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Learning by Doing in Contests*

Derek J. Clark and Tore Nilssen†

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Abstract

We introduce learning by doing in a dynamic contest. Contestants compete in an early round and can use the experience gained to reduce effort cost in a subsequent contest. A contest designer can decide how much of the prize mass to distribute in the early contest and how much to leave for the later one in order to maximize total efforts. We show how this division affects effort at each stage, and present conditions that characterize the optimal split. The results are indicative of the fact that the designer weakly prefers to leave most of the prize mass for the second contest to reap the gains from the learning by doing effect.

Keywords: Learning by doing; dynamic contest; prize division

JEL Codes: D74, D72

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1 Introduction

In many contest situations, the same set of participants meet repeatedly in a series of contests. One example is applicants for grants from a research council that awards projects over several years. Other examples are in sports, where teams meet several times. In many such circumstances, there are interesting connections between the contests in the sequence. One such connection, explored in the present paper, is learning by doing: A contestant may, by exerting effort in early rounds, gain expertise and therefore be a more efficient contestant in later rounds.

Such learning by doing naturally has two effects on a contestant's behavior: His incentives to exert effort in later rounds increase since he now gets more out of his late-round effort with learning effects from early-round efforts. He also has incentives to increase efforts in early rounds, since such effort not only provides a chance to win early-round prizes but also makes him a more efficient contestant for late-round prizes.

Clearly, then, learning by doing makes contestants work harder, which is good for a principal aiming at making contestants' total efforts as high as possible. But many principals of repeated contests are budget constrained and need to make tough priorities. In such cases, the question is how to spread prizes across the sequence of contests. With learning-by-doing effects, the principal faces an interesting trade off: By having the higher prizes early on, the principal induces extra effort in early rounds, thereby providing the contestants with a lot of experience that can be used in later rounds. On the other hand, this leaves the participants with smaller prizes to fight over in those later rounds when they can expend effort more efficiently, and to counter this, the principal may want to leave more of the total prize money to later rounds.

To study this trade off, we set up a simple model of a sequence of two contests where two contestants compete for prizes, one in each round. The total prize over the two contests is assumed given, and we ask how the principal, or contest designer, should split the total over the two contests. We model learning by doing by assuming that, in the second contest, a participant's cost of effort is decreasing in the amount of effort put into the first contest. The aim of the contest designer is to maximize the two participants' total efforts over the two periods. The model is difficult to analyze with a general formulation of the learning-by-doing effect. We do find, however, that the trade-off described above is decided in the favour of the second period in cases where there are decreasing returns to scale in learning by doing: In such cases, the principal's choice is always to have the larger prize in the second period, and more so the greater is the learning-by-doing effect.

This paper contributes to a growing literature on contest design.[1] But whereas the current literature almost exclusively discusses the design of a single contest, we consider a sequence of contests, the crucial question being how the total prize should be split among the contests in the sequence.

The paper is related to what Konrad (2009) calls multi-stage battles, such as races and tugs-of-war;[2] One way of viewing the difference from that line of work, is that these authors discuss various forms of *dynamic win advantages* in repeated contests: winning an early round gives you some sort of advantage in later rounds, no matter how much effort you put in to win that early round. Here, on the other hand, we discuss a form of *dynamic effort advantages*: exerting effort in an early round, whether you win it or not, gives you an advantage in later rounds. A related analysis, also featuring dynamic effort advantages, is Grossmann and Dietl (2009) who model a contestant's early-round effort as an investment that has a positive effect on win chances in later rounds.[3] They do not discuss any contest-design issues.

Our work is of course also related to earlier studies on the economics of learning by doing, in particular by Arrow (1962) and Fudenberg and Tirole (1983). Of particular interest is Fudenberg and Tirole's discussion of a balanced-budget tax-subsidy scheme, which resembles our contest design problem. Their remark that incentives should be shifted to the mature phase of an industry when learning by doing is present, is reflected in our result that the optimal late prize is larger than the early prize.

The paper is organized as follows. Section 2 presents the general framework for the analysis as well as conditions that characterize the equilibrium. Section 3 demonstrates the results in an analysis of three specific learning-by-doing functions, and a discussion of the results and applications of the

[1] Examples include Moldovanu and Sela (2006) and Fu and Lu (2009).

[2] See Konrad (2009, ch. 8), as well as papers by Klumpp and Polborn (2006), Konrad and Kovenock (2009), and Clark *et al.* (2010). These papers do not typically discuss contest design, an exception being Clark, *et al.*'s (2010) analysis of how a principal should split prizes across rounds in a situation with repeated contests and dynamic win advantages.

[3] One can, of course, also envision there being effort disadvantages, i.e., today's effort having a negative effect on tomorrow's win chances, for example because of fatigue. This is discussed in single-contest, multi-stage settings by Harbaugh and Klumpp (2005) and Ryvkin (2009).

model are presented in Section 4.

2 The framework

Two participants compete over two periods for prizes with a total value of 1. Before the two rounds of contests, the principal determines how to split the total. In particular, the principal chooses the prize $M \in [0,1]$ available in the second period, leaving $(1-M)$ as prize for the first period. Participant i's effort in the first period is a_i and in the second period b_i, $i \in \{1,2\}$. Write $A := a_1 + a_2$ for total effort in period 1, and similarly $B := b_1 + b_2$ for total effort in period 2. The cost of expending effort a in period 1 equals a. However, the cost of expending effort in period 2 depends on the activity in period 1. In particular, a contestant with activity a in period 1 has a cost of expending effort b in period 2 equal to $c(a)\,b$, where $c(0) = 1$, and $c(a) > 0$, $c'(a) < 0$, and $c''(a) \geq 0$, all $a \geq 0$. Thus, the higher activity in period 1, the lower is the cost of effort in period 2. We write $c_i := c(a_i)$, and $C := c_1 + c_2$.

In each of the two periods, the winner of the contest is determined by a standard Tullock contest function. We disregard discounting and let the principal maximize participants' total efforts over the two periods, $A + B$.

We start by analyzing the period-2 game, following the participants' efforts (a_1, a_2) in period 1. The expected profit of player i in period 2 is then:

$$\pi_{i2} = \frac{b_i}{B}M - c(a_i)\,b_i, \quad i \in \{1,2\}. \tag{1}$$

The second-period subgame has an equilibrium determined by the following set of first-order conditions:

$$\frac{b_j}{B^2}M - c_i = 0, \quad i,j \in \{1,2\}, i \neq j.$$

Solving, we obtain:

$$b_i = \frac{c_j}{C^2}M, \quad i,j \in \{1,2\}, i \neq j.$$

We see that a player's effort in period 2 is proportional to the prize in that period; and own effort is increasing in the rival's cost level if the rival is more competitive in period 2, *i.e.*, $sign\frac{\partial b_i}{\partial c_j} = sign(c_i - c_j)$. Expected period-2 profit of player i in equilibrium can now be found by insertions in (1). We obtain:

$$\pi_{i2} = \left(\frac{c_j}{C}\right)^2 M, \quad i,j \in \{1,2\}, i \neq j.$$

Moving to period 1, a contestant takes into account the effect his effort today has on his expected profit tomorrow. Thus, in period 1, each player maximizes

$$\pi_{i1} = \left[\frac{a_i}{A}(1-M) - a_i\right] + \left(\frac{c(a_j)}{c(a_1)+c(a_2)}\right)^2 M, \qquad i,j \in \{1,2\}, i \neq j, \tag{2}$$

where $(1-M)$ is the prize in period 1. The first two elements, inside square brackets, are first-period profit while the third term is second-period profit. Players' equilibrium efforts in period 1, supposing second-order conditions are satisfied, are found through the first-order condition:[4]

$$\frac{a_j}{A^2}(1-M) - 1 - 2\frac{c_i' c_j^2}{C^3} M = 0, \quad i,j \in \{1,2\}, i \neq j. \tag{3}$$

Because of the symmetric nature of the problem, we can focus on a symmetric equilibrium, in which $a_1 = a_2 = a$, $c_1 = c_2 = c$, and $C = 2c$. This means that the above equation can be written as:

$$\frac{1}{4a}(1-M) - 1 - \frac{c'}{4c}M = 0,$$

which can be solved implicitly for a:

$$a = \frac{c(a)(1-M)}{4c(a) + Mc'(a)} \tag{4}$$

It is not in general clear that second-order conditions are satisfied, though. Note, in particular, that second-period profits may be convex in a player's first-period effort a_i. To see this, twice differentiate the last term of (2) to obtain

$$\frac{\partial}{\partial a_i}\left(\left(\frac{c(a_j)}{c(a_1)+c(a_2)}\right)^2 M\right) = -2\frac{c_i' c_j^2}{C^3} M > 0,$$

[4] We disregard cases where players choose their only available corner solution, $a = 0$, as well as cases where their optimum efforts are not well defined. This means that a player's equilibrium action is found through the first-order condition.

and

$$\frac{\partial^2}{\partial a_i^2}\left(\left(\frac{c\left(a_j\right)}{c\left(a_1\right)+c\left(a_2\right)}\right)^2 M\right)=-2\frac{c_j^2}{C^4}\left[Cc_i''-3\left(c_i'\right)^2\right]M.$$

This last expression is negative only if $c_i'' > 3\frac{\left(c_i'\right)^2}{c_1+c_2}$. If it is positive, then second period profit is increasing and convex, in which case an interior solution from (3) can be obtained only if that convexity is not too strong.

The complete second-order condition for player i's choice of first-period effort is:

$$-(1-M)\frac{2a_j}{(a_i+a_j)^3}-2c_j^2\left(\frac{\left(c_i+c_j\right)c_i''-3c_i'^2}{\left(c_i+c_j\right)^4}\right)M<0,$$

where the first (second) term stems from the effect of a_i on first-period (second-period) profit. Evaluated at a symmetric situation, this amounts to:

$$-(1-M)\frac{1}{4a^2}-\left(\frac{2cc''-3\left(c'\right)^2}{8c^2}\right)M<0$$

Inserting from (4) and simplifying, we obtain:

$$-2\left(4c+Mc'\right)^2-\left[2cc''-3\left(c'\right)^2\right]M(1-M)<0 \tag{5}$$

First-period profit is increasing and strictly concave in a_i, and second-period profit is increasing and may be concave or convex in a_i. If it is too convex, then the player will want a corner solution for a_i. To stop this, we need a learning-by-doing function that is sufficiently convex in a. The details of the conditions necessary will be discussed in the specific cases analyzed below.

Second-order conditions are not the only possible problem, however. As alluded to in footnote 2, we also need to ensure that the equilibrium first-period effort, given implicitly in (4), is feasible, in particular that it is non-negative. For this to hold requires that the denominator on the right-hand side of (4) be positive, or that $4c(a)+Mc'(a)>0$, for all $M\in[0,1]$. Since the second term here is negative, because $c'<0$, this condition holds for all a and M if it holds for all a at $M=1$, i.e., if

$$\frac{c'(a)}{c(a)}>-4,\text{ all } a\geq 0.$$

This amounts to an assumption that $c(a)$, even though it is falling, never falls too steeply.

We let $a(M)$ denote the equilibrium period-1 effort of the contestants as a function, given in (4), of the second-period prize M.

There will be symmetry also in period 2, implying $b_1 = b_2 = b$, and

$$b = \frac{M}{4c(a)} \tag{6}$$

where $a = a(M)$. The solution is denoted $b(M)$. Note that effort cost in the second period will be constant, given the level of M set by the principal: $c(a)b = \frac{M}{4}$. The dissipation rate in the second period, given by $\frac{2c(a)b}{M} = \frac{1}{2}$, is independent of the form of the learning-by-doing function and the division of the prize between rounds.

Claim 1 *The rate of rent dissipation in the second contest is independent of the prize division and the learning-by-doing function:* $\frac{2c(a)b}{M} = \frac{1}{2}$.

The principal aims at maximizing total effort $A + B$, which in the symmetric equilibrium is $2(a + b)$, with respect to the second-period prize M. This gives rise to the first-order condition:

$$\frac{da}{dM} + \frac{db}{dM} = 0. \tag{7}$$

Expressions for $\frac{da}{dM}$ and $\frac{db}{dM}$ are found by differentiating (4) and (6). Note that we need to implicitly differentiate (4) since both sides vary in a. Thus, from (4), we have:

$$da = -\frac{c(4c + c')}{(4c + Mc')^2}dM + \frac{M(1 - M)}{(4c + Mc')^2}\left((c')^2 - cc''\right)da,$$

implying

$$\frac{da}{dM} = -\frac{c(4c + c')}{M(2M - 1)(c')^2 + 8c(2c + Mc') + M(1 - M)cc''} \tag{8}$$

When we find the change in round-2 efforts, from (6), we need to account for both the direct effect on b of an increase in M, and the indirect effect working through the round-1 effort a:

$$\frac{db}{dM} = \frac{\partial b}{\partial M} + \frac{\partial b}{\partial a}\frac{da}{dM} = \frac{1}{4c}\left(1 - \frac{c'}{c}M\frac{da}{dM}\right), \tag{9}$$

where $\frac{da}{dM}$ is given in (8).

The effect of shifting the prize mass to the second contest on contestants' early effort can be signed unambiguously.

Claim 2 *For any optimal $M \in (0,1]$, it is the case that $\frac{da}{dM} < 0$.*

Proof. From (9) we have that

$$\frac{da}{dM} + \frac{db}{dM} = \frac{da}{dM} + \frac{1}{4c}\left(1 - \frac{c'}{c}M\frac{da}{dM}\right) = \frac{da}{dM}(1 - \frac{c'}{c}M) + \frac{1}{4c}$$

An optimal solution for M implies that

$$\frac{da}{dM}(1 - \frac{c'}{c}M) + \frac{1}{4c} \leq 0,$$

or

$$\frac{da}{dM}(1 - \frac{c'}{c}M) \leq -\frac{1}{4c}.$$

with strict equality for an internal solution $M \in (0,1)$. The right hand side is negative, and $(1 - \frac{c'}{c}M) > 0$ since $c' < 0$. For this inequality to hold requires $\frac{da}{dM} < 0$. ■

Moving prize mass to the second contest exerts different effects on early effort. First, early effort makes it cheaper to compete for the larger second contest prize and this tends to increase early efforts. On the other hand, the reduction in first contest prize tends to reduce first round efforts directly. By Claim 2 this first order effect always dominates. Furthermore, for an internal $M \in (0,1)$, it is the case that $\frac{db}{dM} > 0$ by Claim 2 and equation (9).[5]

The principal's first-order condition is then:

$$\frac{da}{dM} + \frac{db}{dM} =$$
$$\left(-\frac{c\,(4c + c')}{M\,(2-M)\,(c')^2 + 8c\,(2c + Mc') + M\,(1-M)\,cc''}\right)(1 - \frac{Mc'}{4c^2}) + \frac{1}{4c} = 0$$

The principal's optimum M is thus found among solutions in $[0,1]$, satisfying a second-order condition, of the following equation:

$$\left[cc'' - 2\,(c')^2\right]M^2 - c\,(12c' + c'')\,M - 4c^2\,[4\,(1-c) - c'] = 0 \qquad (10)$$

[5]Note that this does not necessarily follow in the case of a corner solution $M = 1$.

Solving this equation in general is not possible. We therefore resort to discussing some specific cases of learning-by-doing effects in order to see how this phenomenon affects the contest design. Discussions of the second-order condition are also relegated to the specific cases below.

3 Three learning-by-doing functions

In order to get a firmer grip on this problem, and in particular to get out results on the optimum contest design under learning by doing, it is necessary to introduce a specific cost function $c(\cdot)$. Such a specific function also has the advantage that we can parameterize the learning-by-doing effect and thus discuss the effect on the optimum contest design of learning by doing being more pronounced. We will discuss three different functions below. From this discussion, some features of the solution will emerge: the late prize is always weakly larger ($M \geq \frac{1}{2}$); and it is always weakly increasing in the extent of the learning-by-doing effect.

3.1 Logistic learning by doing

We start out with the logistic learning-by-doing function

$$c(a) = \frac{1}{1+sa}, \ \ s > 0 \tag{11}$$

Here, the parameter s is a measure of the learning-by-doing effect: the higher is s, the larger is the reduction in period-2 effort costs from a given effort in period 1. We have:

$$c'(a) = -\frac{s}{(1+sa)^2} < 0; \ \ c''(a) = \frac{2s^2}{(1+sa)^3} > 0 \tag{12}$$

This function has the interesting property that $cc'' = 2(c')^2$. This means that the quadratic term in (10) vanishes. Therefore, a solution of the principal's problem is found implicitly from that equation as

$$M = \frac{4c(a)\left[4(1-c(a)) - c'(a)\right]}{-12c'(a) - c''(a)}, \text{ with } a = a(M);$$

or, with insertions from (11) and (12),

$$M = 2\frac{1+4a(M)\left[1+sa(M)\right]}{6\left[1+sa(M)\right]-s}.$$

The next step is to find $a(M)$. By combining (4) and (11), we find an implicit expression for $a(M)$:

$$a = \frac{(1-M)(1+sa)}{4(1+sa) - Ms}$$

Solving for an explicit expression, we find two solutions, out of which one is always negative and one is always positive. We want the positive solution, implying:

$$a(M) = \frac{1}{8s}\left(s - 4 + \sqrt{(s+4)^2 - 16Ms}\right) \tag{13}$$

Note that $\frac{da}{dM} < 0$ for all parameter values; this is in accordance with Claim 2 above. Moreover, it turns out that the second-period profit of a player is always concave in this case, implying that the second-order condition for a contestant's choice of first-period effort is always satisfied.[6]

We can now use (6) and (13) to find an expression for contestants' second-period effort in this case. We have

$$b(M) = \frac{1}{32}M\left(s + 4 + \sqrt{(s+4)^2 - 16Ms}\right) \tag{14}$$

Note that a necessary condition for the effort functions to be defined is that the square root expression in $a(M)$ and $b(M)$ is defined, i.e. $\frac{(s+4)^2}{16s} \geq M$. This is always fulfilled since the left-hand side has a minimum value of 1.

The principal aims at maximizing total expected effort, that is maximizing $a(M) + b(M)$, where $a(M)$ and $b(M)$ are given by (13) and (14). This gives rise to the first-order condition[7]

$$24Ms - 8s - s^2 + 16 - (s+4)\sqrt{(s+4)^2 - 16Ms} = 0$$

Heeding the restriction that $M \in [0,1]$, we thus have:

$$M = \begin{cases} \frac{1}{36s}\left[s^2 + 8s - 32 + (s+4)\sqrt{s^2 + 8s + 64}\right], \\ \qquad\qquad\qquad\qquad \text{if } 0 < s < 4\left(1+\sqrt{2}\right) \approx 9.66, \\ 1, \qquad\qquad\qquad\qquad \text{if } s \geq 4\left(1+\sqrt{2}\right). \end{cases} \tag{15}$$

[6] The concavity of the second-period profit is ensured by noting that $c_i'' - 3\frac{(c_i')^2}{C} = \frac{s^2}{2(1+sa)^3} > 0$.

[7] The second order condition is satisfied for $\frac{s^2+8s+32}{12s} > M$ which always holds since the left-hand side is always above 1.

The solution is depicted in Figure 1.

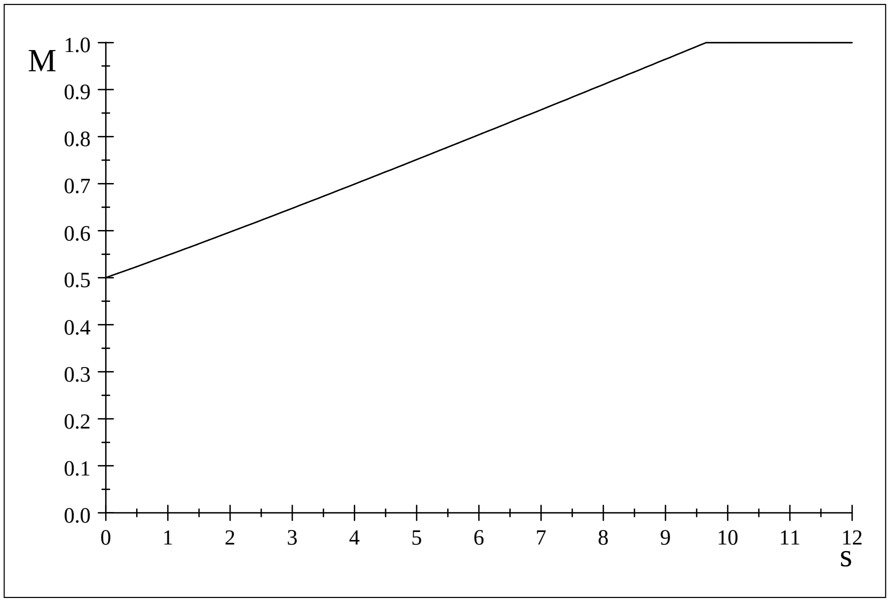

Figure 1: Optimal M with logistic learning by doing

As the learning by doing effect diminishes ($s \to 0$), the optimal prize split is $\frac{1}{2}$ in each round. A more pronounced learning by doing effect means that progressively more of the prize mass is moved to the second round. The initial relationship between optimal M and the parameter s is convex, so that prize mass is moved at an increasing rate. When the whole prize is distributed in the second period, then only the corner solution obtains.

We can now put (15) into (13) to find how contestants' effort depends on the extent of learning by doing under the principal's optimal contest design. We have:

$$a = \begin{cases} \frac{1}{8s}\left(s-4+\frac{1}{3}\sqrt{5\left(s+4\right)^2+192-4\left(s+4\right)\sqrt{s^2+8s+64}}\right), & \text{if } 0 < s < 4\left(1+\sqrt{2}\right) \\ \frac{s-4}{4s}, & \text{if } s \geq 4\left(1+\sqrt{2}\right). \end{cases}$$

$$b = \begin{cases} \frac{\left(s+\sqrt{(s+4)^2-\frac{4}{9}(s+4)\sqrt{s^2+8s+64}-\frac{4}{9}(8s+s^2-32)}+4\right)\left(8s+(s+4)\sqrt{s^2+8s+64}+s^2-32\right)}{1152s}, & \text{if } 0 < s < 4\left(1+\sqrt{2}\right) \\ \frac{1}{32}\left(s+4+\sqrt{-8s+s^2+16}\right), & \text{if } s \geq 4\left(1+\sqrt{2}\right). \end{cases}$$

Thus, efforts are positive for all $s > 0$, and first-period effort is increasing in the learning-by-doing effect: $\frac{da}{ds} > 0$. Both parts of this function are concave, and as s becomes very large, first-period effort converges to $\frac{1}{4}$ per player. Efforts per player are depicted in Figure 2, where the whole line is a and the dashed one b.

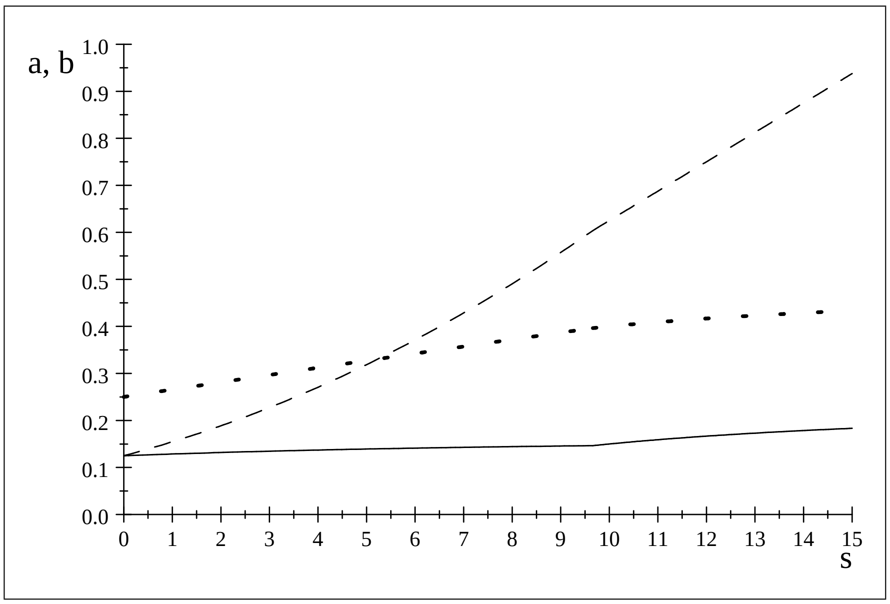

Figure 2: Optimal efforts and rent dissipation with logistic learning by doing

Effort in the second period lies over that of the first period, and is always increasing in the learning by doing parameter; the total cost of this exploding effort converges to $\frac{1}{4}$, however, since this cost is $\frac{M}{4}$, as noted previously, and $M = 1$ for large enough values of s. In Figure 2, rent dissipation per player is depicted as the dotted line. This reflects a player's total cost of seeking the prize of one unit: $a + c(a)b$. As the learning-by-doing parameter gets larger, the dissipation rate per player increases, converging to $\frac{1}{2}$ in the limit.

3.2 Linear learning by doing

Consider next a linear learning-by-doing function:

$$c(a) = 1 - ta, \; t > 0, \tag{16}$$

where now t is the parameter measuring the strength of the learning-by-doing effect. This linear formulation is appropriate in situations where there is no reason to believe that there is a diminishing return from learning by doing as first-period effort increases, since here $c''(a) = 0$. In this case, the principal's first-order condition in (10) writes

$$t^2 M^2 - 6t(1 - ta)M + 2t(1 - ta)^2(1 + 4a) = 0, \text{ with } a = a(M). \tag{17}$$

Combining (4) and (16), we find that $a(M)$ now solves:

$$8ta^2 - 2[4 + t(1 - 2M)]a + 2(1 - M) = 0.$$

This equation has two solutions,

$$a = \frac{1}{8t}\left[4 + t(1 - 2M) \pm \sqrt{(4 - t)^2 - 4t^2 M(1 - M)}\right], \tag{18}$$

when the term under square root in (18) is non-negative, i.e., when

$$\left(\frac{4 - t}{2t}\right)^2 \geq M(1 - M), \tag{19}$$

and none otherwise. The right-hand side of (19) is never greater than $\frac{1}{4}$, since $M \in [0, 1]$. A sufficient condition for the left-hand side to be no less than $\frac{1}{4}$, so that players' equilibrium efforts are well defined, is $t \in (0, 2]$. We therefore impose such an upper bound on the learning-by-doing effect t. With this restriction, we can subject both the two solutions in (18) to the second-order condition for a player's first-period effort in a symmetric situation, given in (5). We find that the higher root in (18) does not satisfy the second-order condition for any $t \in (0, 2]$, whereas the lower root satisfies the second-order condition if and only if

$$t < \frac{4\left[5\sqrt{6M(1 - M)} - 6\right]}{25M(1 - M) - 6} \tag{20}$$

Subject to this constraint, we thus have

$$a(M) = \frac{1}{8t}\left[4 + t(1 - 2M) - \sqrt{(4 - t)^2 - 4t^2 M(1 - M)}\right] \tag{21}$$

Inserting this back into the first-order condition for M in (17) and solving, we find a simple solution:

$$M = \frac{1}{2}. \tag{22}$$

Thus, when the learning-by-doing effect is linear, the optimal policy for the principal is to split the total prize evenly between the two rounds of contest.

We can now put $M = \frac{1}{2}$ back into the constraint in (20) to find that the second-order condition on players' first-period effort requires

$$t < \bar{t} := 8\left(5\sqrt{6} - 12\right) \approx 1.98$$

Finally, given this restriction on t, one can show that the principal's second-order condition for her choice of M is satisfied for all $t \in \left(0, \bar{t}\right)$. And for the permitted range of the learning by doing parameter, one can verify that $\frac{\partial a}{\partial M} < 0$.

Putting (22) back into (21) and then using (6), we find that, when the principal exercises her optimal policy in the linear case, the contestants exert the following efforts:

$$a = a\left(\frac{1}{2}\right) = \frac{1}{2t}\left(1 - \sqrt{1 - \frac{t}{2}}\right) = b\left(\frac{1}{2}\right),\ t \in \left(0, \bar{t}\right).$$

When the prize is divided equally between periods, the contestants have equal effort in each period, and this effort is increasing in the learning-by-doing effect: $\frac{da}{dt} > 0$, $\frac{db}{dt} > 0$ $\forall t \in \left(0, \bar{t}\right)$. These efforts are depicted as the whole line in Figure 3.

Although effort is equal in each period, the cost of efforts in round 2 is lower than round 1 due to the learning by doing effect. In the linear case, this cost is constant at $c(a)b = \frac{1}{8}$. The amount of rent dissipation per player is $a + c(a)b = \frac{4+t-2\sqrt{2(2-t)}}{8t}$, depicted as the dotted line in Figure 3. Rent dissipation per player increases and is convex in the learning-by-doing parameter, reaching the maximum level of 0.352 for $t = \bar{t}$.

3.3 Exponential learning by doing

Finally, we study the case of an exponential learning-by-doing effect, with

$$c\left(a\right) = e^{-ua}, \tag{23}$$

implying

$$c'\left(a\right) = -ue^{-ua};\ c''\left(a\right) = u^2 e^{-ua}. \tag{24}$$

In this case, the first-order condition of the principal, (10), implies

$$u^2 M^2 - u\left(12 - u\right) M + 4\left[4 - \left(4 - u\right) e^{-ua}\right] = 0, \text{ with } a = a\left(M\right). \tag{25}$$

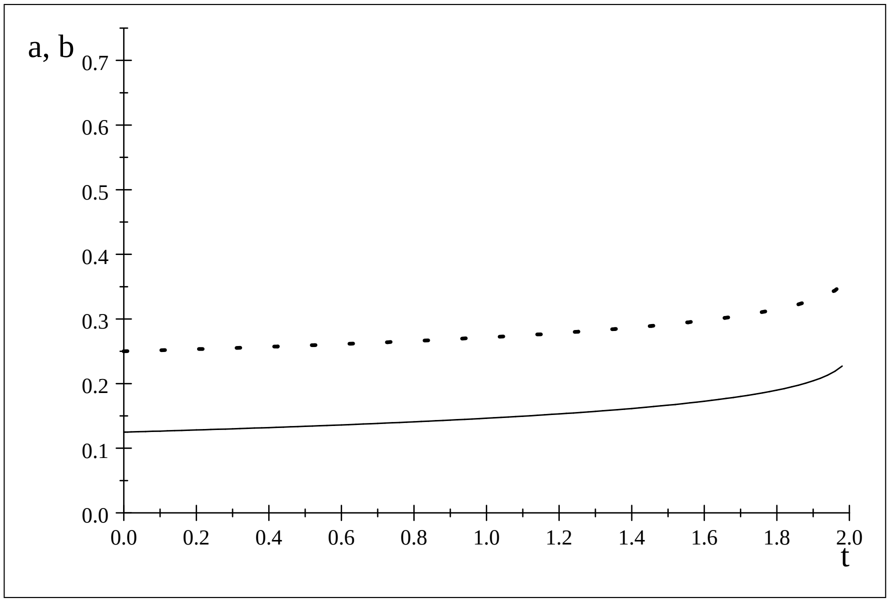

Figure 3: Optimal effort and rent dissipation with linear learning by doing ($a = b$)

Combining (4) and (23), we have:

$$a(M) = \frac{1-M}{4-uM}. \tag{26}$$

To make sure that effort in the first period is positive we require the learning-by-doing parameter to be $u < 4$. In this range, it is easily verified that $\frac{\partial a}{\partial M} < 0$. The second-order condition for players' choice of first-period effort, given in (5), becomes, after insertions from (23), (24), and (26),

$$\frac{u^2 M}{2} - \frac{(4-uM)^2}{1-M} < 0. \tag{27}$$

Inserting first-period efforts from (26) in (25), we have the optimal M as an implicit function of u, the extent of the learning-by-doing effect:

$$u^2 M^2 - u(12-u)M + 4\left[4 - (4-u)\,e^{-u\frac{1-M}{4-uM}}\right] = 0 \tag{28}$$

We have not succeeded in obtaining an explicit solution for M from this equation. However, in Figure 4 we plot combinations of M and u that solve the equation in (28), obtained from a numerical analysis. In the same figure, we also plot the second-order condition from (27). In particular, combinations (u, M) below and to the left of the dashed line in Figure 4 satisfy the second-order condition for first-period effort. A check of the second-order condition for the principal's choice of M shows that it is satisfied whenever the second-order condition on players' first-period efforts, given in (27), holds. The whole line gives then the optimal value of M as a function of the learning-by-doing parameter u; from the figure, we see that the principal's optimal choice of M, the second-round prize, is increasing in the learning-by-doing effect u, and at an increasing rate, for any value of $u \in (0, 3.74)$.

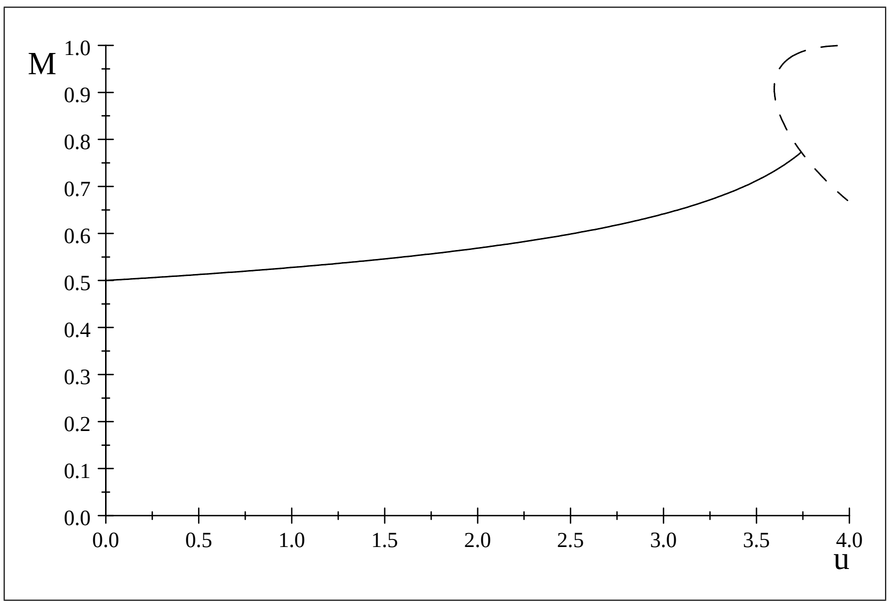

Figure 4: Optimal M in exponential case.

First-period effort cannot be computed directly for this case as a closed-form solution. Numerical analysis reveals the relationships depicted in Figure 5 between efforts and the learning-by-doing parameter. This indicates that $\frac{da}{du} > 0$, and $\frac{db}{du} > 0$, $\forall u \in (0, 3.74)$. The same figure also plots the level of rent dissipation per player. The larger effort in the second period occurs at a significantly lower cost than the efforts in the preceding one.

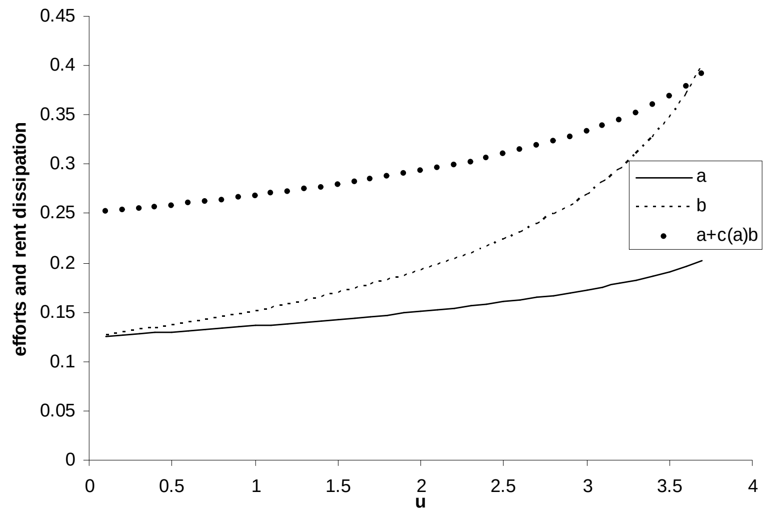

Figure 5: Optimal efforts and rent dissipation with exponential learning by doing

4 Discussion and applications

We have analyzed a simple contest design problem in which a principal faces agents who compete against each other twice, and who reduce future effort costs through experience gained in the first confrontation. The instrument of design for the principal is how much of the prize mass to distribute early, and how much to leave for the late contest, in order to maximize total effort. Effort in the first period has a twofold effect since it can secure the early contest prize, at the same time as it reduces the costs of competing for the later prize. In distributing the prize mass, the principal must be aware of the trade-off involved: A large early prize stimulates contestants' effort in the first period, but makes effort in the second period less attractive. Since effort in the second period is less costly due to the learning by doing effect, this loss of later effort can outweigh the effort stimulated in the first contest, leading to more weight being given to the later prize.

Even in the context of the very simple model that we have developed, it would seem that general results for the distribution of the prize mass are hard to obtain. The amount of rent dissipation in the second contest is shown to be half of the prize on offer at that stage, and we have shown generally that shifting the prize mass to the second period will decrease effort at the first stage.

To move the analysis further to investigate the optimal division of the prize between periods, we have had to assume different forms for the learning by doing function. The results are indicative of a general pattern. The principal will always prefer to distribute at least as much of the prize in the second contest as the first. This reduces effort in the first contest, however, but encourages effort in the second contest when this activity is cheaper. In the logistic and exponential cases, in which the learning-by-doing function is convex, the principal distributes more to the second contest,[8] the stronger is the learning-by-doing effect, and therefore the larger is the cost reduction achieved from a given first round effort. The linear case is different, with half of the prize being distributed to each contest, independent of the effectiveness of learning by doing. With the learning-by-doing function being linear, there is no increased effect on the margin from pushing more of the prize into the second contest.

The phenomenon of gaining experience in contests that affects the ability to compete later on would appear to have widespread applications. Research tournaments in which labs compete for the best invention have been shown to have a close connection to the Tullock rent-seeking game that we have analyzed here.[9] One can imagine that there are several stages to the invention process which may make it natural to divide up an overall prize into stage prizes. The principal may for example grant an intermediate prize to the lab that makes the best prototype, and then a prize to the team making the best marketable product. Experience gained in the early stage will likely affect the cost of efforts later on.

The aim of maximizing efforts is also sensible in this context, since the European Union's Lisbon strategy of 2000 pinpointed a target level for research and development activity.[10] The process of applying for research money fits well into our framework, where early applications give experience that aid later ones. A key insight from our study with a clear bearing on the running of large research programs is that, in the presence of learning-by-doing effects, the larger funds should be kept for late rounds. And the stronger the learning-by-doing effects are, the more research money should be saved for

[8] Until a corner solution is reached in the logistic case.

[9] See Baye and Hoppe (2003) and Fullerton and McAfee (1999).

[10] See Johansson, et al. (2007) for more on the Lisbon agenda.

later rounds.

Sports contests provide another area of application in which a natural goal is to maximize the effort contributed by the competitors (see Szymanski, 2003). The structure that we have analyzed resembles that of two teams that compete with each other first in one competition (such as a league), and then again in another such as the playoff finals or a cup competition. Experience gained from the first encounter affects efforts levels in the second. Again, our analysis indicates that the playoff prize should be larger than the prize for winning the league, and the more so the stronger learning-by-doing effects are present.

Industrial organization provides several applications of our framework and analysis. Two firms that compete for shares of different market segments for example, or firms that compete to have their product adopted as the standard in which the initial competition is to establish a product, and then the later one to determine the one chosen as the standard.

With respect to procurement contracts, our analysis shows that a principal may be well advised to divide them up into smaller prizes so as to take advantage of the learning by doing effect. More effort will be expended in the competition to win the contract if the contestants are allowed to reap the gains from their experience in competing for an intermediate prize, than if the prize is awarded at once.

More generally, our analysis points to the importance of seeing sequences of contests in context. When there are dynamic effort effects, such as the learning-by-doing effects highlighted in this paper, the players will take these effects into account when planning for their participation in the contest sequence. And so should also the principal do, whether she is running a research program or a sports league.

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