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Modelling Addiction in Life-Cycle Models: Revisiting the Treatment of Latent Stocks and Other Unobservables

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MODELLING ADDICTION IN LIFE-CYCLE MODELS: REVISITING THE TREATMENT OF LATENT STOCKS AND OTHER UNOBSERVABLES

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Abstract: Dynamic modeling of demand for goods whose cumulated stocks enter an intertemporal utility function as latent variables, is discussed. The issues include: how represent addiction, how handle unobserved expectations and changing plans, how deal with 'dynamic inconsistency'? Arguments are put forth to give all optimizing conditions attention, not only those in which all variables are observable. If the latter, fairly common, 'limited information-reduced dimension' strategy is pursued, problems are shown to arise in attempting to identify coefficients of the preference structure and to test for addictive stocks. Examples, based on quadratic utility functions, illustrate the main points and challenge the validity of testing the 'rational addiction' hypothesis, by using linear, single-equation autoregressive models, as suggested by Becker, Grossman, and Murphy (1994) and adopted in several following studies.

Keywords: Life-cycle model. Addiction. Identification. Latent stocks. Perfect foresight. Rational expectations. Dynamic inconsistency.

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1 Introduction

In modelling demand for commodities like tobacco, alcohol, caffeine, and drugs, but also certain other goods and even for services, addiction reflecting cumulated past consumption receives may be important. A characteristic of addictive goods is the increase in current consumption induced by an increase in past consumption. Such stock variables are rarely observable to the econometrician in his endeavour to quantify the theory by estimating equations connecting observed variables. In a dynamic model of individual behaviour, habit formation in general and addiction in particular may be represented by a time-varying stock variable representing the individual's consumption history [cf., *e.g.*, Lluch (1974), Dixon and Lluch (1977), Becker and Murphy (1988), and Wangen (2004)]. However, unobserved habit effects can also be considered individual 'properties', represented within a static model by individual specific latent variables [cf. Wangen and Biørn (2001)].

This paper discusses critically the *translation* of a dynamic theory-model containing unobservable variables into an econometric model expressed in terms of observable variables. Of particular concern will be the translation of a life-cycle model with unobserved stocks and flows into an econometric model to be confronted with time-series or panel data. Two approaches are available: (a) reduce the model's dimension by disregarding all equations which contain unobserved variables, or (b) retain the model's size and the latent variables in the structure and represent them by additional relationships which connect the latent variables to observed ones. The frequently used 'Euler equation approach' exemplifies (a), Imposing perfect foresight or rational expectations exemplifies (b). Previous related literature includes: (i) Becker, Grossman, and Murphy (1994) ['rational addiction' for cigarettes analyzed by time-series data], (ii) Labeaga (1999), Wangen and Biørn (2001), Baltagi and Griffin (2001, 2002), Bretteville-Jensen and Biørn (2003), and Jones and Labeaga (2003) [similar studies for cigarettes, alcoholic beverages, and heroin, using panel data for micro or more aggregate units], (iii) Pollak (1970), Phlips (1972) and Pashardes (1986) [myopic behaviour formalized through one-period utility functions; (Pashardes also considering forward-looking behaviour), incorporating stocks or habit formation], and (iv) Diewert (1974), and Muellbauer (1981) [inter-temporal models with stocks of consumer 'durables'].

Translating neo-classical demand theory for goods subject to habit formation into an econometric model raises several questions: First, should one-period (myopic) or multi-period (dynamic) utility functions be used, and in the latter case, how represent addiction? Second, should geometric discounting, starting from any decision time, be assumed? Third, should perfect foresight be assumed or should expectations (exogenous) and plans (endogenous) be revised as new information becomes available, and if so, how should unobserved expectations, plans, and addictive stocks be represented? Fourth, should the econometric implementation be based on a complete system with as many equations as endogenous variables, or would a 'limited information' approach, picking a subset of equation with mainly observable variables, be sufficient?

In this paper, we reconsider some of these questions. We depart from previous literature on addiction and habit formation at crucial points: First, we abandon perfect foresight, *i.e.*, allow for revisions of expectations and plans at any time. Second, the complete equation system as derived from the optimizing conditions is in focus. Variables with leads have two time indices: the current decision time and the future time of realization. We argue that this is a recommendable procedure even though observations on such double-indexed variables are rare. The length of the consumer's horizon relative to the length of the time series becomes important. A perfect foresight approach may, under reasonable assumptions, lead to an overdetermined equation system when all theory-restrictions are exploited.

We further demonstrate that an equation with lags and leads in X_t, say the purchases of the addictive good cigarettes in period t of the form

$$X_t = a_{-1}X_{t-1} + a_{+1}X_{t+1} + \text{price terms} + \text{intercept} + \text{disturbance},$$

as considered in Becker, Grossman, and Murphy (1994), and reexamined by improved econometric methods in Labeaga (1999), Baltagi and Griffin (2001, 2002), and Jones and Labeaga (2003), can be given widely different interpretations relative to an underlying structural model. In other words, we argue that *identification problems* prevail. Using five examples, we discuss which specific parts of the model contribute to the lags and the leads. This casts doubts on the attraction of testing 'rational-addiction' life-cycle theories by means of the kind of equations used in the mentioned articles.

The rest of the paper is disposed as follows: In Section 2 the theoretical model and its three elements, the utility function, the accumulation process for the addictive stock and the intertemporal budget constraint, are presented. In Section 3 we take the first steps towards an econometric model version, by formulating the equation system which represents the optimizing conditions from the theory, by means of lag- and lead-polynomials and a general form for the instantaneous utility function. Next, in Section 4, five examples of increasing complexity, based on a quadratic utility function are considered, the first two being baseline examples with a degenerate dynamic structure. In all cases except the very simplest ones, autoregressive equations in the purchase of the addictive good arise. The lessons we draw from these examples are that there are obvious problems in distinguishing econometrically between rational addiction theory-models and other less strongly theory-based dynamic models of the purchases. Section 5 concludes.

2 Theoretical framework

A consumer with a *horizon of H periods*, who is in period t, 'today', is considered. The model describing his behaviour has three elements.

First, the preference ordering is represented by a *utility function* which is additive in period utilities. The instantaneous utility, is a twice differentiable concave function, $U(C_t, X_t, S_t)$, in consumption of a non-addictive good, C_t, consumption of an addictive good, X_t, and the stock of the latter as perceived in period t, S_t. The prospective utility in period $t+i$, as seen from period t, $U(C_{t,t+i}, X_{t,t+i}, S_{t,t+i})$, depends on planned consumption of the non-addictive good, $C_{t,t+i}$, and of the addictive good, $X_{t,t+i}$, as well as the stock of the latter, $S_{t,t+i}$. The two time subscripts denote the current period and a future period to which the plans refer. Further, $\beta_{t,i}$ is the subjective discounting factor by means of which utility in period t is discounted i periods ahead. Letting $\rho_{t,j}$ be the subjective interest rate for utility in period $t+j$ $(j = 0, 1, 2, \dots)$, we can write $\beta_{t,i} = \prod_{j=0}^{i-1}(1+\rho_{t,j})^{-1}$ $(i = 1, \dots, H)$, $\beta_{t,0} = 1$. The *multi-period utility* then is

$$V_t = U(C_t, X_t, S_t) + \sum_{i=1}^{H} \beta_{t,i} U(C_{t,t+i}, X_{t,t+i}, S_{t,t+i}). \quad (1)$$

Second, the *accumulation of the addictive good* is described as a weighted sum of past purchases as follows: Let $d_{t,j}$ (≥ 0) be the share of the purchase of the addictive good with contributes to the stock giving utility j periods later, the *survival function* for short. In particular, $d_{t,0}$ may be zero, so that the addictive stock, $S_{t,t+i}$, or addiction, is determined by decisions made before time $t+i$.[1] Hence,

$$S_{t,t+i} = \sum_{j=0}^{\infty} d_{t,j} X_{t,t+i-j} = \sum_{j=0}^{i} d_{t,j} X_{t,t+i-j} + \bar{S}_{t,t+i}, \quad (2)$$

where $\sum_{j=0}^{i} d_{t,j} X_{t,t+i-j}$ represents the part of the addictive stock that is determined by the purchase flow in the time interval $[t, t+i]$, and

$$\bar{S}_{t,t+i} = \sum_{j=i+1}^{\infty} d_{t,j} X_{t+i-j}, \qquad i = 0, 1, \dots, H,$$

is *predetermined*. Addiction can be said to occur whenever $d_{t,0} \geq 0$, $d_{t,1} > 0$. For $i = 0$ we have

$$S_t = d_{t,0} X_t + \bar{S}_{t,t} = d_{t,0} X_t + \sum_{j=1}^{\infty} d_{t,j} X_{t-j}.$$

Equation (2) includes as special cases several formulations in the literature, not only for allegedly addictive goods, but also for (services from) 'durable' goods. One parametrization is geometric decay: $d_{t,j} = (1-\delta_t)^j$, $\delta_t \in (0,1)$, a second is 'hyperbolic discounting' [see, *e.g.*, Laibson (1997), Azfar (1998), Harris and Laibson (2003), Diamond and Köszegi (2003)], a third is the simplistic specification of Becker, Grossman, and Murphy (1994): $d_{t,0} = 1$, $d_{t,1} = d_1 > 0$, $d_{t,2} = d_{t,3} = \cdots = 0$. The latter implies

$$S_t = X_t + d_1 X_{t-1},\ S_{t,t+1} = X_{t,t+1} + d_1 X_t,\ S_{t,t+i} = X_{t,t+i} + d_1 X_{t,t+i-1},\ i = 2, \dots, H,$$

[1] Usually, the *survival function* $d_{t,j}$ is non-negative and non-increasing in j, but these assumptions are not obvious. For instance $d_{t,j+1} > d_{t,j}$ would represent a case where a purchase made $j+1$ periods ago has stronger impact on current addiction than has a purchase made j periods ago.

so that (1) takes the form

$$V_t = U^*(C_t, X_t, X_{t-1}) + \sum_{i=1}^{H} \beta_{t,i} U^*(C_{t,t+i}, X_{t,t+i}, X_{t,t+i-1}),$$

assuming that the parameters of the survival function are absorbed in the functional form U^*.

The third element is the *intertemporal budget constraint.* We assume, for simplicity and without essentially restricting the argument, that both goods have prices equal to one in all periods. Time-varying relative prices can be allowed for by a straightforward extension, attaching prices to the C and X variables.

Let $W_{t,t+H}$ be the difference between terminal wealth and initial wealth, as planned at time t, let $I_{t,t+i}$ be the exogenous (non-wealth) income and the interest income accruing from the initial wealth which in period t is expected for period $t{+}i$, and let $r_{t,j}$ be the one-period interest rate which in period t is expected for period $t{+}j$ and $\alpha_{t,i} = \prod_{j=0}^{i-1}(1{+}r_{t,j})^{-1}$. In Appendix A it is shown that the full equation system derived from utility maximization becomes

$$
\begin{aligned}
&(3) && C_t + X_t + \sum_{j=1}^{H} \alpha_{t,j}(C_{t,t+j} + X_{t,t+j}) + \alpha_{t,H} W_{t,t+H} = I_t + \sum_{j=1}^{H} \alpha_{t,j} I_{t,t+j},\\
&(4) && U_{C,t} = \beta_{t,1}\alpha_{t,1}^{-1} U_{C,t,t+1} = \beta_{t,2}\alpha_{t,2}^{-1} U_{C,t,t+2} = \cdots = \beta_{t,H}\alpha_{t,H}^{-1} U_{C,t,t+H},\\
&(5) && U_{C,t} - U_{X,t} - U_{S,t} = \sum_{j=1}^{H} \beta_{t,j} d_{t,j} U_{S,t,t+j},\\
&(6) && \beta_{t,i}(U_{C,t,t+i} - U_{X,t,t+i} - U_{S,t,t+i}) = \sum_{j=1}^{H-i} \beta_{t,i+j} d_{t,j} U_{S,t,t+i+j}, \quad i{=}1,\dots,H,\\
&(7) && S_t = d_{t,0} X_t + \bar{S}_{t,t},\\
&(8) && S_{t,t+i} = \sum_{j=0}^{i-1} d_{t,j} X_{t,t+i-j} + d_{t,i} X_t + \bar{S}_{t,t+i}, \qquad i{=}1,\dots,H,
\end{aligned}
$$

where $U_{Q,t} = \partial U(C_t, X_t, S_t)/\partial Q_t$, $U_{Q,t,t+i} = \partial U(C_{t,t+i}, S_{t,t+i})/\partial Q_{t,t+i}$ $(Q = C, X, S)$. Equation (3) is the budget constraint, (4) connects the marginal utilities of ordinary non-addictive consumption in the current and future periods, (5) and (6) connect the marginal utilities of ordinary consumption with the marginal utilities of the addictive good, and (7) and (8) connect the time path of the addictive stocks to past expenditures. Altogether, (3)–(8) determine – for given income path I_t, $\{I_{t,t+i}\}_{i=1}^{i=H}$, discount factors $\{\alpha_{t,i}\}_{i=1}^{i=H}$, and terminal wealth $W_{t,t+H}$ – the consumption and the addictive stock *in the current period* t, (C_t, X_t, S_t) as well as their planned time paths $\{C_{t,t+i}, X_{t,t+i}, S_{t,t+i}\}_{i=1}^{i=H}$ thereafter – in total $3(H{+}1)$ variables. Equations (3)–(8) will form the fundament of the econometric model.

3 Towards an econometric model formulation

A reformulation using lead- and lag-polynomials

We introduce the *backward shift (lag) operator*, B, and the *forward shift (lead) operator*, F, defined by, respectively, $\mathsf{B}^n C_t = C_{t-n}$ $(n = 1, 2, \dots,\ )$, and $\mathsf{F}^m C_t = C_{t,t+m}$ $(m = 1, 2, \dots\ )$. Notice that F^m defines a variable with two time subscripts;

F^m is not the same as B^{-m}. The following lead- and lag-polynomials will be needed:[2]

$$
(9)\qquad
\begin{aligned}
&\phi_t(\mathsf{F}) = \alpha_{t,1}\mathsf{F} + \alpha_{t,2}\mathsf{F}^2 + \cdots + \alpha_{t,H}\mathsf{F}^H, \\
&\psi_{t,0}(\mathsf{F}) = \beta_{t,1}d_{t,1}\mathsf{F} + \beta_{t,2}d_{t,2}\mathsf{F}^2 + \cdots + \beta_{t,H}d_{t,H}\mathsf{F}^H, \\
&\psi_{t,i}(\mathsf{F}) = \frac{\beta_{t,i+1}}{\beta_{t,i}}d_{t,1}\mathsf{F} + \frac{\beta_{t,i+2}}{\beta_{t,i}}d_{t,2}\mathsf{F}^2 + \cdots + \frac{\beta_{t,i+H}}{\beta_{t,i}}d_{t,H-i}\mathsf{F}^{H-i}, \qquad && i=1,\dots,H, \\
&\zeta_{t,i}(\mathsf{F}) = d_{t,0}\mathsf{F}^i + d_{t,1}\mathsf{F}^{i-1} + d_{t,2}\mathsf{F}^{i-2} + \cdots + d_{t,i-1}\mathsf{F}, && i=1,\dots,H, \\
&\xi_{t,0}(\mathsf{B}) = d_{t,1}\mathsf{B} + d_{t,2}\mathsf{B}^2 + \cdots, \\
&\xi_{t,i}(\mathsf{B}) = \frac{d_{t,i+1}}{d_{t,i}}\mathsf{B} + \frac{d_{t,i+2}}{d_{t,i}}\mathsf{B}^2 + \cdots && i = 1,2,\dots .
\end{aligned}
$$

Substituting $\eta_{t,i} = \beta_{t,i}\alpha_{t,i}^{-1}$ and using (9), we can rewrite (3)–(8) as

$$(10)\qquad (C_t + X_t - I_t) + \phi_t(\mathsf{F})(C_t + X_t - I_t) + \alpha_{t,H}\mathsf{F}^H W_t = 0,$$

$$(11)\qquad U_{C,t} = \eta_{t,1}\mathsf{F}U_{C,t} = \eta_{t,2}\mathsf{F}^2 U_{C,t} = \cdots = \eta_{t,H}\mathsf{F}^H U_{C,t},$$

$$(12)\qquad U_{C,t} - U_{X,t} - U_{S,t} = \psi_{t,0}(\mathsf{F})U_{S,t},$$

$$(13)\qquad \mathsf{F}^i(U_{C,t} - U_{X,t} - U_{S,t}) = \mathsf{F}^i\psi_{t,i}(\mathsf{F})U_{S,t},$$

$$(14)\qquad S_t = d_{t,0}[X_t + \xi_{t,0}(\mathsf{B})X_t],$$

$$(15)\qquad \mathsf{F}^i S_t = \zeta_{t,i}(\mathsf{F})X_t + d_{t,i}[X_t + \xi_{t,i}(\mathsf{B})X_t], \qquad i=1,\dots,H.$$

Note that:
[1] The coefficients of $\psi_{t,0}(\mathsf{F})$ and $\psi_{t,i}(\mathsf{F})$ depend on both the subjective discounting factors and the survival function of the stock.
[2] $\psi_{t,i}(\mathsf{F})$ and $\zeta_{t,i}(\mathsf{F})$ have $H-i$ and i terms, respectively.
[3] If the stock remembers purchases P periods backwards (P finite): $d_{t,j}=0$ $(j>P)$, then $\xi_{t,i}(\mathsf{B})$ has $P-i$ terms.

> EXAMPLE: In the special case with *geometric, time invariant depreciation and geometrically declining discount factors and infinite horizon, i.e.,*
>
> $\alpha_{t,i} = \alpha^i$ $(0 \le \alpha < 1)$, $\beta_{t,i} = \beta^i$ $(0 \le \beta < 1)$, $d_{t,i} = d^i$ $(0 \le d < 1)$ $\forall\, i\,\&\, t$; $H \to \infty$,
>
> the polynomials in (9) are simplified to
>
> $$\begin{aligned} \phi_t(\mathsf{F}) &= \phi(\mathsf{F}) = \textstyle\sum_{j=0}^{\infty}\alpha^j\mathsf{F}^j = \frac{\alpha\mathsf{F}}{1-\alpha\mathsf{F}}, \\ \psi_{t,i}(\mathsf{F}) &= \psi(\mathsf{F}) = \textstyle\sum_{j=0}^{\infty}(\beta d)^j\mathsf{F}^j = \frac{\beta d\mathsf{F}}{1-\beta d\mathsf{F}}, \\ \xi_{t,i}(\mathsf{B}) &= \xi(\mathsf{B}) = \textstyle\sum_{j=0}^{\infty} d^j\mathsf{B}^i = \frac{d\mathsf{B}}{1-d\,\mathsf{B}}, \\ \zeta_{t,i}(\mathsf{F}) &= \zeta_i(\mathsf{F}) = \textstyle\sum_{j=0}^{i-1} d^j\mathsf{F}^{i-j} = \frac{\mathsf{F}^i(1-d\mathsf{F}^{-i})}{1-d\mathsf{F}^{-1}}. \end{aligned}$$

[2]Their t-subscripts indicate that both sequences of discounting factors as well as the weights of the survival function of the addictive stock are allowed to change over time. The lead polynomial $\phi_t(\mathsf{F})$ generates the budget constraint, $\psi_{t,0}(\mathsf{F})$ and $\psi_{t,i}(\mathsf{F})$ generate the relationships between the marginal utilities for the consumption flows C and X and the stock S. The lag polynomials $\xi_{t,0}(\mathsf{B})$ and $\xi_{t,j}(\mathsf{B})$ generate the relationships between addictive stocks and previous expenditures, whereas $\zeta_{t,j}(\mathsf{F})$ describes the dependence of the stock planned in period $t+i$ on the expenditures in periods $t+1,\dots,t+i$.

The system (10)–(15) contains, for $t = 1, \dots, T$, $3(H+1)$ equations in $Q_t \equiv (C_t, X_t, S_t)$ and $\{Q_{t,t+i}\}_{i=1}^{i=H} \equiv \{C_{t,t+i}, X_{t,t+i}, S_{t,t+i}\}_{i=1}^{i=H}$. If no further restrictions are imposed, we therefore have $T(H+1)$ equations in $(C_t, C_{t,t+i})$, $(X_t, X_{t,t+i})$ and $(S_t, S_{t,t+i})$, *i.e.*, a determined system of $3T(H+1)$ equations:

$$
(16) \qquad \begin{array}{lllll}
Q_1, & Q_{1,2}, & Q_{1,3}, & \dots, & Q_{1,H}, \\
Q_2, & Q_{2,3}, & Q_{2,4}, & \dots, & Q_{2,1+H}, \\
 & \vdots & & & \\
Q_T, & Q_{T,T+1}, & Q_{T,T+2}, & \dots, & Q_{T,T+H}
\end{array} \qquad Q = (C, X, S).
$$

The treatment of unobservable variables

Translating (10)–(15) into a model to be confronted with time series data raises the obvious problem that the agents' income expectations and plans for future consumption and the current and future addictive stocks are both unobserved. The only observable variables are: (I_t, C_t, X_t), $(I_{t+1}, C_{t+1}, X_{t+1})$, $(I_{t+2}, C_{t+2}, X_{t+2}), \dots$. The double-indexed variables are only (at best) 'in the mind of' the consumer and unobservable to the econometrician.

The $3(H+1)$ equations in (10)–(15), for any current time t, *have different econometric status*. All of them contain at least one unobservable variable: (i) Equation (14), although having only current and lagged variables, cannot be estimated since its left-hand side variable is unobservable. (ii) The H equations in (13) contain *only* leads, all of them unobservable. (iii) The $2(H+1)$ equations in (10)–(12) and (15) combine leads, current values and lags, and hence include both observable and unobservable variables.

In econometric studies of addiction based on life-cycle theory, it is almost always assumed that the consumer (i) has perfect foresight with respect to the exogenous variables, which *annihilates the first subscript on the interest factors* $\alpha_{t,i}$, and (ii) always sticks to his original plan [see Becker, Grossman, and Murphy (1994) and the successors mentioned above]. Formally, (i)–(ii) may be interpreted as *imposing additional restrictions which annihilates the first subscript on* $(C_{t,t+i}, X_{t,t+i}, S_{t,t+i})$. That this practice is so common is surprising, in view of the vast econometric literature that exists on the modelling of agents' formation of expectations and on the modelling of equation systems with *latent structural variables*. It may indeed be appealing to formally consider all variables in (10)–(15) with two subscripts as latent.

Before addressing the question of how to model the latter variables we consider *implications of the perfect foresight assumption*. We have assumed that the consumer's planning horizon spans H periods *from any current time* t. The econometrician's sample period may be longer or shorter than the length of the consumers' horizon. It is not unlikely that drug-addicted persons, in particular, have short horizons. On the other hand, the time window of time series data, and especially panel data, may be short. So both $T > H$ and $T < H$ should be of interest.

Perfect foresight and overdeterminatedness

If (10)–(15) are (mathematically) *independent* equations and if we also impose perfect foresight, we get an *overdetermined equation system for any* $H > 1$. This conclusion has crucial econometric implications. Perfect foresight when $T > H$ is different from perfect foresight when $T < H$, as it implies (see Appendix B):

$$\text{If } T > H: \quad \begin{cases} Q_{1,2} = Q_2, \\ Q_{1,3} = Q_{2,3} = Q_3, \\ \quad \vdots \\ Q_{1,H} = Q_{2,H} = Q_{3,H} = \cdots = Q_{H-1,H} = Q_H, \\ Q_{2,H+1} = Q_{3,H+1} = Q_{4,H+1} = \cdots = Q_{H,H+1} = Q_{H+1}, \\ Q_{3,H+2} = Q_{4,H+2} = Q_{5,H+2} = \cdots = Q_{H+1,H+2} = Q_{H+2}, \\ \quad \vdots \\ Q_{T-H+1,T} = Q_{T-H+2,T} = Q_{T-H+3,T} = \cdots = Q_{T-1,T+2} = Q_T. \end{cases}$$

$$\text{If } T < H: \quad \begin{cases} Q_{1,2} = Q_2, \\ Q_{1,3} = Q_{2,3} = Q_3, \\ \quad \vdots \\ Q_{1,T} = Q_{2,T} = Q_{3,T} = \cdots = Q_{T-1,T} = Q_T. \end{cases}$$

The conclusion of overdeterminatedness is conceptually closely related to 'dynamic inconsistency' in planning, as expressed as a theoretical problem in discounting 'future pleasures' by a constant interest rate more than fifty years ago:

> "An individual who because he does not discount all future pleasures at a constant rate of interest finds himself continuously repudiating his past plans may learn to distrust his future behaviour, and may do something about it. Two kinds of action are possible. (1) He may try to precommit his future activities either irrevocably or by contriving a penalty for his future self if he should misbehave. This we call the *strategy of precommitment*. (2) He may resign himself to the fact of inter-temporal conflict and decide that his "optimal" plan at any date is a will-o'the-wisp which cannot be attained, and learn to select the present action which will be best in the light of future disobedience. This we call the *strategy of consistent planning*." [Strotz (1956, p. 173)].

How could we in formulating an econometric model treat the overdeterminatedness which follows? In the next section we give examples to clarify this.

4 Four econometric models compared

In this section, we consider *five examples*, of increasing complexity and realism, although with rather simple parametric forms for $U(\cdot)$, $\alpha_{t,i}$, $\beta_{t,i}$, and $d_{t,i}$. The intention is first, to illustrate econometric implementations of the system (10)–(15) and demonstrate that equations containing lag and lead distributions in the expenditures on the addictive good, similar to those considered in Becker, Grossman, and

Murphy (1994), and further examined by improved methods in Labeaga (1999), Baltagi and Griffin (2001, 2002), and Jones and Labeaga (2003), can be given widely different interpretations and hence involve *identification problems*. The second intention is to put into focus from which parts of the model contribute to the lags and the leads. Altogether, these examples cast doubt on the attraction of using equations of the form used in the above mentioned papers for testing 'rational-addiction' life-cycle theories econometrically.

Throughout we make the following assumptions:

(i) $W_{t,t+H}=0$, *i.e.*, terminal wealth equals initial wealth,

(ii) the addictive stock S is unobservable,

(iii) the $\alpha_{t,i}$, $\beta_{t,i}$ and $d_{t,i}$ sequences are invariant to t, and

(iv) quadratic utility functions.

The latter assumption implies that the three marginal utilities which occur in (4)–(6) can be written as:

$$\begin{aligned} U_{Ct} &= a_C + a_{CC}C_t + a_{CX}X_t + a_{CS}S_t, \\ U_{Xt} &= a_X + a_{XC}C_t + a_{XX}X_t + a_{XS}S_t, \\ U_{St} &= a_S + a_{SC}C_t + a_{SX}X_t + a_{SS}S_t, \\ U_{Ct} - U_{Xt} - U_{St} &= A_0 + A_C C_t + A_X X_t + A_S S_t, \end{aligned} \tag{17}$$

where the as are constants ($a_{CX}=a_{XC}, a_{CS}=a_{SC}, a_{SX}=a_{XS}$) and $A_0=a_C-a_X-a_S$ and $A_Q=a_{CQ}-a_{XQ}-a_{SQ}$ ($Q=C,X,S$).[3]

The five examples are:

a: *One-period optimization. Geometric stock.*

b: *Multi-period optimization. Constant interest rates. No stock accumulation.*

c: *Multi-period optimization. Constant interest rates. Geometric stock.*

d: *Multi-period optimization. Variable interest rates. No stock accumulation.*

e: *Multi-period optimization. Variable interest rates. Non-geometric stock.*

Examples **a** and **b** act as baseline cases, being strictly inappropriate for addictive goods. Example **c** generalizes **a** and **b**, **d** generalizes **b**, and **e** generalizes **c** and **d**.

a: One period. Geometric stock

Consider first the simple case with a one-period (myopic) utility function $V_t = U(C_t, X_t, S_t)$ and assume that the survival function of the addictive stock declines geometrically at the rate $1-\delta$ ($0<\delta\leq 1$) over an infinite period, so that δ can be

[3] Several econometric implementations of 'rational addiction' models [*e.g.*, Becker, Grossman, and Murphy (1994), Labeaga (1999), Baltagi and Griffin (2001, 2002) and Jones and Labeaga (2003) rely on assumptions (ii)–(iv), without, however, specifying the point of departure as in (10)–(15).

interpreted as a retirement rate for the stock. Then the $\alpha_{t,i}$s are irrelevant and

$$\begin{aligned} &\beta_{t,0} = 1, \quad \beta_{t,1} = \beta_{t,2} = \cdots = 0, \\ &d_{t,0} = 1, \quad d_{t,j} = (1-\delta)^j \quad (0<\delta\leq 1), \qquad\qquad j=1,2,\dots\ , \end{aligned}$$

so that (9) is simplified to

$$\phi_t(\mathsf{F}) = \psi_{t,i}(\mathsf{F}) = 0, \qquad \xi_{t,i}(\mathsf{B}) = \xi(\mathsf{B}) = \frac{(1-\delta)\mathsf{B}}{1-(1-\delta)\mathsf{B}}, \qquad \forall\, t \,\&\, i,$$

because the lag-polynomial $1-(1-\delta)\mathsf{B}$ is invertible. Then (10)–(15) degenerate to three equations in C_t, X_t, S_t:

$$\begin{aligned} &\text{(a.1)} \qquad && C_t + X_t = I_t, \\ &\text{(a.2)} \qquad && U_{C,t} - U_{X,t} - U_{S,t} = 0, \\ &\text{(a.3)} \qquad && S_t - (1-\delta)S_{t-1} = X_t, \end{aligned}$$

with I_t exogenous and S_{t-1} predetermined. Inserting (17) in (a.2) and adding disturbances $u^b_{1t}, u^a_{2t}, u^a_{3t}$, – letting 'con' symbolize an unspecified constant – we get an equation system with a one-period lag and no leads:

$$\begin{aligned} &\text{(A.1)} \qquad && C_t + X_t = I_t + u^a_{1t}, \\ &\text{(A.2)} \qquad && A_C C_t + A_X X_t + A_S S_t = \text{con} + u^a_{2t}, \\ &\text{(A.3)} \qquad && [1-(1-\delta)\mathsf{B}]S_t = X_t + u^a_{3t}. \end{aligned}$$

Multiplying (A.2) by $[1-(1-\delta)\mathsf{B}]$ and using (A.1) and (A.3), we obtain

$$\begin{aligned} [1-(1-\delta)\mathsf{B}][A_C(I_t - X_t + u^a_{1t}) + A_X X_t] + A_S(X_t + u^a_{3t}) \\ = \text{con} + [1-(1-\delta)\mathsf{B}]u^a_{2t}. \end{aligned}$$

This yields the following first-order autoregressive equation in X, with a one-period lag distribution in I and a disturbance:

$$\text{(A.4)} \qquad \begin{aligned} &(A_X + A_S - A_C)X_t \\ &\quad = (A_X - A_C)(1-\delta)X_{t-1} - A_C I_t + A_C(1-\delta)I_{t-1} + \text{con} + v^a_t, \end{aligned}$$

where $v^a_t = (u^a_{2t} - A_C u^a_{1t}) - (1-\delta)(u^a_{2,t-1} - A_C u^a_{1t-1}) - A_S u^a_{3t}$. Both X_t and X_{t-1} are correlated with the composite disturbance v^a_t, as the latter has a memory of at least one period. If u^a_{1t}, u^a_{2t} or u^a_{3t} have a memory the memory of v^a_t will be longer than the memory of either of them.

Equation (A.4) can be normalized either by division by $(A_X + A_S - A_C)$, to give an equation *with a one-period lag* in X_t, or by division by $(A_X - A_C)(1-\delta)$, to give an equation *with a one-period lead* when we in the latter case increase $(t-1,t)$ to $(t,t+1)$, and put X_t at the left-hand side. Regardless of which normalization we adopt, we will for estimation need an instrument for X_{t-1}, respectively X_{t+1}, in view of the memory of v^a_t.

If the geometric stock accumulation equation (A.3) had been generalized to a rational lag distribution, say $\xi_S(\mathsf{B})S_t = \xi_X(\mathsf{B})X_t + u^a_{3t}$, where $\xi_S(\mathsf{B})$ and $\xi_X(\mathsf{B})$ are finite-order lag-polynomials, the former invertible, we would obtain

$$\xi_S(\mathsf{B})[A_C(I_t - X_t + u^a_{1t}) + A_X X_t] + A_S[\xi_X(\mathsf{B})X_t + u^a_{3t}] = \text{con} + \xi_S(\mathsf{B})u^a_{2t}.$$

This would expand (A.4) to a higher-order autoregressive equation in X, with a disturbance process with longer memory.

Hence we can conclude: *A multi-period horizon is not necessary for obtaining an autoregressive equation in the purchase of the addictive good, of order at least two.*

b: Multi-period. Constant interest rates. No stock

Next we assume an arbitrary, *finite* horizon, but let the stock evaporate in only one period, so that $S_t = X_t$ ($\Longrightarrow \mathsf{F}^i S_t = \mathsf{F}^i X_t\ \forall i$). Also, we assume that $\alpha_{t,i}$ and $\beta_{t,i}$ are geometrically declining at the rates $(1+r)^{-1}$ and $(1+\rho)^{-1}$, respectively, so that

$$\begin{array}{lll}
\alpha_{t,i} = \alpha^i, & \alpha = (1+r)^{-1} \quad (r > 0), & \\
\beta_{t,0} = 1, \quad \beta_{t,j} = \beta^j, & \beta = (1+\rho)^{-1} \quad (\rho > 0), & \\
\eta_{t,j} = \eta^j, & \eta = (1+r)/(1+\rho), & \\
d_{t,0} = 1, \quad d_{t,j} = 0, & & j = 1, \dots, H.
\end{array}$$

Then (9) degenerates to

$$\begin{array}{ll}
\phi_t(\mathsf{F}) = \phi(\mathsf{F}) = \sum_{j=1}^{H} \alpha^j \mathsf{F}^j = \frac{\alpha\mathsf{F}(1-\alpha^H\mathsf{F}^H)}{1-\alpha\mathsf{F}}, & \\
\psi_{t,i}(\mathsf{F}) = 0, & i = 0, 1, \dots, H, \\
\zeta_{t,i}(\mathsf{F}) = 0, & i = 1, \dots, H, \\
\xi_{t,0}(\mathsf{B}) = 0, \quad \xi_{t,i}(\mathsf{B}) \text{ undefined and irrelevant}, & i = 1, \dots, H,
\end{array}$$

and (10)–(15) collapse into $3(H+1)$ equations in $(C_t, X_t, S_t), \{C_{t,t+i}, X_{t,t+i}, S_{t,t+i}\}_{i=1}^{i=H}$:

$$\begin{array}{lll}
\text{(b.1)} & (C_t + X_t - I_t) + \sum_{j=1}^{H} \alpha^j \mathsf{F}^j (C_t + X_t - I_t) = 0, & \\
\text{(b.2)} & U_{C,t} = \eta^i \mathsf{F}^i U_{C,t}, & \\
\text{(b.3)} & U_{C,t} - U_{X,t} - U_{S,t} = 0, & \\
\text{(b.4)} & \mathsf{F}^i [U_{C,t} - U_{X,t} - U_{S,t}] = 0, & \\
\text{(b.5)} & S_t = X_t, & \\
\text{(b.6)} & \mathsf{F}^i S_t = \mathsf{F}^i X_t, & i = 1, \dots, H.
\end{array}$$

Inserting (17) in (b.2)–(b.4) and adding disturbances $u^b_{1t}, u^b_{2it}, u^b_{3t}, u^b_{4it}, u^b_{5t}, u^b_{6it}$ gives

$$\begin{array}{lll}
\text{(B.1)} & C_t + X_t - I_t + \sum_{j=1}^{H} \alpha^j (C_{t,t+j} + X_{t,t+j} - I_{t,t+j}) = u^b_{1t}, & \\
\text{(B.2)} & a_{CC}C_t + a_{CX}X_t + a_{CS}S_t = \eta^i [a_{CC}C_{t,t+i} + a_{CX}X_{t,t+i} + a_{CS}S_{t,t+i}] + \text{con} + u^b_{2it}, & \\
\text{(B.3)} & A_C C_t + A_X X_t + A_S S_t = \text{con} + u^b_{3t}, & \\
\text{(B.4)} & A_C C_{t,t+i} + A_X X_{t,t+i} + A_S S_{t,t+i} = \text{con} + u^b_{4it}, & \\
\text{(B.5)} & S_t = X_t + u^b_{5t}, & \\
\text{(B.6)} & S_{t,t+i} = X_{t,t+i} + u^b_{6it}, & i = 1, \dots, H.
\end{array}$$

This equation system has H leads and no lag.

From (B.1)–(B.4), after elimination of S_t, S_{t+1} by using (B.5)–(B.6), we get

$$
\text{(B.7)} \qquad
\begin{aligned}
& C_t + X_t - I_t + \textstyle\sum_{j=1}^{H} \alpha^j (C_{t,t+j} + X_{t,t+j} - I_{t,t+j}) = u^b_{1t}, \\
& a_{CC} C_t + (a_{CX} + a_{CS}) X_t + a_{CS} u^b_{5t} \\
& \quad = \eta^i [a_{CC} C_{t,t+i} + (a_{CX} + a_{CS}) X_{t,t+i} + a_{CS} u^b_{6it}] + \text{con} + u^b_{2it}, \\
& A_C C_t + (A_X + A_S) X_t + A_S u^b_{5t} = \text{con} + u^b_{3t}, \\
& A_C C_{t,t+i} + (A_X + A_S) X_{t,t+i} + A_S u^b_{6it} = \text{con} + u^b_{4it}, \qquad i = 1, \dots, H,
\end{aligned}
$$

This system has $2(H+1)$ equations determining the time path of the expenditures on the two commodities from the current and expected income flow. How could it be transformed to be suitable for confrontation with time series data?

A suggestion may be to *exploit the reduced form*. The reduced form of (B.7) expresses X_t, C_t, $\{X_{t,t+i}\}_{i=1}^{i=H}$, and $\{C_{t,t+i}\}_{i=1}^{i=H}$ as linear functions of the discounted current and expected income flow $I_t + \sum_{j=1}^{H} \alpha^i I_{t,t+i}$ and the disturbance components. To represent the income process we may *replace* $I_{t,t+i}$ *by* $\mathsf{E}(I_{t,t+i}) = \mathsf{E}(I_{t+i}|\Omega^I_t)$ *where* Ω^I_t *is the consumer's information set at time* t, containing the history of income, and possibly of other exogenous variables, up to period t. In the equation to be confronted with data, the parameters which describe this expectation process will then become part of the reduced-form equation and be intermingled with the structural coefficients in (B.7). Although this strategy may work – technically – it may be difficult to see how it can lead to an equation for X_t useful for testing the 'rational addiction' hypothesis.

We will here, and at similar places for the following examples, consider two strategies for constructing econometric models:

[A]: *Imposing perfect foresight.*

[B]: *Modelling expectations.*

[A] To formalize *perfect foresight* in Example **b** we delete the first subscript on all leaded variables, *i.e.*, replace $(C_{t,t+i}, X_{t,t+i}, I_{t,t+i})$ by $(C_{t+i}, X_{t+i}, I_{t+i})$ in the model version intended for estimation. Then (B.7) collapses into

$$
\text{(B.8)} \qquad
\begin{aligned}
& C_t + X_t - I_t + \textstyle\sum_i \alpha^i (C_{t+i} + X_{t+i} - I_{t+i}) = \text{dis}, \\
& a_{CC} C_t + (a_{CX} + a_{CS}) X_t = \eta [a_{CC} C_{t+1} + (a_{CX} + a_{CS}) X_{t+1}] + \text{con} + \text{dis}, \\
& A_C C_t + (A_X + A_S) X_t = \text{con} + \text{dis},
\end{aligned}
$$

where 'dis' is an abbreviation for an unspecified disturbance. The three-equation system (B.8) is still overdetermined in (C_t, X_t) for $t = 1, \dots, T$. Imposing perfect foresight therefore forces us to disregard one equation. If we suspend the budget constraint, solve for C_t and C_{t+1} from the third equation, which gives

$$
C_t = \frac{A_X + A_S}{A_C} X_t + \text{con} + \text{dis}, \qquad C_{t+1} = \frac{A_X + A_S}{A_C} X_{t+1} + \text{con} + \text{dis},
$$

and insert the result into the second equation, we get an equation which connects X_t with X_{t+1}, but it contains no income variable. Alternatively, if we suspend the second equation in (B.8) and insert the above expressions for C_t and C_{t+1} into the budget constraint an equation which connects X_t with $X_t, X_{t+1}, \dots, X_{t+H+1}$ and the discounted income flow $I_t + \sum_{j=1}^{H} \alpha^i I_{t,t+i}$ follows. An equation with the same basic structure, but with different interpretation of the coefficients, will follow if the third equation is suspended. Such an *ad hoc* strategy cannot be recommended as a feasible way of identifying structural coefficients.

[B] To formalize *the formation of plans and expectations* in Example **b** we introduce expectations conditional on the consumer's information set at the planning time, *i.e.*. Letting Ω_t denote the information set at time t, we replace $(C_{t,t+i}, X_{t,t+i})$ by

$$\mathsf{E}(C_{t,t+i}) = \mathsf{E}(C_{t+i}|\Omega_t), \qquad \mathsf{E}(X_{t,t+i}) = \mathsf{E}(X_{t+i}|\Omega_t),$$

where Ω_t can contain the history of the exogenous (and maybe also endogenous) variables up to period t. This gives (B.7) the form

$$\begin{aligned}
&C_t + X_t - I_t + \textstyle\sum_i \alpha^i[\mathsf{E}(C_{t+i}|\Omega_t) + \mathsf{E}(X_{t+i}|\Omega_t) - \mathsf{E}(I_{t+i}|\Omega_t)] = \text{dis},\\
&a_{CC}C_t + (a_{CX} + a_{CS})X_t\\
&\qquad = \eta^i[a_{CC}\mathsf{E}(C_{t+i}|\Omega_t) + (a_{CX} + a_{CS})\mathsf{E}(X_{t+i}|\Omega_t)] + \text{con} + \text{dis},\\
&A_C C_t + (A_X + A_S)X_t = \text{con} + \text{dis},\\
&A_C\mathsf{E}(C_{t+i}|\Omega_t) + (A_X + A_S)\mathsf{E}(X_{t+i}|\Omega_t) = \text{con} + \text{dis}, \qquad i = 1, \dots, H,
\end{aligned} \tag{B.9}$$

which is a system of $2(H+1)$ equations in C_t, X_t and $\{\mathsf{E}(C_{t+i}|\Omega_t), \mathsf{E}(X_{t+i}|\Omega_t)\}_{i=1}^{i=H}$. It resembles an econometric rational expectation model, where $I_{t,t+i}$ can be represented by $\mathsf{E}(I_{t,t+i}) = \mathsf{E}(I_{t+i}|\Omega_t^I)$, with Ω_t^I containing, *inter alia*, the income history up to period t. This system, in contrast to (B.8), will not be overdetermined because $\mathsf{E}(C_{t+i}|\Omega_t), \mathsf{E}(X_{t+i}|\Omega_t)$ depend on t. We can from the two last equations of (B.9) derive

$$C_t = \frac{A_X + A_S}{A_C} X_t + \text{con} + \text{dis}, \qquad \mathsf{E}(C_{t+i}|\Omega_t) = \frac{A_X + A_S}{A_C}\mathsf{E}(X_{t+i}|\Omega_t) + \text{con} + \text{dis},$$

insert them into the first two equations, to obtain a $(H+1)$-equation system which determines X_t and $\{\mathsf{E}(X_{t+1}|\Omega_t)\}_{i=1}^{i=H}$ as linear functions of $I_t + \sum_{j=1}^{H} \alpha^i \mathsf{E}(I_{t+i}|\Omega_t)$ and the disturbances.[4] The period-to-period revision of the information set gives rise to autoregressive equations with leads and lags.

Hence we conclude: *The occurrence of an addictive stock is not a necessary condition for obtaining an autoregressive equation in the purchase of the additive good, of order at least two.* Once again, maintaining that an equation of this form can serve to identify coefficients in the preference structure or for testing for a latent addictive stock is not justified.

Examples **a** and **b** – although not useful for mimicking addiction within a life-cycle theory – serve to demonstrate that scalar autoregressive equations in observed

[4] For a discussion of the solution to, and identification of, single- and multi-equation rational expectation econometric models with forward-looking expectations, see Pesaran (1987, Sections 5.3, 6.6, 7.7).

purchase of a good – or equations with (at least) a one-period lag and a one-period lead – will follow if the consumer *either* accumulates a stock of the good under geometrically declining weights *or* has a one-period horizon. Altogether, these baseline examples challenge the validity of testing the 'rational addiction' hypothesis, under quadratic period utility functions, by means of linear, single-equation autoregressive models, as suggested by Becker, Grossman, and Murphy (1994) and adopted by several followers.

c: Multi-period. Constant interest rates. Geometric stock

The third example combines the stock accumulation assumption in Example **a** with the multi-period horizon assumption in Example **b**. We specifically assume

$$
\begin{array}{llll}
\alpha_{t,i} = \alpha^i, & \alpha = (1+r)^{-1} & (r > 0), & \\
\beta_{t,i} = \beta^i, & \beta = (1+\rho)^{-1} & (\rho > 0), & i = 0,1,\dots, \\
\eta_{t,i} = \eta^i, & \eta = (1+r)/(1+\rho), & & i = 1,2,\dots,H, \\
d_{t,i} = d^i & d = 1-\delta & (0 < \delta \leq 1), & i = 0,1,\dots, \\
\Longrightarrow \quad \beta_{t,i}d_{t,i} = \psi^i, \quad \psi = (1-\delta)/(1+\rho), & & & i = 0,1,\dots .
\end{array}
$$

Then (9) take the form

$$
\begin{array}{ll}
\phi_t(\mathsf{F}) = \sum_{j=1}^{H} \alpha^j \mathsf{F}^j = \frac{\alpha\mathsf{F}(1-\alpha^H\mathsf{F}^H)}{1-\alpha\mathsf{F}}, & \\
\psi_{t,i}(\mathsf{F}) = \sum_{j=1}^{H-i} \psi^j \mathsf{F}^j = \frac{\psi\mathsf{F}(1-\psi^{H-i}\mathsf{F}^{H-i})}{1-\psi\mathsf{F}}, & i = 0,1,\dots,H, \\
\zeta_{t,i}(\mathsf{F}) = \sum_{j=0}^{i-1}(1-\delta)^j\mathsf{F}^{i-j} = \mathsf{F}^i \sum_{j=0}^{i-1}(1-\delta)^j\mathsf{B}^j = \frac{F^i[1-(1-\delta)^i\mathsf{B}^i]}{1-(1-\delta)\mathsf{B}}, & i = 1,\dots,H, \\
\xi_{t,i}(\mathsf{B}) = \frac{(1-\delta)\mathsf{B}}{1-(1-\delta)\mathsf{B}}, & i = 0,1,\dots .
\end{array}
$$

When $\delta = 1$, they collapse into the polynomials given above for Example **b**. Combining these polynomials with (10)–(15), we find that (b.1)–(b.2) are unchanged while the right-hand sides of (b.3)–(b.6) change. The system now reads:

$$
\begin{array}{lll}
\text{(c.1)} & (C_t + X_t - I_t) + \sum_{i=1}^{H} \alpha^i\mathsf{F}^i(C_t + X_t - I_t) = 0, & \\
\text{(c.2)} & U_{C,t} = \eta^i\mathsf{F}^iU_{C,t}, & \\
\text{(c.3)} & U_{C,t} - U_{X,t} - U_{S,t} = \sum_{j=1}^{H} \psi^j\mathsf{F}^jU_{S,t}, & \\
\text{(c.4)} & \mathsf{F}^i[U_{C,t} - U_{X,t} - U_{S,t}] = \sum_{j=1}^{H-i} \psi^j\mathsf{F}^{i+j}U_{S,t}, & \\
\text{(c.5)} & S_t - (1-\delta)S_{t-1} = X_t, & \\
\text{(c.6)} & \mathsf{F}^i[1 - (1-\delta)\mathsf{B}]S_t = \mathsf{F}^iX_t - (1-\delta)^i, & i = 1,\dots,H.
\end{array}
$$

Consider again modelling strategies [A] and [B]. Comparing the conclusions with those from Example **b** serves to put into focus how inclusion of an addictive stock with geometrically declining weights affects the possibilities for deriving an econometric model.

[A] We can *implement a perfect foresight assumption* in Example **c** by deleting the first subscript on all leaded variables. The three-equation system (B.7) will now be extended to the $(H+3)$-equation system:

$$
\text{(C.1)}\qquad
\begin{aligned}
&C_t+X_t-I_t+\textstyle\sum_i \alpha^i \mathsf{F}^i(C_t+X_t-I_t) = \text{dis},\\
&[1-(1-\delta)\mathsf{B}][a_{CC}C_t+a_{CX}X_t]+a_{CS}X_t\\
&=\eta\mathsf{F}\{[1-(1-\delta)\mathsf{B}][a_{CC}C_t+a_{CX}X_t]+a_{CS}X_t\}+\text{con}+\text{dis},\\
&[1-(1-\delta)\mathsf{B}][A_CC_t+A_XX_t]+A_SX_t\\
&=\textstyle\sum_{j=1}^{H}\psi^j\mathsf{F}^j\{[1-(1-\delta)\mathsf{B}][a_{SC}C_t+a_{SX}X_t]+a_{SS}X_t\}+\text{con}+\text{dis},\\
&\mathsf{F}^i[1-(1-\delta)\mathsf{B}][A_CC_t+A_XX_t]+A_SX_{t,t+i}\\
&=\textstyle\sum_{j=1}^{H-i}\psi^j\mathsf{F}^{i+j}\{[1-(1-\delta)\mathsf{B}][a_{SC}C_t+a_{SX}X_t]+a_{SS}X_t\}+\text{con}+\text{dis},\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad i=1,\dots,H,
\end{aligned}
$$

see Appendix C, in particular (*C7), for detailed derivations. This extension reflects the stock accumulation and the assumed H period horizon. However, only two equations are needed for explaining $\{C_t, X_t\}_{t=1}^{t=T}$, which means that $H+1$ equations can be suspended. This strategy therefore suffers from an arbitrariness similar to that in Example **b**.

[B] Instead of deleting the first subscript on the leaded variables, we can introduce equations to *simulate the formation of expectations*. Replacing $(C_{t,t+i}, X_{t,t+i})$ and $I_{t,t+i}$ in (*C7) by $[\mathsf{E}(C_{t,t+i})=\mathsf{E}(C_{t+i}|\Omega_t), \mathsf{E}(X_{t,t+i})=\mathsf{E}(X_{t+i}|\Omega_t)]$ and $\mathsf{E}(I_{t,t+i})=\mathsf{E}(I_{t+i}|\Omega_t)$, we get a system of $2(H+1)$ equations in (C_t, X_t), $\{\mathsf{E}(C_{t+i}|\Omega_t), \mathsf{E}(X_{t+i}|\Omega_t)\}_{i=1}^{i=H}$ of the form

$$
\text{(C.2)}\qquad
\begin{aligned}
&C_t+X_t-I_t+\textstyle\sum_{i=1}^{H}\alpha^i[\mathsf{E}(C_{t+i}|\Omega_t)+\mathsf{E}(X_{t+i}|\Omega_t)-\mathsf{E}(I_{t+i}|\Omega_t)] = \text{dis},\\
&[1-(1-\delta)\mathsf{B}][a_{CC}C_t+a_{CX}X_t]+a_{CS}X_t\\
&\quad=\eta^i\{[1-(1-\delta)\mathsf{B}][a_{CC}\mathsf{E}(C_{t+i}|\Omega_t)+a_{CX}\mathsf{E}(X_{t+i}|\Omega_t)]\\
&\qquad\qquad\qquad\qquad+a_{CS}\mathsf{E}(X_{t+i}|\Omega_t)\}+\text{con}+\text{dis},\\
&[1-(1-\delta)\mathsf{B}][A_CC_t+A_XX_t]+A_SX_t\\
&\quad=\textstyle\sum_{j=1}^{H}\psi^j\{[1-(1-\delta)\mathsf{B}][a_{SC}\mathsf{E}(C_{t+j}|\Omega_t)+a_{SX}\mathsf{E}(X_{t+j}|\Omega_t)]\\
&\qquad\qquad\qquad\qquad+a_{SS}(\mathsf{E}(X_{t+j}|\Omega_t)\}+\text{con}+\text{dis},\\
&[1-(1-\delta)\mathsf{B}][A_C\mathsf{E}(C_{t+i}|\Omega_t)+A_X\mathsf{E}(X_{t+i}|\Omega_t)]+A_S\mathsf{E}(X_{t+i}|\Omega_t)\\
&\quad=\textstyle\sum_{j=1}^{H-i}\psi^j\{[1-(1-\delta)\mathsf{B}][a_{SC}\mathsf{E}(C_{t+i+j}|\Omega_t)+a_{SX}\mathsf{E}(X_{t+i+j}|\Omega_t)]\\
&\qquad\qquad\qquad\qquad+a_{SS}\mathsf{E}(X_{t+i+j}|\Omega_t)\}+\text{con}+\text{dis},\quad i=1,\dots,H,
\end{aligned}
$$

which generalizes (B.9). By manipulating this linear system so that the $2H$ variables $\{\mathsf{E}(C_{t+i}|\Omega_t), \mathsf{E}(X_{t+i}|\Omega_t)\}_{i=1}^{i=H}$ are eliminated, we end up with a VAR system in (C_t, X_t) which includes the discounted income path $I_t+\sum_{i=1}^{H}\alpha^i\mathsf{E}(I_{t+i}|\Omega_t)$ and linear combinations of the disturbances. From this the 'final equations' for C_t and X_t, of AR form with exogenous variables and identical AR-coefficients can be derived [see, for example Lütkepohl (1991, Section 8.2)].

Hence we conclude: *The occurrence of an addictive stock with geometric weights and a multi-period horizon may be manipulated to give an autoregressive equation* *in the purchase of the additive good of a higher order than 2, depending on the length of the horizon.* Once again, maintaining that an equation of the AR(2) form for the purchase of the addictive good can serve to identify coefficients in the quadratic preference structure or for testing for the presence of such a stock in the way suggested by Becker, Grossman, and Murphy (1994) is not justified. We cannot exclude that identification and testing *may be possible*, but it remains to be seen and will require a more elaborate dynamic analysis.

d: Multi-period. Variable interest rates. No stock

This, fourth example also extends Example **b**, now by relaxing the assumption that $\beta_{t,j}$ and $\alpha_{t,j}$ are geometrically declining in j, *i.e.*, *relaxing constancy of the market and the subjective interest rates.* Again, the stock is assumed to evaporate within one period: $S_t = X_t$ ($\Longrightarrow \mathsf{F}^i S_t = \mathsf{F}^i X_t$, $\forall i$). Precisely, we assume

$$\begin{aligned}
&\beta_{t,0} = 1, \quad \beta_{t,i} = \beta_i, \\
&\alpha_{t,i} = \alpha_i \\
&\eta_{t,i} = \eta_i = \beta_i/\alpha_i, \\
&d_{t,0} = 1, \quad d_{t,i} = 0, \qquad\qquad i=1,\dots,H,
\end{aligned}$$

which imply that (9) is simplified to

$$\begin{aligned}
&\phi_t(\mathsf{F}) = \textstyle\sum_{i=1}^{H} \alpha_i \mathsf{F}^i, \\
&\psi_{t,i}(\mathsf{F}) = 0, && i = 0, 1, \dots, H, \\
&\zeta_{t,i}(\mathsf{F}) = 0, && i = 1, \dots, H, \\
&\xi_{t,i}(\mathsf{B}) = 0, && i = 1, \dots, H.
\end{aligned}$$

Then (10)–(15) are reduced to $3(H{+}1)$ equations in (C_t, X_t, S_t), $\{C_{t,t+i}, X_{t,t+i}, S_{t,t+i}\}_{i=1}^{i=H}$, similar to (b.1)–(b.6), *except that the geometrically declining* (α^j, β^j) *in (b.1) and (b.2) are replaced by unrestricted* (α_j, β_j). See (*C8)–(*C13) in Appendix C.

Let us once again consider modelling strategies [A] and [B]. Comparing the results with those for Example **b** serves to put into focus how relaxation of the geometric succession assumption for the weights of the addictive stock affects the possibilities for deriving an econometric model.

[A] Instead of the three-equation system (B.7), we now get – by dropping the first subscript on all leaded variables in (*C14) in Appendix C – the $(H+2)$-equation system

$$\text{(D.1)}\quad \begin{aligned}
&C_t + X_t - I_t + \textstyle\sum_{i=1}^{H} \alpha_i (C_{t+i} + X_{t+i} - I_{t+i}) = \text{dis}, \\
&a_{CC} C_t + (a_{CX} + a_{CS}) X_t = \eta_1 [a_{CC} C_{t+1} + (a_{CX} + a_{CS}) X_{t+1}] + \text{con} + \text{dis}, \\
&a_{CC} C_t + (a_{CX} + a_{CS}) X_t = \eta_2 [a_{CC} C_{t+2} + (a_{CX} + a_{CS}) X_{t+2}] + \text{con} + \text{dis}, \\
&\qquad \vdots \\
&a_{CC} C_t + (a_{CX} + a_{CS}) X_t = \eta_H [a_{CC} C_{t+H} + (a_{CX} + a_{CS}) X_{t+H}] + \text{con} + \text{dis}, \\
&A_C C_t + (A_X + A_S) X_t = \text{con} + \text{dis}.
\end{aligned}$$

Since $\eta_1, \eta_2, \ldots, \eta_H$ do not form a geometric succession, Equations 2 through $H+1$ no longer collapse into one equation when the equation system is designed for time series data for (C_t, X_t):[5] *The econometric implications of the $H+2$ equations (D.1) differ crucially from those of the 3 equations (B.8).* When the ratios between any two succeeding η_is are not the same, the H middle equations equations in (D.1) are in general in conflict, their left-hand sides being equal, their right-hand sides not. If we were to select two equations from this overdetermined system, we would have to select two which are not in conflict, which again signifies arbitrariness. We may also have to include equations mimicking the consumer's prediction of η_i.

[B] Replacing again $(C_{t,t+i}, X_{t,t+i})$ by $[\mathsf{E}(C_{t,t+i}) = \mathsf{E}(C_{t+i}|\Omega_t), \mathsf{E}(X_{t,t+i}) = \mathsf{E}(X_{t+i}|\Omega_t)]$ and $I_{t,t+i}$ by $\mathsf{E}(I_{t+i}|\Omega_t)$, we end up with get the following generalization of (B.9):

$$
\begin{aligned}
& C_t + X_t - I_t + \textstyle\sum_i \alpha_i[\mathsf{E}(C_{t+i}|\Omega_t) + \mathsf{E}(X_{t+i}|\Omega_t) - \mathsf{E}(I_{t+i}|\Omega_t)] = \text{dis}, \\
& a_{CC}C_t + (a_{CX} + a_{CS})X_t \\
& \qquad = \eta_1[a_{CC}\mathsf{E}(C_{t+1}|\Omega_t) + (a_{CX} + a_{CS})\mathsf{E}(X_{t+1}|\Omega_t)] + \text{con} + \text{dis}, \\
& a_{CC}C_t + (a_{CX} + a_{CS})X_t \\
& \qquad = \eta_2[a_{CC}\mathsf{E}(C_{t+2}|\Omega_t) + (a_{CX} + a_{CS})\mathsf{E}(X_{t+2}|\Omega_t)] + \text{con} + \text{dis}, \\
& \qquad \vdots \\
& a_{CC}C_t + (a_{CX} + a_{CS})X_t \\
& \qquad = \eta_H[a_{CC}\mathsf{E}(C_{t+H}|\Omega_t) + (a_{CX} + a_{CS})\mathsf{E}(X_{t+H}|\Omega_t)] + \text{con} + \text{dis}, \\
& A_C C_t + (A_X + A_S)X_t = \text{con} + \text{dis}, \\
& A_C \mathsf{E}(C_{t+1}|\Omega_t) + (A_X + A_S)\mathsf{E}(X_{t+1}|\Omega_t) = \text{con} + \text{dis}, \\
& A_C \mathsf{E}(C_{t+2}|\Omega_t) + (A_X + A_S)\mathsf{E}(X_{t+2}|\Omega_t) = \text{con} + \text{dis}, \\
& \qquad \vdots \\
& A_C \mathsf{E}(C_{t+H}|\Omega_t) + (A_X + A_S)\mathsf{E}(X_{t+H}|\Omega_t) = \text{con} + \text{dis}.
\end{aligned} \tag{D.2}
$$

This determined system has $2(H+1)$ equations in C_t, X_t and $\{\mathsf{E}(C_{t+i}|\Omega_t), \mathsf{E}(X_{t+i}|\Omega_t)\}_{i=1}^{i=H}$. To solve it we can proceed by first using the last $H+1$ equations to express C_t by means of X_t and $\mathsf{E}(C_{t+i}|\Omega_t)$ by means of $\mathsf{E}(X_{t+i}|\Omega_t)$ $(i = 1, \ldots, H)$ and second insert the result into the first $H+1$ equations, etc.

e: Multi-period. Variable interest rates. Non-geometric stock

The fifth and final example generalizes all the previous ones. Not only are a finite multi-period horizon and a stock accumulation allowed for, but also general, non-geometric paths for $\beta_{t,j}$, $\alpha_{t,j}$ and $d_{t,i}$ are assumed. Precisely, our assumptions are

[5] This is an aspect of the 'dynamic inconsistency' referred to in Section 3.

now

$$
\begin{aligned}
d_{t,0} &= 1, \quad d_{t,i} = d_i, && i=1,2,\dots,P,\\
\alpha_{t,i} &= \alpha_i, &&\\
\beta_{t,0} &= 1, \quad \beta_{t,i} = \beta_i, &&\\
\eta_{t,i} &= \eta_i = \beta_i/\alpha_i, && i=1,\dots,H,\\
\beta_{t,i}d_{t,i} &= \psi_i = \beta_i d_i, && i=1,\dots,K=\min[H,P].
\end{aligned}
$$

The lead- and lag-polynomials are time invariant and take the form[6]

$$
\begin{aligned}
\phi_t(\mathsf{F}) &= \textstyle\sum_{j=1}^{H} \alpha_j \mathsf{F}^j,\\
\psi_{t,i}(\mathsf{F}) &= \psi_i(\mathsf{F}) = \textstyle\sum_{j=1}^{K-i} \psi_j \mathsf{F}^j,\\
\zeta_{t,i}(\mathsf{F}) &= \zeta_i(\mathsf{F}) = \textstyle\sum_{j=1}^{i-1} d_j \mathsf{F}^j,\\
\xi_{t,0}(\mathsf{B}) &= \xi_0(\mathsf{B}) = \textstyle\sum_{j=1}^{P} d_j \mathsf{B}^j, \qquad \xi_{t,i}(\mathsf{B}) = \xi_i(\mathsf{B}) = \textstyle\sum_{j=1}^{P} (d_{i+j}/d_i) \mathsf{B}^j,
\end{aligned}
$$

The system (c.1)–(c.6) is now generalized to (*C15)–(*C20) given in Appendix C.

From here on we can proceed as in Example **c**. First we substitute the expressions for the marginal utilities, (17), eliminating all stock variables from (*C16)–(*C18) by using (*C19)–(*C20). We then get autoregressive equations in (C_t, X_t) which generalize (c.1)–(c.6). Again, the perfect foresight modelling strategy [A], implying omission of the t subscript from all double indexed variables, would give an overdetermined system, while the 'rational expectation' strategy [B], although being econometrically feasible and giving a system determining, *inter alia*, (C_t, X_t), is more complicated than (C.2) and (D.2). It is obvious that the interpretation of their coefficients departs considerably from the interpretation of the coefficients of the Becker-Grossman-Murphy (1994) 'rational addiction' equations.

We therefore conclude that Example **e** – maybe the one that most adequately models accumulation of latent addictive habits in a life-cycle quadratic period utility context – strengthens, once again, the conclusions we drew from the baseline Examples **a** and **b**.

5 Conclusions and extensions

In this paper, a life-cycle theory-model containing an unobserved stock of a habit-related good and its translation into an econometric model of observed expenditure flows, have been considered. We have given arguments to support the view that econometric modelling in such cases should exploit the *full equation system obtained from the theory*, not only pick one or a few of its equations which appear as 'econometrically simple'. The fact that some equations contain variables which are

[6]When $d_i = d^i$, $\alpha_i = \alpha^i$, $\beta_i = \beta^i$, $\eta = (\beta/\alpha)^i$ and $P \to \infty$, we revert to Example **c**. When $d_{t,i} = 0$, $(i \geq 1)$, we revert to Example **d**.

non-observable to the econometrician and at best only 'in the mind of' the respondents, including stock variables as well as plans and expectations related to flow variables, should not, in principle, be an objection against this line of attack. The equations introduced to eliminate the unobservable variables will then become part of the econometric model.

We have shown that when stock accumulation occurs and the consumer's horizon exceeds one period, the implied expenditure function for the addictive good contains *both lags and leads*. The former are consequences of the stock accumulation, the latter follow from the forward-looking utility and budget constraint. The latent addictive stock variables can be eliminated from econometric equations by *exploiting the stock-flow relationship* via lag-distribution representations.

Imposing perfect foresight on a systems of equations derived a from consumer's optimizing behaviour give, in general, an equation system which is mathematically *overdetermined*. The only exceptions occur in the extreme case with geometrically declining coefficients in lag and lead distributions and infinitely long horizon for the consumption plans as well as infinitely long memory in the accumulation of the addictive stock. How to cope with 'dynamic inconsistency' of this kind when constructing econometric model versions in general is an unsettled question. Pursuing a strategy in which expectations are modelled, exploiting ideas from the 'rational expectation' literature, may be a feasible approach. This brings lags into the equations to be estimated.

By several examples, based for simplicity on quadratic period utility functions and in this respect following a lot of previous literature, we have shown that multiperiod models in which addictive stocks are disregarded completely, lead to autoregressive equations in the expenditures on the addictive good which have essentially the same form as those following from models including unobserved addictive stocks and then eliminating them afterwards. From this we conclude that attempts to identify the parameters in utility functions containing latent stocks are very likely to run into problems. Hence, testing the 'rational addiction' hypothesis from time series of purchases of the addictive good, income, etc. is also likely to become problematic. Qualitatively dissimilar theories give rise to similar econometric equations, making it difficult for econometricians to distinguish between them.

Several problems are left for further investigation. *First*, a reconsideration of the model's econometric implications under financial restrictions (*e.g.*, borrowing constraints) may be worthwhile. *Second*, the model may be redesigned from a time-series to a *panel data* format. Latent individual heterogeneity – with respect to *e.g.*, subjective discounting factors, survival rates, memory, length of horizon, etc., as well as observed heterogeneity related to age, cohort, gender, education, employment status, etc. – can then be modelled and analyzed, and hopefully be better understood. *Third*, exploring the implications of replacing the time invariance (stationarity) assumption for the lag- and lead-distributions with non-stationarity may be interesting since changes in interest rates are often announced in advance, *e.g.* by central banks, to affect expectations. *Fourth*, if in compiling micro data for addictive and other habit-affected goods, the data collectors could motivate the respondents to give information about addictive stocks, length of memory, length of planning horizon, size of discount rates, etc., we may obtain more precise inference about the role played by addictive stocks in consumers' life-cycle behaviour. Anyway, before modelling the purpose of the research should be clarified, whether it is to test theories about addiction, to understand factors which determine purchases of addictive goods or to provide the best tool for predicting such purchases.

Appendix A: The optimizing conditions

In this appendix we explain the derivation of (3)–(8) from the optimizing conditions.

Let $W^*_{t,t+i}$ denote net wealth which in period t is planned for period $t+i$, $I^*_{t,t+i}$ denote the exogenous (non-wealth) income which in period t is expected for period $t+i$, $r_{t,j}$ denote the one-period interest rate which in period t is expected to apply in period $t+j$ ($j = 0, 1, 2, \dots$), and for convenience define

(*A1) $$\begin{aligned} W_{t,t+i} &= W^*_{t,t+i} - W^*_{t-1}, \\ I_{t,t+i} &= I^*_{t,t+i} + r_{t,i} W^*_{t-1}, \qquad i = 1, \dots, H. \end{aligned}$$

The latter are, respectively, wealth in excess of initial wealth and predetermined income, *i.e.*, exogenous income plus interests on initial wealth. The *period budget constraint* for period $t+i$ as planned in period t then is $W^*_{t,t+i} = (1+r_{t,i})W^*_{t,t+i-1} + I^*_{t,t+i} - C_{t,t+i} - X_{t,t+i}$, or, equivalently,

(*A2) $$W_{t,t+i} = (1+r_{t,i})W_{t,t+i-1} + I_{t,t+i} - C_{t,t+i} - X_{t,t+i}, \qquad i = 0, 1, 2, \dots, H.$$

Defining

(*A3) $$R_{t,j,k} = \prod_{g=j}^{k-1} (1+r_{t,g}). \qquad j = 0, \dots, k; \; k = 0, \dots, H,$$

and eliminating $W_{t,t+1}, \dots, W_{t,t+H-1}$, we obtain, since $W_{t-1} = 0$,

$$\begin{aligned} W_{t,t+H} = R_{t,0,H}(I_t - C_t - X_t) &+ R_{t,1,H}(I_{t,t+1} - C_{t,t+1} - X_{t,t+1}) \\ &+ R_{t,2,H}(I_{t,t+2} - C_{t,t+2} - X_{t,t+2}) + \cdots + (I_{t,t+H} - C_{t,t+H} - X_{t,t+H}). \end{aligned}$$

Multiplication by $R^{-1}_{t,0,H}$ yields the *inter-temporal budget constraint*

(*A4) $$I_t - C_t - X_t + \sum_{i=1}^{H} \alpha_{t,i}(I_{t,t+i} - C_{t,t+i} - X_{t,t+i}) = \alpha_{t,H} W_{t,t+H},$$

where $\alpha_{t,i} = R^{-1}_{t,0,H}$. The planned terminal wealth, $W_{t,t+H}$, is taken as exogenous.

The *Lagrangian* for maximization of (1) subject to (2) and (*A4) is

(*A5) $$\begin{aligned} \mathcal{L}_t = U(C_t, X_t, S_t) &+ \sum_{i=1}^{H} \beta_{t,i} U(C_{t,t+i}, X_{t,t+i}, S_{t,t+i}) \\ &- \lambda_t[(C_t + X_t - I_t) + \sum_{i=1}^{H} \alpha_{t,i}(C_{t,t+i} + X_{t,t+i} - I_{t,t+i}) + \alpha_{t,H} W_{t,t+H}] \\ &- \mu_{t,0}(S_t - d_{t,0} X_t - \bar{S}_{t,t}) - \sum_{i=1}^{H} \mu_{t,i}(S_{t,t+i} - \sum_{j=1}^{i-1} d_{t,j} X_{t,t+i-j} - d_{t,i} X_t - \bar{S}_{t,t+i}), \end{aligned}$$

where λ_t and $\mu_{t,0}, \mu_{t,1}, \dots, \mu_{t,H}$ are Lagrange multipliers. The first-order conditions *for the current period, i.e.,* $i = 0$, are

$$\frac{\partial \mathcal{L}_t}{\partial C_t} = U_{C,t} - \lambda_t = 0,$$
$$\frac{\partial \mathcal{L}_t}{\partial S_t} = U_{S,t} - \mu_{t,0} = 0,$$
$$\frac{\partial \mathcal{L}_t}{\partial X_t} = U_{X,t} - \lambda_t + \mu_{t,0} d_{t,0} + \mu_{t,1} d_{t,1} + \cdots + \mu_{t,H} d_{t,H} = 0,$$

where $U_{Q,t} = \partial U(C_t, X_t, S_t)/\partial Q_t$ $(Q = C, X, S)$, while *for the future periods* they read

$$\frac{\partial \mathcal{L}_t}{\partial C_{t,t+i}} = \beta_{t,i} U_{C,t,t+i} - \lambda_t \alpha_{t,i} = 0,$$
$$\frac{\partial \mathcal{L}_t}{\partial S_{t,t+i}} = \beta_{t,i} U_{S,t,t+i} - \mu_{t,i} = 0,$$
$$\frac{\partial \mathcal{L}_t}{\partial X_{t,t+i}} = \beta_{t,i} U_{X,t,t+i} - \lambda_t \alpha_{t,i} + \mu_{t,i} d_{t,0} + \mu_{t,i+1} d_{t,1} + \cdots + \mu_{t,H} d_{t,H-i} = 0, \quad i = 1, 2, \dots, H,$$

where $U_{Q,t,t+i} = \partial U(C_{t,t+i}, S_{t,t+i})/\partial Q_{t,t+i}$ $(Q = C, X, S)$. Eliminating $\mu_{t,1}, \dots, \mu_{t,H}$ and λ_t we obtain from these conditions in combination with (2))

(*A6) $\quad C_t + X_t + \sum_{j=1}^{H} \alpha_{t,j}(C_{t,t+j} + X_{t,t+j}) + \alpha_{t,H} W_{t,t+H} = I_t + \sum_{j=1}^{H} \alpha_{t,j} I_{t,t+j},$

(*A7) $\quad U_{C,t} = \beta_{t,1} \alpha_{t,1}^{-1} U_{C,t,t+1} = \beta_{t,2} \alpha_{t,2}^{-1} U_{C,t,t+2} = \cdots = \beta_{t,H} \alpha_{t,H}^{-1} U_{C,t,t+H},$

(*A8) $\quad U_{C,t} - U_{X,t} - U_{S,t} = \sum_{j=1}^{H} \beta_{t,j} d_{t,j} U_{S,t,t+j},$

(*A9) $\quad \beta_{t,i}(U_{C,t,t+i} - U_{X,t,t+i} - U_{S,t,t+i}) = \sum_{j=1}^{H-i} \beta_{t,i+j} d_{t,j} U_{S,t,t+i+j}, \quad i = 1, \dots, H,$

(*A10) $\quad S_t = d_{t,0} X_t + \bar{S}_{t,t},$

(*A11) $\quad S_{t,t+i} = \sum_{j=1}^{i-1} d_{t,j} X_{t,t+i-j} + d_{t,i} X_t + \bar{S}_{t,t+i}, \qquad i = 1, \dots, H.$

Appendix B: Length of horizon versus length of time series

In this appendix, we elaborate the relationships between the $3T(H+1)$ variables in (16) under perfect foresight when the time series length exceeds the consumer's horizon, and when the opposite is the case.

A. If $T > H$, we could formalize perfect foresight (although not, in general, 'time consistency') by imposing *additional restrictions* on $Q = (C, X, S)$ of two kinds:

(a) *Relating to the sample period* $1, \dots, H, H+1, \dots, T$ and restricting the observable variables:

(*B1)
$$\begin{aligned}
&Q_{1,2} = Q_2,\\
&Q_{1,3} = Q_{2,3} = Q_3,\\
&\quad\vdots\\
&Q_{1,H} = Q_{2,H} = Q_{3,H} = \cdots = Q_{H-1,H} = Q_H,\\
&Q_{2,H+1} = Q_{3,H+1} = Q_{4,H+1} = \cdots = Q_{H,H+1} = Q_{H+1},\\
&Q_{3,H+2} = Q_{4,H+2} = Q_{5,H+2} = \cdots = Q_{H+1,H+2} = Q_{H+2},\\
&\quad\vdots\\
&Q_{T-H+1,T} = Q_{T-H+2,T} = Q_{T-H+3,T} = \cdots = Q_{T-1,T+2} = Q_T.
\end{aligned}$$

(b) *Relating to the post-sample period*, $T+1, \dots, T+H$, including only latent variables:

(*B2)
$$\begin{aligned}
&Q_{T-H+2,T+1} = Q_{T-H+3,T+1} = Q_{T-H+4,T+1} = \cdots = Q_{T,T+1} = Q_{T+1},\\
&Q_{T-H+3,T+2} = Q_{T-H+4,T+2} = Q_{T-H+5,T+2} = \cdots = Q_{T+1,T+2} = Q_{T+2},\\
&\quad\vdots\\
&Q_{T+1,T+H} = Q_{T+2,T+H} = Q_{T+3,T+H} = \cdots = Q_{T+H-1,T+H} = Q_{T+H}.
\end{aligned}$$

B. If $H>T$, the additional equations expressing perfect foresight are of two kinds:
(a) *Relating to the sample period* $1,\dots,T$ and restricting the observable variables:

$$\text{(*B3)}\qquad \begin{array}{l} Q_{1,2}=Q_2,\\ Q_{1,3}=Q_{2,3}=Q_3,\\ \qquad\vdots \\ Q_{1,T}=Q_{2,T}=Q_{3,T}=\cdots=Q_{T-1,T}=Q_T. \end{array}$$

(b) *Relating to the post-sample period*, $T{+}1,\dots,H,H{+}1,\dots,H{+}T$, including only latent variables:

$$\text{(*B4)}\qquad \begin{array}{l} Q_{1,T+1}=Q_{2,T+1}=Q_{3,T+1}=\cdots=Q_{T,T+1}=Q_{T+1},\\ Q_{1,T+2}=Q_{2,T+2}=Q_{2,T+2}=\cdots=Q_{T+1,T+2}=Q_{T+2},\\ \qquad\vdots\\ Q_{1,H}=Q_{2,H}=Q_{3,H}=\cdots=Q_{H-1,H}=Q_H,\\ Q_{2,H+1}=Q_{3,H+1}=Q_{4,H+1}=\cdots=Q_{H,H+1}=Q_{H+1},\\ \qquad\vdots\\ Q_{T+1,H+T}=Q_{T+2,H+T}=Q_{T+3,H+T}=\cdots=Q_{H+T-1,H+T}=Q_{H+T}. \end{array}$$

This case, of course, includes the infinite horizon case ($H\to\infty$).

Appendix C: Detailed derivations for Examples c, d and e

Example c: Inserting (17) in (c.2)–(c.4) and adding disturbances we get

$$\text{(*C1)}\qquad C_t+X_t-I_t+\textstyle\sum_{i=1}^{H}\alpha^i(C_{t,t+i}+X_{t,t+i}-I_{t,t+i})+\text{dis},$$

$$\text{(*C2)}\qquad \begin{array}{l} a_{CC}C_t+a_{CX}X_t+a_{CS}S_t\\ \quad=\eta^i[a_{CC}C_{t,t+i}+a_{CX}X_{t,t+i}+a_{CS}S_{t,t+i}]+\text{con}+\text{dis}, \qquad i=1,\dots,H, \end{array}$$

$$\text{(*C3)}\qquad \begin{array}{l} A_CC_t+A_XX_t+A_SS_t\\ \quad=\sum_{j=1}^{H}\psi^j[a_{SC}C_{t,t+j}+a_{SX}X_{t,t+j}+a_{SS}S_{t,t+j}]+\text{con}+\text{dis}, \end{array}$$

$$\text{(*C4)}\qquad \begin{array}{l} A_CC_{t,t+i}+A_XX_{t,t+i}+A_SS_{t,t+i}\\ \quad=\sum_{j=1}^{H-i}\psi^j[a_{SC}C_{t,t+i+j}+a_{SX}X_{t,t+i+j}+a_{SS}S_{t,t+i+j}]+\text{con}+\text{dis}, \qquad i=1,\dots,H, \end{array}$$

$$\text{(*C5)}\qquad S_t-(1-\delta)S_{t-1}=X_t+u^c_{5t},$$

$$\text{(*C6)}\qquad \begin{array}{l} S_{t,t+1}-(1-\delta)S_t=X_{t,t+1}-(1-\delta)+\text{dis},\\ S_{t,t+i}-(1-\delta)S_{t,t+i-1}=X_{t,t+i}-(1-\delta)^i+\text{dis}, \end{array} \qquad i=2,\dots,H.$$

This equation system, which generalizes (B.1)–(B.6), has H leads and one lag. Multiplying (*C2)–(*C4) by $[1-(1-\delta)\mathsf{B}]$ and inserting for S_t and S_{t+1} from (*C5)–(*C6) we obtain

$$\text{(*C7)}\qquad \begin{array}{l} C_t+X_t-I_t+\sum_{i=1}^{H}\alpha^i(C_{t,t+i}+X_{t,t+i}-I_{t,t+i})=\text{dis},\\[4pt] [1-(1-\delta)\mathsf{B}][a_{CC}C_t+a_{CX}X_t]+a_{CS}X_t\\ =\eta^i\{[1-(1-\delta)\mathsf{B}][a_{CC}C_{t,t+i}+a_{CX}X_{t,t+i}]+a_{CS}X_{t,t+i}+\text{con}+\text{dis}, \quad i=1,\dots,H,\\[4pt] [1-(1-\delta)\mathsf{B}][A_CC_t+A_XX_t]+A_SX_t\\ =\sum_{j=1}^{H}\psi^j\{[1-(1-\delta)\mathsf{B}][a_{SC}C_{t,t+j}+a_{SX}X_{t,t+j}]+a_{SS}X_{t,t+j}\}+\text{con}+\text{dis},\\[4pt] [1-(1-\delta)\mathsf{B}][A_CC_{t,t+i}+A_XX_{t,t+i}]+A_SX_{t,t+i}\\ =\sum_{j=1}^{H-i}\psi^j\{[1-(1-\delta)\mathsf{B}][a_{SC}C_{t,t+i+j}+a_{SX}X_{t,t+i+j}]+a_{SS}X_{t,t+i+j}\}+\text{con}+\text{dis},\\ \qquad\qquad i=1,\dots,H. \end{array}$$

From this system we can derive (C.1) and (C.2) in the main text.

Example d: Inserting (17) in (b.2)–(b.4), after having replaced (α^i, η^i) by (α_i, η_i) and having added disturbances, we get

(*C8) $C_t + X_t - I_t + \sum_{i=1}^{H} \alpha_i (C_{t,t+i} + X_{t,t+i} - I_{t,t+i}) = \text{dis},$

(*C9) $a_{CC} C_t + a_{CX} X_t + a_{CS} S_t = \eta_i [a_{CC} C_{t,t+i} + a_{CX} X_{t,t+i} + a_{CS} S_{t,t+i}] + \text{con} + \text{dis},$

(*C10) $A_C C_t + A_X X_t + A_S S_t = \text{con} + \text{dis},$

(*C11) $A_C C_{t,t+i} + A_X X_{t,t+i} + A_S S_{t,t+i} = \text{con} + \text{dis},$

(*C12) $S_t = X_t + \text{dis},$

(*C13) $S_{t,t+i} = X_{t,t+i} + \text{dis}, \qquad i = 1, \dots, H.$

This equation system has H leads, but no lag. From (*C8)–(*C11), after having eliminated S_t and $S_{t,t+i}$ by using (*C12)–(*C13), it follows that

(*C14)
$$\begin{aligned} & C_t + X_t - I_t + \textstyle\sum_{i=1}^{H} \alpha_i (C_{t,t+i} + X_{t,t+i} - I_{t,t+i}) = \text{dis}, \\ & a_{CC} C_t + (a_{CX} + a_{CS}) X_t = \eta_i [a_{CC} C_{t,t+i} + (a_{CX} + a_{CS}) X_{t,t+i} + \text{con} + \text{dis}, \\ & A_C C_t + (A_X + A_S) X_t + A_S u_{5t}^d = \text{con} + \text{dis}, \\ & A_C C_{t,t+i} + (A_X + A_S) X_{t,t+i} + A_S = \text{con} + \text{dis}, \qquad i = 1, \dots, H, \end{aligned}$$

From this system we can derive (D.1) and (D.2) in the main text.

Example e: Inserting the relevant lag- and lead polynomials in (10)–(15) we get

(*C15) $(C_t + X_t - I_t) + \sum_{i=1}^{H} \alpha_i F^i (C_t + X_t - I_t) = 0,$

(*C16) $U_{C,t} = \eta_i \mathsf{F}^i U_{C,t}, \qquad i = 1, \dots, H,$

(*C17) $U_{C,t} - U_{X,t} - U_{S,t} = \sum_{j=1}^{H} \psi_j \mathsf{F}^j U_{S,t},$

(*C18) $\mathsf{F}^i [U_{C,t} - U_{X,t} - U_{S,t}] = \sum_{j=1}^{K-i} \psi_j \mathsf{F}^{i+j} U_{S,t}, \qquad i = 1, \dots, K = \min[H, P],$

(*C19) $S_t = X_t + \sum_{j=1}^{P} d_j \mathsf{B}^j X_t,$

(*C20) $\mathsf{F}^i S_t = \mathsf{F}^i X_t + \sum_{j=1}^{P} d_j \mathsf{F}^i \mathsf{B}^j X_t, \qquad i = 1, \dots, H.$

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