $b_{j,t+1} = B\left(b_{j,t}; \theta_j, R\right)$ *is Ljapunov-stable, where* R *is the equilibrium interest rate.*

Inelastic Labor Supply. In this section, we provide an analytical characterization of equilibrium under the assumption that agents' labor supply is inelastic. In particular, we set $H(\tau) = 1$ and $A(\tau; R) = (1-\tau)\, w(R)$, implying that $\bar{\tau} = 1$. Given R, each country's MPE (part 1 of Definition 1) is characterized by a system of two functional equations:

$$\frac{(1-\omega)(1+\beta)}{w(R)\,(1-\tau_j)} = (1+\omega(\lambda-1))\,\frac{\theta}{g_j}, \tag{6}$$

$$\frac{g'_j}{g_j} = \underbrace{-\frac{(1-\omega)\lambda\beta}{1+\omega(\lambda-1)}\,\frac{\partial G\left(b'_j; \theta_j, R\right)}{\partial b'_j}}_{\text{Disciplining effect}}, \tag{7}$$

where $g_j = G\left(b_j; \theta_j, R\right), g'_j = G\left(b'_j; \theta_j, R\right)$, $\tau_j = T\left(b_j; \theta_j, R\right)$, and $b'_j = g_j + Rb_j - \tau_j \equiv B\left(b_j; \theta_j, R\right)$. Equation (6) yields the trade-off between the marginal cost of taxation, due to the reduction in private consumption suffered by the young, and the marginal benefit of public good provision. Such a trade-off reveals a conflict of interest between young and old voters. The old want higher taxes and current spending on public goods. Thus, the more power held by the old (i.e., higher ω) the greater the reduction in c/g. The preference for public good provision affects this trade-off: a higher θ or a higher λ reduces c/g. Equation (7) is a generalized Euler equation (GEE) for public good consumption. Its right-hand side (and in particular the derivative $\partial G/\partial b'$) captures the *disciplining effect* exercised by the young voters who anticipate that increasing debt will prompt a fiscal adjustment reducing their future public good consumption. Such an effect hinges on the old's taste for public good. If $\lambda = 0$, then all voters would choose $b'_j = \bar{b}$. Thus, the young's concern for *future* public good provision is key to sustaining a tight fiscal policy, given the lack of intergenerational altruism.

We guess and verify that the equilibrium policy functions are linear (see the proof of Proposition 1). Substituting the guesses into (4)-(6)-(7) and solving for the undetermined coefficients yields:

$$g_j = G(b_j;\theta_j,R) = R\gamma(\theta_j)\left(\bar{b}-b_j\right),\ b'_j = B(b_j;\theta_j,R) = \bar{b} - \frac{(1-\omega)\lambda\beta R}{1+\omega(\lambda-1)}\gamma(\theta_j)\left(\bar{b}-b_j\right) \quad (8)$$

$$\tau_j = T(b_j;\theta_j,R) = 1 - \frac{(1-\omega)(1+\beta)}{(1+\omega(\lambda-1))\theta_j}\frac{R}{w(R)}\gamma(\theta_j)\left(\bar{b}-b_j\right),$$

where $\gamma(\theta) \equiv (1+(1-\omega)((1+\beta)+\theta\beta\lambda)/(\theta(1+\omega(\lambda-1))))^{-1} > 0$ and $\gamma'(\theta) > 0$. Note that $B(b;\theta,R)$ is decreasing in θ, i.e., a larger weight on public goods reduces debt accumulation. Moreover, $\partial G/\partial b' = -R\gamma(\theta) < 0$, so public good provision is falling in b.

Next, we turn to the determination of the world interest rate and the associated debt distribution (parts 2-3 of Definition 1). To this end, rewrite the law of motion of debt in (8) as $\left(\bar{b}-b'_j\right) = (R/R^*(\theta_j))\left(\bar{b}-b_j\right)$, where $R^*(\theta) \equiv (1+\omega(\lambda-1))/(\gamma(\theta)\cdot\beta(1-\omega)\lambda) > 1$, $R^{*\prime}(\theta) < 0$. Note that (i) there can be no $\theta_j \in \Theta$ such that $R > R^*(\theta_j)$. Otherwise, a positive measure of countries would accumulate an ever-increasing surplus, whereas the rest of the world can at most run a debt equal to $\bar{b}$, thereby preventing world asset-market clearing.[7] (ii) It is impossible that $R^*\left(\bar{\theta}\right) > R$. Otherwise, all countries would converge to the debt limit, and agents would hold no private wealth since their first-period income is fully taxed away. Again, the world asset market would not clear. Given (i) and (ii), the SSMPPE must feature $R = R^*\left(\bar{\theta}\right)$, namely, the market clearing interest rate is determined by the countries with the strongest preference for the public good. All other countries have $R < R^*(\theta_j)$ and converge to public poverty. Eq. (5) then pins down the average debt level for countries with $\theta = \bar{\theta}$.[8]

Proposition 1 *A SSMPPE is characterized by the set of policy functions (8) and by the following steady-state equilibrium conditions:*
(i) $R = R^*\left(\bar{\theta}\right) \equiv \left(\frac{(1-\omega)\lambda}{1+\omega(\lambda-1)}\beta\gamma\left(\bar{\theta}\right)\right)^{-1} > 1$
(ii) $\int_{j|_{\theta_j=\bar{\theta}}} b_j = \upsilon\cdot b\left(\bar{\theta}\right) \equiv \upsilon\cdot\bar{b} - \frac{\bar{\theta}\lambda}{1+\bar{\theta}\lambda}\left(\bar{b}+K(R,1)\right) < \upsilon\cdot\bar{b}$, $\int_{j|_{\theta_j=\bar{\theta}}} g_j = \upsilon\cdot G\left(b\left(\bar{\theta}\right);\bar{\theta},R\right) > 0$,
$\int_{j|_{\theta_j=\bar{\theta}}} \tau_j = \upsilon\cdot T\left(b\left(\bar{\theta}\right);\bar{\theta},R\right) < \upsilon$, *where* $R = R^*\left(\bar{\theta}\right)$.
(iii) $b_j = \bar{b}$, $g_j = 0$, *and* $\tau_j = 1$, *for almost all* $j|_{\theta_j<\bar{\theta}}$.

The SSMPPE has stark properties: Even small cross-country differences in θ's lead to divergence: in all countries except those with the highest θ, private and public consumption are

[7] A SMPPE such that all countries have $b = \bar{b}$ exists, but violates our stability criterion.

[8] In Song et al. (2011), we provide a full characterization of the MPE in a non-stationary environment where the debt distribution, the capital stock, and the interest rate are time varying. We show that for any initial distribution of debt and capital, such MPE converges to the stationary equilibrium of Proposition 1.

crowded out by debt repayment to foreign lenders. The fiscally disciplined (high-θ) countries hold the entire world wealth and are the only ones that can provide public goods.

This proposition is fundamentally different from the well-known result that in an economy where agents (or countries) have different discount factors, the most patient agents end up owning all assets in the economy. Indeed, our agents have finite lives and do not save beyond their old age. Rather, the result hinges on the lack of commitment inherent under repeated voting. Suppose, for instance, that voters at time t could commit to fiscal policy in period t and $t+1$. Then, irrespective of θ, the young and the old would agree to set $b_{t+2} = \bar{b}$, inducing public poverty ever after. It is therefore the dynamic game that empowers the future generations and averts immiseration in the high-θ economies. We return to this point in Section 4.

Elastic Labor Supply. We introduce elastic labor supply by assuming that young agents share their time endowment between market (h) and household production ($1-h$). Market earnings are subject to a linear tax rate, $\tau \in [0,1]$. Old agents can only do household production.[9] The household production technology is given by $y_H = F(h)$, where $F' < 0$, $F'' \leq 0$, $F''' \leq 0$, and $F(1) = 0$. Since household production cannot be taxed, taxation distorts labor allocation. Now, $A(\tau; R) = \max_{h\in[0,1]} \{(1-\tau)\, w(R)\, h + F(h) + F(0)/R\}$, and $H(\tau) = -(F')^{-1}((1-\tau)\, w(R))$, where $H_\tau \leq 0$, $H_{\tau\tau} \leq 0$ and $A_\tau = -w(R)\, H(\tau)$, by the envelope theorem. Let $e(\tau) \equiv -(dH/d\tau)(\tau/H(\tau))$ denote the tax elasticity of labor supply. The properties of F ensure that $e_\tau \geq 0$. Hence, $e(\bar{\tau}) = 1$, and $e(\tau) < 1$ for all $\tau < \bar{\tau}$. The functional equation (6) becomes

$$\frac{(1-\omega)(1+\beta)}{A(\tau_j; R)} = (1 - e(\tau_j))(1 + \omega(\lambda - 1))\frac{\theta_j}{g_j}. \tag{9}$$

Equation (9) encompasses the case of lump-sum taxes, (6), as a particular case, where $A(\tau; R) = 1-\tau$ and $e(\tau) = 0$. The key difference is that, while under lump-sum taxes the equilibrium c/g ratio was constant, it grows with taxes when labor is elastic (since $e_\tau > 0$). Intuitively, the convex tax distortion makes it more expensive to finance g when b, and, hence, interest payments, is larger. This has interesting implications for the GEE, (7). Under inelastic labor supply, $\partial G/\partial b'$ is constant, hence, the disciplining effect is independent of b. In contrast, passing on the bill becomes increasingly hard when taxes are distortionary and distortions are convex.[10] As debt accumulates, future taxes raise less and less revenue, inducing future governments to make more than proportional cuts in g. The young perceive fiscal adjustments

[9] The qualitative results are unchanged if one assumes that the agents receive no labor income in the second period. The second-period income facilitates the quantitative exercise, since in the real world there are transfers to the old from which we abstract for simplicity, and which affects the personal saving behavior.

[10] This means that the slope $\partial G/\partial b'$ is not constant along the transition. In steady state, $\partial G/\partial b' = -(1+\omega(\lambda-1))/((1-\omega)\lambda\beta) < 0$, i.e., increasing b' reduces next-period public good provision.

as increasingly painful and therefore demand more fiscal discipline as b increases. As we will see below, this growing fiscal discipline can halt debt accumulation and sustain a steady state with an interior debt level even for economies with $\theta < \bar{\theta}$.

A full analytical characterization of the SSMPPE under elastic labor supply is not available; therefore, we must resort to numerical analysis. We solve the model numerically by two different methods. First, we use a standard projection method with Chebyshev collocation to approximate T and G, exploiting the equilibrium conditions (7) and (9). Second, we use the algorithm of Krusell et al. (2002) – see Appendix B. The results are essentially identical.

2 Quantitative Analysis

This section illustrates the properties of the model with elastic labor supply and shows that a reasonably calibrated version of the model is consistent with key features of OECD economies. We then use the calibrated economy to run some quantitative experiments. As above, we analyze a stationary equilibrium where the world interest rate is constant. The length of a period is 30 years. We assume a capital share of output of $\alpha = 1/3$ and an annualized capital-output ratio of 3. Firms' optimization then implies an annualized interest rate of 4%, which is standard in quantitative macro (cf. Trabandt and Uhlig, 2010). We normalize w to unity and parameterize the household production technology by the production function $F(h) = \xi/(1+\xi) \cdot X \cdot \left(1 - h^{1+1/\xi}\right)$, where $\xi > 0$ is the Frisch elasticity of labor supply. Since $\bar{\tau} = 1/(1+\xi)$, we set $\xi = 2/3$ so that the top of the Laffer curve is at $\bar{\tau} = 60\%$, in line with Trabandt and Uhlig (2010). Moreover, we set X to target a ratio of market labor earnings to total income of the young (including the value of home production) of 33/51, which is the ratio of market hours worked to total hours for US working-age households (Aguiar and Hurst, 2007). We set $\omega = 0.25$ to reflect the political influence of the old, measured by voters' turnout.[11] The parameters θ and λ determine fiscal policy. We let Θ capture the empirical debt distribution. To keep the analysis simple, we focus on two types of countries, half of which have a high $\bar{\theta}$ and half of which have a low $\underline{\theta}$.[12] We set $\lambda = 2.2$ to match the average OECD labor taxes in steady state.[13] Note that this calibration implies that the old care more for

[11] In the real world, there are fewer retirees than workers, but their turnout rate is higher. We try to resolve this tension in the two-period model by setting ω equal to the share of aggregate votes cast by voters 61 years and older in the 2004 US election. Below we explore the effect of increasing ω to 0.5.

[12] To focus on small open economies, we exclude the US, Japan, and Germany and order the remaining OECD countries according to their debt-GDP ratio. The average during 2002-2008 was 56%, and the 50% countries with the largest (smallest) debt had an average debt-to-output ratio of 75% (36%). Since one period in the model corresponds to 30 years, we set $\bar{\theta}$ and $\underline{\theta}$ to target steady-state debt-output ratios of $75\%/30 = 2.5\%$ and 1.2% in the high- and low-debt economies, respectively.

[13] During 2002-2008, labor taxes accounted for 27.1% of GDP in small OECD countries. This includes all unambiguous taxes on labor income, plus taxes on goods and services, plus 2/3 of taxes on individual income

public goods than the young, which we view as a reasonable feature (e.g., parks, safety, etc.). Finally, aggregate capital and government debt determine the world wealth, and β is set so that the world market for savings clears. The average annual debt-output ratio in small OECD countries is 0.56, which implies a world wealth-output ratio of $(3 + 0.56)/30$ and, hence, an annualized $\beta = 0.973$. Table 1 summarizes the parameters.

Figure 1 plots the SSMPPE equilibrium policy functions B, G, and T for the case of inelastic labor supply of Proposition 1 (panels a-b-c) and for calibrated economies (panels a'-b'-c'). The two curves in each panel represent high- and low-θ economies, respectively. Consider, first, debt. When labor is inelastic (panel a), B coincides with the 45-degree line for the $\bar{\theta}$ economies, so every b is a steady state. For the low-θ economies, the slope of B is smaller than unity, and the dynamics converge to $\bar{b}$. In the calibrated equilibrium with elastic labor supply (panel a'), both high- and low-θ economies have a strictly convex $B(b)$ function, crossing the 45-degree line twice: at an interior steady-state level and at the natural debt limit. The intuition for such convexity is that the disciplining effect strengthens as b grows, implying less than proportional increases in b' as b gets larger. Second, both high- and low-θ economies converge to the interior steady state, as long as $b_0 < \bar{b}$. Thus, differences in θ drive differences in steady-state debt levels, but there is no immiseration for even small positive levels of θ.

Panels b-b' and c-c' plot the equilibrium functions G and T, respectively, for the case of inelastic and elastic labor supply. Under inelastic labor supply (panels b and c), G and T are, respectively, decreasing and increasing linear functions of b, and the c/g ratio remains constant along the transition. Under elastic labor supply, both T and G are concave in b (panel b'-c'). The value of $|\partial G/\partial b'|$ and, hence, the disciplining effect, is increasing in b, which is the reason why each economy has a unique and stable interior steady state. For instance, if an economy starts with b above steady state, the public to private consumption will increase over time as b falls. The crux is the convex nature of the tax distortion that makes it more costly to finance g as an economy comes closer to the Laffer curve.

Figure 2 shows that the SSMPPE converges smoothly to the equilibrium with inelastic labor supply as $\xi \to 0$. The figure displays steady-state values of R, b, g, and τ, for $\xi \in [0, 1]$ holding all other parameters as in Table 1. At $\xi = 0$, low-θ economies are immiserized, whereas the governments of high-θ countries run on average a surplus ($b = -0.05$), and can afford both a high public good provision ($g = 0.30$) and low taxes ($\tau = 0.10$). For low ξ's, results are similar, although there is no full immiseration. As ξ increases, the difference in taxes shrinks (for $\xi > 0.2$ the tax differences are very small). A larger distortion strengthens fiscal discipline

and profits. With $\alpha = 1/3$, this amounts to 0.407 of labor income.

in the low-θ economies, so the world interest rate falls. Thus, high-θ economies enjoy about the same private consumption as do low-θ economies, but have more public goods and lower debt. For the benchmark calibration of $\xi = 0.6$, the high-θ economies have an expenditure-to-GDP ratio 10% larger than that of low-θ economies (36.1% vs. 32.8%) whereas taxes are about the same (41.2% vs. 40.2%). Debt is non-monotonic in ξ in high-θ economies. This is due to two opposing forces: On the one hand, a larger ξ reduces R, making public saving less attractive. On the other hand, the larger distortion strengthens the drive to cut taxes. At lower levels of ξ, the former effect prevails, whereas at higher levels of ξ, the latter is stronger.

Some limitations of our quantitative analysis should be acknowledged. First, the model should ideally feature a finer partition of time to capture more nuanced political shifts over the life cycle than the coarse old-young partition of a two-period model. Second, as in the previous literature on dynamic games, we cannot appeal to general existence or uniqueness results. Nevertheless, several aspects of the numerical solutions are reassuring: when lowering the Frisch elasticity ξ toward zero – in which case existence can be proven – the equilibrium policy rules and equilibrium R converge smoothly to the analytical equilibrium. Moreover, two different numerical methods yield the same solution. Third, to focus on government debt, we abstracted from some dimensions of fiscal choices such as transfers, capital income taxes, and sovereign debt default. Finally, although we make progress on the general equilibrium by focusing on stationary equilibria, we have abstracted from transitions of the world economy.

Table 1: Calibration

Target observation		*Parameter*	
Capital-output ratio (annualized)	3	R	$(1.04)^{30}$
Aggregate world wealth-output ratio (annualized)	3.56	β	$(0.973)^{30}$
Capital's share of output	1/3	α	1/3
Average tax on labor	40.7%	λ	2.22
Tax rate corresponding to the top of the Laffer curve	60%	ξ	2/3
Debt-GDP ratio for high-debt countries	75%	$\bar{\theta}$	0.39
Debt-GDP ratio for low-debt countries	36%	$\underline{\theta}$	0.37
Ratio of labor income to total income for young	33/51	X	1.225
Relative voter turnout for the old (61+) in the US	25%	ω	0.25

3 Empirical Implications

Fiscal shocks . The model features interesting fiscal policy dynamics. Suppose that the economy is hit by a one-time fiscal shock (e.g., a surprise war) requiring an exogenous spending of Z units. The shock is equivalent to an exogenous increase in debt from b to $b+Z/R$. Since T is increasing in b and G is decreasing in b, the government reacts by increasing τ and decreasing

g in wartime. After the war, debt, taxes, and expenditure revert slowly to the original steady state. These predictions accord well with the empirical evidence of Bohn (1998), who finds the US debt-to-output ratio to be highly persistent, but mean reverting.

Government efficiency and cross-country debt distribution. In our model, high- and low-θ economies can be interpreted as economies whose governments provide public goods more and less efficiently. One interpretation is that θ is a stand-in for the quality of public goods. For an equal g, high-θ (low-θ) governments provide public goods of higher (lower) quality, implying a larger (smaller) marginal utility (θ/g) of government expenditure. Another interpretation is that θ measures the elasticity of effective public good provision to government expenditure.[14]

Thus, the theory predicts that the steady-state debt-GDP ratio is decreasing in the efficiency of public good provision. This prediction is consistent with data for industrialized countries. We proxy the efficiency of governments by the index of corruption perception provided by Transparency International (TI, where a high index means low corruption), and measure government debt by the ratio of central government debt to GDP (source: OECD). We calculate national averages of both measures over the sample period 1990-2008 for the set of twenty-four countries that were OECD members over the entire period.[15] With the exception of Turkey (which in fact turns out to be a strong outlier), this yields a homogeneous sample of financially integrated industrialized countries. The cross-country correlation between debt and the TI corruption index is negative and significant, $\rho = -0.51$ ($p-$value 0.01). The result is not driven by outliers: Excluding Turkey strengthens the correlation, $\rho = -0.68$.[16] The correlation is robust to alternative measures of the efficiency of governments, such as the corruption measure from the International Country Risk Guide ($\rho = -0.46$) and measures of "effectiveness of governance," "quality of regulation," and "control of corruption" from the World Bank's Worldwide Governance Indicators ($\rho = -0.41$, $\rho = -0.44$, and $\rho = -0.49$, respectively).

The existing measures of corruption have little time variation. Instead, one can use political shifts within countries to assess the implications for debt *dynamics*. In an earlier version of this paper (Song et al. 2007), we assumed shocks to political preferences. Suppose left-leaning

[14] Suppose $\tilde{g} = g^{\theta}$, where $\tilde{g}$ denotes the effective provision and g denotes the expenditure. The utility agents earn from an expenditure level g would then be $\theta \log(g)$ as postulated in (2).

[15] When early data are missing, we use the available data. We exclude post-2008 years, since the debt-efficiency correlation is a steady-state prediction, whereas the post-2008 debt dynamics are affected by the response to a large shock. Including 2009-10 does not change the results significantly.

[16] Most countries that accessed the OECD after 1990 are former socialist economies, some of which were not separate entities before 1990. Many started from low debt levels and are still in a transition toward steady state. We exclude them since we compare data with the theory's steady-state predictions.

If we include all current OECD countries, the correlation remains significant as long as one conditions on a dummy for countries entering after 1990. The partial correlation coefficient between debt and corruption is negative and significant at the 5% level: $\rho_P = -0.37$, and $\rho_P = -0.49$ if Israel, a high-debt outlier, is excluded.

(right-leaning) governments have stronger preferences for public (private) good consumption. The theory then predicts that debt should grow more under right-wing governments. In line with this prediction, we find that in a panel of OECD countries 1980-2005, a political shift from left to right increases the debt-GDP ratio by an average of 0.7 percentage points per year.

Demographic changes. The model is sensitive to the political power of the old. Consider increasing ω to 0.5 in all countries.[17] This change has dramatic effects: the lower fiscal discipline yields an average (annualized) steady-state debt-GDP ratio of 216%! The annualized interest rate increases to 5% and taxes increase to 55.5% on average, implying a massive crowd-out of physical capital and a 33% reduction in world (market) GDP. Most of the tax revenue is now devoted to servicing the debt, and government expenditures fall by 62% compared to the benchmark steady state, even though the old care more about g than the young.

Another interesting application is the (partial) equilibrium effect of a demographic transition. Here, we consider a single small economy in steady state at $t = 0$. We assume that the resource cost of public good provision is proportional to the population size. Unexpectedly, at $t = 1$ the birth rate rises and reverts to normal in $t \geq 2$ (i.e., a baby boom followed by a baby bust). Let N_t denote the size of the cohort born in t. The political weights of the young and old are now adjusted by age group sizes, being respectively $(1-\omega) N_t$ and ωN_{t-1}. The initial increase in N_t/N_{t-1} implies larger political weight of the young and, hence, a stronger fiscal discipline and low debt. Taxes are low and public good provision is high due to the large tax base. Then, N_t/N_{t-1} falls in $t = 2$. This has two consequences: the ageing population increases the political influence of the old and reduces the tax base. Both effects weaken fiscal discipline. Debt grows and converges gradually to steady state. Meanwhile, taxes increase and public good provision falls. These predictions are consistent with the observation that since the 1980s an increasing share of old voters has been accompanied by rising government debt, especially in rapidly ageing societies like Japan. The U-shaped debt behavior in the baby boom-baby bust example also resembles the post-war pattern for debt in many OECD countries. A simulated demographic transition is shown in Figure 3 in Appendix B.

4 Social Planner

In this section, we consider the choice of a Ramsey planner of a small country who sets the domestic fiscal policy sequence so as to maximize the discounted utility of all generations, subject to the competitive equilibrium conditions and R, as determined by the SSMPPE. Fol-

[17] The vote share of people age 61+ is expected to increase to 50% by 2050. Moreover, Mulligan and Sala-i-Martin (1999) argue that the political clout of the old has increased even beyond demographics. For example, since 1968 the overall US voting turnout has fallen by 10%, whereas it has increased 3% for those age 65+.

lowing Farhi and Werning (2007), the planner attaches geometrically decaying Pareto weights $\psi_t \equiv \psi^{t+1}$ (with $\psi \in [0,1)$) on the discounted utility of each generation t. The sequential formulation of the planner's problem is

$$\mathcal{W} = \max_{\{g_t,\tau_t,b_{t+1}\}_{t=0}^{\infty}} \left\{ \beta U_{O,t} + \sum_{t=0}^{\infty} \psi^{t+1} U_{Y,t} \right\}, \tag{10}$$

subject to (1), $b_t \leq \bar{b}\ \forall t$, and the initial debt b_0. Proposition 2 characterizes the allocations.

Proposition 2 *Given R, the Ramsey allocation of an individual economy is characterized by the Euler equation for public good consumption, $g'/g = \psi R$, and the following intratemporal trade-off between private and public goods:*

$$\frac{\psi(1+\beta)}{A(\tau;R)} = (1-e(\tau))\frac{(\beta\lambda+\psi)\theta}{g}. \tag{11}$$

Suppose $\psi R < 1$. Then, the fiscal policy sequence converges to "public poverty": $\lim_{t\to\infty} b_t = \bar{b}$, $\lim_{t\to\infty} g_t = 0$, *and* $\lim_{t\to\infty} \tau_t = \bar{\tau}$.

The Ramsey allocation differs starkly from the SSMPPE: the long-run debt is entirely determined by the planner's discount factor ψ and is independent of her θ. This feature has interesting implications. Consider, for example, an SSMPPE with inelastic labor supply and recall that in that case, the equilibrium interest rate is falling in $\bar{\theta}$. Therefore, for any $\bar{\theta} > 0$ and $\omega < 1$, there exists a threshold discount factor $\bar{\psi} > 0$ such that a planner with a discount factor $\psi \in \left[0, \bar{\psi}\right)$ would start depleting resources, plunging the country into immiseration, even if it were initially rich and had $\theta = \bar{\theta}$.[18] The paternalistic planner has concerns for future generations, but R might be so low that she chooses to grab resources from them to benefit more recent generations, inducing what we call long-run immiseration. Conversely, in political equilibrium, selfish voters would like to extract resources from future generations, but the lack of commitment limit their ability of doing so. In turn, the repeated political representation of young voters averts immiseration.[19]

This result extends to the Ramsey allocation where the fiscal policies of *all* countries are determined by a universal planner with discount factor ψ. To see this, consider the symmetric case with identical countries and inelastic labor supply. In this case the world interest rate

[18] Conversely, for ψ sufficiently large, the planner would choose ever-increasing g and c.

Another interesting benchmark is a non-paternalistic planner who only cares about the two generations alive at $t = 0$. In this case, public poverty is attained after two periods. This would also be the political equilibrium outcome if the initial generations of voters could choose fiscal policy with commitment over two periods.

[19] This result is related to Sleet and Yeltekin (2008), who show that in a dynamic private information model, the lack of commitment arising from political constraints can avert immiseration in an environment where this would arise under commitment. In their paper this is due to an ever-increasing concentration of wealth.

is $R = \psi^{-1}$ and there is no immiseration in the long run. However, in the corresponding SSMPPE, the world interest rate would be $R^*\left(\bar{\theta}\right) > 1$. Hence, there exists a range of low (high) discount factors ψ such that R is higher (lower) in the planning allocation than in the corresponding SSMPPE, implying a lower (higher) world capital stock and wages in steady state. The possibility that a paternalistic planner may treat future generations worse than do selfish short-lived agents who set fiscal policy subject to a sequence of political constraints runs against the standard intuition that politico-economic forces (political rent-seeking, pork barrel, etc.) lead to an excessive public debt.[20] We have abstracted from such distortions, here, in order to highlight how the intergenerational conflict shapes debt. Assessing the relative importance of the different mechanisms is left to future research.

APPENDIX: Proof of Proposition 1

Using equation (1) to eliminate g_j, rewrite the problem as

$$\begin{aligned} & \max_{\left\{b'_j \in [\underline{b},\bar{b}], \tau_j \in [0,1]\right\}} U\left(\tau_j, b'_j - Rb_j + \tau_j\, w\left(R\right), G\left(b'_j; \theta_j, R\right); \theta_j, R\right) \\ = \; & (1-\omega)\left((1+\beta)\log\left(1-\tau_j\right) + \theta_j \log\left(b'_j - Rb_j + \tau_j\, w\left(R\right)\right) + \beta\lambda\theta_j \log\left(G\left(b'_j; \theta_j, R\right)\right)\right) \\ & +\omega\left(\log\left(1-\tau_{-1,j}\right) + \lambda\theta_j \log\left(b'_j - Rb_j + \tau_j\, w\left(R\right)\right)\right). \end{aligned}$$

The FOC's with respect to τ_j and b'_j yield, respectively, (6) and (7). Next, guess that $g_j = \gamma_j R\left(\bar{b} - b_j\right)$, where γ_j is an undetermined coefficient. Then, (6) and (7) yield the expressions for $\tau_j = T\left(b_j; \theta_j, R\right)$ and $b'_j = B\left(b_j; \theta_j, R\right)$ in (8). Standard algebra shows then that (1), (6), and (7) are verified if and only if $\gamma_j = \gamma\left(\theta_j\right) \equiv \left(1 + \left(1-\omega\right)\left(\left(1+\beta\right) + \theta_j\beta\lambda\right)/\theta_j\left(1+\omega\left(\lambda-1\right)\right)\right)^{-1}$, establishing (i).

Next, consider the steady-state world market clearing condition. Recall that $C_Y = \left(1+\beta\right)^{-1}\left(1-T\right)w\left(R\right)$. Stationarity implies that, for a unit measure of countries, $b = b'$ and $K' = K\left(R,1\right)$. The argument in the text establishes that $R = R^*\left(\bar{\theta}\right)$. Since $R < R^*\left(\theta\right)$ for all $\theta < \bar{\theta}$, then (8) implies that a measure $1-\upsilon$ of countries has $b = \bar{b}$ and $\tau = 1$. Hence, (5) simplifies to

$$\upsilon \frac{\beta}{1+\beta}\left(1 - T\left(b; \bar{\theta}, R\right)\right) w\left(R\right) = \left(1-\upsilon\right)\bar{b} + \int_{j|_{\theta_j = \bar{\theta}}} b_j + K\left(R,1\right),$$

where $R = R^*\left(\bar{\theta}\right)$ and $\bar{b} = w\left(R\right)/\left(R-1\right)$. Substituting $T\left(b; \bar{\theta}, R\right)$ into this expression and simplifying terms yields $\int_{j|_{\theta_j=\bar{\theta}}} b_j = \bar{b} - \upsilon^{-1}\bar{\theta}\lambda/\left(1+\bar{\theta}\lambda\right) \cdot \left(\bar{b} + K\left(R,1\right)\right)$. Finally, Ljapunov stability holds for any b in high-θ economies, since $B\left(b; \bar{\theta}, R^*\left(\bar{\theta}\right)\right) = b$. For low-$\theta$ economies, $b' < b$ for all $b < \bar{b}$, and thus $\bar{b}$ is asymptotically stable. QED

[20] The possibility that future generations are better off in a competitive equilibrium than in the allocation chosen by a paternalistic planner who discounts the utilities of future generations arises also in standard OLG models. Here, we prove that this result extends to a model with endogenous debt and fiscal policy.

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Appendix B (not for publication)

In this appendix we present some supplementary material for the paper "Rotten Parents and Disciplined Children: A Politico-Economic Theory of Public Expenditure and Debt" by Zheng Song, Kjetil Storesletten, and Fabrizio Zilibotti. Section B1 provides the politico-economic microfoundations of the model. Section B2 provides the derivation of equation (9). Section B3 provides the proof of Proposition 2. Section B4 extends the analysis under CRRA utility. Section B5 shows the result of numerical analysis using the alternative method of Krusell et al. (2002). Finally, Section B6 shows the details of the numerical analysis of a demographic transition such as the one discussed in the text.

B1. PROBABILISTIC VOTING MODEL

The political equilibrium discussed in the paper has an explicit politico-economic microfoundation in terms of a politico-economic voting model, based on Lindbeck and Weibull (1987). In this appendix, we describe the voting model, which is an application of Persson and Tabellini (2000) to a dynamic voting setting.

The population has a unit measure and consists of two groups of voters, young and old, of equal size (we discussed below the extension to groups of different sizes). The electoral competition takes place between two office-seeking candidates, denoted by A and B. Each candidate announces a fiscal policy vector, b', τ, and g, subject to the government budget constraint, $b' = Rb + g - w(R)\,\tau\,H(\tau)$, and to $b' \leq \bar{b}$.[21] Since there are new elections every period, the candidates cannot make credible promises over future policies (i.e., there is lack of commitment beyond the current period). Voters choose either of the candidates based on their fiscal policy announcements and on their relative *appeal*, where the notion of appeal is explained below. In particular, a young voter prefers candidate A over B if, given the inherited debt level b, preference parameter θ, the world interest rate, and the equilibrium policy functions $\langle B, G, T\rangle$ which apply from tomorrow and onwards,

$$U_Y\left(\tau_A, g_A, G\left(b'_A\right); b, \theta, R\right) > U_Y\left(\tau_B, g_B, G\left(b'_B\right); b, \theta, R\right).$$

Likewise, a young voter prefers candidate A over B if

$$U_O\left(g_A; b, \theta, R\right) > U_O\left(g_B; b, \theta, R\right).$$

[21] Note that the announcement over the current fiscal policy raises no credibility issue, due to the assumption that the politicians are pure office seekers and have no independent preferences on fiscal policy.

σ^{iJ} (where $J \in \{Y, O\}$) is an individual-specific parameter drawn from a symmetric group-specific distribution that is assumed to be uniform in the support $[-1/\left(2\phi^J\right), 1/\left(2\phi^J\right)]$. Intuitively, a positive (negative) σ^{iJ} implies that voter i has a bias in favor of candidate B (candidate A). Note that the distributions have density ϕ^J and that neither group is on average biased towards either candidate. The parameter δ is an aggregate shock capturing the ex-post average success of candidate B whose realization becomes known after the policy platforms have been announced. δ is drawn from a uniform i.i.d. distribution on $[-1/(2\psi), 1/(2\psi)]$.[22] The sum of the terms $\sigma^{iJ} + \delta$ captures the relative appeal of candidate B: it is the inherent bias of individual i in group J for candidate B irrespective of the policy that the candidates propose. The assumption of uniform distributions is for simplicity (see Persson and Tabellini (2000), for a generalization).

Note that voters are rational and forward looking. They take into full account the effects of today's choice on future private and public-good consumption. Because of repeated elections, they cannot decide directly over future fiscal policy. However, they can affect it through their choice of next-period debt level (b'), which affects future policy choices through the equilibrium policy functions B, T, and G.

It is at this point useful to identify the "swing voter" of each group, i.e., the voter who is ex-post indifferent between the two candidates:

$$\begin{aligned}
\sigma^Y\left(b'_A, \tau_A, g_A, b'_B, \tau_B, g_B; b, \theta, R\right) &= U_Y\left(\tau_A, g_A, G\left(b'_A\right); b, \theta, R\right) - U_Y\left(\tau_B, g_B, G\left(b'_B\right); b, \theta, R\right) - \delta \\
\sigma^O\left(g_A, g_B; b, \theta, R\right) &= U_O\left(g_A; b, \theta, R\right) - U_O\left(g_B; b, \theta, R\right) - \delta.
\end{aligned}$$

Conditional on δ, the vote share of candidate A is

$$\begin{aligned}
& \pi_A\left(b'_A, \tau_A, g_A, b'_B, \tau_B, g_B; b, \theta, R\right) \\
= \; & 1 - \pi_B\left(b'_A, \tau_A, g_A, b'_B, \tau_B, g_B; b, \theta, R\right) \\
= \; & \frac{1}{2}\phi^Y\left(\sigma^Y\left(b'_A, \tau_A, g_A, b'_B, \tau_B, g_B; b, \theta, R\right) + \frac{1}{2\phi^Y}\right) \\
& + \frac{1}{2}\phi^O\left(\sigma^O\left(g_A, g_B; b, \theta, R\right) + \frac{1}{2\phi^O}\right) \\
= \; & \frac{1}{2} + \frac{1}{2}\left(\phi^Y \times \left(U_Y\left(\tau_A, g_A, G\left(b'_A\right); b, \theta, R\right) - U_Y\left(\tau_B, g_B, G\left(b'_B\right); b, \theta, R\right)\right) - \delta\right) \\
& + \frac{1}{2}\left(\phi^O \times \left(U_O\left(g_A; b, \theta, R\right) - U_O\left(g_B; b, \theta, R\right)\right) - \delta\right).
\end{aligned}$$

[22] The realization of δ can be viewed as the outcome of the campaign strategies to boost the candidates' popularity. Such an outcome is unknown *ex ante*.

Note that π_A and π_B are stochastic variables, since δ is stochastic. The probability that candidate A wins is then given by

$$\begin{aligned} p_A &= \text{Prob}_\delta \left[\pi_A \left(b'_A, \tau_A, g_A, b'_B, \tau_B, g_B; b, \theta, R \right) \geq \frac{1}{2} \right] \\ &= \text{Prob} \left[\begin{array}{c} \delta < \frac{\phi^Y}{\phi^Y + \phi^O} \left(U_Y \left(\tau_A, g_A, G\left(b'_A \right); b, \theta, R \right) - U_Y \left(\tau_B, g_B, G\left(b'_B \right); b, \theta, R \right) \right) \\ + \frac{\phi^O}{\phi^Y + \phi^O} \left(U_O \left(g_A; b, \theta, R \right) - U_O \left(g_B; b, \theta, R \right) \right) \end{array} \right] \\ &= \frac{1}{2} + \psi \left(1 - \omega \right) \left(U_Y \left(\tau_A, g_A, G\left(b'_A \right); b, \theta, R \right) - U_Y \left(\tau_B, g_B, G\left(b'_B \right); b, \theta, R \right) \right) \\ &\quad + \psi \omega \left(U_O \left(g_A; b, \theta, R \right) - U_O \left(g_B; b, \theta, R \right) \right), \end{aligned}$$

where $\omega \equiv \phi^O / \left(\phi^Y + \phi^O \right)$.

Since both candidates seek to maximize the probability of winning the election, the Nash equilibrium is characterized by the following equations:

$$\begin{aligned} \left(b'^*_A, \tau^*_A, g^*_A \right) &= \max_{b'_A, \tau_A, g_A} \left(1 - \omega \right) \left(U_Y \left(\tau_A, g_A, G\left(b'_A \right); b, \theta, R \right) - U_Y \left(\tau_B, g_B, G\left(b'_B \right); b, \theta, R \right) \right) \\ &\quad + \omega \left(U_O \left(g_A; b, \theta, R \right) - U_O \left(g_B; b, \theta, R \right) \right), \\ \left(b'^*_B, \tau^*_B, g^*_B \right) &= \max_{b'_B, \tau_B, g_B} \left(1 - \omega \right) \left(U_Y \left(\tau_B, g_B, G\left(b'_B \right); b, \theta, R \right) - U_Y \left(\tau_A, g_A, G\left(b'_A \right); b, \theta, R \right) \right) \\ &\quad + \omega \left(U_O \left(g_B; b, \theta, R \right) - U_O \left(g_A; b, \theta, R \right) \right). \end{aligned}$$

Hence, the two candidates' platform converge in equilibrium to the same fiscal policy maximizing the weighted-average utility of the young and old,

$$\left(b'^*_A, \tau^*_A, g^*_A \right) = \left(b'^*_B, \tau^*_B, g^*_B \right) = \max_{b' \tau, g} \left(\left(1 - \omega \right) U_Y \left(\tau, g, G\left(b' \right); b, \theta, R \right) + \omega U_O \left(g; b, \theta, R \right) \right),$$

subject to the government budget constraint. This is the political objective function given in the body of the paper.

Note that the parameter ω has a structural interpretation: it is a measure of the relative variability within the old group of the candidates' appeal. As shown above, ϕ^Y / ϕ^O (and, hence, ω) affects the number of swing voters in each group. For instance, suppose that $\phi^O > \phi^Y$. Intuitively, this means that the old are more "responsive" in electoral terms to fiscal policy announcements in favor of or against them. An alternative interpretation is that $1/\phi^J$ measures the extent of group J heterogeneity with respect to other policy dimensions that are orthogonal to fiscal policy. For example, the young might work in different sectors and cast their votes also based on the sectoral policy proposed by each candidate. As a result, the vote of the young is less responsive to fiscal policy announcements, and the young have effectively less political power than the old. This interpretation is consistent with Mulligan and Sala-i-Martin (1999)

and Hassler *et al.* (2005). In the extreme case of $\omega = 1$, the old only care about fiscal policy ($\phi^O \to 0$) and the distribution has a mass point at $\sigma^O = 0$. In this case, the young have no influence and the old dictate their fiscal policy choice (as in the commitment solution with $\alpha = 0$).

Suppose, next, that the groups have different relative size, and that there are N_O old voters and N_Y young voters. Proceeding as above, the political objective function is then modified to

$$\begin{aligned}\left(b_A'^*, \tau_A^*, g_A^*\right) &= \left(b_B'^*, \tau_B^*, g_B^*\right) = \\ &= \max_{b'\tau,g} \left\{(1-\omega) N_Y U_Y \left(\tau, g, G\left(b'\right); b, \theta, R\right) + \omega N_O U_O \left(g; b, \theta, R\right)\right\}\end{aligned}$$

We conclude by noting that the probabilistic voting outlined in this appendix applies equally to both static and dynamic models (under the assumption of MPE). The political model entails some important restrictions. First, agents only condition their voting strategy on the payoff-relevant state variable (here, debt). Second, the shock δ is i.i.d. over time – otherwise, the previous realization of δ becomes a state variable, complicating the analysis substantially. Third, although the assumption of uniform distributions can be relaxed, it is necessary to impose regularity conditions on the density function in order to ensure that the maximization problem is well behaved.

B2. DERIVATION OF EQUATION (9)

Following the logic of the proof of Proposition 1, write the problem as

$$\begin{aligned}&\max_{\left\{b' \in \left[\underline{b}, \overline{b}\right], \tau \in [0, \bar{\tau}]\right\}} U\left(\tau, b' - Rb + \tau\, w\left(R\right)\, H\left(\tau\right), G\left(b'; \theta, R\right); \theta, R\right) \\ = \;& (1-\omega)\left(\left(1+\beta\right)\log\left(A\left(\tau; R\right)\right) + \theta \log\left(b' - Rb + \tau\, w\left(R\right)\, H\left(\tau\right)\right) + \beta\lambda\theta \log\left(G\left(b'; \theta, R\right)\right)\right) \\ & + \omega\left(\log\left(A\left(\tau_{-1}; R\right)\right) + \lambda\theta \log\left(b' - Rb + \tau\, w\left(R\right)\, H\left(\tau\right)\right)\right).\end{aligned}$$

This yields

$$-\frac{(1-\omega)(1+\beta)}{A\left(\tau; R\right)} A_\tau\left(\tau; R\right) = \frac{\theta}{g}\left(1 + \omega\left(\lambda - 1\right)\right) \frac{\mathrm{d}}{\mathrm{d}\tau}\left(w\left(R\right)\, \tau\, H\left(\tau\right)\right).$$

Hence, using that fact that $A_\tau\left(\tau; R\right) = -w\left(R\right)\, H\left(\tau\right)$ and the definition of $e\left(\tau\right)$, we obtain

$$\frac{(1-\omega)(1+\beta)}{A\left(\tau; R\right)} = \left(1 - e\left(\tau\right)\right) \frac{\theta}{g}\left(1 + \omega\left(\lambda - 1\right)\right),$$

which is expression (9) in the text.

B3. PROOF OF PROPOSITION 2

Ignoring irrelevant terms, the planning problem can be expressed as

$$\mathcal{W} = \max_{\{g_t,\tau_t,b_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \left(\beta\left(\psi^t\lambda + \psi^{t+1}\right)\theta g_t + (1+\beta)\,\psi^{t+1}A\left(\tau_t;R\right)\right)$$

subject to a period budget constraint,

$$b_{t+1} = g_t + Rb_t - \tau_t\, w\left(R\right)\, H\left(\tau_t\right),$$

a debt limit ($b_t \leq \bar{b}$ for all t), and a given b_0.

Write the problem as a standard Lagrange problem with multipliers ζ_t associated with the budget constraints. The first-order conditions for g_t , τ_t, and b_{t+1} yield

$$0 = \psi^t\frac{\beta\lambda\theta}{g_t} + \psi^{t+1}\frac{\theta}{g_t} - \zeta_t \tag{12}$$

$$0 = -\psi^{t+1}\left(1+\beta\right)\frac{\frac{dA(\tau_t;R)}{d\tau_t}}{A\left(\tau_t;R\right)} + w\,\zeta_t\left(H\left(\tau_t\right) + \tau_t\frac{dH\left(\tau_t\right)}{d\tau_t}\right) \tag{13}$$

$$0 = \zeta_t - \psi R\zeta_{t+1}. \tag{14}$$

Combining (12)-(13) and exploiting that $A_\tau = -w\left(R\right)\,H\left(\tau\right)$ and $e\left(\tau\right) = -\left(dH/d\tau\right)\left(\tau/H\left(\tau\right)\right)$ yields (11). Combining (12) (for period t and $t+1$) and (14) yields $g'/g = \psi R$ as in the Proposition. It is clear from this expression that $\lim_{t\to\infty} g_t = 0$ since the growth rate of g is constant and negative (i.e., $\psi R < 1$). In the case of elastic labor supply, the maximum tax rate is smaller than 100% ($\bar{\tau} < 1$), so the left-hand side of (11) must be bounded away from zero. Since $\lim_{t\to\infty} g_t = 0$, it follows that $\lim_{t\to\infty} e\left(\tau_t\right) = 1 = e\left(\bar{\tau}\right)$ and, hence, $\lim_{t\to\infty}\tau_t = \bar{\tau}$. The budget constraint (1) then implies $\lim_{t\to\infty} b_t = \bar{b}$. In the case of inelastic labor supply, $e\left(\tau\right) = 0$ for all τ, so $\lim_{t\to\infty} g_t = 0$ and (11) imply $\lim_{t\to\infty}\tau_t = 1$ and, hence, $\lim_{t\to\infty} b_t = \bar{b}$.

B4. CALIBRATED ECONOMY: ROBUSTNESS USING THE KRUSELL ET AL. (2002) METHOD

The main numerical approach to compute the equilibrium in the calibrated economies are based on a projection method with Chebyshev collocation to approximate the policy function based on equations (4), (9), and (7). In order to assess the robustness of our quantitative results, we also solved for the equilibrium using an alternative algorithm proposed by Krusell, Kuruscu, and Smith (2002, KKS henceforth) to compute the equilibrium policy functions. As opposed to the global nature of the projection method, KKS is based on the calculation of

higher-order derivatives of (4), (9), and (7) around steady state. We find that when using derivatives up to the fourth order, the KKS algorithm identifies – up to the fourth decimal point for debt – the same (internal) steady state as the one we found using projection methods. As illustrated in Figure 3 (the analogue of Figure 1), even outside of the steady state the two alternative solutions for the policy rule are quantitatively similar.

FIGURE 3 HERE

B5. CRRA UTILITY

In this section, we provide a complete characterization of the equilibrium under general CRRA utility. We consider the case of inelastic labor supply. The analysis can be extended to the case of elastic labor supply, though, as in the case of logarithmic utility, a full analytical solution is not available in this case.

Proposition 3 *Assume that agents have CRRA utility:*

$$U_Y = \frac{c_Y^{1-\sigma}-1}{1-\sigma} + \theta\frac{g^{1-\sigma}-1}{1-\sigma} + \beta\left(\frac{(c_O')^{1-\sigma}-1}{1-\sigma} + \lambda\theta\frac{g^{1-\sigma}-1}{1-\sigma}\right),$$

where $\sigma > 1/2$. Then there exists a SSMPPE equilibrium with policy functions given by

$$\tau = T(b;\theta,R) = 1 - \left(1+\beta(\beta R)^{\frac{1-\sigma}{\sigma}}\right)\left(\frac{1-\omega}{\theta(1+\omega(\lambda-1))}\right)^{\frac{1}{\sigma}}\frac{\gamma(\theta,R)}{w(R)}\left(\bar{b}-b\right), \tag{15}$$

$$b' = B(b;\theta,R) = \bar{b} - \left(\frac{(1-\omega)\lambda\beta}{1+\omega(\lambda-1)}\gamma(\theta,R)\right)^{\frac{1}{\sigma}}\left(\bar{b}-b\right), g = G(b;\theta,R) = \gamma(\theta,R)\left(\bar{b}-b\right),$$

where $\gamma(\theta,R)$ is the unique non-negative solution to the polynomial

$$\left(\frac{(1-\omega)\lambda\beta}{1+\omega(\lambda-1)}\gamma(\theta,R)\right)^{\frac{1}{\sigma}} = R - \gamma(\theta,R)\left(1+\left(1+\beta^{\frac{1}{\sigma}}(R)^{\frac{1-\sigma}{\sigma}}\right)\left(\frac{1-\omega}{\theta(1+\omega(\lambda-1))}\right)^{\frac{1}{\sigma}}\right). \tag{16}$$

The world interest rate is pinned down by the unique solution of the following equation:

$$R = 1 + \Gamma(R;\omega,\lambda,\beta,\sigma,\theta), \tag{17}$$

where

$$\Gamma(R;\omega,\lambda,\beta,\sigma,\theta) \equiv \frac{1+\omega(\lambda-1)}{(1-\omega)\lambda\beta}\left(1+\left(1+\beta(\beta R)^{\frac{1-\sigma}{\sigma}}\right)\left(\frac{1-\omega}{\theta(1+\omega(\lambda-1))}\right)^{\frac{1}{\sigma}}\right).$$

In the SSMPPE, $R = R^*\left(\bar{\theta}\right)$, namely, the world interest rate is set by the countries with the strongest preference for the public good. All other countries have $R < R^*(\theta)$ and converge to immiseration. Finally, the average debt level for countries with $\theta = \bar{\theta}$ is unique and is given by

$$b\left(\bar{\theta}\right) = \bar{b} - \frac{(Q\alpha)^{\frac{1}{1-\alpha}}}{\upsilon} \cdot \frac{\frac{1-\alpha}{\alpha}\frac{R}{(R-1)}(R)^{-\frac{\alpha}{1-\alpha}} + (R)^{-\frac{1}{1-\alpha}}}{R + \left(\frac{(1-\omega)}{(1+\omega(\lambda-1))}\right)^{\frac{1-\sigma}{\sigma}} \frac{1}{\lambda\beta}\left(\frac{\beta}{\theta}\right)^{\frac{1}{\sigma}} R^{\frac{1}{\sigma}}},$$

where $R = R^*\left(\bar{\theta}\right)$.

Proof. The optimal savings decision yields

$$\begin{aligned} c_Y &= \frac{1}{1+\beta^{\frac{1}{\sigma}}R^{\frac{1-\sigma}{\sigma}}} w(R)\,(1-\tau) \\ c'_O &= \frac{\beta^{\frac{1}{\sigma}}R^{\frac{1-\sigma}{\sigma}}}{1+\beta^{\frac{1}{\sigma}}R^{\frac{1-\sigma}{\sigma}}} Rw(R)\,(1-\tau). \end{aligned}$$

Thus, ignoring irrelevant terms, we can write the political objective function as

$$\begin{aligned} U\left(\tau, b' - Rb + w\,\tau, G\left(b'\right); \theta, R\right) &= \frac{1}{1-\sigma}\Big[(1-\omega)\left(1+\beta\left(\beta R\right)^{\frac{1-\sigma}{\sigma}}\right)^{\sigma}\left(w(R)\,(1-\tau)\right)^{1-\sigma} \\ &\quad +\theta\left(1+\omega\left(\lambda-1\right)\right)\left(b' - Rb + w(R)\,\tau\right)^{1-\sigma} \\ &\quad +(1-\omega)\,\beta\lambda\theta\left(G\left(b';\theta,R\right)\right)^{1-\sigma}\Big]. \end{aligned}$$

The FOCs yield

$$\begin{aligned} (1-\omega)\left(1+\beta\left(\beta R\right)^{\frac{1-\sigma}{\sigma}}\right)^{\sigma}\left(w(R)\,(1-\tau)\right)^{-\sigma} w(R) &= \theta\left(1+\omega\left(\lambda-1\right)\right) g^{-\sigma} w(R), \\ \theta\left(1+\omega\left(\lambda-1\right)\right) g^{-\sigma} + (1-\omega)\,\beta\lambda\theta\left(g'\right)^{-\sigma}\frac{\partial G\left(b';\theta,R\right)}{\partial b'} &= 0. \end{aligned}$$

Rearranging terms yields

$$\frac{w(R)\,(1-\tau)}{g} = \left(1+\beta\left(\beta R\right)^{\frac{1-\sigma}{\sigma}}\right)\left(\frac{1-\omega}{\theta\left(1+\omega\left(\lambda-1\right)\right)}\right)^{\frac{1}{\sigma}} \tag{18}$$

$$\left(\frac{g'}{g}\right)^{\sigma} = -\frac{(1-\omega)\,\lambda\beta}{1+\omega\left(\lambda-1\right)}\frac{\partial G\left(b';\theta,R\right)}{\partial b'}. \tag{19}$$

Next, the government budget constraint implies

$$\bar{b} - b' = \bar{b} - g - Rb - w\,(1-\tau) + w.$$

Plugging in (18), recalling that $w = (R-1)\,\bar{b}$, and rearranging terms yields

$$\bar{b} - b' = -g\left(1 + \left(1+\beta\left(\beta R\right)^{\frac{1-\sigma}{\sigma}}\right)\left(\frac{1-\omega}{\theta\left(1+\omega\left(\lambda-1\right)\right)}\right)^{\frac{1}{\sigma}}\right) + R\left(\bar{b} - b\right).$$

Guess that $g = \gamma(\theta, R)\left(\bar{b} - b\right)$.

The GEE implies that

$$\begin{aligned}\left(\bar{b} - b'\right) &= \left(\frac{(1-\omega)\lambda\beta}{1+\omega(\lambda-1)}\frac{\partial G(b';\theta,R)}{\partial b'}\right)^{\frac{1}{\sigma}}\left(\bar{b} - b\right) \\ &= \left(\frac{(1-\omega)\lambda\beta}{1+\omega(\lambda-1)}\gamma(\theta,R)\right)^{\frac{1}{\sigma}}\left(\bar{b} - b\right).\end{aligned}$$

Hence, we can rewrite the budget constraint as

$$\begin{aligned}&\left(\frac{(1-\omega)\lambda\beta}{1+\omega(\lambda-1)}\gamma(\theta,R)\right)^{\frac{1}{\sigma}}\left(\bar{b} - b\right) \\ = \; &-\gamma(\theta,R)\left(1+\left(1+\beta(\beta R)^{\frac{1-\sigma}{\sigma}}\right)\left(\frac{1-\omega}{\theta(1+\omega(\lambda-1))}\right)^{\frac{1}{\sigma}}\right)\left(\bar{b} - b\right) + R\left(\bar{b} - b\right).\end{aligned}$$

Hence, $\gamma(\theta, R)$ is the solution to the polynomial equation (16). To see why (16) has a unique solution such that $\gamma > 0$, note that the left-hand side of (16) is monotone increasing in γ (for $\gamma \geq 0$), while the right-hand side is monotone decreasing in γ. Second, for $\gamma = 0$, the left-hand side of (16) is zero while the right-hand side is positive.

Moreover, as long as $\sigma \geq 1$ (sufficient condition), the solution for γ is decreasing in θ. To see why, note that the RHS of (16) is in this case increasing in R. Moreover, the LHS is increasing in γ while the RHS is decreasing in γ. Thus, an increase in R would increase the RHS of (16), which requires an offsetting increase in γ. Then, the generalization of (8) to CRRA utility yields (15).

Next, we consider the stationary GE solution. In a steady state, $\bar{b} - b' = \bar{b} - b$ implying that

$$\begin{aligned}1 &= \left(\frac{(1-\omega)\lambda\beta}{1+\omega(\lambda-1)}\gamma(\theta,R)\right)^{\frac{1}{\sigma}} \\ &\Rightarrow \\ \gamma(\theta,R) &= \frac{1+\omega(\lambda-1)}{(1-\omega)\lambda\beta}.\end{aligned}$$

Substituting this expression of $\gamma(\theta, R)$ into equation (16) yields an implicit expression for the steady-state level of R (given θ),

$$R = 1 + \frac{1+\omega(\lambda-1)}{(1-\omega)\lambda\beta}\left(1+\left(1+\beta^{\frac{1}{\sigma}}(R)^{\frac{1-\sigma}{\sigma}}\right)\left(\frac{1-\omega}{\theta(1+\omega(\lambda-1))}\right)^{\frac{1}{\sigma}}\right),$$

which is equivalent to (17). Consider the above definition of $\Gamma(R;\omega,\lambda,\beta,\sigma,\theta)$. Note that $\Gamma > 0$. If $\sigma > 1$, then Γ is decreasing in R. If $\sigma \in (1/2, 1)$, then Γ is increasing and strictly concave in R. If $\sigma \in (0, 1/2)$, then Γ is increasing and convex in R. Thus, $\sigma > 1/2$ is a

sufficient condition for the existence and uniqueness of a SSMPPE. Moreover, Γ is decreasing in θ. This ensures that (assuming $\sigma > 1/2$), $R^*(\theta)$ is monotone decreasing in θ. The proof that $R = R^{*(\bar{\theta})}$ is the SSMPPE equilibrium interest rate, proceeds as in the case of logarithmic utility discussed in Section 1. Finally, to compute the steady-state debt in high-θ economies (in order to obtain fiscal policy), note that given the fiscal policy, savings can be computed as a function of b:

$$\begin{aligned} s &= w(1-\tau) - \frac{w(1-\tau)}{1+\beta^{\frac{1}{\sigma}} R^{\frac{1-\sigma}{\sigma}}} \\ &= \left(1 - \frac{1}{1+\beta^{\frac{1}{\sigma}} R^{\frac{1-\sigma}{\sigma}}}\right)\left(\frac{(1-\omega)}{(1+\omega(\lambda-1))\theta}\right)^{\frac{1}{\sigma}}\left(1+\beta^{\frac{1}{\sigma}}(R)^{\frac{1-\sigma}{\sigma}}\right) g \\ &= \left(\frac{(1-\omega)}{(1+\omega(\lambda-1))}\right)^{\frac{1-\sigma}{\sigma}} \frac{1}{\lambda\beta}\left(\frac{\beta}{\theta}\right)^{\frac{1}{\sigma}} R^{\frac{1-\sigma}{\sigma}}\left(\bar{b}-b\right). \end{aligned}$$

Recalling the equilibrium expression for τ yields an expression for savings in the high-θ countries:

$$s(T(b^*,R),R) = \left(\frac{(1-\omega)}{(1+\omega(\lambda-1))}\right)^{\frac{1-\sigma}{\sigma}} \frac{1}{\lambda\beta}\left(\frac{\beta}{\theta}\right)^{\frac{1}{\sigma}} R^{\frac{1-\sigma}{\sigma}}\left(\bar{b}-b^*\right).$$

Imposing that $s_j = 0$ and $b_j = \bar{b}(R) = w(R)/(R-1)$ for all countries with $\theta_j < \bar{\theta}$, equilibrium condition 2 can be expressed as

$$\upsilon \cdot s(T^*,R) = \upsilon b^* + (1-\upsilon)\bar{b} + K(R,1).$$

Substituting in the savings yields an expression for the average debt in the high-θ economies:

$$\left(1 + \left(\frac{(1-\omega)}{(1+\omega(\lambda-1))}\right)^{\frac{1-\sigma}{\sigma}} \frac{1}{\lambda\beta}\left(\frac{\beta}{\theta}\right)^{\frac{1}{\sigma}} R^{\frac{1-\sigma}{\sigma}}\right)\upsilon\left(\bar{b}-b^*\right) = \bar{b} + K(R,1).$$

Rearranging and substituting in the expressions for $K(R,1)$ and $w(R)$ yields

$$\begin{aligned} b^* &= \bar{b} - \frac{1}{\upsilon}\left(1 + \left(\frac{(1-\omega)}{(1+\omega(\lambda-1))}\right)^{\frac{1-\sigma}{\sigma}} \frac{1}{\lambda\beta}\left(\frac{\beta}{\theta}\right)^{\frac{1}{\sigma}} R^{\frac{1-\sigma}{\sigma}}\right)^{-1} \cdot \left(\frac{(1-\alpha)Q^{\frac{1}{1-\alpha}}(\alpha)^{\frac{\alpha}{1-\alpha}}}{(R)^{\frac{\alpha}{1-\alpha}}(R-1)} + \left(\frac{Q\alpha}{R}\right)^{\frac{1}{1-\alpha}}\right) \\ &= \bar{b} - \frac{(Q\alpha)^{\frac{1}{1-\alpha}}}{\upsilon} \cdot \frac{\frac{1-\alpha}{\alpha}\frac{R}{(R-1)}(R)^{-\frac{\alpha}{1-\alpha}} + (R)^{-\frac{1}{1-\alpha}}}{R + \left(\frac{(1-\omega)}{(1+\omega(\lambda-1))}\right)^{\frac{1-\sigma}{\sigma}} \frac{1}{\lambda\beta}\left(\frac{\beta}{\theta}\right)^{\frac{1}{\sigma}} R^{\frac{1}{\sigma}}}. \end{aligned} \tag{20}$$

To see that the average steady-state bond holdings b^* is unique given $R = R^*\left(\bar{\theta}\right) > 1$, note that the numerator in the second term (on the right-hand side) of equation (20) is monotone decreasing in R for $R > 1$, while the denominator is monotone increasing in R. ■

B6. CALIBRATED ECONOMY: DEMOGRAPHIC TRANSITION

In this section, we consider a fully anticipated demographic transition such that at $t = 0$ the economy is in the steady state described in the benchmark calibration of Table 1 with $N_0 = 1$, where N_t denotes the size of the cohort born at t. Then, at $t = 1$ there is an unexpected baby boom, with the size of the young population growing to $N_1 = 1.35$. This corresponds to an annualized population growth rate of 1%. Afterwards, the population growth returns to zero ("baby bust"), so the cohort size stays constant at $N_t = 1.35$ for all $t \geq 2$. A falling population growth has two effects in the model. First, by increasing the relative size of the old cohort, it increases their political influence (recall that the political weight of the young and old are, respectively, $(1 - \omega) N_t$ and ωN_{t-1}). This causes a reduction in fiscal discipline and an increase in the current taste for the public good (since $\lambda \geq 1$). Second, the fall in population growth reduces the share of working population and the size of the tax base. Formally, the government budget constraint must be rewritten as $b_{t+1} (N_t + N_{t+1}) / (N_{t-1} + N_t) = g_t + Rb_t - (N_t / (N_{t-1} + N_t)) \tau_t w H (\tau_t)$, where b and g are now debt and public good per capita, respectively.

FIGURE 4 HERE

Figure 4 shows the impulse response of debt per capita, public good per capita, and taxes along the demographic transition. For illustrative purposes, we assume initial debt per capita to be equal to the final steady-state debt per capita. Clearly, this choice is arbitrary, but the main qualitative results do not hinge on it. At $t = 1$, when the share of young agents is large, debt falls, as anticipated above. Taxes are low and public good provision is high due to the large tax base. From $t = 2$ onward, taxes grow and public good provision falls. Most interestingly, debt starts growing and eventually converges to the steady state. Thus, our theory predicts that ageing societies increase debt accumulation, in line with the empirical observation that since the 1980s an increasing share of old voters has been accompanied by a rising government debt, especially in quickly ageing societies like Japan.

The U-shaped behavior of debt in the example also resembles the post-war pattern for debt in the US and most Western European countries. Debt was initially high due to the war shock (in 1946, the US federal debt-GDP ratio was 122%) and fell gradually until the end of the 1970s, reaching a trough of 33% in 1981. During this period, the population share over 40 went from 35% in 1948 to 36% in 1981. Thereafter, debt increased reaching 68% in 2008, while the population share over 40 went up to 46%. In the same period, taxation grew and the share of government purchase of goods and services fell. Both facts are consistent with the impulse response of Figure A2.

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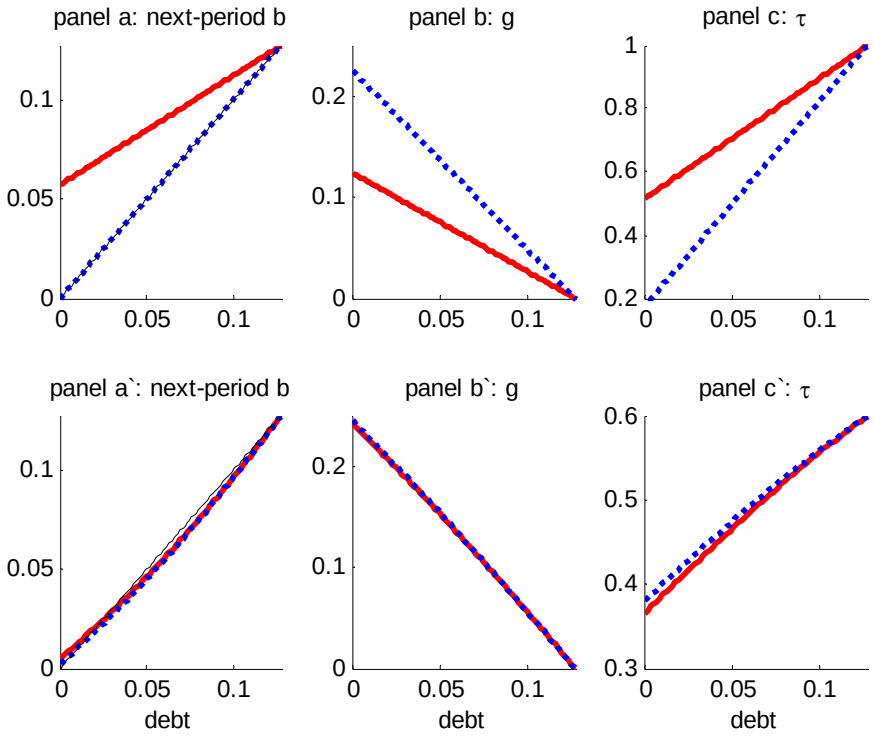
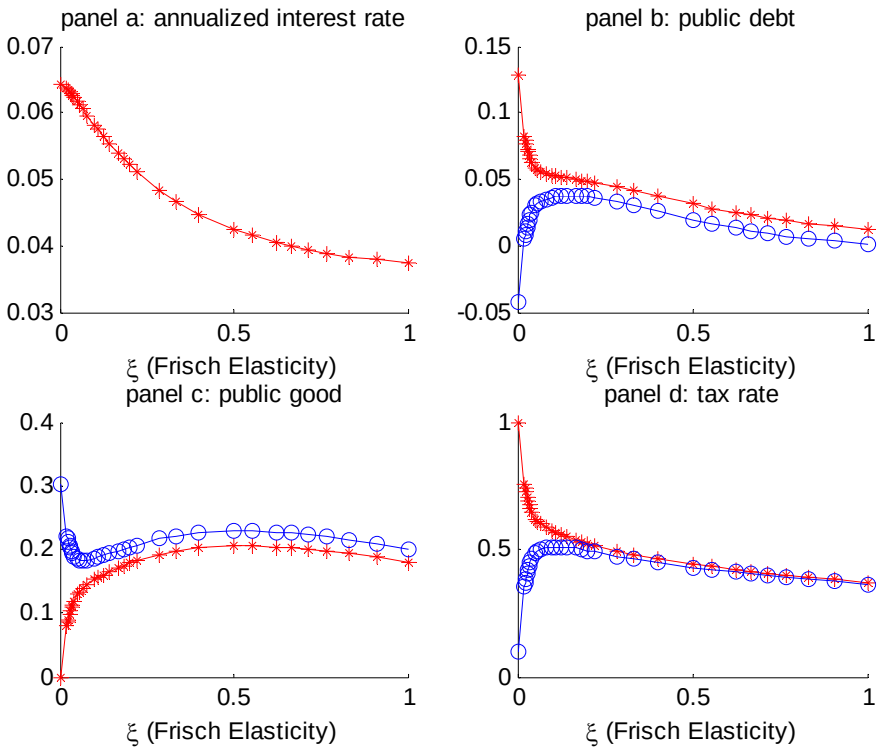
Figure 1: Equilibrium Policy Functions



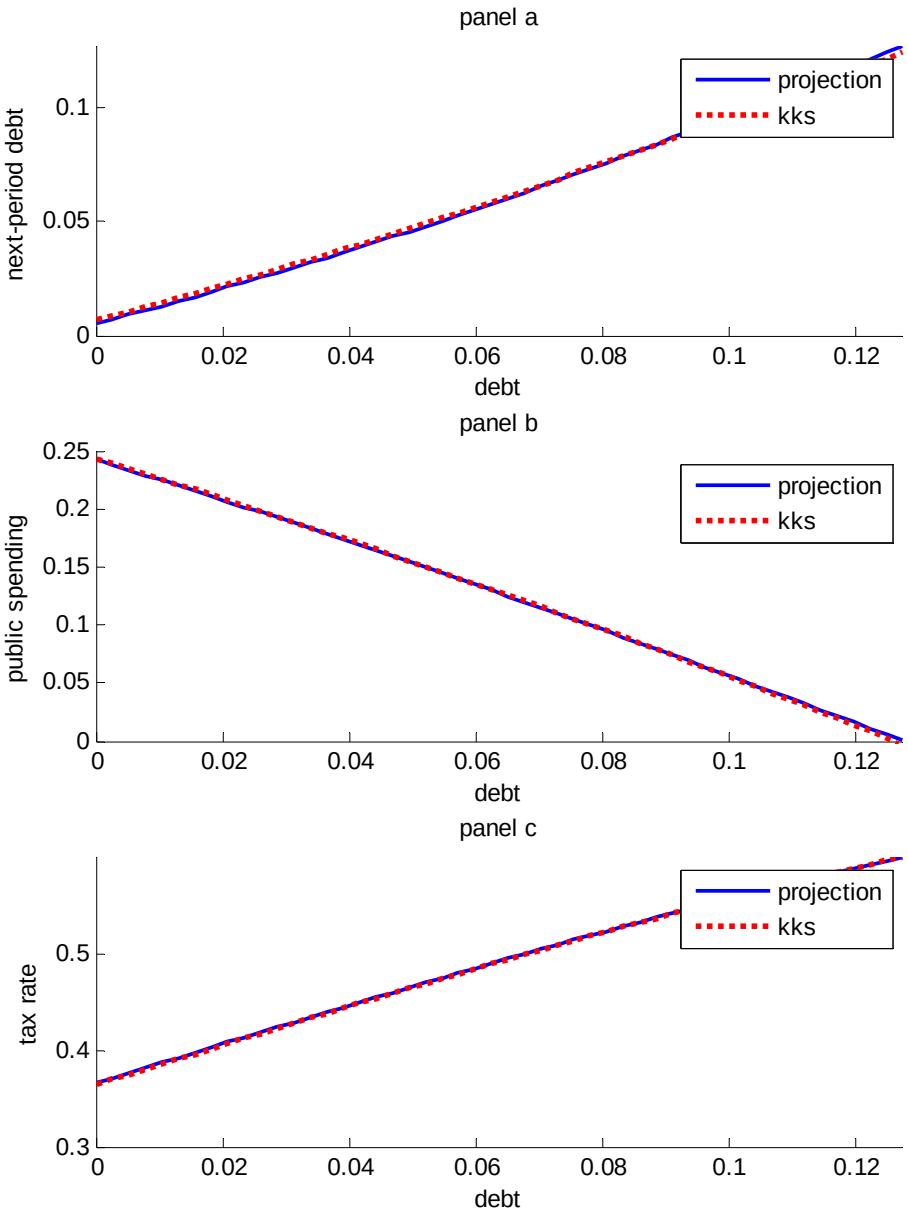
Panels a-c plot the equilibrium policy functions B(b), G(b), and T(b), respectively, for the inelastic labor economy (Proposition 1, qualitative graphs). The solid red (dotted blue) line denotes the low-θ (high-θ) economy. Panels a'-c' plot the corresponding equilibrium policy functions for the calibrated economy with elastic labor supply (parameter values are as given in Table 1).

Figure 2: Continuity of the SMPPE



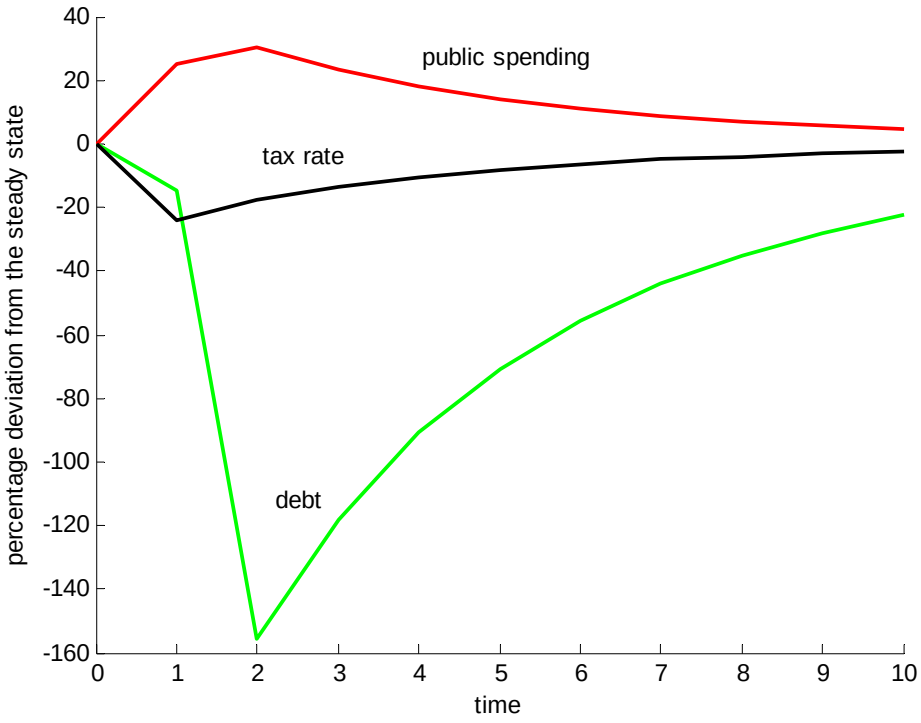
Panel a plots the (annualized) equilibrium interest rate for economies with Frisch elasticity of labor supply ξ ranging from 0 to 1. All other parameters are as given in Table 1. Panels b, c, and d plot the corresponding steady-state allocations of debt, public goods, and taxes. The solid red (dotted blue) line denotes the low-θ (high-θ) economies. Stars and diamonds show the 24 values of ξ for which the SSMPPE is computed numerically. The values for ξ=0 correspond to the equilibrium computed analytically in Proposition 1.

Figure 3 (Appendix B):
Projection Method vs. Krusell, Kuruscu, and Smith (2002)



The figure shows policy rules computed using two different numerical methods: the projection method (solid line) and the Krusell-Kuruscu-Smith (KKS) method (dotted line), respectively. Panels a, b, and c show the equilibrium policy rules for debt B(b), public good provision G(b), and taxes T(b), respectively. Parameter values are the same as in the calibrated economy (see Table 1).

Figure 4 (Appendix B): Demographic Transition



The figure shows impulse-response functions of a demographic boom-bust shock. The annualized population growth increases from zero to 1% in period t=1 and reverts to zero thereafter. The initial debt at period 0 is that of steady state. The remaining parameter values are listed in Table 1.

Figure 5 (Appendix B): Government Efficiency and Corruption versus Government Debt

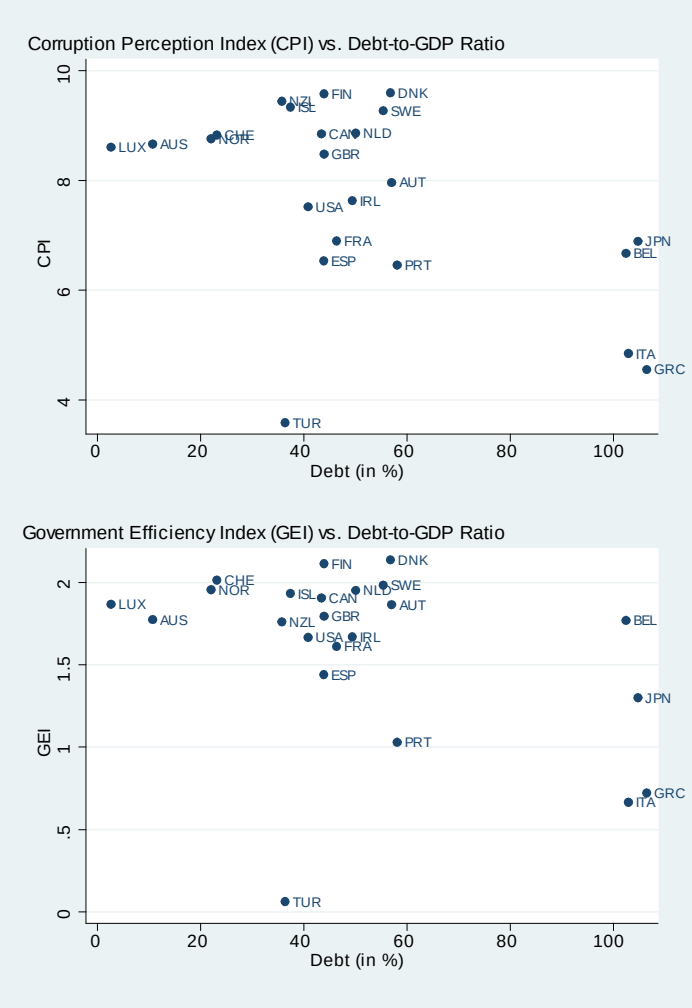


Figure 5: The upper panel plots the index of corruption perception provided by Transparency International against the ratio of central government debt to GDP (source: OECD). The lower panel plots the index of effectiveness of governance from the World Bank's Worldwide Governance Indicators against the ratio of central government debt to GDP (source: OECD). The observations are averages over 1990-2008 and includes all countries that were OECD members over the entire period.