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Energy in a Bathtub: Electricity Trade between Countries with Different Generation Technologies

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ENERGY IN A BATHTUB: ELECTRICITY TRADE BETWEEN COUNTRIES WITH DIFFERENT GENERATION TECHNOLOGIES*

by

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Abstract: Many countries have followed a policy of being self-sufficient in electricity. However, in the last two decades exchange of electricity across borders has become more widespread, and the European Union's policy is to encourage a gradual expansion of cross-border trading and integration of electricity markets. It is therefore of interest to study what happens with the price formation in home markets when borders are opened up for trade in electricity and generating technologies differ. There is a common international market, Nord Pool, between the Nordic countries since 1996, and trade now takes place between many European countries on a bilateral basis. A stylised general equilibrium model of trade of electricity between two countries; Hydro and Thermal, with hydro and thermal technologies, is used to investigate price and quantity consequences going from autarky to trade in a competitive market, as revealed by using a social planning perspective for cooperation between countries.

JEL classification: F14, Q40

Keywords: Electricity; hydropower; thermal power; international trade

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1. Introduction

Self-sufficiency in electricity generation creates country-specific prices that may influence the structure of industry and, e.g., choice of space-heating technology. This has been the case for Norway developing a huge metal smelting industry after World War II, also in an international context, and basing a significant share of space heating on direct use of electricity. It is therefore of interest to study what happens with the price formation in domestic markets when borders are opened up for trade in electricity. Efficiency gains may be substantial. There is a common international market, Nord Pool, between the Nordic countries since 1996 and international trade now takes place between many European countries on a bilateral basis, e.g., France – England, France – Italy (Italy imports about 20% of its electricity), Germany – Holland, etc. The energy policy of the European Union is encouraging a gradual expansion of cross-border trading and integration of electricity markets (Jamasb and Pollit, 2005).

Trade between countries with electricity generation based on hydro power, like Norway, and countries using thermal plants is of special interest due to the flexibility of hydropower when such a system has a sizeable water storage capacity. In the media Norway has been launched as an "electricity battery" for continental Europe, even using pumped storage on a large scale within a daily cycle of trade in electricity. The increasing share of wind power in countries like Denmark and Germany implies increased demand for short-run flexibility for other types of generation due to the intermittent character of wind power. Hydro power with reservoirs is the most flexible form of generation and well suited to complement wind power. An additional point of interest is that hydro power is quite clean from a climate gas point of view (there may be minor releases of methane gas from dams), so a question is what impact trade between hydro power countries and thermal countries may have on climate gas emissions.

The objective of this paper is to study, within a highly stylised model of trade in electricity between two countries, the price formation and quantities of electricity following trade, and contrast this with the situation of autarky for both countries. Because hydropower has some unique features and because most systems are dominated by thermal power, price formation in markets dominated by hydropower is not much analysed. Moreover, most such attempts are not easily accessible for non-specialists. Thus, we will also try to simplify the analysis as much as possible, but still making the relevant points. Although the formal analysis is carried out as a social planning problem, appealing to the second welfare theorem the characteristics of the solutions for prices and quantities are directly relevant for what may happen in a competitive wholesale market with many independent electricity producers.

However, we will only study the management problem of utilising existing capacities optimally, and will not address the investment problem of expanding production capacity. We introduce two stylised countries; one country has only regulated hydro power (e.g., Norway), while the other country has only fossil fuel thermal power (e.g., Denmark). For increased realism we will extend the model discussion later to include unregulated hydro and in the thermal country wind power (wind power together with hydro is studied in Førsund (2007); Førsund et al., 2008). For convenience the countries are called Hydro and Thermal. The two electricity-producing sectors will be treated at an aggregate level. At this level it suffices to specify a single interconnector between the countries. Thus, we are analysing trade between a hydropower sector in one country and a thermal power sector in another country with a single transmission line between the countries.

The literature on the specific topic of electricity trade between a hydro and a thermal country is very small, but, of course, the literature on gains by trade in general is vast. Of the few (academic) papers studying the effects of trade in electricity between the Nordic countries, where differences in generation technologies are large, ranging form hydro power in Norway and Sweden, nuclear power in Sweden and Finland, conventional coal- and gasfired thermal power in Denmark and Finland to wind power in Denmark, there are some recent studies of the effect of the integration into a common Nordic market for electricity, the Nord Pool market. Amundsen and Bergman (2007) outline the integration of the Nordic wholesale market for electricity and give some information about price development, interconnector capacities, etc. Amundsen and Bergman (2006) and von der Fehr et al. (2005) explain why the Nord Pool market functioned quite well and proved itself to be robust during the episode of 2002- 2003 of unusually low inflows to the reservoirs of Norway and Sweden (a situation that probably had resulted in rationing in the previously regulated markets). von der Fehr and Sandsbråten (1997), inspired by the Nordic market, study trade between two countries, one with hydro power and the other with thermal power. Four periods are distinguished, day and night for summer and winter periods, respectively, and demand-and inflow configurations are assumed to be cyclically repeated. A steady state analysis, before and after opening up for trade, is conducted solving both the current management problem of optimal use of given capacities, and finding the optimal capacities. Four thermal technologies with different variable- and capital-cost characteristics are specified corresponding to a typical load pattern for the four time periods. A graphical analysis is performed of the implications of trade regimes. The analysis may be a little complicated to follow. The present paper is based on a simpler, but may be more transparent, approach representing a further development of the analysis in Førsund (2007). The simpler approach is still sufficient to bring out the most interesting possibilities opened up by trade.

Models of regional electricity markets, having features in common with the model used in this paper, specifying both hydro power and thermal plants, have been used to study trade between California and nearby states in Borenstein et al. (2002) and Bushnell (2003), focussing on market power issues on the backdrop of the crisis in California in year 2000.

In Section 2 the situation of autarky for Hydro is studied, using a discrete time model of a single hydro power system with a single reservoir, and assuming full certainty about demand and inflows. The existence of a reservoir and the mismatch between inflows and demand necessitates a dynamic analysis. A rule for setting prices is derived. Autarky for Thermal is also studied. Making the simplifying assumption acceptable at an aggregate level of no start-up or close-down cost of thermal units it suffices with a static analysis to derive the price rule and determine quantities. In Section 3 trade between Hydro and Thermal is introduced, and the conditions for a common equilibrium price derived, together with the shift in consumption between the two countries. A new type of graphical illustration is provided, and day trade, unregulated hydro and wind power, and climate gas emissions are discussed. Section 4 concludes.

2. Autarky

Autarky in Hydro

The optimal management of the hydro system at an aggregate level is studied using a model bringing out the importance of the reservoir, but disregarding engineering details about optimal management of single hydro power plants (Wood and Wollenberg, 1984). We will follow a partial equilibrium approach, so no interaction with the rest of the economy is modelled. The social planner maximises a standard objective function of (aggregate) consumer plus producer surplus, thus simulating a perfectly competitive market. Transmission between producer and consumer nodes within each country is neglected, together with restriction on power in the hydro country, and ramping up or down (see Førsund (2007) for extensions). It is assumed (following actual conditions) that operating marginal costs are zero (do not depend on the current output level for hydro plants), and that fixed costs are sunk cost and are neglected, so the objective function is simply the gross area under the demand curve. Discounting is not introduced for convenience because the horizon is usually so short that the effect will be negligible (or does not contribute to our understanding of the qualitative nature of the solutions). The length of the time period may range from one hour to a week, month and year. The time periods are considered consecutively.

The social planning problem for the T periods within the given planning horizon can then be expressed in the following way (Førsund, 2007):

$$
\begin{aligned}
&Max \sum_{t=1}^{T} \int_{z=0}^{e_t^H} p_t^H(z)dz \\
&s.t. \\
&R_t \le R_{t-1} + w_t - e_t^H \\
&R_t \le \overline{R} \\
&R_t, w_t, e_t^H \ge 0 \ , \ t = 1,..,T \\
&T, R_o, \overline{R} \text{ given}, R_T \text{ free}
\end{aligned}
\qquad (1)
$$

All variables are non-negative. Consumption of electricity, e_t^H, is equal to production of electricity. The demand function on price form, $p_t^H(e_t^H)$, is assumed to decrease in quantity in the standard way. The first condition is the water accumulation equation. The reservoir at the end of period t is R_t, water from the previous period is R_{t-1}, inflow during period t is w_t and use of water e_t^H. The reservoir limit is $\overline{R}$, measured as the amount of water that can maximally be utilised (i.e. usually there is a positive minimum level of the reservoir due to environmental considerations). It is assumed that water in the reservoir measured in m^3 of water can be converted to electricity measured in kWh by applying a fixed conversion factor given by the hydro system's physical characteristics (e.g., height of heads). Using this conversion factor we can then measure all quantity variables in the energy unit kWh. However, we will still refer to the variables originally measured in water units as "water." For convenience no scrap value function for water in the reservoir or minimum level in the last period are introduced, so the amount at the end of period T is technically a free variable.

The optimisation problem (1) is a discrete-time dynamic programming problem, and special solution procedures (Bellman's backward induction) have been developed for this class of problems (Sydsæter et al., 2005). However, due to the simple structure of the problem we shall treat it as a non-linear programming problem and use the Kuhn – Tucker conditions for a problem with non-negative variables to discuss qualitative characterisations of the optimal solution.

The Lagrangian function is:

$$\begin{aligned} L = & \sum_{t=1}^{T} \int_{z=0}^{e_t^H} p_t(z)dz \\ & -\sum_{t=1}^{T} \lambda_t (R_t - R_{t-1} - w_t + e_t^H) \\ & -\sum_{t=1}^{T} \gamma_t (R_t - \bar{R}) \end{aligned} \tag{2}$$

Necessary first-order conditions are:

$$\begin{aligned} & \frac{\partial L}{\partial e_t^H} = p_t(e_t^H) - \lambda_t \le 0 \ (= 0 \text{ for } e_t^H > 0) \\ & \frac{\partial L}{\partial R_t} = -\lambda_t + \lambda_{t+1} - \gamma_t \le 0 \ (= 0 \text{ for } R_t > 0) \\ & \lambda_t \ge 0 (= 0 \text{ for } R_t < R_{t-1} + w_t - e_t^H) \\ & \gamma_t \ge 0 (= 0 \text{ for } R_t < \bar{R}) \ , \qquad\qquad t = 1,..,T \end{aligned} \tag{3}$$

We will assume that a unique solution exists, and limit our discussion to the conditions above.

Now, our general objective is that the model should tell us something qualitatively about optimal production and consumption of electricity that has real world interest. We will then limit the number of potential optimal solutions by making reasonable assumptions. One such assumption is that we require positive production in all periods yielding the conditions

$$p_t^H(e_t^H) = \lambda_t \ , t = 1,..,T \tag{4}$$

The shadow price λ_t of the stored water may be termed the *water value*. Note that we have not ruled out the possibility that the water value is zero. This may happen if we have overflow in the system, i.e. water has to be spilt and inequality holds in the water accumulation equation.

We see that the second equation in (3) is the essential one for the dynamics of our system. If the reservoir capacity is not constrained, then we have from the complementary slackness condition in (3) that the shadow price on the reservoir constraint is zero, implying that the water values for periods t and $t+1$ are equal, and hence the prices are equal. The period prices may also be different although the shadow price on the reservoir constraint is zero. We see from (3) that this may happen if it is optimal that the reservoir at the end of period $t+1$ is empty. We then have that $-\lambda_t + \lambda_{t+1} \leq 0$, i.e., we may have that $\lambda_t > \lambda_{t+1}$. For this to be optimal the reservoir level must also be run down to the allowed minimum level in period t, because a transfer from period t to period $t+1$ cannot be optimal when the water value is greater in period t than in period $t+1$. Such a situation implies that the periods are disconnected; there is no recursiveness present for these two periods in determining the shadow prices. Already Hveding (1968) pointed out that prices in a hydro system only change if the system runs up against constraints.

The bathtub diagram for two periods

There are only two successive periods involved in the equation of motion in (3). This means that a sequence of two period diagrams may capture the main features of the general solution. We will use the development of shadow prices to give insights into the qualitative characteristics of an optimal solution. We will not try to derive an actual price path, but illustrate briefly main possibilities.

Adding together the water-storage equations in (1) for $t=1,2$ we always have, assuming no spill of water

$$e_1^H + e_2^H = R_o + w_1 + w_2 \qquad (5)$$

The total electricity produced over the two periods is equal to the available water from period $t = 0$ (R_o) and the inflows in period 1 and 2 (measured in kWh). The solution for two periods can be illustrated in a *bathtub diagram*, Figure 1, showing the total available water as the floor of the bathtub, and the period-demand functions anchored on each wall. Inflow plus the initial water R_o in period 1 is AC, and inflow in period 2 is CD. The maximal storage is now introduced as BC. The storage is measured from C toward the axis for period 1 because the decision of how much water to transfer to period 2 is made in period 1.

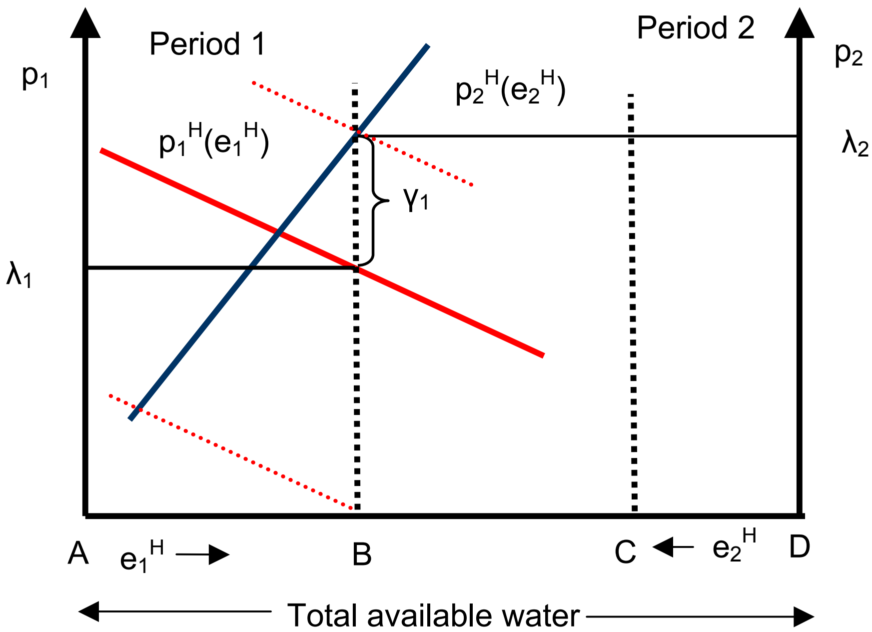

Figure 1. Bathtub diagram. Social optimum with reservoir constraint binding

The intersection of the demand curves illustrated implies that without the reservoir constraint, BC, more water would be used in period 2. However, since this is not permitted the best that can be done is to transfer the maximal amount, BC, to period 2. Therefore, the amount AB will be used in period 1, this water is *locked in*, and in period 2 the transferred water is used together with period 2 inflow, CD. Thus the water value for period 1, λ_1, becomes lower than the water value for period 2, λ_2. Using (3) above we have that $\lambda_1 = \lambda_2 - \gamma_1$. The period prices are equal to the water values, and the difference between them is equal to the shadow price on the reservoir constraint.

Notice that the water allocation will be the same for a wide range of period 1- demand curves keeping the same period 2- curve, or vice versa. (The period 1- curve can be shifted down to passing through B and shifted up to passing through the level for the period 2 water value, as indicated by the dotted lines.) The price differences between the periods may correspondingly vary considerably.

If the intersection of the period-demand curves takes place inside the vertical lines from B and C marking the reservoir size, the period prices will become equal. Enough water will be transferred from period 1 to period 2 to make this happen.

Autarky in Thermal

If a merit order ranking of individual conventional thermal plants according to marginal costs is unique, we may aggregate over individual plants with increasing marginal costs by using this ranking as the sector's supply curve. It may formally be approximated by postulating a (merit order) relationship between total output and total fuel costs:

$$c_t = c(e_t^{Th}), c' > 0, c'' > 0,\ e_t^{Th} \leq \overline{e}^{Th} \tag{6}$$

where e_t^{Th} is the amount of electricity produced in period t, and $\overline{e}^{Th}$ is the total capacity. The c(.)-function may represent different thermal technologies as to primary fuel (coal, gas bio). For simplicity it is assumed that primary fuel prices remain constant and that there is no disembodied technical change.

At the thermal plant level it is well known from the engineering literature that there are start-up costs and close-down costs, or in general costs of ramping up and down. However, for an aggregate modelling of the sector, overlooking these costs may serve as a good enough approximation for our purpose of investigating trade in electricity. The social planning problem in Thermal is then quite simple and can be solved period for period for $t\epsilon[1,T]$:

$$\begin{aligned}
&Max \int_{z=0}^{e_t^{Th}} p_t^{Th}(z)dz - c(e_t^{Th}) \\
&s.t. \\
&e_t^{Th} \leq \overline{e}^{Th} \\
&e_t^{Th}, \overline{e}^{Th} \geq 0\ , \overline{e}^{Th} \text{ given}
\end{aligned} \tag{7}$$

The Lagrangian for the problem is:

$$L = \int_{z=0}^{e_t^{Th}} p_t^{Th}(z)dz - c(e_t^{Th}) - \theta_t(e_t^{Th} - \overline{e}^{Th}) \tag{8}$$

Necessary first-order conditions are:

$$\begin{aligned}
&\frac{\partial L}{\partial e_t^{Th}} = p_t^{Th}(e_t^{Th}) - c'(e_t^{Th}) - \theta_t \leq 0\ (= 0 \text{ for } e_t^{Th} > 0) \\
&\theta_t \geq 0 (= 0 \text{ for } e_t^{Th} < \overline{e}^{Th})
\end{aligned} \tag{9}$$

We get the well-known result that the period price is set equal to the marginal cost. If the production is constrained, then the marginal cost gets an addition equal to the shadow price on the capacity. There is no explicit definition of base load or peak load, but we may say that the minimum part of the cost curve actively used in all T periods represents base load, and that the closer we come to the exhaustion of the total capacity the more typical peak is the additional capacity taken into use.

3. Trade between Hydro and Thermal

Within an international market like the Nordic Nord Pool market equilibrium prices are determined at the intersection between aggregated demand and aggregated supply curves. As indicated above, the opening up of trade between the neighbours Hydro – Norway - and Thermal - Denmark - may provide the basis for a stylised case. In 2007 Norway had a hydro share exceeding 98%, and Denmark had a thermal share of 81% (and a wind power share of 19%; wind power will be introduced later). In a common market between Hydro and Thermal, the production capacity of Thermal is given, and so is the total amount of water within the planning horizon and reservoir capacity for Hydro.

In the electricity market with just two countries, net trade in electricity must balance in the sense that export from one country is the other county's import (and vice versa). The energy balance for each country can then be written:

$$\begin{aligned} x_t^H &= e_t^H + e_{Th,t}^{XI} - e_{H,t}^{XI} \\ x_t^{Th} &= e_t^{Th} + e_{H,t}^{XI} - e_{Th,t}^{XI}, \ t = 1,...,T \end{aligned} \quad (10)$$

The quantities of electricity consumed in each country and exported, respectively imported, are now identified by the country sub- and superscripts, "*H*" and "*Th*" for Hydro and Thermal, respectively. Thus, x_t^H and x_t^{Th} are the electricity consumptions in the two countries. The superscript "*XI*" denotes export or import. Import expands a country's consumption possibility while export contracts it. When one country exports the other country cannot, but must import the identical volume (and vice versa), so we assume that only one of the trade variables for each country is positive for the same period, *t*.

The transmission system will not be dealt with explicitly. However, it is assumed that the interconnector has a limited fixed capacity. (We could also introduce a fixed cost per unit transmitted, as found in the literature, but this will not change our analysis qualitatively, so it is dropped for simplicity.)

Introducing trade within an integrated market for cooperating countries means that identifying trade flows between the countries and accounting for export and import on a country basis are not relevant for the joint optimisation problerm of how to utilise existing generating capacities. Our model is partial concerning just electricity, and we have no constraints on the balance of trade in electricity. It may well be an optimal solution to import for more than what is earned in export over the planning horizon. The model formulation thus conforms to a competitive market covering both countries.

The cooperative social planning problem for Hydro and Thermal can then be expressed in the following way:

$$\begin{aligned}
&\max \ \sum_{t=1}^{T}[\int_{z=0}^{x_t^H} p_t^H(z)\,dz + \int_{z=0}^{x_t^{Th}} p_t^{Th}(z)\,dz - c(e_t^{Th})] \\
&\text{subject to} \\
&x_t^H = e_t^H + e_{Th,t}^{XI} - e_{H,t}^{XI} \\
&x_t^{Th} = e_t^{Th} - e_{Th,t}^{XI} + e_{H,t}^{XI} \\
&R_t \le R_{t-1} + w_t - e_t^H \\
&R_t \le \overline{R} \\
&e_{H,t}^{XI} \le \overline{e}^{XI},\ e_{Th,t}^{XI} \le \overline{e}^{XI} \\
&e_t^{Th} \le \overline{e}^{Th} \\
&x_t^H, x_t^{Th}, e_t^H, e_t^{Th}, e_{Th,t}^{XI}, e_{H,t}^{XI} \ge 0 \\
&T, w_t, R_o, \overline{R}, \overline{e}^{XI}, \overline{e}^{Th} \text{ given, } R_T \text{ free },\ t = 1,..,T
\end{aligned} \tag{11}$$

Value terms are expressed in the same money unit (Euro is used for Nord Pool transactions). The benefit terms are the areas under the demand curves up to the actually consumed levels including eventual import.

We could add to the realism of the model by also considering restriction on hydropower production (also taking care of power restriction) and on internal country transmissions. It is straightforward to introduce such constraints. We focus on the constraint on the interconnector capacity between the countries, but will offer some comments on impacts of additional constraints below.

Substituting for country consumptions from the energy balances in the objective function, the Lagrangian is:

$$
\begin{aligned}
L = &\sum_{t=1}^{T}[\int_{z=0}^{e_t^H + e_{Th,t}^{XI} - e_{H,t}^{XI}} p_t^H(z)\,dz + \int_{z=0}^{e_t^{Th} - e_{Th,t}^{XI} + e_{H,t}^{XI}} p_t^{Th}(z)\,dz - c(e_t^{Th})] \\
&-\sum_{t=1}^{T}\lambda_t(R_t - R_{t-1} - w_t + e_t^H) \\
&-\sum_{t=1}^{T}\gamma_t(R_t - \overline{R}) \\
&-\sum_{t=1}^{T}\alpha_{H,t}(e_{H,t}^{XI} - \overline{e}^{XI}) \\
&-\sum_{t=1}^{T}\alpha_{Th,t}(e_{Th,t}^{XI} - \overline{e}^{XI}) \\
&-\sum_{t=1}^{T}\theta_t(e_t^{Th} - \overline{e}^{Th})
\end{aligned}
\tag{12}
$$

Because export from one country is the other country's import we solve only for export for each country.

The first-order necessary conditions are:

$$
\begin{aligned}
&\frac{\partial L}{\partial e_t^H} = p_t^H(x_t^H) - \lambda_t \le 0 \quad (= 0 \text{ for } e_t^H > 0) \\
&\frac{\partial L}{\partial e_{H,t}^{XI}} = -p_t^H(x_t^H) + p_t^{Th}(x_t^{Th}) - \alpha_{H,t} \le 0 \ (= 0 \text{ for } e_{H,t}^{XI} > 0) \\
&\frac{\partial L}{\partial R_t} = -\lambda_t + \lambda_{t+1} - \gamma_t \le 0 \ (= 0 \text{ for } R_t > 0) \\
&\frac{\partial L}{\partial e_t^{Th}} = p_t^{Th}(x_t^{Th}) - c'(e_t^{Th}) - \theta_t \le 0 \ (= 0 \text{ for } e_t^{Th} > 0) \\
&\frac{\partial L}{\partial e_{Th,t}^{XI}} = p_t^H(x_t^H) - p_t^{Th}(x_t^{Th}) - \alpha_{Th,t} \le 0 \ (= 0 \text{ for } e_{Th,t}^{XI} > 0) \\
&\quad t = 1,..,T
\end{aligned}
\tag{13a}
$$

The complementary slackness conditions involving the Lagrangian parameters are

$$
\begin{aligned}
&\lambda_t \ge 0 \ (= 0 \text{ for } R_t < R_{t-1} + w_t - e_t^H) \\
&\gamma_t \ge 0 \ (= 0 \text{ for } R_t < \overline{R}) \\
&\theta_t \ge 0 \ (= 0 \text{ for } e_t^{Th} < \overline{e}^{Th}) \\
&\alpha_{H,t} \ge 0 \ (= 0 \text{ for } e_{H,t}^{XI} < \overline{e}^{XI}) \\
&\alpha_{Th,t} \ge 0 \ (= 0 \text{ for } e_{Th,t}^{XI} < \overline{e}^{XI}), \quad t = 1,..,T
\end{aligned}
\tag{13b}
$$

Whenever reservoir constraints are involved we get a time-specific water value as shown in the first condition in (13a), and an equation of motion for the reservoir shadow prices, here the third condition. If hydropower is produced the first condition holds with equality, and the period price in Hydro is equal to the water value. Furthermore, if hydropower is exported we have from the second condition in (13a) that the socially optimal prices in the countries must be the common equilibrium price as long as the export capacity is not constrained, because according to the complementary slackness condition, the shadow price on the interconnector capacity is zero. If hydropower export is zero, then the shadow price, $\alpha_{H,t}$, on the constraint for export of hydropower is still zero. According to the second condition in (13a) the prices in Hydro and Thermal may then differ, with Thermal price being less than or equal to the hydropower price. The question is if such a difference can be part of an optimal solution in our model. With a lower Thermal price (than Hydro) the objective function could be increased by transferring a unit of thermal production to Hydro, i.e., exporting thermal power. But looking at the fifth condition, for thermal export, and the complementary slackness condition, when Thermal export is positive, we have that the prices again have to be equal.

If the capacity constraint in Thermal is not binding, i.e. $\theta_t = 0$, then the common equilibrium price that was established to be equal the equilibrium price, is also equal to the marginal production cost in Thermal.

If trade constraints are binding, both export and import will be binding for the same period. The second and fifth conditions in (13a) tell us that in such a situation it may be optimal to have different prices between the countries. The price will be lower in the country that is export-constrained than in the country that is import-constrained. An active export constraint forces the country to use more electricity at home, and to realise this, the price has to decrease. For an importing country the home price has to increase as a response to being rationed on imports.

When there is a period-price difference between the countries, a rent is earned on the interconnector that is usually termed *congestion rent*. It is calculated as the price difference times the traded volume. If Hydro is exporting to the capacity limit the amount of rent is

$$\alpha_{H,t} e_{H,t}^{XI} = \alpha_{H,t} e_{Th,t}^{XI} = \alpha_{H,t} \bar{e}^{XI} \qquad (14)$$

If Thermal is exporting we get a similar expression but with the shadow price $\alpha_{Th,t}$. These shadow prices will in general differ, and thus the congestion rent, depending on the demand functions for the countries. In Nord Pool there is no redistribution scheme in place for such rents, as it should not be, because the rent is dissipated by the market.

An extended energy bathtub

The impact of a reservoir constraint can be illustrated for two periods as in Figure 2, extending Figure 1 to also include thermal capacity. (In Crampes and Moreaux (2001, p. 980) an illustration is offered of the joint use of a hydro plant and a thermal plant by a social planner. However, the principles behind the illustrations are rather different.) In order not to complicate the figure too much, trade is not constrained. Hydro is described by a hydro bathtub in the middle covering both periods (cf. Figure 1), and then the bathtub is extended by thermal capacity in Thermal with one period on each side. The Hydro bathtub floor is *AD*, and available water in period 1 is *AC* and inflow is *CD* in period 2. The maximal amount *BC* can be stored in period 1 and transferred to period 2.

The dotted demand curves for Hydro and the hydro bathtub walls with solid vertical lines erected from *A* and *D* show the autarky solution for Hydro. The country-specific autarky equilibria in price and quantities are indicated by the dotted lines. We have that for Hydro the autarky prices are equal for the periods. The reservoir capacity *BC* is not fully utilised in Hydro transferring water from period 1 to period 2 to obtain the optimal social autarky solution.

Turning to Thermal, the period 1- situation is shown on the left-hand side of Figure 2. The demand function is anchored on the Hydro bathtub wall up from *A*, and falling downward to

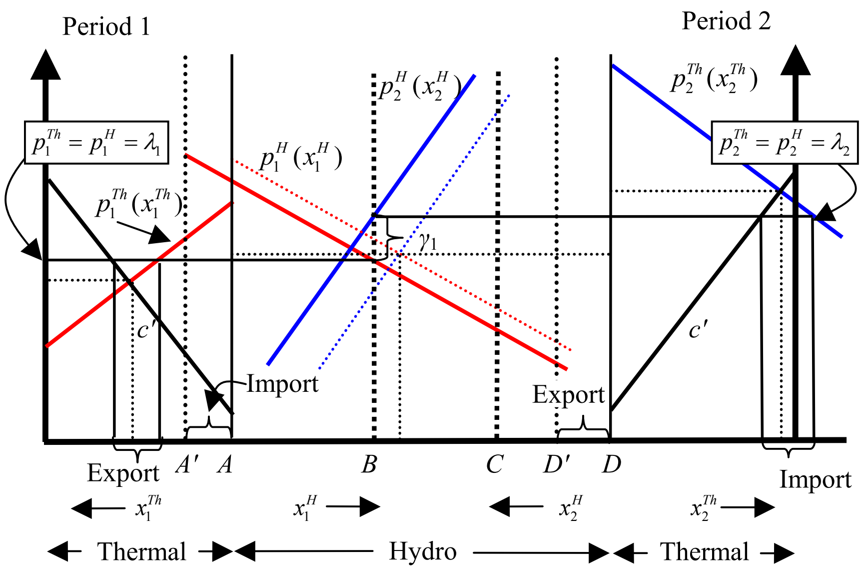

Figure 2. Energy bathtub. Trade between Hydro and Thermal with a reservoir constraint. Autarky solutions indicated by thin dotted lines.

the left. The marginal cost curve is also anchored on the wall up from *A* and rising to the left, ending on the general left-hand axis when the thermal capacity is exhausted. The autarky period 1- price for Thermal, shown by the dotted line, is lower than the autarky price in Hydro. The situation for Thermal in period 2 is portrayed on the right-hand side of the diagram. The demand curve is anchored at the Hydro bathtub wall up from *D* and falling to the right.The marginal cost curve is also anchored on this wall and rising to the right and ending when capacity is exhausted on the right-hand axis. The autarky period 2 - price is higher than the autarky period price in Hydro. The capacity in Thermal is almost exhausted in period 2.

Opening up for trade we have a common equilibrium price forming for period 1 for Hydro and Thermal. The bathtub wall for period 1 for Hydro gets a horizontal shift to the left, indicated by the dotted vertical line erected from *A'*, equal to the import to Hydro in period 1. The demand curve is correspondingly shifted to the left. The equilibrium price is just slightly lower than the autarky price. What is remarkable is that the water use is changed markedly between the two periods compared with autarky. Now a full reservoir *BC* is transferred to period 2. Since the equilibrium price is slightly lower in period 1, with trade the total electricity consumption in Hydro is also slightly greater. But notice that the use of water in period 1 goes down. In Thermal the price increase and the consumption goes markedly down. But note that total production is increased.

The autarky price for Thermal in period 2 suggested export possibilities for Hydro since the Hydro autarky price was considerably lower. A maximal amount in the reservoir is now saved for use in period 2. The common equilibrium price in period 2 is found after shifting, for Hydro, the demand curve and bathtub wall from the right-hand bathtub wall erected from *D* to the left, indicated by the dotted vertical line from *D'*. The horizontal shift is equal to the export of hydropower to Thermal. Then the price is determined by the intersection of the shifted demand curve and the broken line erected from *B* representing the maximal reservoir, and marking the start of water available for period 2. The difference in prices between the two periods is expressed by the shadow price γ_1 on the reservoir constraint. The price in period 2 in Thermal falls compared with the autarky price resulting in less production, but the total consumption increases due to import from Hydro. This may indicate a long-term benefit for Thermal, since expanding capacity may be postponed. For Hydro we note that the equilibrium price is higher than the autarky price, leading to lower electricity consumption with trade, i.e., less water is used at home due to export.

The trade benefits Thermal in period 2 with lower price and higher consumption compared with autarky. In period 1 the pattern is reversed. Since the trades are almost equal in the example, Thermal gets a deficit on the electricity trade, and Hydro a corresponding surplus since the equilibrium price is lower when Thermal exports than when it imports, and vice versa for Hydro.

We did not impose constraints on production and internal country transmission capacity in the model above. We can use Figure 2 to indicate possible influences of such constraints when they are binding. If Thermal has a domestic transmission network constraint that does not allow the full consumption in period 2 as shown (due to the fact that total consumption now is greater than the production in Thermal) then the constraint will force a lower consumption, lower import, and a higher price in period 2. The prices will now differ between the countries in period 2. Hydro will export less. The motivation for storing maximal water in period 1 is weakened, and the constraint may lead to the reservoir storage not being completely filled. The implication is that Hydro may consume more water in both periods; the equilibrium price in period 1 will decrease and reduce the export from Thermal and increase consumption there. If the available water in period 2, using the greatest share of total water, is greater than what can be produced in Hydro, exports in period 2 is reduced and we get more water used in period 1 lowering the common equilibrium price and lowering export from Thermal and total production, while the price in period 2 will increase. Thus the price difference between the periods widens.

Day trade

The day trade between Norway and Denmark is often mentioned as an example of gains by trade when hydropower with storage is coupled with a thermal system (von der Fehr and Sandsbråten, 1997), and this is also the idea behind launching Norway as the electricity battery of Europe. Norway can import thermal power in the night time and accumulate water in the reservoirs when demand in both countries is low and only the most cost-efficient thermal plants are generating power, and then export hydropower in daytime and save Denmark for taking into use the most cost-inefficient thermal plants. If we think about one hour as the period definition in model (11), Figure 2 may illustrate this development of trade over day and night. If period 1 represents night time and period 2 daytime, then we just have export from Thermal during the night and import to Hydro, accumulating more water than in autarky, and the reverse in daytime: export of hydro and import to Thermal. The two flows are about equal, but the flows may, of course, differ with other configurations. Since more capacity is used in Thermal in night time the marginal cost is pushed up, and there is a reduction in marginal costs during daytime in Thermal due to the lower price. In our example the capacity utilisation in Thermal is about the same for the two periods seen together, but this is just the result of the design of the example.

In the example, water is also used in night time. The condition for not using water at all in such day trade is, inspecting the conditions in (13a), that it is possible to store the total inflow of water to the reservoir in night time, and that the result of trading is a higher price in day time. This may be realistic for a number of periods.

Pumped storage may enhance the scope for day trade. If planned when a hydro plant is constructed the turbine can even be of a type that can be reversed and run as a pump. The general requirement for pump storage to be economic is that there is enough reservoir capacity to receive the pumped water in addition to the period inflow, and that the differences in period prices between night and day are big enough to outweigh efficiency losses in lifting water up to the reservoir and releasing water on to the turbine again.

Unregulated hydro and wind power

It is usually the case that it is not possible to regulate all hydropower. A part of the capacity is based on rivers with no or quite limited storage capacity of water. This is also the case for Norway. The consequence of unregulated hydro is that this power has to be used in order not to loose the electricity production. Storable hydro will have to adapt to the natural variation in unregulated hydro to the extent reservoir capacity allows. Unregulated hydro leads to greater price variations in the short run, and without external trade possibilities may contribute to especially low price in periods with low demand. There are capacity limits on the unregulated power, but this is unnecessary to bring into the model.

Denmark, that is our example of a thermal country, also has a sizeable share of wind power (19%). This is also intermittent, and in order not to be lost when increasing other generating capacity has to be ramped down, and corresponding ramping up when the wind subsides. In

Nord Pool this regulation is most economically done by storable hydropower. Wind power is, as hydro power, characterised by so small variable costs that these can be set to zero. Maintenance can be regarded as fixed costs. There is an upper limit on wind power given by the maximal wind speed that the mills can operate under and the maximal duration of this wind condition. As for unregulated hydro power we can neglect this capacity constraint.

It is straightforward to incorporate these two generating capacities into our trading model framework. Let us denote unregulated hydro production by $e_t^{H,u}$ and wind power by $e_t^{Th,w}$. These exogenous (and known) quantities will appear in their respective energy balances. Extending eq. (10) we have:

$$\begin{aligned} x_t^H &= e_t^H + e_t^{H,u} + e_{Th,t}^{XI} - e_{H,t}^{XI} \\ x_t^{Th} &= e_t^{Th} + e_t^{Th,w} + e_{H,t}^{XI} - e_{Th,t}^{XI}, \ t = 1,...,T \end{aligned} \tag{15}$$

The system of first-order conditions (13a,b) remains unaffected by the unregulated power terms. It is only the level of solutions for the endogenous variables that will change.

The effects on the objective function in (11) of marginal changes in unregulated electricity can be found straightforwardly by applying the envelope theorem, assuming that we have an optimal solution. We have by differentiating the Lagrangian function in (12) partially:

$$\frac{\partial L}{\partial e_t^{H,u}} = p_t^H(x_t^H)\ ,\ \frac{\partial L}{\partial e_t^{Th,w}} = p_t^{Th}(x_t^{Th}), t = 1,...,T \tag{16}$$

The value of a marginal increase (decrease) in the unregulated electricity production is equal to the market price in each country. The connection between the country prices remains the same as discussed above.

An increase in unregulated power will *cet. par.* lead to a lower price and a higher quantity. Using Figure 2 and considering only an increase in wind power in the thermal country in period 1, the vertical energy bathtub wall on the left-hand side will move to the left, and so will the thermal marginal cost curve. Thus there is a tendency for increased export from Thermal in period 1, but this can only happen if the price in hydro is reduced, so the common equilibrium price in the two countries will be lower. Hydro continues to save the maximal amount of water to period 2. If unregulated hydro increase in the first period we have *cet. par.* that the equilibrium price will be lower, thus leading to less import from Thermal and a lower production, but higher consumption since the price is lower. This way of reasoning can be extended to cover changes in period 2 and scenarios of change in both types of unregulated power. If increases or decreases happen at the same time we will obviously have greater price variations between periods.

Climate gas emissions

An especially interesting effect of trade in these times of awareness of climate change is the consequence for total emissions. In our example above illustrated in Figure 2, trade leads to increased thermal production in period 1, but reduced production in period 2 with about the same production. If we assume that high-cost, peak-load thermal capacity has higher emission factors than base-load capacity, then the total emissions will be reduced. Less fluctuation in the short run, e.g. a daily cycle, will also contribute to reducing emissions even for the same total electricity production due to higher emission factors when starting up or closing down coal-fired plants (Rosnes, 2008). However, if gas-based thermal capacity constitutes peak load, this capacity has lower emission factor than base load that would most likely be coal-based plants. The emissions then increase. This is the result obtained in Noorland (2007), simulating scenarios for the introduction of a cable between Norway and the Netherlands, NorNed, which came on in 2008. (See also Holland and Mansur, 2006).

In the case of existence of unregulated hydro and wind power there may be a tendency for increased variation in the price level in the countries in the short run. To the extent that regulated hydro cannot be a perfect swing producer and even out all price differences, then utilisation of thermal capacity will also fluctuate and have the potential of increasing emissions due to ramping up or down. These ramping effects will be offset by volume effects if unregulated power increases, but the opposite happens when unregulated power decreases.

4. Conclusions

It is important to realise that the results we have obtained for prices and quantities when trading across national borders within a stylised model of a social planner seeking common optimal solutions, will, subject to some conditions (Førsund, 2007), also represent the pricing and quantity rules in a competitive market. We have firmly established the results that opening up for trade between two countries leads to equalisation of prices for the same time period when no constraint on the interconnector capacity appears; the law of one price when a homogenous good is traded. Exhausting the thermal capacity does not change this and neither will constraining hydro output. But introducing internal transmission constraints will, if binding, affect the price levels in the same way as the constraint on the interconnector does above. When transmission constraints affect trade flows, export price will be lower than import prices. This result also holds for regional trade flows. A reasonable conjecture is that this result is also valid in a model with more countries.

The Nord Pool market is a wholesale market. There may not be a similar law of one price in the retail market due to different organisational and regulatory set-ups in the Nordic countries (Amundsen and Bergman, 2007).

The total hydro output within the horizon is given. Thus trade implies a shifting of the hydro production over the periods compared with the autarky situation. The thermal output will adjust accordingly and in general the variation in capacity utilisation will be more even. The change in the electricity consumption in each country in each period follows the sign in the change of prices. However, consumption of electricity may go down in the exporting country in such a period as a consequence of more equal prices also over time.

When talking about the Nord Pool market the viewpoint that marginal thermal capacity determines the market price is often encountered. However, this is not how a simultaneous equilibrium solution should be interpreted. The water value is equal to the marginal cost of thermal in equilibrium. Another matter is that shift in marginal costs of thermal, e.g., introduction of carbon quotas on emissions, will generate a new equilibrium price. But so will variations in inflows to hydro reservoirs. In Figure 2 these events may be simulated by shifting marginal cost curves for Thermal and shifting the length of the bathtub floor for Hydro.

In the case autarky prices differ between countries, there are in general gains by trade. However, this is subject to the standard caveat that redistribution may be necessary to realise benefits for all participating countries within reasonable time periods. Opting for a common wholesale-market solution like in Nord Pool no specific redistribution mechanism is used outside the market. However, as to domestic acceptance of opening up for trade in electricity within the European Union, the distributional problem may be a discussion point. Some industries may loose access to cheaper electricity than foreign competitors enjoy. However, increased security of supply realised by extending integrated electricity markets is a public good for all parties.

The analysis underlined that it is crucial for obtaining price equalisation between countries on the average to invest sufficiently in interconnector capacities. It is a question how decisions are made to expand commonly used interconnector capacity. In principle a system-wide cost-benefit analysis should be carried out and costs distributed in such a way that demand is not unduly influenced once capacities are in place.

The challences of incorporating unregulated hydro and wind power is better seen when this generation is modelled as stochastic. Especially wind power has a relatively large variability that it is difficult to predict even in the short run. Both inflows to regulated hydro and demand are depending on weather conditions, and incorporating all this uncertainty is a challenging task.

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