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Employee Stock Options

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Employee Stock Options*

Øystein Børsum†

June 29, 2010

Abstract

An entrepreneur with information about firm quality seeks financing from an uninformed investor in order to pay a worker. I show that if the worker, too, knows the true quality of the firm, then certain long term wage agreements can credibly signal firm quality. Such wage agreements have a low initial wage and are equity-like in the sense that future pay is tied to firm performance, because only a worker in a good quality firm would be willing to defer compensation to an uncertain future, getting paid only if the firm succeeds. Moreover, in an important pooling equilibrium, all firms use equity-like wage contracts. The model provides an economic rationale for the use of stock options among regular, non-executive employees, in particular in small, knowledge-intensive firms (such as in the "new economy") where workers are more likely to have information about the true quality of the firm.

JEL classification: D82; G32; J33; M52

Keywords: Financing; Asymmetric information; Signaling; Employees; Compensation; Stock options

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1 Introduction

Stock options grants to regular employees are less about providing work incentives than they are about firm financing. While agency theory gives a rationale for tying the remuneration of executives to the performance of the firm, the link between the effort of each regular employee and firm outcomes is at best weak, and ridden with free rider problems.[1] The trouble with stock options compared to other forms of remuneration such as fixed pay, is that risk averse employees value them below their cost to the firm (Hall and Murphy, 2000). Absent incentive effects, therefore, their use seems inefficient (Hall and Murphy, 2003).

This paper argues that asymmetric information, and in particular the case when workers know the true quality of the firm, is central to understand stock options granted to regular employees. For this purpose, I build a model in which an entrepreneur with information about firm quality seeks financing from an uninformed investor in order to pay a worker. Due to limited liability and lack of own funds, the entrepreneur cannot, by himself, credibly signal firm quality to external investors. But if the worker, too, knows the true quality of the firm, then *certain long term wage agreements can credibly signal firm quality*. Such wage agreements have a low initial wage and are equity-like in the sense that future pay is tied to firm performance, because only a worker in a good quality firm would be willing to defer compensation to an uncertain future, getting paid only if the firm succeeds.[2] Because the worker prefers stable wages, this is a costly way for the entrepreneur to remunerate the worker. But it may be the best the entrepreneur can achieve given the need for financing and the asymmetry of information.

Under certain conditions, the model admits a pooling equilibrium that Pareto dominates (in an interim sense) the least cost separating equilibrium, in which *all* firms use equity-like wage agreements. Here, a good quality firm always prefers to defer some of the worker's compensation, because on the margin, this is a less expensive source of funds than external financing.[3] This suggests that equity-like wage agreements could become the standard in an industry or business segment where workers are known to have inside information about the quality of the firm (and not only a characteristic of good quality firms).

This work inherits from a large literature on corporate financing under asymmetric information. The idea that an entrepreneur can signal firm quality to external investors through his financial position was initially proposed by Leland and Pyle (1977), and

[1]The incentive effect from individual-based performance pay is likely to be much stronger than firm-based performance pay.

[2]Throughout the paper, the words "wage" and "compensation" are used interchangeably to mean the total compensation to the worker, including the payoff from any stock options.

[3]For bad quality firms the opposite is true, but they must use the same wage agreement as good quality firms in order to avoid being revealed as bad.

the present model has much in common with the ideas expressed in their model.[4] In particular, the essence of the signalling mechanism is the same: the insider (in a good quality firm) must adopt a payoff structure that covaries with firm performance in such a way that it would not be rational to do so, were the firm less likely to succeed (the case of a bad quality firm). It also inherits from the work of Myers (1984) and Myers and Majluf (1984) in that external financing may be "too expensive", leading firms to prefer internal sources of funds. In contrast to their work, however, the present model does not give rise to a strict "pecking order".

The idea that workers may have inside information and therefore play a role in firm financing, partly comes out of these classical articles. Nevertheless, to the best of my knowledge, no-one has explicitly modeled the financing problem with three parties, as I do. An explicit model will help to solve the debate in the inconclusive empirical literature on the use of broad-based stock option plans (See Core and Guay, 2001, Ittner et al., 2003, and Oyer and Schaefer, 2005). One of the striking facts exposed in this literature is that broad-based stock option plans are used much more in firms in the "new economy" than in firms in other sectors. The present model seems particularly well suited for "new economy" firms. A typical such firm (e.g. a software start-up) is small and knowledge intensive. This likely provides workers with information about firm quality that external investors do not have. It also increases the likelihood that the entrepreneur depends crucially on the workers' willing participation in order to run the firm, and that wages are a major part of the firm's expenses. These are, in addition to the assumption of a cash constrained entrepreneur, the key assumptions and decisive factors in the model.

The model is one of multidimensional contract design by an informed principal (the entrepreneur) facing two agents, one informed (the worker) and one uninformed (the investor). Early models of multidimensional signalling are Wilson (1984) and Milgrom and Roberts (1985). As these authors demonstrate, the major difference from unidimensional signalling models is the need for a careful examination of all combinations of choice variables that can serve as signals, and deviations from them.[5] Following their approach, I assume that the entrepreneur proposes a single contract.[6] A rather different alterna-

[4]Tirole (2006) develops the Leland-Pyle model in an optimal contracting framework similar to that used here.

[5]The issue is more complicated when the information hidden from one party is multidimensional, i.e. when types cannot be ordered along a line, as shown by Quinzii and Rochet (1985).

[6]Maskin and Tirole (1992) loosen this assumption using a mechanism design approach. In the context of the present model, their approach would amount to letting the entrepreneur propose a *menu* of contracts from which he is eventually allowed to choose a specific contract, given that the worker and the investor accept the menu. Intuitively, proposing a menu of contracts yields at least as high payoff to the entrepreneur as proposing a single contract, and may sometimes strictly improve it. However, Maskin and Tirole (1992) show that the least cost separating equilibrium, which plays an important role in my analysis, is also the unique equilibrium outcome of their more general mechanism, under certain parameter ranges.

tive, perhaps less appealing to the corporate finance context, would be to assume that the investor, rather than the entrepreneur, proposes the contract, to obtain a screening model.[7]

Signalling models with more than two parties are not uncommon, but usually posit several uninformed agents generating diverging interests.[8] In the present model, the informed principal (the entrepreneur) faces one informed agent (the worker) and one uninformed agent (the investor). The entrepreneur and the worker have partially aligned interests. One the one hand, they both would like to obtain financing, a condition for realising any potential surplus in the firm. On the other hand, each of them would like as much of the surplus for himself. To handle this, I let the entrepreneur propose a take-it-or-leave it offer to the worker, who chooses between this and an outside option. A alternative but presumably more cumbersome approach would be to explicitly model a bargaining process between the two as a first step before financing is requested.

The paper proceeds as follows: Section two lays out the model. Section three contains the analysis and is the main part of the paper. It starts by characterising the (least cost) separating equilibrium, paying close attention to how wage contracts can credibly signal firm quality when the worker is informed. Next, it characterises an important pooling equilibrium which Pareto dominates the least cost separating equilibrium. These are but two in a large set of perfect Bayesian equilibria for the model, but I show that imposing a set of reasonable restrictions on out-of-equilibrium beliefs implies uniqueness. Section four considers how the equilibrium wage profile could be implemented and argues that stock options are perfectly suited for this purpose, because they provide commitment and are easily observable. It also discusses the use of stock options to attract and retain employees in light of worker inside information. Section five concludes with a discussion of the efficiency of employee stock options.

[7]See Rothchild and Stiglitz (1976), and Wilson (1977), for screening approaches to the description of insurance markets.

[8]For instance, Gertner, Gibbons and Scharfstein (1988) study a model in which one informed party (a firm) has opposing interests with respect to the signal it would like to send to two uninformed parties (a regulator and a potential competitor).

2 Model

2.1 Agents, technology, and the capital market

I consider a contractual relationship over two periods between an entrepreneur and a worker (together forming a firm), and the capital market.

The *entrepreneur* is the owner of the firm, and this entitles him to the residual cash flow after workers' and creditors' claims to the firm's output. The entrepreneur cares about expected cash flows. He is protected by limited liability, so he will never operate the firm if it isn't profitable. Closing the firm brings zero residual cash flow. Consequently, for the firm to operate in period one, the expected discounted cash flow as of period one must be positive (else, closing the firm immediately would be preferable). And for the firm to operate in period two, the cash flow in period two must also be positive (else, closing the firm in period two would be preferable).

To operate the firm, the entrepreneur needs the *worker*. The worker cares about expected lifetime utility of wages, where the utility derived in each period is represented by the function $u(\cdot)$ which is strictly increasing and strictly concave for all wage levels, i.e. $u' > 0$ and $u'' < 0$.[9] In order to ensure that the wage in period one is strictly positive in any contract, I assume that $u(\cdot)$ satisfies the Inada condition $\lim_{w\to 0} u'(w) = \infty$. To save on notation, I will omit subscripts for variables referring to period one (a convention maintained throughout the paper). The worker's objective is

$$U = u(w) + \beta E[u(w_2)], \tag{1}$$

where β denotes the time discount factor (strictly positive, finite, and the same for all agents in the model) and E denotes the expectations operator. Because of the strict concavity of $u(\cdot)$, there is a rationale for the entrepreneur to provide the worker with stable wages across both time and states.

The worker can quit for some alternative job providing a wage $\bar{w} > 0$. I assume that the entrepreneur cannot easily replace the worker, so if the worker chooses to quit for the alternative job, the firm is closed. It is convenient to normalise the utility value of the alternative job to zero, i.e. $u(\bar{w}) = 0$. Then, for the firm to operate in both periods, wages must satisfy two lower bounds on the worker's utility: $U \geq 0$ (else quitting immediately would give higher expected discounted utility) and $u(w_2) \geq 0$ (else quitting in period two would give higher expected utility).

The *production technology* of the firm is as follows. Output in period one is zero. The firm must operate in period one in order to live to see period two. Output in period two is

[9] The worker is therefore averse to all forms of variation in wages, be it intratemporal (risk) or intertemporal.

stochastic. I consider a scenario with only two possible outcomes, dubbed "success" and "failure". To simplify notation, I will let S (for success) or F (for failure) replace the time subscript 2 whenever appropriate. The conditional period two output is $Y_2 \in \{Y_S, Y_F\}$ with

$$Y_S > 0 \quad \text{and} \quad Y_F = 0.$$

I assume labour is the sole input factor for the firm and wages the sole cost. The Inada condition implies that the wage to the worker must be strictly positive. But the firm generates zero output in period one. So operating the firm in period one implies a loss equal to the wage cost. Thus period one can be thought of as an investment phase, whereby the wage cost is "invested" with the hope to reap benefits in period two. In a sense, the model is of the "fixed investment" type, as the scale of the firm is fixed. Contrary to such models, however, the cost of this investment is not fixed, because the wage is an endogenous outcome of contract formation.[10] In this sense, the model resembles a "variable investment" type model.[11]

To cover the period one operating loss, the firm needs *financing*, and the interesting case is when the entrepreneur depends on the capital market (which will be the uninformed party) to obtain it. Hence, I will assume that the entrepreneur has no own funds. Let B denote the amount lent to the firm. The cash flow constraint is

$$B - w \geq 0. \tag{2}$$

Note that I allow the entrepreneur to borrow more than he needs to finance wages, i.e. $B > w$, although eventually in equilibrium $B = w$. In some cases, it would be without loss of generality to impose $B = w$ from the onset, but it is of interest to see when this is not the case and why. On the other hand, in order to simplify the analysis, I will assume that lending is to the firm, not to the entrepreneur personally, and that the entrepreneur retains the control rights of the firm.

There is a competitive capital market consisting of many small risk neutral, limited liability investors, which I for simplicity will refer to as the *investor*. Because of competition, the investor is always willing to lend when doing so has a non negative expected net discounted cash flow, i.e. when

$$-B + \beta E[R_2] \geq 0, \tag{3}$$

where $R_2 \in \{R_S, R_F\}$ denotes the payoff the investor receives in period two, in the case of success and failure respectively, in return for lending B in period one. Here, R_S and

[10]Positive period one output $Y > 0$ could be added, as well as a required fixed investment cost Q to be paid on top of the wage cost. Indeed, an interpretation of the model in the text is the case $Y = Q \geq 0$.

[11]See Tirole (2006, chapter 3) for a discussion of "fixed" versus "variable" investment models.

R_F are choice variables, but R_2 is a stochastic variable because which of the two values it will take is a stochastic outcome. Note that neither R_S nor R_F can be negative, because under limited liability, the investor will never pay out any amount to the entrepreneur in the last period.

Using this notation, the entrepreneur's objective can now be stated explicitly as

$$V = B - w + \beta E[Y_2 - R_2 - w_2]. \tag{4}$$

Note that limited liability of the entrepreneur requires

$$Y_2 - R_2 - w_2 \geq 0, \tag{5}$$

because the entrepreneur will never operate the firm unless it is profitable. This implies that the firm will be closed in the case of failure, as no wage can be paid. To see this, note that $Y_F = 0$ implies that any wage would have to be financed with additional money from the investor, i.e. $w_F > 0$ would require $R_F < 0$, but this would violate the limited liability of the investor. The worker will never work for free, i.e. $w_F = 0$, because he will be strictly better off quitting for the alternative wage $\bar{w} > 0$. Consequently, the outcome in case of failure is always firm closure (and zero payments $w_F = R_F = Y_F = 0$).[12]

2.2 Contracts and information

A *contract* is a four dimensional vector

$$c = (w, w_S, B, R_S) \in R^4_+.$$

A contract is *feasible* if it satisfies the cash flow constraint (2) and the limited liability constraint (5). The set of feasible contracts

$$C = \left\{ c : \begin{array}{ll} \text{(CF)} & B - w \geq 0 \\ \text{(LL)} & Y_S - R_S - w_S \geq 0 \end{array} \right\} \tag{6}$$

is compact and convex, and includes the origin $c_0 = (0, 0, 0, 0)$ (which I take to mean immediate firm closure).

Evidently, the contract has two parts. The first part is a (long term) wage arrangement between the entrepreneur and the worker, specifying a wage w in period one, and a wage w_S in period two, contingent on the firm's success. I will call this the contract's *wage*

[12]Observe that if investors could commit, it might be in their interest to sell an insurance contract whereby the worker would receive a severance payment in the case of failure (i.e. $w_F > 0$), to compensate for the inconvenience of job loss.

profile. The second part is a borrowing arrangement between the entrepreneur and the investor concerning financing, specifying an amount B lent to the firm in period one, and a payoff R_S to the investor in period two, contingent on the firm's success. I will call this the contract's *borrowing terms*. The payoffs to the entrepreneur from the contract are the residual cash flows $B - w$ in period one and $Y_S - R_S - w_S$ in period two.

The *information problem* is as follows. There are two types of firms, dubbed "good" and "bad", that differ only in terms of the firm's probability of achieving success in period two. Using a tilde to decorate parameters and variables pertaining to bad firms (a convention maintained throughout the paper), these probabilities are denoted π and $\tilde{\pi}$, respectively, with

$$0 < \tilde{\pi} < \pi < 1,$$

Both the entrepreneur and the worker know the true type of their firm, but the investor does not. This could be interpreted as e.g. some specific competence or knowledge held jointly by the entrepreneur and the worker, about the product or the market the firm operates in, which is relevant to achieve success. This information is non-verifiable, so it cannot simply be disclosed or requested proven. For the entrepreneur and worker in a good firm, this is unfortunate, because the investor will cast doubt on whether they tell the truth about their type, given that the entrepreneur and worker in bad firms would be tempted to pretend that they, too, constitute a good firm.

Since the entrepreneur is the owner of the firm, I find it natural to assume he proposes a contract to the two other parties. The timing is as follows:

1. The entrepreneur offers a contract $c \in C$.

2. The worker accepts or rejects to work for the contract c. In case of rejection, the worker leaves for the alternative wage $\bar{w}$, and the firm is closed, giving payoff 0 to the entrepreneur.

3. The investor, having observed that the worker has accepted to work for the contract c, accepts or rejects to provide financing for c. In case of rejection, since wages cannot be paid, the firm is closed and the worker turns to the alternative wage $\bar{w}$.

4. The contract c, accepted both by the worker and the investor, is enacted.

Whether the worker accepts or rejects a given contract will depend on his outside option. I assume that the worker can make alternative use of his inside information in the market. Imagine, for example, that a software programmer could take the ideas his firm is developing and use it at a competing firm. To capture this in a generic way, I introduce a parameter x, defined as the expected utility the worker could obtain from the use of his information outside the firm by leaving in period one. The parameter x effectively serves as a sharing rule for any surplus that exists in the firm. The valid range of x is a compact interval $[0, \bar{x}]$, where $x = 0$ corresponds to the case where the entire surplus goes to the entrepreneur, i.e. the worker obtains no more utility than the utility of the alternative wage $\bar{w}$, since $u(\bar{w}) = 0$. $x = \bar{x}$ corresponds to the case where the entire surplus goes to the worker. If there is no surplus in the firm, then $\bar{x} = 0$.[13] This amounts to a worker participation constraint

$$U \geq x. \tag{7}$$

The investor's beliefs are key to whether he accepts or rejects the contract. The investor has a *prior belief* that a firm is good with probability μ. From the contract proposed, and the worker's response to it, he will update this belief before deciding whether to accord financing or not. Suppose in stage three above, the investor observes that the entrepreneur has proposed a specific contract $c \in C$, and that the worker has accepted it. Let $\mu(c)$ denote the investor's *posterior belief*, i.e. the updated probability that a firm is good, given that the entrepreneur has proposed and the worker has accepted c.[14] The investor break even constraint (3) then becomes

$$I(c) = -B + \beta[\pi\mu(c) + \tilde{\pi}(1 - \mu(c))]R_S \geq 0. \tag{8}$$

Consider the entrepreneur in a good firm. His *contract problem* is to find a feasible contract $c \in C$ (see eq. 6) maximising his net expected discounted cash flow subject to the worker participation constraint (7) and the investor break even constraint (8). (I use F, WP and BE as abbreviations for these restrictions.) This amounts to identifying a contract $c = (w, w_S, B, R_S)$ that solves

$$\begin{aligned} V(x) &= \max\{B - w + \beta\pi[Y_S - R_S - w_S]\} \qquad (9) \\ & \quad s.t. \\ (\text{F}) &: \quad c \in C \\ (\text{WP}) &: \quad u(w) + \beta\pi u(w_S) \geq x \\ (\text{BE}) &: \quad -B + \beta[\pi\mu(c) + \tilde{\pi}(1 - \mu(c))]R_S \geq 0 \end{aligned}$$

for values of x in a valid range $[0, \bar{x}]$. The contract problem for the entrepreneur in a bad firm is defined correspondingly, replacing π with $\tilde{\pi}$ in the objective and in the participation constraint (WP). By construction of $\bar{x}$, solutions to the contract problems (9) always

[13] Since any surplus in the firm will be finite, the upper bound $\bar{x}$ must also be finite. The exact value of $\bar{x}$ depends on the parameters of the problem, and must be established case by case. In section three, I study the case with positive surplus only in good type firms.

[14] This is shorthand notation for the posterior belief function $\mu(c, r)$, where $r \in \{$"accept","reject"$\}$ is the response of the worker. The shorthand notation is convenient because worker acceptance is required for the firm to operate, and therefore a prerequisite for financing.

exist, but they may be trivial in the sense that the entrepreneur proposes to close the firm immediately, i.e. $c = c_0$, for both types. If a non-trivial solution exists, however, then it is always weakly preferable to the entrepreneur, because feasibility requires $V(x) \geq 0$.

2.3 Equilibrium definition

Definition 1 *A pure strategy perfect Bayesian equilibrium is a pair of contracts $(c, \tilde{c}) \in C^2$, one for each type of entrepreneur, a pair of worker response functions $(r_W(c), \tilde{r}_W(c))$ mapping pairs of contracts $(c, \tilde{c}) \in C^2$ into {"accept", "reject"}^{2}, an investor belief function $\mu(c)$ mapping contracts $c \in C$ accepted by the worker into a posterior probability that the firm in question is of the good type, and an investor response function $r_I(c)$ mapping contracts $c \in C$ accepted by the worker into {"accept", "reject"}, such that:*

1. *c is optimal for the entrepreneur in a good firm, and $\tilde{c}$ is optimal for the entrepreneur in a bad firm, each taking as given the worker response functions $(r_W(c), \tilde{r}_W(c))$, the investor's response function $r_I(c)$, and the other type's optimal choice of contract.*

2. *The worker accepts only if he obtains expected utility at least equal to his outside option, taking as given the investor's response function $r_I(c)$, i.e. for all $c = (w, w_S, B, R_S) \in C$,*

$$
\begin{aligned}
r_W(c) = \text{"accept"} &\Rightarrow u(w) + \beta\pi u(w_S) \geq x \\
\tilde{r}_W(c) = \text{"accept"} &\Rightarrow u(w) + \beta\tilde{\pi} u(w_S) \geq 0.
\end{aligned}
$$

3. *The investor accepts only if he expects to break even according to his posterior beliefs, i.e. for all $c = (w, w_S, B, R_S) \in C$,*

$$
r_I(c) = \text{"accept"} \Rightarrow -B + \beta[\mu(c)\pi + (1 - \mu(c))\tilde{\pi}]R_S \geq 0.
$$

4. *The investor's belief function $\mu(c)$ obeys Bayes' rule whenever it applies.*

2.4 Symmetric information benchmark

It is instructive to see the outcomes of the model when information is symmetric. In this case, the investor knows just as well as the entrepreneur and the worker whether their firm is good or bad. Let π denote the probability of success for any generic firm. The *symmetric information contract problem* for the entrepreneur of this firm is

$$\begin{aligned} V^{SY}(\Pi, x) &= \max\{B - w + \beta\pi[Y_S - R_S - w_S]\} \\ & s.t. \\ \text{(F)} &: c \in C \\ \text{(WP)} &: u(w) + \beta\pi u(w_S) \geq x \\ \text{(BE)} &: -B + \beta\pi R_S \geq 0 \end{aligned} \tag{10}$$

for values of x in a valid range $[0, \bar{x}^{SY}]$. The properties of solutions to this problem are as follows:

Proposition 1 (Symmetric information) *Suppose* $\beta\pi Y_S - (1+\beta\pi)\bar{w} > 0$. *Then there exists an* $\bar{x}^{SY} > 0$ *such that for* $x \in [0, \bar{x}^{SY}]$,

1. *There exists a contract* $c^{SY} = (w^{SY}, w_S^{SY}, B^{SY}, R_S^{SY})$ *solving the symmetric information contract problem (10) with* $V^{SY}(\pi, x) \geq 0$
2. *The wage profile* (w^{SY}, w_S^{SY}) *is uniquely determined by*
$$w^{SY} = w_S^{SY} = u^{-1}\left(\frac{x}{(1+\beta\pi)}\right). \tag{11}$$
3. *The borrowing terms* (B^{SY}, R_S^{SY}) *satisfy*
$$\frac{R_S^{SY}}{B^{SY}} = \frac{1}{\beta\pi}, \tag{12}$$
but any B^{SY} *in the interval* $[w^{SY}, \beta\pi(Y_S - w_S^{SY})]$ *is a solution.*

Proof. See appendix. ■

The optimal contract under symmetric information has a constant wage $w^{SY} = w_S^{SY}$, because this is the most efficient way to provide utility to the worker when $u(\cdot)$ is strictly concave. This is illustrated in Figure 1 for the case of a good firm (probability of success π). The solid curve shows all wage combinations that provide expected utility $U = x$ to the worker, where some $x > 0$ is taken as given. From the worker's objective (1) we see that this indifference curve has slope

$$\frac{dw}{dw_S} = -\beta\pi\frac{u'(w_S)}{u'(w)} \tag{13}$$

in (w_S, w) space, and is strictly convex due to the strict concavity of $u(\cdot)$. From the entrepreneur's objective (4) we see that, holding other variables constant, all wage combinations along a straight line with slope (minus) $\beta\pi$ provide the same expected profit. The

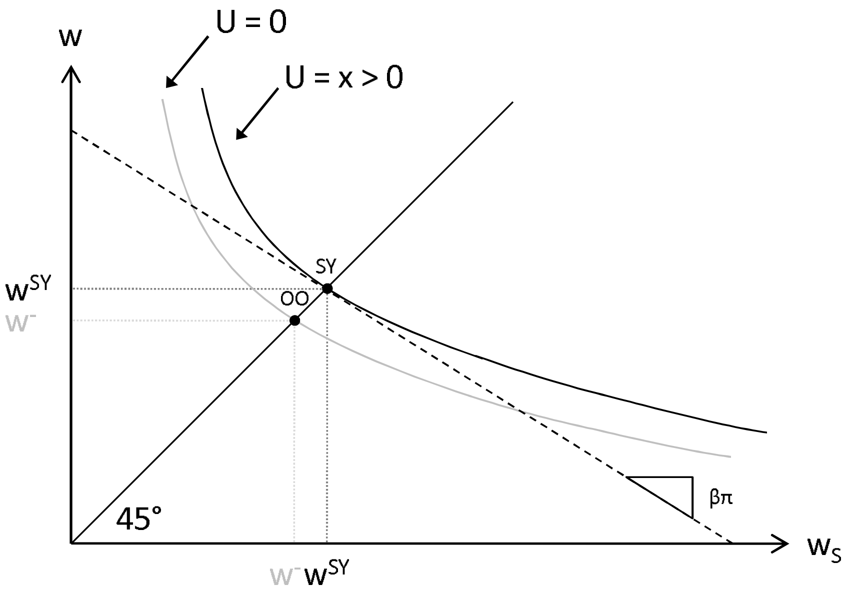

Figure 1: Wage profile in the symmetric information contract.

dashed line shows one such iso profit curve. Assuming feasibility, all the wage combinations on the dashed line can be achieved through financing, and since the investor and the entrepreneur both discount the future success outcome with $\beta\pi$, all these combinations can be achieved at the same expected cost to the entrepreneur. In optimum the worker's indifference curve and the entrepreneur's iso profit line must be tangent, because only then is there nothing to gain for either party in rearranging the wage profile. It is clear from (13) that the worker's indifference curve has slope (minus) $\beta\pi$ only when $w_S = w$, i.e. it must lie on the 45 degree line from the origin. For $U = x$, this is the point SY. Any other point on the worker's indifference curve would require moving to the north-east of the iso profit line, strictly lowering the expected profits of the entrepreneur.

Because the entrepreneur and the investor have identical preferences (both are risk neutral, and discount the future with $\beta\pi$), there is more than one level of borrowing that attains the optimum. To see this, note that the investor's break even constraint BE always binds (else, lower R_S to increase the objective), and use it to substitute for B in the objective function. Then all R_S terms cancel, reflecting that the maximum value of V^{SY} obtains from all feasible contracts on the schedule $B = \beta\pi R_S$. Without loss of generality, I restrict attention to the smallest level of borrowing satisfying the cash flow constraint CF, which is $B^{SY} = w^{SY}$.

3 Equilibrium analysis

For the remainder of the paper, I consider the case where a good firm is (strictly) profitable, while a bad firm is not profitable, i.e. the probability of success for a good and a bad firm, π and $\tilde{\pi}$ respectively, are such that

$$\beta\pi Y_S - (1+\beta\pi)\bar{w} > 0 \tag{14}$$

$$\beta\tilde{\pi} Y_S - (1+\beta\tilde{\pi})\bar{w} < 0. \tag{15}$$

This condition holds for a sufficiently high $\pi \in [0,1]$ and a sufficiently low $\tilde{\pi} \in [0,\pi)$ as long as $Y_S > \frac{1+\beta}{\beta}\bar{w}$. Under symmetric information, a good firm will obtain financing and operate, while a bad firm will be closed. The *symmetric information outcome* $(c,\tilde{c}) = (c^{SY}, c_0)$, where $c^{SY} = (w^{SY}, w_S^{SY}, B^{SY}, R_S^{SY})$ is the solution to the symmetric information contract problem (10) for a good firm, provides an efficiency benchmark both in terms of efficient investment and in terms of efficient risk sharing between the entrepreneur and the worker.

The symmetric information outcome cannot constitute an equilibrium when there is private information about type, because it is always in the interest of an entrepreneur and a worker in a bad firm to mimic the behaviour of a good firm.

Lemma 1 *Both the entrepreneur and the worker in a bad firm always weakly prefer c^{SY} to c_0, and at least one of them strictly prefers it. Consequently, (c^{SY}, c_0) cannot constitute a perfect Bayesian equilibrium when there is private information about type.*

Proof. See appendix. ■

There are only two kinds of pure strategy perfect Bayesian equilibria, separating and pooling.[15] The following sections consider them in turn. Models that use the perfect Bayesian equilibrium concept commonly have a large set of equilibria, and my model is not an exception to this rule. The reason for the multiplicity is that Bayes' rule do not restrict out-of-equilibrium beliefs. Constructing these in different ways help sustaining different equilibria. Towards the end of the section, however, I provide reasonable restrictions on these beliefs that imply a unique solution (in pure strategies).

3.1 Least cost separation

A *separating contract* is contract from which the investor uniquely identifies firm type. Can the entrepreneur in a good firm somehow design a contract that is profitable for him,

[15] To keep the analysis simple, I do not consider hybrid equilibria. In such equilibria, the entrepreneur in one or both types of firms randomise over several contracts to which they are indifferent.

and at the same time unprofitable for the entrepreneur in a bad firm? Interestingly, when the entrepreneur has no own funds and is protected by limited liability, the answer is no.

Lemma 2 *There exists no feasible contract c such that $\tilde{V}(c) < \tilde{V}(c_0) = 0$.*

Proof. By definition (see cond. 6), a feasible contract satisfies the cash flow constraint $B - w \geq 0$ and the limited liability constraint $Y_S - R_S - w_S \geq 0$. But then, the expected profits $\tilde{V}(c)$ for the entrepreneur is non-negative, since $\tilde{V}(c) = B - w + \beta\tilde{\pi}[Y_S - R_S - w_S]$. ■

Lemma (2) says that the entrepreneur in a good firm cannot separate from the entrepreneur in a bad firm by inducing him to prefer closing. The reason is clear: Under limited liability, an entrepreneur cannot lose more than any initial funds he invests. When he has no funds, there is nothing to invest, and nothing to lose. The penniless entrepreneur protected by limited liability is a classical case in the corporate finance literature.[16,17] Different suggestions have already been brought forward to solve it.[18] I consider exploiting that the worker has inside information. This can be done by designing a contract that the worker would not accept if his firm were bad. Consider the following *least cost separating contract problem*

$$\begin{aligned} V^{SE}(x) &= \max_c \{B - w + \beta\pi[Y_S - R_S - w_S]\} \qquad (16) \\ & \quad s.t. \\ \text{(F)} &: \quad c \in C \\ \text{(WP)} &: \quad u(w) + \beta\pi u(w_S) \geq x \\ \text{(ICW)} &: \quad u(w) + \beta\tilde{\pi} u(w_S) \leq 0 \\ \text{(BE)} &: \quad -B + \beta\pi R_S \geq 0 \end{aligned}$$

for values of x in a valid range $[0, \bar{x}^{SE}]$. The difference between this problem and the symmetric information contract problem is the incentive compatibility constraint for the worker (denoted ICW). To see why it has this form, recall that if the worker rejects the proposed contract, the firm must close. Consequently, the worker would obtain his outside

[16] See Tirole (2006, chapter 6) for an overview of firm financing under asymmetric information built around this case. Clearly, if the entrepreneur had sufficient own funds, he could achieve the first best symmetric information payoffs without external financing. With some own funds, it may be possible for the entrepreneur to signal firm quality by investing a large stake in his own firm.

[17] Limited liability, though very common, is not the only way to organise enterprise. But even with so-called unlimited liability of owners as in e.g. sole proprietorships and partnerships, liability is in practice always limited. In the United States, for instance, chapter 7 in the Bankruptcy Code (on the right to basic liquidation for individuals and businesses) limits the personal liability for debt. See White (2008) for a discussion of the economics of corporate and personal bankruptcy law.

[18] For instance, if the firm possessed machinery or a building with alternative uses even when the firm fails, i.e. the firm has a positive salvation value, then it should be possible to borrow up to this amount at a risk free rate by posting the valuables as collateral.

option (which from Condition 1 also is the symmetric information outcome for the worker in a bad firm). Therefore, if the contract includes a wage profile (w, w_S) that the worker in a bad firm would reject, i.e. if

$$\tilde{U}(c) = u(w) + \beta\tilde{\pi}u(w_S) < 0 = \tilde{U}(c_0), \tag{17}$$

then such a contract *accepted by the worker* credibly signals firm quality.[19] The solution to problem (16) and a corresponding perfect Bayesian equilibrium is given in the following proposition.

Proposition 2 (Least Cost Separation) *For any prior belief μ, there exists an $\bar{x}^{SE} > 0$ such that for any $x \in (0, \bar{x}^{SE}]$,*

1. *There exists a contract $c^* = (w^*, w_S^*, B^*, R_S^*)$ solving the least cost separating contract problem (16) with $V^{SE}(x) \geq 0$.*

2. *The wage profile (w^*, w_S^*) is uniquely determined by*

$$\left.\begin{array}{r} u(w^*) + \beta\pi u(w_S^*) = x \\ u(w^*) + \beta\tilde{\pi}u(w_S^*) = 0 \end{array}\right\} \Rightarrow w^* < w_S^*. \tag{18}$$

3. *The borrowing terms (B^*, R_S^*) satisfy*

$$\frac{R_S^*}{B^*} = \frac{1}{\beta\pi}, \tag{19}$$

but any B^* in the interval $[w^*, \beta\pi(Y_S - w^*)]$ is a solution.

4. *The strategies $(c, \tilde{c}) = (c^*, c_0)$ constitute a perfect Bayesian equilibrium for appropriately chosen posterior beliefs for actions off the equilibrium path, e.g. for all $c \in C$, $\mu(c) = 0$ whenever $c \neq c^*$.*

Proof. See appendix. ■

The defining characteristic of the separating contract is the strictly increasing wage profile, $w^* < w_S^*$. This is illustrated in figure 2.[20] The curve indicated $\tilde{U} = 0$ is the indifference curve for the worker in a bad firm corresponding to the value of his outside option, given by the point OO. Because $\tilde{\pi} < \pi$, we see from eq. (13) that an indifference

[19] Because signalling is costly for the entrepreneur, the incentive constraint (17) will be binding in equilibrium. The solution to the least cost separating contract problem (16), being the supremum, therefore satisfies the constraint with equality.

[20] For comparison, I use the same set up as in figure 1. That is, the separating contract provides expected utility $U = x$ to the worker in a good firm, where $x > 0$ is taken as given and identical to the one considered in figure 1.

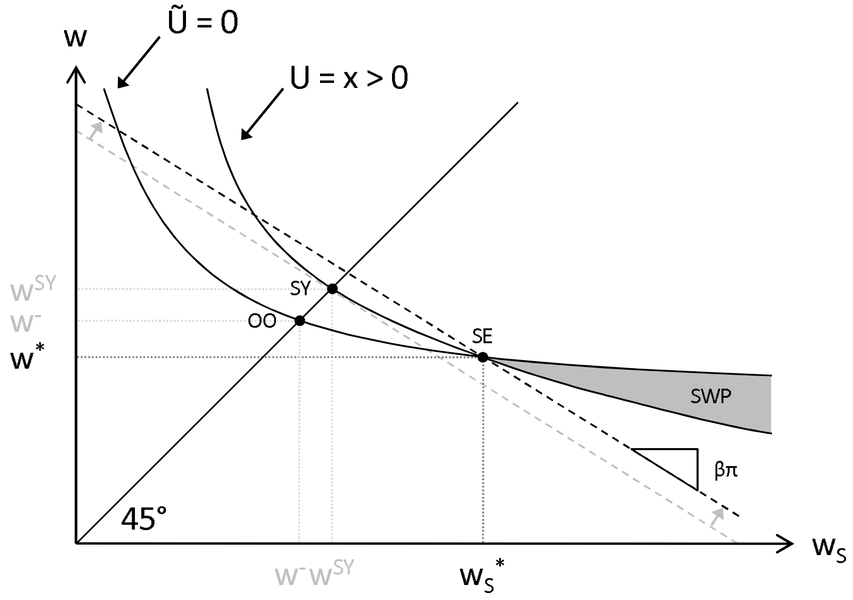

Figure 2: Wage profile in the least cost separating contract.

curve for $\tilde{U}$ is always less steep than an indifference curve for U at any given point in (w_S, w)-space.[21] Lemma 2 showed that the entrepreneur in a bad firm had nothing to lose from proposing any contract profitable to the entrepreneur in a good firm. By contrast, the *worker* in a bad firm may have something to lose from accepting it. If the proposed contract includes a wage profile (w, w_S) located anywhere above the curve $\tilde{U} = 0$, the worker would accept the contract. But if the wage profile is located (strictly) below $\tilde{U} = 0$, then he is (strictly) better off rejecting the contract. Consequently, separation can be achieved with any wage profile located in the shaded area above $U = x$ and below $\tilde{U} = 0$. These wage profiles credibly signal firm quality because *only a worker in a good firm would be willing to defer compensation to an uncertain future, getting paid only if the firm succeeds.*

The dashed lines with slope (minus) $\beta\pi$ in figure 2 are iso expected profit lines for the entrepreneur in a good firm, conditional on achieving separation. The closer to the origin, the lower the expected wage cost, so the higher the expected profit. The point SE, determined by the intersection of $U = x$ and $\tilde{U} = 0$, thus pinpoints the (limit of the) most profitable wage profile that achieves separation. I will call this the *least cost separating wage profile.*[22] Shortly, I will return to discuss equilibrium refinements, but it

[21]Hence, the Spence-Mirrlees (or "single crossing") condition is satisfied.

[22]This wage profile is uniquely determined because $\tilde{\pi} < \pi$, implying that the indifference curves of workers in different type firms cross only once, and because $u(\cdot)$ is strictly concave, implying that the worker's indifference curve is strictly convex.

is appropriate to point out the following already: As long as the investor understands that the worker is rational, he should infer that any contract c with a wage profile such that $U \geq x$ and $\tilde{U} < 0$ that has been accepted by the worker, must pertain to a good firm. In this case, the least cost separating equilibrium is the unique separating equilibrium of the model. Moreover, $V^{SE}(x)$ becomes a lower bound on the equilibrium payoff to the entrepreneur in a good firm, as it can always be attained. Finally, because the least cost separating contract always exists, this assumption guarantees the existence of an equilibrium of the model when there is private information about type.

Signalling through deferred worker compensation is parallel to the signalling by undiversified entrepreneurs in Leland and Pyle (1977). The key is that something is at risk, since this is what distinguishes a good firm from a bad (it has a higher probability of success). Hence, the optimal wage profile is equity-like. Put differently, the worker co-invests in the firm through the wage profile.[23]

Because the worker is averse to wage variation, signalling through deferred compensation increases the expected wage cost for the firm compared to the symmetric information outcome. This cost of signalling can be seen in figure 2 by noting that the iso profit curve passing through SE lies further away from the origin than the one passing through SY.[24] Intuitively, the less averse the worker is to wage variation, the less convex the indifference curves, and so the smaller the cost of signalling, for any given value of x.[25]

3.2 Pooling

In a pooling equilibrium, the entrepreneur proposes the same contract, and the worker responds in the same way, whether the firm is good or bad. Consequently, the investor cannot identify firm type, and from Bayes' rule, posterior beliefs $\mu(\cdot)$ must equal prior beliefs μ along the equilibrium path, hence the investor break even constraint (8) becomes

$$-B + \beta[\mu\pi + (1-\mu)\tilde{\pi}]R_S \geq 0.$$

[23]Deferred remuneration as in condition (18) is sometimes referred to as "borrowing from workers". In the context of signalling, "worker co-investment" is more appropriate, because it emphasises the important point that something is at risk.

[24]These curves can be compared because in the separating equilibrium, the firm's true type is revealed, and so the gross rate of interest $\frac{R_S^*}{B^*}$ is the same as when information is symmetric, cf. eqs. (12) and (19).

[25]Had the worker been risk neutral (as the entrepreneur), the indifference curve corresponding to $U = x$ would have been identical to the iso profit line going through the point SY. Consequently, both the entrepreneur and the worker in a good firm would have been equally well off as in the symmetric information outcome. The least cost separating wage profile would still require $w^* < w_S^*$, but would be indeterminate because any feasible point on the iso profit line would provide $U = x$ and attain the optimum.

Suppose the entrepreneur in a good firm could choose freely among contracts that could satisfy this constraint. This is the *pooling contract problem*

$$
\begin{aligned}
V^{PO}(x) &= \max\{B - w + \beta\pi[Y_S - R_S - w_S]\} \\
& \quad s.t. \\
\text{(F)} &: \quad c \in C \\
\text{(WP)} &: \quad u(w) + \beta\pi u(w_S) \geq x \\
\text{(BE)} &: \quad -B + \beta[\mu\pi + (1-\mu)\tilde{\pi}]R_S \geq 0
\end{aligned} \tag{20}
$$

for values of x in a valid range $[0, \bar{x}^{PO}]$. Motivated by the argument that the entrepreneur in a good firm should be able to guarantee himself the least cost separating payoff (under the weak condition that the investor understands that the worker is rational), I look for solutions to the pooling contract problem with $V^{PO}(x) \geq V^{SE}(x)$. They are as follows:

Proposition 3 (Pooling) *Fix any $x \in (0, \bar{x}^{SE}]$, and let c^* denote the solution to the least cost separating contract problem (16) for this x. Then there exists a $\mu^* \in [0, 1)$ such that for any prior beliefs $\mu \geq \mu^*$,*

1. *There exists a contract $\mathring{c} = (\mathring{w}, \mathring{w}_S, \mathring{B}, \mathring{R}_S)$ solving the pooling contract problem (20) with $V^{PO}(x) \geq V^{SE}(x) \geq 0$.*

2. *The contract $\mathring{c}$ is uniquely determined by*

$$
\frac{u'(\mathring{w}_S)}{u'(\mathring{w})} = \frac{\mu\pi + (1-\mu)\tilde{\pi}}{\pi} < 1 \Rightarrow \mathring{w} < \mathring{w}_S \tag{21}
$$

$$
u(\mathring{w}) + \beta\pi u(\mathring{w}_S) = x \tag{22}
$$

$$
\frac{\mathring{R}_S}{\mathring{B}} = \frac{1}{\beta(\mu\pi + (1-\mu)\tilde{\pi})} > \frac{1}{\beta\pi} \tag{23}
$$

$$
\mathring{B} = \mathring{w}. \tag{24}
$$

3. *The strategies $(c, \tilde{c}) = (\mathring{c}, \mathring{c})$ constitute a perfect Bayesian equilibrium for appropriately chosen posterior beliefs for actions off the equilibrium path, e.g. for all $c \in C$, $\mu(c) = 0$ if $c \neq \mathring{c}$.*

4. *The outcome of the equilibrium $(c, \tilde{c}) = (\mathring{c}, \mathring{c})$ strictly Pareto dominates (in an interim sense) the outcome of the least cost separating equilibrium $(c, \tilde{c}) = (c^*, c_0)$.*

Proof. See appendix. ■

Interestingly, the pooling contract has an equity-like wage profile $\mathring{w} < \mathring{w}_S$. The reason for this is illustrated in figure 3. Compared with the case when information is symmetric,

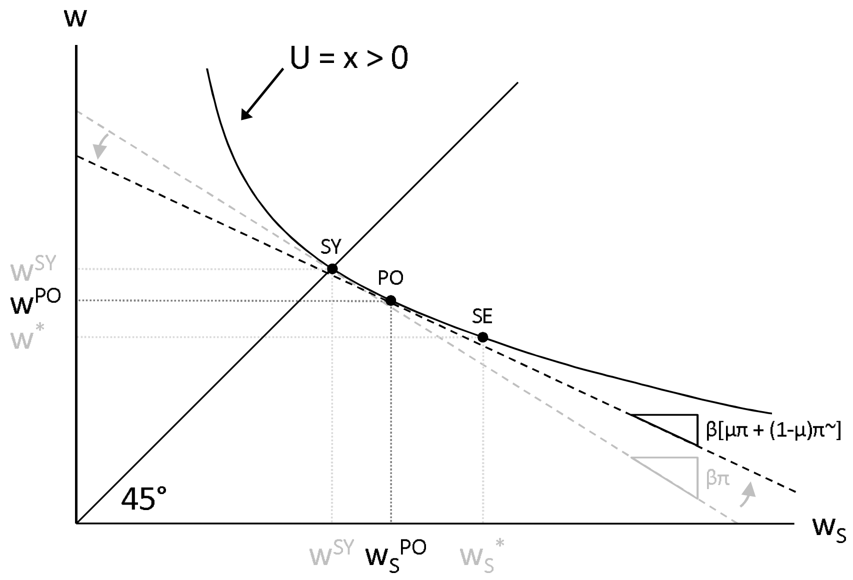

Figure 3: Wage profile in the simple pooling contract.

the iso (expected) profit line for the entrepreneur (the dashed line) is tilted and flatter in (w_S, w)-space. This is because the entrepreneur must borrow to finance the current wage w, and since $\tilde{\pi} < \pi$ and $\mu \in (0,1)$, borrowing is strictly more expensive under the terms of the simple pooling contract than when information is symmetric (see condition 23). The *pooling wage profile* $(\mathring{w}, \mathring{w}_S)$ is given by the point PO where the iso profit line is tangent to the worker's indifference curve.[26] Intuitively, as $\tilde{\pi}$ and μ become smaller, the cost of borrowing increases, and this increases the expected cost to the entrepreneur of paying w relative to w_S. Consequently, the pooling wage profile moves down along the $U = x$ line. In the pooling contract, therefore, the entrepreneur *defers the worker's compensation to the uncertain future in order to avoid expensive external financing.*

Holding worker utility constant (at x), the simple pooling wage profile yields strictly lower expected profits than the symmetric information wage profile SY (as PO lies further away from the origin than the gray, dashed symmetric information iso profit line passing through SY). Again, this is because the strict concavity of $u(\cdot)$ makes the indifference curve $U = x$ strictly convex. Importantly, however, with $(\mathring{w}, \mathring{w}_S)$, the entrepreneur exploits a cost advantage of internal finance implied by the fact that the worker knows the true quality of the firm. To see why, suppose we start from the constant wage case given by the point SY, and consider deferring compensation on the margin, holding worker utility constant. From eq. (13) we see that this is strictly less expensive than external

[26]It is uniquely determined because of the strict concavity of $u(\cdot)$.

financing, because for any $\tilde{\pi} < \pi$ and $\mu < 1$,

$$\left.\frac{dw_S}{dw}\right|_{w=w_S=w^{SY}} = \frac{1}{\beta\pi}\left.\frac{u'(w)}{u'(w_S)}\right|_{w=w_S=w^{SY}} = \frac{1}{\beta\pi} < \frac{1}{\beta(\mu\pi + (1-\mu)\tilde{\pi})} = \left.\frac{dR_S}{dB}\right|_{\mu(c)=\mu}. \quad (25)$$

Consequently, it is *always optimal to defer some of the worker's compensation to the uncertain future, no matter how averse he is to wage variation.* As more and more compensation is deferred, the cost advantage of internal finance is gradually eroded, as $\frac{u'(w)}{u'(w_S)}$ increases. The optimal solution is the unique, interior point given by condition (21). Therefore, even though external financing is "too expensive" from the point of view of the entrepreneur in a good firm, there is no strict "pecking order" in the sense of Myers (1984) and Myers and Majluf (1984). Rather, the equilibrium always combines external and internal financing.

Note that the welfare criterion used in this paper is interim, i.e. the expected payoff for each agent (entrepreneur, worker, or investor) is evaluated given his own private information only. This is the most relevant criterion when agents know their own type at the time when decisions are made.[27] Hence, the last part of proposition 3 means that no entrepreneur or worker of any type is worse off, and at least one is strictly better off, in the pooling equilibrium relative to the least cost separating equilibrium.

3.3 Equilibrium refinement and efficiency

Models applying the perfect Bayesian equilibrium concept often obtain a plethora of equilibria, and my model is not an exception to this rule. The multiplicity arises because Bayes' rule gives no guidance to beliefs about actions off the equilibrium path, and these beliefs are often key to sustaining the equilibrium itself. Which of these equilibria should be chosen?

I have already argued in favour of the relatively weak condition that the investor understands the worker is rational, so that whenever a contract with a separating wage profile is accepted by the worker, the investor should infer that the firm is of the good type. Then the least cost separating equilibrium outcome is always attainable for the entrepreneur in a good firm. This puts a lower bound on his equilibrium payoff, eliminating all other separating equilibria as well as pooling equilibria with a lower payoff. For sufficiently large values of μ, however, there exist pooling equilibria that Pareto dominate the least cost separating equilibrium (of which the equilibrium in proposition 3 is an example), and these will not be eliminated.

As a next step, I impose Cho and Kreps' (1986) "intuitive criterion". This criterion eliminates all pooling equilibria in which the entrepreneur borrows more than he needs

[27] See Holmstrom and Myerson (1983) and the references therein.

in order to finance the current wage w, i.e. equilibria with $B > w$. In the previous section we saw that the entrepreneur in a good firm finds the pooling rate of interest "too expensive", and therefore never wants to borrow more than he needs. An entrepreneur in a bad firm, however, would want to borrow as much as possible at this rate, finding it "too cheap". If the investor understands these differing incentives, he should infer that an entrepreneur proposing to reduce B when $B > w$, belongs to a good type firm.

In many applications, Cho and Kreps' (1986) "intuitive criterion" eliminates *all* pooling equilibria. This is not the case here, because conditional on $B = w$, the entrepreneur in a good firm and the entrepreneur in a bad firm order all possible contracts in the same way.[28] The "intuitive criterion" has no bite then, because it is impossible to construct a deviation from equilibrium that is dominated for the entrepreneur in a bad firm, but not for the one in a good firm. In other words, there is *nothing for the investor to learn* from any particular choice among such contracts: the worker would always accept, and the entrepreneur would always order them in the same way, no matter the type of firm. I therefore propose to impose

$$\mu(c) = \mu \tag{26}$$

for such contracts. The following proposition shows that these three conditions together imply uniqueness.

Proposition 4 (Equilibrium uniqueness and efficiency) *Suppose the following three conditions hold:*

1. *The investor understands that the worker is rational, so that if a contract $c = (w, w_S, B, R_S) \in C$ satisfying $u(w) + \beta\pi u(w_S) \geq x$ and $u(w) + \beta\tilde{\pi}u(w_S) < 0$ has been accepted by the worker, then the investor infers the firm is good, i.e.*

$$\mu(c) = 1. \tag{27}$$

2. *Equilibria satisfy the "intuitive criterion" of Cho and Kreps (1986).*

3. *The investor learns nothing from observing pooling contracts that the entrepreneur would always order in the same manner, and the worker would always accept, no matter the type of firm, i.e. if $c = (w, w_S, B, R_S)$ is a contract satisfying $B = w$, $V(c) \geq V^{SE}(x)$, $\tilde{V}(c) \geq 0$, $u(w) + \beta\pi u(w_S) \geq x$ and $u(w) + \beta\tilde{\pi}u(w_S) \geq 0$, then*

$$\mu(c) = \mu. \tag{28}$$

[28] Conditional on $B = w$, the expected payoff to the entrepreneur in a good firm is $\beta\pi[Y_S - R_S - w_S]$. For the entrepreneur in a bad firm it is $\frac{\tilde{\pi}}{\pi}$ times this. The ordering of contracts is preserved through this monotone transformation.

Then there is a unique equilibrium of the model with private information about type: the pooling equilibrium $(c, \tilde{c}) = (\mathring{c}, \mathring{c})$ *whenever* $V^{PO}(x) \geq V^{SE}(x)$*, and the least cost separating equilibrium* $(c, \tilde{c}) = (c^*, c_0)$ *otherwise. The equilibrium is efficient (in an interim sense) in the set of all possible perfect Bayesian equilibria of the model.*

Proof. See appendix. ■

Proposition 4 establishes that the equilibrium is efficient in the set of all possible equilibria of the model. This is evaluated in an interim sense, i.e. the expected payoff for each agent (entrepreneur, worker, or investor) is evaluated given his own private information only. This does not, however, imply that the equilibrium always constitutes an interim efficient allocation in the sense following Holmstrom and Myerson (1983). The reason for this discrepancy is that one can improve on the pooling equilibrium given in proposition 3 by allowing the entrepreneur to propose a *menu* of contracts, from which he is eventually allowed to choose a specific contract, given that the worker and the investor accept the menu. The source of improvement is that, by included in this menu a contract in which the entrepreneur can receive a lump sum transfer in return for closing the firm immediately, one can avoid the costly overinvestment implied by conventional pooling contracts.[29] On the other hand, one can show that the least cost separating equilibrium does indeed constitute an interim efficient allocation in the sense following Holmstrom and Myerson (1983), for a non-empty interval of prior beliefs μ.

[29] Tirole (2006) describes mechanisms of this kind in related examples, and discusses the reasonability of the assumption in the corporate finance context. The main issue is whether a lump sum payment would not attract imposter entrepreneurs, without any project at all, eventually driving down the prior probability μ of a good firm until the least cost separating equilibrium prevails.

4 The case for stock options

What does it take to implement the optimal wage profile? The model is based on two key assumptions about w_S that are interesting to discuss in light of employee stock options: First, the entrepreneur is *committed* to pay the worker w_S in case of success, and second, w_S is *observable* to the external investor.

4.1 Stock options provide commitment

If the entrepreneur lacked commitment towards the worker, the worker would disbelieve he would actually get w_S in the case of success, and consequently reject the optimal wage profile.[30] Of course, real world firms may have other ways of building credibility about their worker payments, such as well established bonus and promotion systems. But stock options are unambiguous: They have clearly defined property rights, and their payoffs are simple functions of the stock price (a measure, albeit imperfect, of firm success). In this sense, a stock option grant paying off $w_S - w$ is perfectly suited for implementing the optimal wage profile, combined with a fixed wage of w.

It is commonly argued that firms use stock options to retain employees.[31] The commitment question gives an interesting interpretation of retaining insider employees. Recall that I have assumed the worker in a good firm can obtain utility x in period one if he leaves the firm and exploits his inside information in the market. Once uncertainty has been revealed in period two, it is too late for the worker to exploit this alternative. For example, using the example of a small software developer, the worker may have important information about the software under development, that would be valuable to a competitor in period one. But once the software has been developed in period two, it will already be patented and the information valueless. Consequently, the worker has no bargaining power in period two. If there is a commitment problem, then, the worker would be better off leaving in period one. In this view, stock options are an important instrument for retaining insider employees, since they provide the entrepreneur with the commitment necessary to ensure them the value of their information.

[30] The same problem of commitment does not arise with respect to the payment of R_S in case of success, as debt contracts are enforceable by law.

[31] See Ittner et al. (2003) for a discussion.

4.2 Stock options are easily observable

Successful signalling occurs when the investor understands that only a worker in a good firm would accept the proposed wage profile. This requires that the investor can observe w_S.[32] The argument for stock options in this case is the same as that for commitment, but even stronger: While it may not be impossible for an external investor to decipher the implications for the worker of internal compensation schemes such as bonus or promotion systems, a stock option grant can easily be disclosed, and its consequences for the worker are transparent.[33]

This argument holds as much for the pooling equilibrium as for separating equilibrium. Indeed, in the case of pooling, the entrepreneur in a bad firm must avoid revealing the true quality of his firm. Precisely for the same reason that outside financing is "too expensive" for the entrepreneur in a good firm, it is "too cheap" for the entrepreneur in a bad firm. At this rate, he would prefer to borrow more in period one, and use it to smooth the wage profile. He cannot do this, however, because it would be taken as a signal of bad quality.[34] Therefore, stock options could be expected to be common across *all firms* in an industry or segment where workers are known to have inside information, not only at firms that are in fact good.

It is commonly argued that firms use stock options to attract employees with particular characteristics, e.g. less risk averse people.[35] The model I have developed suggests an alternative interpretation of how stock options can attract workers. Recall that all along, I have considered an *existing* worker-insider. A potential new hire must be considered as uninformed. Imagine such a worker receiving job offers from firms in this industry or segment, and that offers have the same conditions for new as for existing workers.[36] He faces an information problem similar to that of the external investor, in that he would like to know whether a firm is good or bad, because working for a good firm provides higher expected utility than working for a bad firm. Stock options for existing employees can therefore be interpreted as way to signal to potential new hires that the firm is good (or, conversely, a way to avoid revealing that the firm is bad).

[32]If w_S is not observable, it is also a question how the investor can verify that the proposed contract is feasible, i.e. whether it satisfies the limited liability constraint $Y_S - R_S - w_S \geq 0$. If the investor's claim has higher priority than the worker's claim, so that any $R_S \leq Y_S$ can be promised to the investor, then one could find some w so small that the largest feasible w_S could never attain $\tilde{U} \geq 0$. This would indeed signal firm quality, but in a very expensive manner.

[33]Standard legal provisions for stock option grants ensure that workers cannot undo their positions. Hence, the worker's payoff is a simple function of the stock and the exercise price.

[34]Recall that the simple pooling contract maximises payoff to the entrepreneur in a good firm, so any deviation from this contract would "signal" bad quality.

[35]Again, Ittner et al. (2003) give a broad discussion.

[36]This must hold if the new hire becomes informed once he starts working for the firm.

5 Conclusion

I have proposed a model in which there is an economic rationale for tying the worker's wage to the performance of the firm. A simple way (though not the only imaginable way) to implement the optimal wage profile is through stock options. The key ingredients of the model seem particularly plausible for knowledge intensive firms such as in the "new economy". An explicit model will help to solve the debate in the inconclusive empirical literature on the use of broad-based stock option plans (See Core and Guay, 2001, Ittner et al., 2003, and Oyer and Schaefer, 2005).

Hall and Murphy (2003) argue that because risk averse employees value stock options below their cost to the firm, then absent incentive effects, their use seems inefficient. The present model does not contest the idea that stock options are a costly way to remunerate risk averse employees. However, it does contest the idea that their use is inefficient. The use of options should be thought of as resolving another, more important problem, namely that of financing under asymmetric information. The concern for risk sharing is traded off against the cost of overinvestment and cross subsidisation implied when good firms do not distinguish themselves from bad firms.[37] Only when this cost is relatively low, i.e. only when the investor's prior belief μ that a firm is good is high, does risk sharing come to the forefront. And even then, risk is never perfectly shared in the sense that the worker's wage is flat.

The welfare cost of asymmetric information in this model depends crucially on how risk averse the worker is. Hall and Murphy (2000) have tried to quantify how much less an undiversified, risk averse employee values a stock option below the cost of the option to the firm (taken to be the option's Black-Scholes value). Interestingly, for moderate holdings of options relative to the employee's total wealth (which seems to be the empirically relevant case), the discount they report is rather small.[38] The welfare cost for a given degree of risk aversion may therefore be modest. Moreover, not all industries or segments experience asymmetric information to the same extent, though they draw from the same pool of workers in the labour market. Consequently, one should expect that over time, workers with different degrees of risk aversion self select into the sectors with the compensation schemes that best fit with their preferences. This sorting would reduce the average degree of worker risk aversion where there are problems of asymmetric information, thus mitigating the welfare cost to society.

[37] There is overinvestment in the sense that the bad firm would not obtain financing under symmetric information. The expected loss implied by financing a bad firm is paid in expectation by the good firm, hence there is cross subsidisation. This possibility was first pointed out by De Meza and Webb (1987).

[38] Typically less than 10 percent when the Black-Scholes value of all options held is one third or less than the employee's total wealth, assuming that the degree of relative risk aversion is constant and equal to 2.

References

Cho, I. K., and D. M. Kreps. 1987. "Signaling Games and Stable Equilibria." *Quarterly Journal of Economics*, 102 (May): 179-221.

Core, John E., and Wayne R. Guay. 2001. "Stock Option Plans for Non-Executive Employees." *Journal of Financial Economics*, 61 (August): 253-287.

De Mesa, David, and David C. Webb. 1987. "Too Much Investment: A Problem of Asymmetric Information." *Quarterly Journal of Economics*, 102 (May): 281-292.

Gertner, Robert, Robert Gibbons and David Scharfstein. 1988. "Simultaneous Signalling to the Capital and Product Markets." *The RAND Journal of Economics*, 19 (Summer): 173-190.

Hall, Brian J., and Kevin J. Murphy. 2000. "Optimal Exercise Prices for Executive Stock Options." *American Economic Review*, 90 (May): 209-214.

Hall, Brian J., and Kevin J. Murphy. 2003. "The Trouble with Stock Options." *Journal of Economic Perspectives*, 17 (Summer): 49-70.

Holmstrom, Bengt, and Roger B. Myerson. 1983. "Efficient and Durable Decision Rules with Incomplete Information." *Econometrica*, 51 (November): 1799-1819.

Ittner, Christopher D., Richard A. Lambert, and David F. Larcker. 2003. "The Structure and Performance Consequences of Equity Grants to Employees of New Economy Firms." *Journal of Accounting and Economics*, 34 (January): 89-127.

Leland, Hayne E., and David H. Pyle. 1977. "Information Asymmetries, Financial Structure and Financial Intermediaries." *Journal of Finance*, 32 (May): 371-387.

Mas-Colell, Andreu, Michael D. Whinston, and Jerry R. Green. 1995. *Microeconomic Theory.* New York and Oxford: Oxford University Press.

Maskin, Eric, and Jean Tirole. 1992. "The Principal-Agent Relationship with an Informed Principal, II: Common Values." *Econometrica*, 60 (January): 1-42.

Milgrom, Paul, and John Roberts. 1986. "Price and Advertising Signals of Product Quality." *Journal of Political Economy*, 94 (August): 796-821.

Myers, Stewart C. 1984. "The Capital Structure Puzzle." *Journal of Finance*, 39 (July): 573-592.

Myers, Stewart C., and Nicholas S. Majluf. 1984. "Corporate Financing and Investment Decisions when Firms Have Information That Investors Do Not Have." *Journal of Financial Economics*, 13 (June): 187-221.

Oyer, Paul, and Scott Schaefer. 2005. "Why Do Some Firms Give Stock Options to All Employees?: An Empirical Examination of Alternative Theories." *Journal of Financial Economics*, 76 (April): 99-133.

Quinzii, Martine, and Jean-Charles Rochet. 1985. "Multidimensional Signalling." *Journal of Mathematical Economics*, 14 (3): 261-284.

Rothschild, Michael, and Joseph Stiglitz. 1976. "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information." *Quarterly Journal of Economics*, 90 (November), 629-649.

Tirole, Jean. 2006. *The Theory of Corporate Finance*. Princeton and Oxford: Princeton University Press.

White, Michelle J. 2008. "Economics of Corporate and Personal Bankruptcy Law." In *The New Palgrave Dictionary of Economics*, ed. Steven N. Durlauf and Lawrence E. Blume. Basingstoke: Palgrave Macmillan.

Wilson, Charles. 1977. "A Model of Insurance Markets with Incomplete Information." *Journal of Economic Theory*, 16 (December): 167-207.

Wilson, Robert. 1985. "Multi-Dimensional Signalling." *Economics Letters*, 19, Issue 1: 17-21.

6 Appendix

Proof of Proposition 1 (Symmetric information). Suppose a solution to the symmetric information contract problem (10) exists. The objective function is weakly concave, and the constraints are quasi-convex. Hence, a solution to the first order conditions and complementary slackness conditions of the associated Lagrangian problem is also a solution to the original problem. The BE constraint must be binding in optimum. Suppose it were not. Then the entrepreneur could marginally lower R_S without violating feasibility, and strictly increase the objective. Use the binding BE constraint to substitute for $B = \beta\pi R_S$ in the objective, attach Lagrange multipliers ϕ and $\beta\pi\psi$ to the feasibility constraints CF and LL (see eq. 6), and the multiplier λ to the WP constraint, and form the Lagrangian function

$$\mathcal{L}^{SY} = (1+\phi)[\beta\pi R_S - w] + (1+\psi)\beta\pi[Y_S - R_S - w_S] + \lambda[u(w) + \beta\pi u(w_S)].$$

The first order conditions for optimality are

$$\begin{aligned}
\frac{\partial \mathcal{L}^{SY}}{\partial w} &= -(1+\phi) + \lambda u'(w) = 0 \Leftrightarrow \lambda u'(w) = 1+\phi \\
\frac{\partial \mathcal{L}^{SY}}{\partial w_S} &= -(1+\psi)\beta\pi + \lambda\beta\pi u'(w_S) = 0 \Leftrightarrow \lambda u'(w_S) = 1+\psi \\
\frac{\partial \mathcal{L}^{SY}}{\partial R_S} &= (1+\phi)\beta\pi - (1+\psi)\beta\pi = 0 \Leftrightarrow \phi = \psi,
\end{aligned}$$

and the complementary slackness conditions are

$$\begin{aligned}
\phi &\geq 0 \text{ and } \phi[B - w] = 0 \\
\psi &\geq 0 \text{ and } \psi[Y_S - R_S - w_S] = 0 \\
\lambda &\geq 0 \text{ and } \lambda[u(w) + \beta\Pi u(w_S) - x] = 0.
\end{aligned}$$

The WP constraint must be binding in optimum. Suppose it were not. Then the entrepreneur could marginally lower either w or w_S without violating feasibility, and strictly increase the objective. Consequently, $\lambda > 0$, and so the optimality conditions yield a constant wage

$$u'(w) = u'(w_S) \Rightarrow w = w_S = u^{-1}\left(\frac{x}{(1+\beta\pi)}\right).$$

The binding BE constraint pins down the gross interest rate on borrowing $\frac{R_S}{B} = \frac{1}{\beta\pi}$, but not the level of borrowing B. Any feasible borrowing terms with this gross interest rate give the same expected payoff to the entrepreneur. The lower bound on borrowing is given by the CF constraint $B^{SY} \geq w^{SY}$, and the upper bound is given by the LL constraint

$Y_S - R_S^{SY} - w_S^{SY} \geq 0 \Leftrightarrow B^{SY} \leq \beta\pi(Y_S - w_S^{SY})$. Evaluating for the optimal (constant) wage, the interval is non-empty if and only if $\beta\pi Y_S - (1+\beta\pi)u^{-1}\left(\frac{x}{(1+\beta\pi)}\right) \geq 0$. Zero is a lower bound on x, and $u^{-1}(0) = \bar{w}$, so a solution to the symmetric information contract problem (10) requires $\beta\pi Y_S - (1+\beta\pi)\bar{w} \geq 0$. Since the wage is strictly increasing in x, and the payoff to the entrepreneur is strictly decreasing in the wage, then whenever the inequality is strict, there exists a non-degenerate interval $[0, \bar{x}^{SY}]$ such that for any x in this interval, a solution to the symmetric information contract problem (10) exists. Feasibility ensures $V^{SY}(\pi, x) \geq 0$. ■

Proof of Lemma 1. From the profitability condition (15) there is some strictly positive surplus in the good firm, so either $U(c^{SY}) > 0$, or $V(c^{SY}) > 0$, or both. From proposition (1) we know that $w^{SY} = w_S^{SY}$, so we have $U(c^{SY}) \geq 0 \Leftrightarrow u(w^{SY}) \geq 0$. The utility to the worker in a bad firm from c^{SY} is therefore

$$\tilde{U}(c^{SY}) = (1+\beta\tilde{\pi})u(w^{SY}) \geq 0 = \tilde{U}(c_0),$$

which holds with strict inequality whenever $U(c^{SY}) > 0$. Similarly, $B^{SY} = w^{SY}$ means we have $V^{SY} \geq 0 \Leftrightarrow [Y_S - R_S - w^{SY}] \geq 0$. The expected cash flow to the entrepreneur in a bad firm from c^{SY} is therefore

$$\tilde{V}(c^{SY}) = \beta\tilde{\pi}[Y_S - R_S - w^{SY}] \geq 0 = \tilde{V}(c_0),$$

which holds with strict inequality whenever $V(c^{SY}) > 0$. (Note that this result holds for any $B^{SY} \geq w^{SY}$, so restricting attention to $B^{SY} = w^{SY}$ is without loss of generality.) If a bad firm proposes the same contract as a good in equilibrium, then Bayes' rule implies $\mu(c) = \mu$. The expected return to the investor from c^{SY} would therefore be

$$-B^{SY} + \beta[\mu\pi + (1-\mu)\tilde{\pi}]R_S^{SY} < 0$$

for any $\mu > 0$), so the investor would not break even. Hence, the symmetric information outcome cannot constitute a perfect Bayesian equilibrium when there is private information about type. ■

Proof of Proposition 2 (Least Cost Separation). Suppose a solution exists to the least cost separating contract problem (16). The BE constraint must be binding in optimum, for the same reason as in the symmetric information contract problem (see the proof of proposition 1). The WP constraint must also be binding in the optimum. Suppose it were not. Then the entrepreneur could marginally lower either w or w_S without violating feasibility or the ICW constraint, and strictly increase the objective. Finally, the ICW constraint must be binding in optimum. Suppose it were not. Then problem (16) would be identical to the symmetric information contract problem (10), so from Proposition 1 the solution would have $w = w_S \geq \bar{w}$, with strict inequality whenever $x > 0$ But this cannot satisfy the ICW constraint for any $x > 0$, hence the constraint must be binding. These two binding constraints suffice to pin down the optimal wage profile. Subtracting ICW from WP yields w_S, and plugging this value back into ICW yields w, where

$$\begin{aligned} w &= u^{-1}\Big(-\frac{\tilde{\pi}}{\pi - \tilde{\pi}}x\Big) \\ w_S &= u^{-1}\Big(\frac{1}{\beta(\pi - \tilde{\pi})}x\Big), \end{aligned}$$

implying a strictly increasing wage profile $w < w_S$ for all $x > 0$. The binding BE constraint pins down the gross interest rate on borrowing $\frac{R_S}{B} = \frac{1}{\beta\pi}$, but not the level of borrowing B. Any feasible borrowing terms with this gross interest rate give the same expected payoff to the entrepreneur. The lower bound on borrowing is given by the CF constraint $B^* \geq w^*$, and the upper bound is given by the LL constraint $Y_S - R_S^* - w_S^* \geq 0 \Leftrightarrow B^* \leq \beta\pi(Y_S - w_S^*)$. $V^{SE}(x)$ is continuous, and strictly decreasing in x since

$$\frac{dV^{SE}(x)}{dx} = \frac{d}{dx}\left[\beta\pi Y_S - u^{-1}\Big(-\frac{\tilde{\pi}}{\pi - \tilde{\pi}}x\Big) - \beta\pi u^{-1}\Big(\frac{1}{\beta(\pi - \tilde{\pi})}x\Big)\right] < 0$$

by the strict concavity of $u(\cdot)$. Strict concavity together with $w < w_S$ implies $V^{SE}(x) < V^{SY}(x)$ for all x, but $\lim_{x\to 0} V^{SE}(x) = \beta\pi Y_S - (1 + \beta\pi)\bar{w} = \lim_{x\to 0} V^{SY}(x) > 0$, given the profitability condition (15) and proposition 1. Hence, there exists a non-degenerate interval $[0, \bar{x}^{SE}]$ such that for any x in this interval, a solution to the least cost separating contract problem (16) exists. Feasibility ensures $V^{SE}(x) \geq 0$. To construct a perfect Bayesian equilibrium from the strategies $(c, \tilde{c}) = (c^*, c_0)$, suppose the investor believes that for any action off the equilibrium path, the posterior probability of being of the good type is zero, i.e. $\mu(c) = 0$ for any $c \neq c^*$. For $V^{SE}(x) \geq 0$, the entrepreneur in a good firm weakly prefers c^*, since the alternative implies no financing and closure. The entrepreneur in a bad firm is indifferent among all possible strategies, which in any case imply closure. ■

Proof of Proposition 3 (Pooling). Suppose a solution exists to the pooling contract problem (20). The objective function is weakly concave, and the constraints are quasi-convex. Hence, a solution to the first order conditions and complementary slackness conditions of the associated Lagrangian problem is also a solution to the original problem. The BE constraint must be binding in optimum, for the same reason as in the symmetric information contract problem (see proposition 1). Use the binding BE constraint to substitute for $B = \beta[\mu\pi + (1-\mu)\tilde{\pi}]R_S$ in the objective, attach Lagrange multipliers ϕ and $\beta\pi\psi$ to the feasibility constraints CF and LL (see eq. 6), and λ to the WP constraint, and form the Lagrangian function

$$\mathcal{L}^{PO} = (1+\phi)\big[\beta[\mu\pi + (1-\mu)\tilde{\pi}]R_S - w\big] + (1+\psi)\beta\pi[Y_S - R_S - w_S] + \lambda[u(w) + \beta\pi u(w_S)].$$

The first order conditions for optimality are

$$\begin{aligned}
\frac{\partial \mathcal{L}^{PO}}{\partial w} &= -(1+\phi) + \lambda u'(w) = 0 \Leftrightarrow \lambda u'(w) = 1+\phi \\
\frac{\partial \mathcal{L}^{PO}}{\partial w_S} &= -(1+\psi)\beta\pi + \lambda\beta\pi u'(w_S) = 0 \Leftrightarrow \lambda u'(w_S) = 1+\psi \\
\frac{\partial \mathcal{L}^{PO}}{\partial R_S} &= (1+\phi)\beta[\mu\pi + (1-\mu)\tilde{\pi}] - (1+\psi)\beta\pi = 0 \Leftrightarrow \frac{1+\psi}{1+\phi} = \frac{\mu\pi + (1-\mu)\tilde{\pi}}{\pi},
\end{aligned}$$

and the complementary slackness conditions are

$$\begin{aligned}
\phi &\geq 0 \text{ and } \phi[B - w] = 0 \\
\psi &\geq 0 \text{ and } \psi[Y_S - R_S - w_S] = 0 \\
\lambda &\geq 0 \text{ and } \lambda[u(w) + \beta\pi u(w_S) - x] = 0.
\end{aligned}$$

The WP constraint must be binding in the optimum, for the same reason as in the symmetric information contract problem (see proposition 1). Consequently, $\lambda > 0$, and so the optimality conditions yield a strictly increasing wage profile

$$\frac{u'(w_S)}{u'(w)} = \frac{1+\psi}{1+\phi} = \frac{\mu\pi + (1-\mu)\tilde{\pi}}{\pi} < 1 \Rightarrow w < w_S.$$

The wage levels are uniquely pinned down from this condition and the binding WP constraint, since $u''(\cdot) < 0$. In terms of Figure 3, the optimal wage profile is the point on the worker's indifference curve $U = x$ where the slope equals the (inverse of the) gross interest rate $\frac{R_S}{B} = \frac{1}{\beta(\mu\pi + (1-\mu)\tilde{\pi})}$ pinned down by the binding BE constraint, because using eq. (13), we have

$$\frac{dw}{dw_S} = -\beta\pi\frac{u'(w_S))}{u'(w)} = -\beta[\mu\pi + (1-\mu)\tilde{\pi}] = \frac{B}{R_S}. \tag{29}$$

The gross interest rate is strictly greater than the (inverse of the) entrepreneur's discounting $\frac{1}{\beta\pi}$. Hence, the CF constraint is binding, and pins down the level of borrowing to $B = w$. To see this, suppose it were not, i.e. suppose $B > w$ in the optimum. Then the entrepreneur could reduce borrowing by a small amount ΔB without violating feasibility, and reduce the payment to investors correspondingly by $\Delta R_S = \frac{1}{\beta(\mu\pi + (1-\mu)\tilde{\pi})}\Delta B$ without changing the BE constraint. But then

$$\begin{aligned}\Delta V^{PO} &= -\Delta B + \beta\pi[-(-\Delta R_S)] \\ &= -\Delta B + \beta\pi[-(-\frac{1}{\beta(\mu\pi + (1-\mu)\tilde{\pi})}\Delta B)] \\ &= \left(\frac{\pi}{\mu\pi + (1-\mu)\tilde{\pi}} - 1\right)\Delta B > 0,\end{aligned}$$

so this could not have been optimal in the first place. Let $\mathring{c}$ denote the solution to the pooling contract problem (20), with payoff $V(\mathring{c}) = V^{PO}(x)$ to the entrepreneur. Consider any x in the interval $(0, \bar{x}^{SE}]$ for which the least cost separating equilibrium c^*, with payoff $V(c^*) = V^{SE}(x)$, is defined. Since B is identically equal to w,

$$\frac{dV^{PO}(x)}{d\mu} = -\beta\pi\left[\frac{dR_S}{d\mu} + \frac{dw_S}{d\mu}\right],$$

which can be evaluated by first differentiating the binding BE constraint using the first order condition (21)) to obtain

$$\begin{aligned}\beta[\mu\pi + (1-\mu)\tilde{\pi}]\frac{dR_S}{d\mu} + R_S\beta(\pi - \tilde{\pi}) &= \frac{dB}{d\mu} = \frac{dw}{d\mu} \\ \Rightarrow -\beta\pi\frac{dR_S}{d\mu} &= \frac{u'(w)}{u'(w_S)}\left[R_S\beta(\pi - \tilde{\pi}) - \frac{dw}{d\mu}\right],\end{aligned}$$

and then differentiating the binding WP constraint holding x constant to obtain

$$u'(w)\frac{dw}{d\mu} + \beta\pi u'(w_S)\frac{dw_S}{d\mu} = 0 \Rightarrow -\beta\pi\frac{dw_S}{d\mu} = \frac{u'(w)}{u'(w_S)}\frac{dw}{d\mu}, \tag{30}$$

implying that $V^{PO}(x)$ is strictly increasing in μ for any given x. Moreover, as $\mu \to 1$, the cost of external financing in the pooling contract tends to the cost of external financing when information is symmetric, so for any x, $\lim_{\mu\to 1} V^{PO}(x) = V^{SY}(x)$. Proposition 2 shows that $V^{SE}(x) < V^{SY}(x)$ for every x, and is independent of μ. Hence, there exists a unique threshold value $\mu^* \in [0,1)$ such that for any prior beliefs $\mu \geq \mu^*$, $V^{PO}(x) \geq V^{SE}(x)$. For $\mathring{c}$ to be a pooling contract, it must be that the entrepreneur in a bad firm is willing to propose it, and the worker in a bad firm is willing to accept it. For $B = w$, the entrepreneur's participation is guaranteed because $\tilde{V}(\mathring{c}) = \frac{\tilde{\pi}}{\pi}V^{PO}(x) \geq 0$. The worker's participation requires $u(w) + \beta\tilde{\pi}u(w_S) \geq 0$. Since the WP constraint is binding, the wage profile for which this constraint binds coincides with the (limit of the) least cost separating wage profile. For $(\mathring{w}, \mathring{w}_S) = (w^*, w_S^*) \Rightarrow \mathring{B} = B^*$, however, the entrepreneur is strictly worse off with the pooling contract, because the gross interest rate is strictly greater than with separation (see eq. 23). Cases with $V^{PO}(x) \geq V^{SE}(x)$ must therefore correspond to strictly higher values of μ. Differentiating $\tilde{U}$ using eq. (30) and the fact that $(\mathring{w}, \mathring{w}_S)$ become less spread out when μ increases, shows that

$$\frac{d\tilde{U}}{d\mu} = u'(w)\frac{dw}{d\mu} + \beta\tilde{\pi}u'(w_S)\frac{dw_S}{d\mu} = \frac{\pi - \tilde{\pi}}{\pi}u'(w)\frac{dw}{d\mu} > 0.$$

Hence, the worker in a bad firm is strictly better off in the pooling contract than in the least cost separating equilibrium. As no-one is worse off, the outcome of the pooling contract therefore strictly Pareto dominates that of the least cost separating equilibrium. Moreover, as $V^{PO}(x)$ is strictly increasing in μ, the entrepreneur in a good firm will also be strictly better off than in the least cost separating contract whenever $\mu > \mu^*$. To construct a perfect Bayesian equilibrium from the strategies $(c, \tilde{c}) = (\mathring{c}, \mathring{c})$, suppose the investor believes that for any action off the equilibrium path, the posterior probability of being of the good type is zero, i.e. for all $c \in C$, $\mu(c) = 0$ if $c \neq \mathring{c}$. For $V^{PO}(x) \geq 0$, the entrepreneur in a either type firm at least weakly prefers $\mathring{c}$, since any alternative $c \in C$ implies no financing and closure. ■

Proof of Proposition 4 (Equilibrium uniqueness and efficiency). The full sets of potential separating and pooling contracts are, respectively

$$C^{SE}(x) = \left\{ c \in C : \begin{array}{ll} \text{(WP)} & u(w) + \beta\pi u(w_S) \geq x \\ \text{(ICW)} & u(w) + \beta\tilde{\pi}u(w_S) \leq 0 \\ \text{(BE)} & -B + \beta\pi R_S \geq 0 \end{array} \right\}$$

$$C^{PO}(x) = \left\{ c \in C : \begin{array}{ll} \text{(WP)} & u(w) + \beta\pi u(w_S) \geq x \\ \text{(WP')} & u(w) + \beta\tilde{\pi}u(w_S) \geq 0 \\ \text{(BE)} & -B + \beta R_S[\mu\pi + (1-\mu)\tilde{\pi}] \geq 0 \end{array} \right\}.$$

For any $\hat{c} \in C^{SE}(x)$, $(c, \tilde{c}) = (\hat{c}, c_0)$ constitutes a perfect Bayesian separating equilibrium, and for any $\hat{c} \in C^{PO}(x)$, $(c, \tilde{c}) = (\hat{c}, \hat{c})$ constitutes a perfect Bayesian pooling equilibrium, for appropriately chosen beliefs for contracts off the equilibrium path, e.g. for any all $c \in C$, $\mu(c) = 0$ if $c \neq \hat{c}$.

Condition 1 (Worker rationality). If the worker has accepted a contract $c \in C^{SE}$, then the investor infers the firm is good, i.e. $\mu(c) = 1$. Consequently, any contract $c \in C^{SE}$ can be attained by the entrepreneur in a good firm. From proposition 2, the least cost separating contract c^* maximises the payoff to the entrepreneur among these contracts, and is therefore the only separating perfect Bayesian equilibrium of the model. Moreover, any pooling equilibrium must yield at least as high payoff to the entrepreneur in a good firm as he gets with the least cost separating contract (else, the entrepreneur would deviate to this contract). The set of contracts $c \in C^{PO}(x)$ such that $V(c) \geq V^{SE}(x)$ may or may not be empty, as illustrated by proposition 3. When the set is empty, the least cost separating equilibrium is the unique perfect Bayesian equilibrium of the model (and by construction, it is efficient in the set of all possible perfect Bayesian equilibria of the model).

Condition 2 (Intuitive criterion). When there are only two types of firms, Cho and Kreps' (1986) "intuitive criterion" is equivalent to an equilibrium dominance test (see Mas-Colell et al., 1995, chapter 13, appendix A), which is somewhat easier to apply. Fix a perfect Bayesian equilibrium $(c, \tilde{c})$, and let $\hat{c}$ denote a contract off the equilibrium path. $\hat{c}$ is *equilibrium dominated* if, for the best possible equilibrium response to $\hat{c}$, the entrepreneur proposing it would be strictly worse off with $\hat{c}$ than he is in the given equilibrium $(c, \tilde{c})$. The *equilibrium dominance test* consists of two steps:

1. Restrict posterior beliefs for contracts off the equilibrium path to $\mu(\hat{c}) = 1$ if $\hat{c}$ is equilibrium dominated for the entrepreneur in a bad firm, but not for the entrepreneur in a good firm.[39]

2. Check whether $(c, \tilde{c})$ still constitutes a perfect Bayesian equilibrium for some posterior belief function $\mu(\cdot)$ satisfying these restrictions.

Because $(V(c) \geq 0$ and $\tilde{V}(c)) \geq 0$ for all $c \in C$, the best possible equilibrium response to any proposed contract is always that both the worker and the investor accept. Given condition 1 above, we are considering equilibria that Pareto dominate the least cost separating equilibrium. Hence, there is no loss of generality in restricting attention to contracts off the equilibrium path that the worker will always accept, and focus on the response of the investor. The investor accepts if and only if he expects to break even, and since the investor's expected profits are strictly increasing in μ, the best possible posterior belief that can be accorded to a contract $\hat{c}$ is always $\mu(\hat{c}) = 1$. Hence, the equilibrium dominance test amounts to checking whether, for a given perfect Bayesian equilibrium $(c, \tilde{c})$, there exists a $\hat{c} \in C^{DEV}(x)$ such that

$$V(\hat{c}) \quad > \quad V(c) \tag{31}$$

$$\tilde{V}(\hat{c}) \quad \leq \quad \tilde{V}(c), \tag{32}$$

where $C^{DEV}(x)$ is the set of feasible contracts that could be met with acceptance from both the worker and the investor, and for which the investor breaks even conditional that

[39] Correspondingly, posterior beliefs are restricted to $\mu(\hat{c}) = 0$ if $\hat{c}$ is equilibrium dominated for the entrepreneur in a good firm, but not for the entrepreneur in a bad firm. But it is never interesting for the entrepreneur to signal bad firm quality, as this leads to firm closure and closure is weakly dominated by any equilibrium outcome, so I can omit it without loss of generality.

he believes the firm is good, i.e.

$$C^{DEV}(x) = \left\{ c \in C : \begin{array}{ll} \text{(WP)} & u(w) + \beta\pi u(w_S) \geq x \\ \text{(WP)} & u(w) + \beta\tilde{\pi} u(w_S) \geq 0 \\ \text{(BE)} & -B + \beta\pi R_S \geq 0 \end{array} \right\}. \quad (33)$$

No pooling equilibrium with $B > w$ passes the equilibrium dominance test. Let c denote the pooling contract in any such equilibrium, and let $\hat{c}$ denote the action $\hat{c}$ off the equilibrium path. Consider keeping the wage profile unchanged, $(\hat{w}, \hat{w}_S) = (w, w_S)$, but reducing both B and R_S so that $\hat{B} = B - \Delta B$, with $\Delta B > 0$ but small, and $\hat{R}_S = R_S - \Delta R_S = R_S - \frac{1}{\beta\pi}(\Delta B + \epsilon)$, with $\epsilon \in (0, \frac{\pi - \tilde{\pi}}{\tilde{\pi}}\Delta B)$. This $\hat{c}$ is feasible since c is feasible, and the worker will accept it since he accepts c. Under the best possible equilibrium response, the entrepreneur in a good firm would be strictly better off with $\hat{c}$ than with c, since $V(\hat{c}) - V(c) = \epsilon > 0$, but the entrepreneur in a bad firm would be strictly worse off, since $\tilde{V}(\hat{c}) - \tilde{V}(c) = \frac{\tilde{\pi}}{\pi}\left[\epsilon - \frac{\pi - \tilde{\pi}}{\tilde{\pi}}\Delta B)\right] < 0$, so $\hat{c}$ is equilibrium dominated for the entrepreneur in a bad firm, but not for the entrepreneur in a good firm. Imposing $\mu(\hat{c}) = 1$ breaks down the initial pooling equilibrium, however, because the entrepreneur in a good firm would then be strictly better off proposing $\hat{c}$ than c, given the investor's response. It remains to verify that the investor breaks even with $\hat{c}$ under these assumptions. Since the investor breaks even with c, a sufficient condition is that profits weakly increase with $\hat{c}$. This holds whenever $\epsilon \leq R_S\beta(1 - \mu)(\pi - \tilde{\pi})$, and such an ϵ is always possible to find. Consequently, no pooling equilibrium with $B > w$ passes the equilibrium dominance test. All pooling equilibria with $B = w$ pass the test, however, because then the expected payoff to the entrepreneur in a bad firm is $\beta\tilde{\pi}[Y_S - R_S - w_S]$, which is just $\frac{\tilde{\pi}}{\pi}$ times the expected payoff for the entrepreneur in a good firm. The ordering of contracts is preserved through this monotone transformation. Hence, it is impossible to find a contract that is equilibrium dominated for the entrepreneur in one type firm, but not the other.

Condition three (No learning from identically ordered contracts). Given conditions 1 and 2 above, the remaining set of pooling contracts satisfy $B = w$, $V(c) \geq V^{SE}(x)$, $\tilde{V}(c) \geq 0$, $u(w) + \beta\pi u(w_S) \geq x$ and $u(w) + \beta\tilde{\pi} u(w_S) \geq 0$. If for any such contract c, the investor learns nothing, i.e. $\mu(c) = \mu$, then any such contract can be attained by the entrepreneur in either type of firm. From proposition 3, the pooling contract $\mathring{c}$ maximises the payoff to the entrepreneur in a good firm among these contracts, and since $B = w$ it also maximises the payoff to the entrepreneur in a bad firm. Hence, whenever the prior belief μ is greater than or equal to the threshold value μ^* given in proposition 3, then $(c, \tilde{c}) = (\mathring{c}, \mathring{c})$ is the unique perfect Bayesian equilibrium of the model (and it is efficient in the set of all possible perfect Bayesian equilibria of the model). ■