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Climate Change and Carbon Tax Expectations

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Climate change and carbon tax expectations[*]

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Abstract

If investors fear that future carbon taxes will be lower than currently announced by policy makers, long-run investments in greenhouse gas mitigation may be smaller than desirable. On the other hand, owners of a non-renewable carbon resource that underestimate future carbon taxes will postpone extraction compared with what they would have chosen had the policymakers been able to commit to the optimal tax path. If extraction costs rise rapidly as accumulated extraction rises, near-term emissions increase as a consequence of a downward bias in the expected future carbon taxes. Whether investments in greenhouse gas mitigation go up or down due to the expectation error depends on the time profile of the returns to the investment.

Keywords: climate change, exhaustible resources, carbon tax

JEL classification: H23, Q30, Q42, Q54

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1 Introduction

The most important contribution to the climate problem is CO_2 from the combustion of fossil fuels. The climate problem is thus to a large extent caused by extracting carbon resources and transferring them to the atmosphere. Logically, any discussion of the climate problem therefore ought to be intimately linked to a discussion of the extraction of carbon resources. In spite of this obvious fact, surprisingly little of the literature makes this link. However, there are important exceptions, such as the early contributions by Sinclair (1992), Ulph and Ulph (1994) and Withagen (1994), and more recent contributions, such as Hoel and Kverndokk (1996), Tahvonen (1997), Chakravorty et al. (2006), Strand (2007), Sinn (2008) and Gerlagh (2010).

One of the insights from this literature is that the principles for setting an optimal carbon tax (or price of carbon quotas) are the same as those one finds when ignoring the resource aspect of the problem. In particular, the optimal carbon tax should at any time be equal to the discounted value of all future marginal climate costs caused by the present emission. At an early stage, when the optimal path of carbon in the atmosphere is rising, increasing marginal climate costs will imply a rise over time in the optimal carbon tax (see e.g. Hoel and Kverndokk, 1996, for details).

An obvious problem with implementing an optimal policy is that policy makers cannot commit to a rising carbon tax. In the policy debate on climate policies it is often argued that long-run investments in greenhouse gas mitigation may be smaller than desirable since investors fear that future carbon prices will be lower than currently announced by policy makers, see e.g. Laffont and Tirole (1996), Montgomery (2005) and Ulph and Ulph (2009). If this is the case, greenhouse gas emissions may be higher than the optimal outcome, which would be achieved if one could commit to the optimal carbon tax path.

In a recent article, Sinn (2008) has implicitly argued that the opposite may be true: If owners of the non-renewable carbon resources believe the

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carbon tax path will grow at a lower rate than what is optimal, they will postpone extraction compared with the extraction path they would have chosen had the policymakers been able to commit to the optimal price path. This argument suggests that although lack of commitment obviously reduces welfare compared with the case of commitment, it is not obvious whether near term emissions will increase or decline as a consequence of lack of commitment.

The present paper uses a simple two-period model of an aggregate economy to analyze how the expected future carbon tax may affect both emissions and investments in substitutes for the carbon resource. Carbon capture and storage is ignored, implying that emissions are identical to carbon extraction. Period 1 in the model may be interpreted as the near future where one has reasonable confidence about the size of the carbon tax, with period 2 being the remaining future. In terms of the number of years, 10-15 years might be a crude estimate of the length of period 1.

In period 1 the government first sets the carbon tax in period 1 and announces its intended carbon tax for period 2. Once the tax is set, carbon resource owners and investors in mitigation capital simultaneously make their choices of period 1 extraction and investment, respectively. Given the outcome of period 1, the government sets the carbon tax for period 2, after which the carbon resource owners decide how much to extract in this period. There is no further investment in mitigation capital in period 2.

The rest of the paper is organized as follows. Sections 2-4 describe the market equilibrium for exogenous carbon taxes in the two periods. Section 5 describes the first-best social optimum, and shows what the carbon taxes must be for the market equilibrium to coincide with the social optimum.

Section 6 derives the consequences of the expected future carbon tax deviating from the optimal tax. The results depend on how sensitive extraction costs are to total carbon extraction. If costs rise rapidly with total extraction, near-term emissions are higher the higher is the expected future carbon tax.

However, if extraction costs only rise slowly as total extraction increases, we may get the opposite result. The effect of the expected future carbon tax on investments in a non-carbon substitute depend on the time profile of the returns to the investment. If most of the returns to the investments come in the near future, investments are increasing in the expected future carbon tax, while the opposite is true if most of the returns to the investments come in the more distant future.

In section 7 it is assumed that climate costs are higher the higher are total emissions, and also higher the higher are near-term emissions for a given value of total emissions. With these assumptions it is shown that if extraction costs rise rapidly with total extraction, climate costs are *higher* the higher is the expected future carbon tax. However, if extraction costs only rise slowly as total extraction increases, we may get the opposite result.

In several countries, there are substantial subsidies offered to investments in renewable energy and energy saving capital. One reason that is often given for such subsidies is the assumed lack of confidence among private agents in a high future carbon tax. An obvious question is whether a subsidy to the carbon substitute brings us closer to the first-best optimum. This is analyzed in Section 8. A small subsidy to non-carbon energy will move near-term emissions and investments in non-carbon energy towards their socially optimal values if a sufficiently large part of the returns to the investments come in the distant future. However, if a sufficiently large part of the the returns to the investments come in the near future, either near-term emissions or investments will be moved in the same direction by a subsidy as by a low expected future carbon tax.

2   The market for the general purpose good

Carbon is used as an input in production of a general purpose good in both periods. The output is increasing in the carbon input and also in a capital

good that is a substitute for carbon energy. An obvious interpretation is that there is a substitute that has high capital costs and low operating costs (such as hydro, wind, and solar energy). Once the investment in capacity of such a substitute is made, it will be operated at full capacity. Alternatively,one could think of the substitute as knowledge capital, i.e. an improved technology that is available at a low cost once it has been developed.

Output in the two periods is $\tilde{f}(x, I)$ and $\tilde{F}(A - x, I)$, where x is carbon extracted and used in period 1 and $A - x$ is carbon extracted and used in period 2. The variable I is the investment in the carbon substitute, which takes place only in period 1. This investment is assumed to affect output in both periods (although the special cases in which either $\tilde{f}_I$ or $\tilde{F}_I$ is zero also will be considered). The functions $\tilde{f}$ and $\tilde{F}$ are assumed to be concave and increasing in both arguments, and it is also assumed that the cross derivatives $\tilde{f}_{xI}$ and $\tilde{F}_{(A-x)I}$ are negative, so that the marginal productivity of using the carbon resource is lower the higher is the capital good I. While this general specification is used in the derivation of the market equilibrium, the formal analysis is restricted to the case in which the carbon resource and the substitute are perfect substitutes in the study of the consequences of errors in carbon tax expectations. For thsi part of the analysis we thus have functions of the type $f(x + \alpha I)$ and $F(A - x + (1 - \alpha)I)$ instead of the more general functions $\tilde{f}(x, I)$ and $\tilde{F}(A - x, I)$. The parameter α tells us what share of the total returns to investment are obtained already during period 1.

The price of the general purpose good is normalized to 1, while the price of the carbon resource that the producers of the general purpose good must pay in the two periods is $p + q$ and $P + Q$, respectively. Here p and P are the prices that the producers of the carbon resource receive in the two periods, while q and Q are the carbon taxes in the two periods. Investment in the carbon substitute uses the general purpose good, and the cost of one unit of

I is c.[1] Finally, the exogenous discount factor is β (equal to $(1+r)^{-1}$, where r is the exogenous discount rate[2]).

Producers of the general purpose good take the resource price in period 1 and 2 as given ($p+q$ and $P+Q$, respectively) and maximize

$$\tilde{f}(x,I) - (p+q)x - cI + \beta\left[\tilde{F}(A-x,I) - (P+Q)(A-x)\right]$$

The maximization gives

$$\tilde{f}_x(x,I) - (p+q) \;\; = \;\; 0 \tag{1}$$
$$\tilde{F}_{A-x}(A-x,I) - (P+Q) \;\; = \;\; 0 \tag{2}$$
$$\tilde{f}_I(x,I) + \beta\tilde{F}_I(A-x,I) - c \;\; = \;\; 0 \tag{3}$$

3   The market for the carbon resource

To extract the carbon resource one needs to use the all purpose good as an input. The input needed per ton of the resource extracted is assumed to be independent of the extraction rate, but increases with accumulated extraction. A special case of this is the case of a constant unit cost of extraction combined with an absolute upper limit $\bar{A}$ on accumulated extraction. The general specification is frequently used in the resource literature, see e.g. Heal (1976) and Hanson (1980). Formally, let each unit of the resource be indexed by a continuos variable z, and let $g(z)$ be the cost of of extracting unit z, with $g' \geq 0$. In the two-period model x is extraction in period 1, and $A-x$ is extraction in

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[1]With the interpretation of I as investment in the capacity to produce a substitute, c includes the present value of the operating costs of the substitute at full capacity.

[2]Notice that r is the consumption discount rate; introducing a utility function and making r endogenous according the the Ramsey rule would add notation but otherwise leave the analysis unchanged.

period 2. The cost of extracting x is thus given by $G(x) = \int_0^x g(z)dz$, and the cost of extracting $A - x$ is $\int_x^A g(z)dz = \int_0^A g(z)dz - \int_0^x g(z)dz = G(A) - G(x)$. Notice that these relationships imply that $G'(x) = g(x)$ and $G'(A) = g(A)$. The limiting case of a constant unit cost g of extraction up to an exogenous limit $\bar{A}$ would imply that $G(x) = gx$ and $G(A) - G(x) = g \cdot (A - x)$ (up to $\bar{A}$). In the subsequent analysis it is assumed that $g'(z) = 0$ for $z \leq \tilde{z}$ and $g'(z) > 0$ for $z > \tilde{z}$, and that $x < \tilde{z} < A$ for all relevant values of x and A.[3] This implies that $g'(x) = 0$, while $G'(A) = g(A) > g(x)$ and $G''(A) = g'(A) > 0$; $g'(A)$ is henceforth denoted g'.

Producers of the carbon resource maximize

$$px - G(x) + \beta \left[P \cdot (A - x) - (G(A) - G(x)) \right]$$

This gives (using $G'(x) = g(x)$ and $G'(A) = g(A)$)

$$p - g(x) = \beta \left[P - g(x) \right] \tag{4}$$

and

$$P = g(A) \tag{5}$$

Using $\beta = (1 + r)^{-1}$, this equation implies that

$$P - p = r(p - g(x))$$

This is simply the Hotelling rule, which formulated this way also holds for the extraction cost assumption we are using.

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[3]This simplifying assumption is not important for the results.

4 The market equilibrium

Equations (1)-(5) are 5 equations determining the 5 variables p, P, x, A, I as functions of the exogenous tax rates q and Q. The tax rate Q is the tax rate that agents expect in period 2. When period 2 arrives, x, I and p are all historically determined, while P and A will be determined by (2) and (5). If Q turns out to be different from what agents expected in period 1, P will also be different from what agents expected, and A will be different from what agents planned.

Eliminating p and P, the market equilibrium (1)-(5) may be rewritten as

$$\tilde{f}_x - \beta \tilde{F}_{A-x} - (1 - \beta)g(x) - (q - \beta Q) = 0 \tag{6}$$

$$\tilde{F}_{A-x} - g(A) - Q = 0 \tag{7}$$

$$\tilde{f}_I + \beta \tilde{F}_I - c = 0 \tag{8}$$

These equations are of course also the first order conditions to the problem of maximizing the total private sector profits given by

$$
\begin{aligned}
\Pi(x, A, I, q, Q) = & \left[\tilde{f}(x, I) - G(x) - cI - qx \right] \\
& + \beta \left[\tilde{F}(A - x, I) - (G(A) - G(x)) - Q(A - x) \right]
\end{aligned} \tag{9}
$$

which is concave in (x, A, I) since $\tilde{f}$, $\tilde{F}$, $-G(A)$ and $-(1-\beta)G(x)$ are concave.

5 The social optimum

Due to the time lag of the climate system, the effect of emissions in period 1 on the climate in period 1 are assumed to be negligible; this is certainly true

if the length of period 1 is no longer than about 10-20 years. Climate costs are therefore assumed to depend on the temperature increase T in period 2 (from some base level). Let the climate costs be given by a damage function $\tilde{D}(T)$, which is assumed to be increasing and strictly convex. The climate depends on emissions in both periods:

$$T = \tilde{T}(x, A - x) = T(x, A) \tag{10}$$

The function $\tilde{T}$ is assumed to be increasing in both its arguments. The variable x in $\tilde{T}$ is due partly to the lagged response of temperature to the stock of carbon in the atmosphere, and partly due to the fact that emissions in period 1 affect the stock of carbon in the atmosphere both in period 1 and 2. It is not obvious that the net affect x on T for a given A is positive, although this seems reasonable if one cares about how rapidly the climate changes.[4] Although T_x has an ambiguous sign, T_A is positive since $\tilde{T}$ is increasing in both arguments.

Inserting (10) into $\tilde{D}(T)$ gives us

$$D(x, A) \equiv \tilde{D}(T(x, A))$$

which is increasing in A, while the sign of D_x will be the same as the sign of T_x. To make the derivations slightly simpler without changing anything of substance, it is assumed that the function $D(x, A)$ takes the simple form

$$D(x, A) \equiv C(A + \gamma x) \tag{11}$$

where γ is a parameter that may be positive ($D_x > 0$) or negative ($D_x < 0$). One interpretation of the case of $\gamma < 0$ is that we only care about the carbon in the atmosphere in period 2, and that a fraction δ of the carbon emitted

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[4]Such a consideration cannot be captured in a 2 period model, but see the discussion in Hoel (2008) in a contionuous time model.

in period 1 is transferred to the ocean and other carbon sinks at the end of period 1, so that only $(1 - \delta)x$ of the emissions in period 1 remain in the atmosphere in period 2. Emissions in period 2 are $A - x$, so climate costs are in this case $C((1 - \delta)x + (A - x)) = C(A - \delta x)$, implying $\gamma = -\delta$ in our notation. In most of the subsequent analysis, it is assumed that $\gamma > 0$, i.e., "a postponement of emissions is good for the environment". The case of $\gamma < 0$ is left to the reader.

Given this climate cost function, the social optimum is found by maximizing

$$\tilde{f}(x, I) - G(x) - I + \beta \left[\tilde{F}(A - x, I) - (G(A) - G(x)) \right] - \beta C(A - \gamma x)$$

The three optimum conditions for the three variables x, A, I are

$$\tilde{f}_x - \beta \tilde{F}_{A-x} - (1 - \beta)g(x) - \beta\gamma C'(A + \gamma x) = 0 \tag{12}$$

$$\tilde{F}_{A-x} - g(A) - C'(A + \gamma x) = 0 \tag{13}$$

$$\tilde{f}_I + \beta \tilde{F}_I - c = 0 \tag{14}$$

Comparing these equations with (6)-(8) it immediately follows that the market outcome coincides with the social optimum if

$$q = \beta(1 + \gamma)C'(A + \gamma x)$$

$$Q = C'(A + \gamma x)$$

Notice that this implies that

$$\frac{Q}{q} = \frac{1}{\beta(1 + \gamma)}$$

If $\gamma = 0$ the carbon tax thus rises at a rate equal to the rate of interest $(= 1 + r = \beta^{-1})$, while it rises at a lower rate if $\gamma > 0$.[5]

6 The effects of a change in the expected future carbon tax

While q is known when decisions are made for the first period, Q has the status of an expected price. To make this clear we use Q^e to denote the expected future carbon tax. Similarly, the variables x and I are decided upon in the first period, while A has the status of a planned variable for the resource owners, and an expected variable for the other agents. We therefore use A^e to denote this variable.

With this modified notation the three equations (6)-(8) define x, I and A^e as functions of q and Q^e. This section describes how changes in Q^e affect x and I. The next section shows how the actual second period value of A is affected by Q^e, via the effect of Q^e on x and I.

As mentioned in the Introduction, the formal analysis is on the case in which the capital good that affects the demand for the resource is a perfect substitute for the resource. However, it is useful first to concider the opposite limiting case, where the cross derivatives between the capital good and the resource are zero, i.e. $\tilde{f}_{Ix} = \tilde{F}_{I(A-x)} = 0$. For this case it follows from (8) that I is independent of Q^e. Moreover, by using (1) and (2), it follows that x is increasing in Q^e. In this case near-term emissions are higher the higher is the expected future carbon tax. In other words, a bias downward in the expected future carbon tax implies a reduction in near-term emissions. It can also be shown that this effect (i.e., the size of the derivative $\frac{\partial x}{\partial Q^e}$) is smaller the smaller is the derivative $g'(A)$.

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[5] If $\gamma < 0$, the carbon tax rises at a rate higher than the rate of interest. This corresponds to what one finds in continuous time models when there is a constraint on the content of carbon in the atmosphere but no climate cost function.

We now turn to the more interesting case in which the cross derivatives $\tilde{f}_{Ix}$ and $\tilde{F}_{I(A-x)}$ are negative. The analysis is restricted to the case where I is a perfect substitute for the resource, i.e.

$$
\begin{aligned}
\tilde{f}(x, I) &= f(x + \alpha I) \\
\tilde{F}(A - x, I) &= F(A - x + (1 - \alpha)I)
\end{aligned}
$$

With this specification the market equilibrium (1)-(5) can be rewritten as

$$
\begin{aligned}
f'(x + \alpha I) - \beta F'(A^e - x + (1 - \alpha)I) - (1 - \beta)g(x) - (q - \beta Q^e) &= 0 & (15) \\
F'(A^e - x + (1 - \alpha)I) - g(A^e) - Q^e &= 0 & (16) \\
af'(x + \alpha I) + \beta(1 - \alpha)F'(A^e - x + (1 - \alpha)I) - c &= 0 & (17)
\end{aligned}
$$

This gives three equations to determine the three variables x, A^e and I as functions of q and Q^e. Just like in the general case, these equations are also the first order conditions to the problem of maximizing the total private sector profits given by

$$
\begin{aligned}
\Pi(x, A^e, I, q, Q^e) &= [f(x + \alpha I) - G(x) - I - qx] & (18) \\
&\quad + \beta[F(A^e - x + (1 - \alpha)I) - (G(A^e) - G(x)) - Q^e(A^e - x)]
\end{aligned}
$$

which is concave in (x, A^e, I) since f, F, $-G(A)$ and $-(1 - \beta)G(x)$ are concave.

Differentiating the three equations (15), (16) and (17) with respect to Q^e gives the following three linear equations (using $g'(x) = 0$ and $g'(A^e) = g'$):

$$M \cdot \begin{pmatrix} \frac{\partial x}{\partial Q^e} \\ \frac{\partial A^e}{\partial Q^e} \\ \frac{\partial I}{\partial Q^e} \end{pmatrix} = \begin{pmatrix} -\beta \\ 1 \\ 0 \end{pmatrix}$$

where

$$\mathbf{M} = \begin{pmatrix} f'' + \beta F'' & -\beta F'' & \alpha f'' - \beta(1-\alpha)F'' \\ -F'' & F'' - g' & (1-\alpha)F'' \\ \alpha f'' - \beta(1-\alpha)F'' & \beta(1-\alpha)F'' & \alpha^2 f'' + \beta(1-\alpha)^2 F'' \end{pmatrix} \qquad (19)$$

Solving the equation system above gives (after some tedious calculations)

$$\frac{\partial x}{\partial Q^e} = \frac{\beta}{H} \left\{ -\alpha(1-\alpha)f'' F'' - \left[\alpha^2 f'' + \beta(1-\alpha)^2 F'' \right] g' \right\} \qquad (20)$$

$$\frac{\partial I}{\partial Q^e} = \frac{\beta}{H} \left\{ (1-\alpha)f'' F'' + \left[\alpha f'' - (1-\alpha)\beta F'' \right] g' \right\} \qquad (21)$$

where $H = -|\mathbf{M}| > 0$ due to the concavity of the function Π defined by (18).

It is easily verified that both these derivatives have ambiguous signs. In particular, the signs depend on the sizes of α and g'; in the Appendix the following result is shown:

Proposition 1: There exists a threshold value $\hat{\alpha}(g') = \frac{f'' F'' - \beta F'' g'}{f'' F'' + (-f'' - \beta F'')g'}$, and for g' sufficiently low also two threshold values $\alpha^*(g')$ and $\alpha^{**}(g')$ where $\alpha^*(g') < \alpha^{**}(g') < \hat{\alpha}(g')$ and where $\alpha^*(g')$ is increasing in g' and $\alpha^{**}(g')$ is decreasing in g', such that

$$
\begin{aligned}
Case\ I\ &:\ \ \alpha < \alpha^*(g') :\ \ \ \frac{\partial x}{\partial Q^e} > 0\ and\ \frac{\partial I}{\partial Q^e} > 0 \\[4pt]
Case\ II\ &:\ \ \alpha^*(g') < \alpha < \alpha^{**}(g') :\ \ \ \frac{\partial x}{\partial Q^e} < 0\ and\ \frac{\partial I}{\partial Q^e} > 0 \\[4pt]
Case\ III\ &:\ \ \alpha^{**}(g') < \alpha < \hat{\alpha}(g') :\ \ \ \frac{\partial x}{\partial Q^e} > 0\ and\ \frac{\partial I}{\partial Q^e} > 0 \\[4pt]
Case\ IV\ &:\ \ \alpha > \hat{\alpha}(g') :\ \ \ \frac{\partial x}{\partial Q^e} > 0\ and\ \frac{\partial I}{\partial Q^e} < 0
\end{aligned}
$$

For g' sufficiently low, case II does not exist, giving case I/III for $\alpha < \hat{\alpha}(g')$.

To interpret this result, we start with cases I/III and IV (which will be the only possible cases if g' is sufficiently large). As Q^e increases, extraction in period 2 becomes less profitable, so $A - x$ goes down and x goes up. The increase in x reduces the period 1 payoff to the investment, while the reduction in $A - x$ increases the period 2 payoff. The former effect will dominate if α is large (case IV), while the latter effect will dominate if α is small (cases I and III).

If g' is sufficiently small, the direct effect of Q^e on x (i.e. holding I constant) is small, cf. the discussion in the beginning of this section. This implies that it is the change in payoff in period 2 that is important for how I is affected by the change in Q^e. If $(1 - \alpha)$ is sufficiently large, I therefore increases with Q^e, since a higher Q^e gives a lower $A - x$. If α is sufficiently large, the increase in αI, which tends to reduce x, will outweigh the direct affect of Q^e on x. In this case x will therefore decline as a response to increased Q^e.

7 Climate costs and carbon tax expectations

Assume that the carbon tax is set optimally in period 2, i.e. so that equation (13) is satisfied, no matter how x and I are determined in period 1. This implies that the actual value of A is determined by

$$F'(A - x(Q^e) + (1 - \alpha)I(Q^e)) = g(A) + C'(A + \gamma x(Q^e))$$

implying that

$$\frac{\partial A}{\partial Q^e} = J^{-1} \left\{ -(F'' + \gamma C'')\frac{dx}{dQ^e} + (1 - \alpha)F''\frac{dI}{dQ^e} \right\} \qquad (22)$$

where $J = C'' + g' - F'' > 0$. Total climate costs depends on $A + \gamma x$, and it follows from (20), (21) and (22) that

$$\frac{\partial(A + \gamma x)}{\partial Q^e} = \frac{\beta}{HJ} \left\{ [\gamma g' - (1 + \gamma)F''] B_x + (1 - \alpha)F''B_I \right\} \qquad (23)$$

where B_x and B_I are the terms in curly brackets in (20) and (21), respectively, i.e.

$$
\begin{aligned}
B_x &= -\alpha(1 - \alpha)f''F'' - \left[\alpha^2 f'' + \beta(1 - \alpha)^2 F''\right]g' \\
B_I &= (1 - \alpha)f''F'' + \left[\alpha f'' - (1 - \alpha)\beta F''\right]g'
\end{aligned}
$$

The term in front of B_x is positive (since $\gamma > 0$), while the sign in front of B_I is negative. If B_x and B_I have opposite signs, $\frac{\partial(A+\gamma x)}{\partial Q^e}$ is therfore unambiguously signed. From Proposition 1 the following therefore follows:

Proposition 2: The sign of $\frac{\partial(A+\gamma x)}{\partial Q^e}$ depends on the size of α in the following manner:

$$
\begin{aligned}
Case\ I\quad&:\quad \alpha < \alpha^*(g'):\quad \frac{\partial(A+\gamma x)}{\partial Q^e}\ \text{has an ambiguous sign}\\[4pt]
Case\ II\quad&:\quad \alpha^*(g') < \alpha < \alpha^{**}(g'):\quad \frac{\partial(A+\gamma x)}{\partial Q^e} < 0\\[4pt]
Case\ III\quad&:\quad \alpha^{**}(g') < \alpha < \hat{\alpha}(g'):\quad \frac{\partial(A+\gamma x)}{\partial Q^e}\ \text{has an ambiguous sign}\\[4pt]
Case\ IV\quad&:\quad \alpha > \hat{\alpha}(g'):\quad \frac{\partial(A+\gamma x)}{\partial Q^e} > 0
\end{aligned}
$$

Going back to equation (23), we see that both the numerator and the denominator contain terms with $(g')^2$. For g' sufficiently high, these terms in the numerator will dominate other terms. The part of the numerator containing $(g')^2$ is $-\gamma\left[\alpha^2 f'' + \beta(1-\alpha)^2 F''\right](g')^2$. The term in square brackets is negative no matter what value α has, implying that the whole expression is positive for $\gamma > 0$. We therefore have the following proposition:

Proposition 3: No matter what value α has, climate costs are increasing in the expected carbon tax, i.e. $\frac{\partial(A+\gamma x)}{\partial Q^e} > 0$, for g' sufficiently large.

Notice that this result follows immediately for the limiting case of A being exogenous, since $\frac{\partial x}{\partial Q^e} > 0$ in this case.

Intuitively, one might think that expectations about a low future carbon tax are bad for the climate. This may be true, and Proposition 2 shows that it is certainly true if $\alpha^*(g') < \alpha < \alpha^{**}(g')$, which may occur if g' is not too high. However, if g' is sufficiently high we get the opposite result: Expectations about a low future carbon tax are *good* for the climate.

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8 The effects of subsidizing investments in the carbon substitute

In several countries, in particular in the EU, there are substantial subsidies offered to investments in renewable energy and energy saving capital. One reason that is often given for such subsidies is the assumed lack of confidence among private agents in a high future carbon tax (or quota price). An obvious question is whether a subsidy to the carbon substitute brings us closer to the first-best optimum, in the sense that it moves x and I in the opposite direction of what the error in the carbon tax expectation moves these variables.

Differentiating the three equations (15), (16) and (17) with respect to investment costs c gives

$$M \cdot \begin{pmatrix} \frac{\partial x}{\partial c} \\ \frac{\partial A^e}{\partial c} \\ \frac{\partial I}{\partial c} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

where M as before is given by (19). Solving these equations we find that $\frac{\partial I}{\partial c} < 0$, while

$$\frac{\partial x}{\partial c} = \frac{1}{H} \left\{ \alpha f'' F'' - \left[\alpha f'' - \beta (1 - \alpha) F'' \right] g' \right\} \tag{24}$$

From (24) it immediately follows that $\frac{\partial x}{\partial c}$ has the same sign as $\alpha - \tilde{\alpha}(g')$, where

$$\tilde{\alpha}(g') = \frac{-\beta F'' g'}{f'' F'' + (-f'' - \beta F'') g'}$$

From the expression for $\hat{\alpha}(g')$ in Proposition 2 it follows that $\tilde{\alpha}(g') < \hat{\alpha}(g')$.

The results above combined with Proposition 2 give us the following proposition:

Proposition 4: No matter what values g' and α have, $\frac{\partial I}{\partial c} < 0$. Moreover, for g' so large that case II in Proposition 2 does not exist, $\frac{\partial x}{\partial Q^e}$ is positive, while the signs of $\frac{\partial I}{\partial Q^e}$ and $\frac{\partial x}{\partial c}$ depend on the size of α in the following manner:

$$Case\ i \quad : \quad \alpha < \tilde{\alpha}(g') : \quad \frac{\partial I}{\partial Q^e} > 0 \text{ and } \frac{\partial x}{\partial c} < 0$$

$$Case\ ii \quad : \quad \tilde{\alpha}(g') < \alpha < \hat{\alpha}(g') : \frac{\partial I}{\partial Q^e} > 0 \text{ and } \frac{\partial x}{\partial c} > 0$$

$$Case\ iii \quad : \quad \alpha > \hat{\alpha}(g') : \quad \frac{\partial I}{\partial Q^e} < 0 \text{ and } \frac{\partial x}{\partial c} > 0$$

For a subsidy (which reduces c) to partly offset the effects on x and I of Q^e being too low, we must have $\frac{\partial x}{\partial Q^e} \frac{\partial x}{\partial c} < 0$ and $\frac{\partial I}{\partial Q^e} \frac{\partial I}{\partial c} < 0$. From Proposition 4 we see that this is the case only for $\alpha < \tilde{\alpha}(g')$. For higher values of α either first period emissions or investments (or both) will be moved in the same direction by a subsidy as by a low expected future carbon tax. Small values of α corresponds to investments where most of the benefits occur in the distant future. Proposition 4 thus gives an argument for subsidizing such investments, provided one believes that market participants typically underestimate the size of future carbon taxes.

9   Concluding remarks

An obvious problem with implementing an optimal climate policy is that policy makers cannot commit to a high future carbon tax. In the policy debate on climate policies it is often argued that long-run investments in greenhouse gas mitigation may be smaller than desirable since investors fear that future carbon prices will be lower than currently announced by policy makers. The

present paper shows that it is not obvious how expectations about future carbon taxes affect important variable such as investments in non-carbon energy and near-term emissions. The effects of expectations about future carbon taxes on these variables depend to a large extent on the properties of the extraction costs of carbon resources. In much of the economics literature on climate policy the resource aspect of carbon is ignored, so that the production of oil, gas and goal is treated in the same manner as the production of other goods. In most of the papers that explicitly treat the resource aspect, it is assumed that there is a strict physical limit of the available resource, and that all of the resources up to this limit sooner or later will be extracted. However, a much more realistic description of the real world is that no such absolute physical limit exists, but that extraction costs are increasing in total extraction. This is the assumption used in the present paper, as well as by Hoel and Kverndokk (1996) and Gerlagh (2010).

The effects of expectations about future carbon taxes on near-term emissions and investments in substitutes for carbon energy depend significantly on how rapidly extraction costs increase with increasing total extraction. In addition, the time profile of the returns to the investment in the non-carbon substitute is important for the effects of expectations about future carbon taxes.

Appendix

The signs of the derivatives (20) and (21) are equal to the signs of the respective curly brackets, i.e., on

$$B_x(\alpha) = -\alpha(1 - \alpha)f''F'' - \left[\alpha^2 f'' + \beta(1 - \alpha)^2 F''\right] g'$$

and

$$B_I(\alpha) = (1-\alpha)f''F'' + \left[\alpha f'' - (1-\alpha)(\beta + (1-\beta)h)F''\right]g'$$

respectively.

Consider first $B_x(\alpha)$. This function is quadratic in α, and is positive for $\alpha = 0$ and for $\alpha = 1$. The first term in this expression is negative (for $0 < \alpha < 1$), while the second term (including the minus sign) is positive. The second term will dominate if g' is sufficiently large. However, for g' sufficiently small there will be a range (α^*, α^{**}) of α-values giving $B_x(\alpha) < 0$. It is straightforward to see that α^* is increasing in g' and that α^{**} is declining in g'.

Consider next $B_I(\alpha)$. This function is declining in α. Moreover, $B_I(0) > 0$ and $B_I(1) < 0$, implying that there exists a value $\hat{\alpha}$ such that $B_I(\hat{\alpha}) = 0$. It is straightforward to see that this value is

$$\hat{\alpha}(g') = \frac{f''F'' - \beta F'' g'}{f''F'' + (-f'' - \beta F'')g'}$$

Consider a value of α giving $B_x(\alpha) < 0$ (when such a value exists). It follows from the definition of $B_x(a)$ that if $B_x(\alpha) < 0$ then

$$(1-\alpha)f''F'' + \left[\alpha f'' + \beta\frac{(1-\alpha)^2}{\alpha}F''\right]g' > 0$$

implying that

$$(1-\alpha)f''F'' + \alpha f''g' > 0$$

From the definition of $B_I(a)$ we immediately see that this last inequality implies that $B_I(a) > 0$. It follows that $\hat{\alpha} > \alpha^{**}$, which concludes our proof of Proposition 1.

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