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The supply side of CO_2 with country heterogeneity

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The supply side of CO_2 with country heterogeneity*

Michael Hoel†

March 3, 2011

Abstract

Several recent articles have analyzed climate policy giving explicit attention to the non-renewable character of carbon resources. In most of this literature the economy is treated as a single unit, which in the context of climate policy seems reasonable to interpret as the whole world. However, carbon taxes and other climate policies differ substantially across countries. With such heterogeneity, the effects on emission paths of changes in taxes, costs and subsidies may be very different from what one finds for a hypothetical world of identical countries.

Keywords: climate change, exhaustible resources, renewable energy, green paradox

JEL classification: Q31, Q41, Q42, Q54, Q58

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1 Introduction

About three quarters of carbon emissions are caused by the combustion of fossil fuels. Policies for reducing carbon emissions must therefore to a large extent be policies that affect fossil fuel markets. In much of the policy discussion and some of the academic literature it is assumed, usually implicitly, that the producer prices of fossil fuels are unaffected by policies directed toward these markets. As shown already by Bohm (1993), endogenizing fuel prices by including the supply side of fossil fuel markets may be important for studying consequences of climate policies. While Bohm's analysis did not explicitly include the dynamic features of the supply side of fossil fuel markets, an early contribution on such dynamic features was given by Sinclair (1992). Sinclair pointed out that "the key decision of those lucky enough to own oil-wells is not so much how much to produce as when to extract it." Since then, there has been a considerable number of contributions discussing optimal climate policy with explicit attention given to the non-renewable character of carbon resources. These contributions either assume a constraint on the amount of carbon in the atmosphere (Chakravorty et al. 2006, 2008, 2011) or explicitly include a climate cost function in the analysis (Ulph and Ulph, 1992; Withagen, 1994; Tahvonen, 1995; Farzin and Tahvonen, 1996; Hoel and Kverndokk, 1996). One of the insights from the literature is that the principles for setting an optimal carbon tax (or price of carbon quotas) are the same as when the limited availability of carbon resources is ignored: At any time, the optimal price of carbon emissions should be equal to the present value of all future climate costs caused by present emissions, often called the social cost of carbon.

During the last couple of years, there has been a renewed interest in analyzing climate policy with explicit attention given to the non-renewable character of carbon resources. Much of this later literature discusses the so-called "green paradox", a term stemming from Sinn (2008a,b). Sinn argues that some designs of climate policy, intended to mitigate carbon emissions, might actually increase carbon emissions, at least in the short run. Sinn's point is that if e.g. a carbon tax rises sufficiently rapidly, profit maximizing resource owners will bring forward the extraction of their resources. Hence, in the absence of carbon capture and storage (CCS), carbon emissions increase.[1] A thorough analysis of the effects of taxation on resource extraction was given by Long and Sinn (1985), but without explicitly discussing climate effects. More recently, Hoel (2010a,b) has studied the relationship between carbon taxes and carbon extraction emphasizing the fact that governments in practise cannot commit to future tax rates.

A rapidly increasing carbon tax is not the only possible cause of a green paradox. A declining price of a substitute, either because of increasing subsidies or technological improvement, can give the same effect: see e.g. Strand (2007), Gerlagh (2011), Grafton et al. (2010), and van der Ploeg and Withagen (2010).

As mentioned above, Sinn used the term "green paradox" to describe a situation where policies intending to mitigate climate change actually increase near-term emissions. Gerlagh (2011) uses the term "weak green paradox" for such a phenomenon, and uses the term "strong green paradox" to describe a situation where policies intending to mitigate climate change increase total climate costs. This distinction is important, since total climate costs depend not only on near-term emissions, but also on all future emissions. One can therefore imagine policies that increase near-term emissions, but that nevertheless reduce future emissions so much that total climate costs decline.

In almost all of the literature referred to above, the economy analyzed is a single unit, which in the context of climate policy seems reasonable to interpret as the whole world.[2] Policies, whether they are optimal or not, are thus implicitly assumed to be the same throughout the world. This is in sharp contrast to reality: Carbon taxes and other climate policies differ

[1] Throuout this paper, CCS is ignored. Discussions of climate policy when there is a possibility of CCS and when the carbon resource scarcity is taken into considereation have been given by Amigues et al. (2010), Le Kama et al. (2009) and Hoel and Jensen (2010).

[2] Papers considering two or more countries in the context of the green paradox include Eichner and Pethig (2009) and Grafton et al. (2010). None of these discuss the effects of exogenous tax changes or cost reductions, as the present paper does.

substantially across countries. In most countries there are no carbon taxes and not much other climate policy. Many countries actually have quite large explicit or implicit subsidies of fossil fuels.[3] In contrast, several states in the US and all EU countries have various types of climate policies. In the EU there is a quota system covering a considerable amount of carbon emissions, which from 2013 will be widened further. The quota price today is about 15 Euro per tonne of CO_2. Several European countries also have carbon taxes for the parts of the economy not covered by the quota system. For instance, Sweden has a carbon tax of up to over 100 Euro per tonne of CO_2, and Norway has a carbon tax varying from 12 to 48 Euro per tonne of CO_2 for a considerable part of the economy not covered by the EU quota system. Moreover, many European countries also have other climate policies that supplement the quota system or the carbon tax, such as subsidies to renewable substitutes for fossil fuels.

With this motivation, the present paper considers a simple two-country economy where countries differ with respect to their climate policies.[4] The differences between countries may be in either carbon taxes (sections 3-6) or in subsidies (section 6). There is a given initial stock of a homogeneous carbon resource with a constant unit extraction cost, set to zero for simplicity. There is also a perfect substitute for the resource, supplied competitively at a constant unit cost. The producer price of the carbon resource increases at the rate of interest in accordance with the Hotelling rule. In each country, the carbon resource is the only energy source as long as the consumer price of the resource is lower than the price of the substitute, while the substitute is the only energy source once the consumer price has reached the price of the substitute.

Section 2 gives a brief discussion of climate costs, and shows that for a specific set of assumptions the social cost of carbon will be constant over

[3] According to a recent IEA report, total world direct subsidies of fossil fuels amount to $312 billion in 2009, and eliminating these subsidies would cut global carbon emissions by about 7% (see http://www.worldenergyoutlook.org/docs/second_joint_report.pdf)

[4] In a previous version of the paper I assumed several countries. However, going from two to many adds to complexity without giving much new insight.

time. While this feature is used in the formal analysis, most results will also hold under the much weaker assumption that the social cost of carbon increases at a rate less than the rate of interest.

Section 3 presents the basic model for the carbon resource, and section 4 analyzes the effects of increased carbon taxes. Whatever their level, carbon tax rates are assumed to be constant. In a world of homogeneous countries an increase in a common tax rate would move resource extraction form the present to the future, and hence reduce climate costs. With heterogeneous countries the effects of increased taxes are not so simple. In particular, we find that if the carbon tax is raised in the country that initially has the lowest tax rate, resource extraction may be speeded up, implying increased climate costs. In this case we hence have a strong green paradox.

Section 5 analyzes the effects of a reduction in the cost of the renewable substitute. In a world of homogeneous countries such a cost reduction will move resource extraction from the future to the present, see Gerlagh (2011). Hence, we get a weak and a strong green paradox in this case. With heterogeneous countries the effects of lower costs for the renewable substitute are not so simple. We get a weak green paradox also in this case. However, if the carbon tax rates differ sufficiently between countries and demand is sufficiently price inelastic, total climate costs may decline as a consequence of the cost reduction. Hence there is no strong green paradox in this case.

Finally, section 6 analyzes the consequences of subsidizing the renewable substitute. With a common subsidy, the effect on climate costs of an increased subsidy is the same as the effect of a cost reduction, while the effects on social welfare differ between the two cases. If countries initially have different subsidies, the effects of increasing a subsidy in one of the countries are different from a cost reduction affecting both countries. In particular, increasing a subsidy always gives a weak green paradox, and also a strong green paradox if the subsidy is increased in the country that initially has the lowest subsidy.

2 Climate costs and carbon resource extraction

In the subsequent analysis, it is assumed that the total amount of carbon resources are given, and that all of this carbon will eventually be extracted and thus emitted into the atmosphere. Total emissions over all future years are hence given. In spite of this, the profile of the carbon extraction is important from a climate point of view. A rapid increase of carbon in the atmosphere will gradually decline over time, as it is transferred to other sinks. However, a significant portion (about 25% according to e.g. Archer, 2005) remains in the atmosphere for ever (or at least for thousands of years). If a fixed amount of carbon, denoted G_0, is extracted over any time period, this will therefore give a long-run increase of about $G_0/4$ in the atmosphere. With a sufficiently slow rate of carbon extraction, carbon in the atmosphere will grow gradually and monotonically until its long-run level S^* is reached. This is illustrated by curve A in Figure 1, where $S(0)$ is the amount of carbon in the atmosphere at our initial date 0 (so $S^* \approx S(0) + G_0/4$)[5]. Clearly, such a development of carbon in the atmosphere will be associated with a gradually changing climate. With a higher rate of extraction, the carbon in the atmosphere will increase more rapidly, and will overshoot its long-run value S^*, as curve B in Figure 1. This will give a considerably faster climate change, probably with temperatures above the slow extraction path for several centuries. One can argue strongly that the climate costs associated with the rapid extraction path are much higher than the climate costs associated with the climate development associated with the slow extraction path. This seems particularly likely if some effects of climate changes are irreversible, and if the speed of climate change is also an important consideration.[6]

To capture the ideas above, let climate costs at time t be an increasing and convex function of the stock of carbon in the atmosphere above preindustrial

[5] Strictly speaking, $S(0)$ is the long-run level of carbon in the atmosphere if emissions were zero for $t \geq 0$.

[6] Tahvonen (1995), Hoel and Isaksen (1995), and Hoel and Kverndokk (1996) explicitly consider the speed of climate change in their analyses.

level, denoted $C(S(t))$. Moreover, we follow Farzin and Tahvonen (1996) and artificially split S into two components $S_1 + S_2$: component 1 that remains in the atmosphere for ever and component 2 that gradually depreciates at a rate δ. For each unit emitted the share that remains in the atmosphere for ever is denoted α. The amount of 1 unit of carbon emissions at time t remaining in the atmosphere at $\tau(> t)$ is thus $\alpha + (1-\alpha)e^{-\delta(\tau-t)}$. If e.g. $\delta = 0.013$ and $\alpha = 0.25$, 45 % of the original emissions will remain in the atmosphere after 100 years, while 27 % still remains after 300 years. These numbers are roughly in line with what is suggested by Archer (2005) and others.

Consider next the climate damage caused by 1 unit of emissions at time t. The total additional damage caused by 1 unit of carbon emissions at time t is the sum of additional damages at all dates from t to infinity caused by the additional stocks from t to infinity. To get from additional stocks at τ to additional damages at τ we must multiply the additional stocks at τ by the marginal damage at τ, which is $C'(S_1(\tau) + S_2(\tau))$. The marginal damage of 1 additional unit of emissions at t, often denoted the social cost of carbon, is thus given by

$$v(t) = \int_t^\infty e^{-r(\tau-t)} \left[\alpha + (1-\alpha)e^{-\delta(\tau-t)}\right] C'(S_1(\tau) + S_2(\tau))d\tau \qquad (1)$$

For $C'' > 0$ the social cost of carbon will vary over time. While $S_1(\tau)$ is increasing as long as emissions are positive, $S_2(\tau)$ may be declining for sufficiently low emissions. In any case , C' and hence $v(t)$ will change over time.[7] To simplify the formal analysis I assume that $C'' = 0$, i.e. that damages are linear in the atmospheric stock . When C' is constant (1) may be rewritten as

$$v(t) = \left[\frac{\alpha}{r} + \frac{1-\alpha}{r+\delta}\right] C' \qquad (2)$$

[7] Farzin and Tahvonen (1996) give a detailed analysis of how $v(t)$ might develop over time when $C'' > 0$.

which is constant over time. While a constant v simplifies the welfare analysis, most of the subsequent results remain valid also under the much weaker assumption that the present value $e^{-rt}v(t)$ is declining over time.

The level and time profile of the carbon tax is not the issue of this paper. Nevertheless, a few points are worth mentioning. If the constant value of v is sufficiently high, none of the resource will be extracted. The optimal outcome is to immediately satisfy the whole energy demand by the renewable substitute. For lower values of v, all of the available resource will be extracted, and extracted more rapidly the lower is v. With a time-varying $v(t)$ this result may be modified. In particular, if the level of a growing $v(t)$ is sufficiently high, the resource rent will be driven to zero (as it will for a constant high v), but some of the resource will nevertheless be extracted before it is optimal to switch to the substitute (Hoel, 2010b). Moreover, even if v is constant total resource extraction may be declining in v if extraction costs are increasing with cumulative extraction (van der Ploeg and Withagen, 2010; Gerlagh, 2011; Hoel ,2010b).

In a optimal world all countries would have a carbon tax equal to v. However, there are many reasons why actual carbon tax rates may be below v. The most obvious reason is that there is little or no international cooperation on climate policy. With n identical countries, the non-cooperative outcome would be for each country to set its carbon tax equal to $\frac{v}{n}$, provided each country acts individually rational as often assumed by economists. There are in addition also various distributional and other policy reasons for actual taxes to differ from their optimal values. These other factors may vary considerably across countries, implying carbon tax rates that differ substantially across countries. It is beyond the scope of this article to discuss the reasons for such tax differences in more detail; I simply assume that tax rates differ across countries, as they do in the real world.

3 The market for fossil fuels

The market for fossil fuels is modeled as a market for a homogeneous nonrenewable carbon resource, given in fixed supply and with no extraction costs. The resource is supplied by competitive owners of the resource, and the equilibrium producer price $p(t)$ therefore rises at the interest rate r as long as there are any remaining reserves.

The demand for carbon is given as the sum of demand from two countries. There is a perfect substitute for the carbon resource, supplied competitively at its unit cost b. Countries have identical gross utility functions depending on the sum of the use of carbon and the substitute, $u(x + y)$, where x and y are the use of carbon and the substitute, respectively. The corresponding demand function is $D(Q)$ satisfying $u'(D(Q)) = Q$, where Q is the consumer price of the resource or substitute. As long as $Q < b$ consumers will consume the resource, but will switch to the substitute when $Q = b$. The producer price of the carbon resource develops according to the Hotelling rule, and is thus $p_0 e^{rt}$. The two countries have exogenous and constant carbon taxes q_1 and q_2, respectively, with $q_1 > q_2$. The consumer price in country i is hence $p_0 e^{rt} + q_i$ until this price reaches b.

To sum up, the demand for the resource and substitute in the two countries is (for $i = 1, 2$)

$$x_i(t) = D\left(p_0 e^{rt} + q_i\right) \text{ and } y_i(t) = 0 \text{ for } t < T_i \tag{3}$$

$$x_i(t) = 0 \text{ and } y_i(t) = D(b) \text{ for } t \geq T_i \tag{4}$$

with T_i determined by

$$p_0 e^{rT_i} + q_i = b \tag{5}$$

Finally, for a given initial resource stock G_0 we must have the equilibrium condition

$$\int_0^\infty [x_1(t) + x_2(t)]\, dt = G_0 \tag{6}$$

The four equations above determine the resource extraction paths for any given values of the exogenous variables q_1, q_2, and b. The next sections show how changes in these variables affect the outcome, and also discuss welfare effects of such changes.

4 Increased carbon taxes

In the Appendix it is shown that differentiation of (3)-(6) w.r.t. q_i gives (for $i \neq j$ and $i, j = 1, 2$)

$$\frac{dp_0}{dq_i} = \frac{rp_0 e^{rT_j}\left[D(b) - rp_0 e^{rT_i} B_i\right]}{H} < 0 \quad (7)$$

$$\frac{dT_j}{dq_i} = \frac{e^{rT_j}\left[-D(b) + rp_0 e^{rT_i} B_i\right]}{H} > 0 \quad (8)$$

$$\frac{dT_i}{dq_i} = \frac{e^{rT_j}\left\{D(b) + rp_0\left[e^{rT_i} B_i - A\right]\right\}}{H} \text{ ambiguous sign} \quad (9)$$

where

$$A = \int_0^{T_1} e^{rt} D'\left(p_0 e^{rt} + q_1\right) dt + \int_0^{T_2} e^{rt} D'\left(p_0 e^{rt} + q_2\right) dt < 0 \quad (10)$$

$$B_i = \int_0^{T_i} D'\left(p_0 e^{rt} + q_i\right) dt < 0 \quad (11)$$

$$H = -rp_0 e^{r(T_1+T_2)}\left[2D(b) - rp_0 A\right] < 0 \quad (12)$$

It is useful to first consider the case in which both countries initially have the same tax, implying $T_1 = T_2 = T$. For this case it follows that

$$\frac{dT}{dq_1} + \frac{dT}{dq_2} = \frac{e^{rT} rp_0 A}{H} > 0$$

Hence, an increase in the common tax rate will for sure extent the period of extraction. This result is well-known from the theory of non-renewable resources: An increased constant tax rate will make the consumer price path flatter, and therefore also the extraction path flatter. Extraction is therefore postponed in time as a consequence of such a tax increase, which in turn reduces climate costs when the present value of the social cost of carbon declines over time.

Consider next the case of increasing the tax in one country, say country i, when $q_1 > q_2$ initially, implying $T_2 > T_1$. It follows from (7) that any tax increase will reduce the resource rent. Moreover, it follows from (8) that a tax increase in one country will always increase the extraction period in the other country. The reason for this is that the tax increase lowers the time path of the producer price. For country j (which is not increasing its tax) it therefore now takes a longer time for the consumer price to move from $p_0 + q_j$ to b, when the country switches from the resource to the substitute. The total resource use in country j therefore also increases, leaving less total resource use to country i that increases its tax. This tends to make the extraction period in country i go down. However, the fact that country i has increased its tax works in the opposite direction: With a higher tax the consumer price path is flattened, tending to move resource use from the present to the future. The net effect on T_i of an increase in q_i is hence ambiguous, as confirmed by (9).

If $\frac{dT_i}{dq_i} > 0$ the effects of a tax increase in one country are similar to an increase in a common tax rate: The use of the resource is postponed in both countries, which in turn reduces climate costs when the present value of the social cost of carbon declines over time.

The case of $\frac{dT_i}{dq_i} < 0$ is more interesting. Assume first that the high-tax country increases its tax, i.e. q_1 is increased. For $\frac{dT_1}{dq_1} < 0$, T_1 goes down while T_2 increases. Since $T_2 > T_1$ initially, the total period of extraction increases, as it did in the case of an increased common tax rate. If instead the low-tax country increases its tax, T_1 will increase and T_2 will decline if $\frac{dT_2}{dq_2} < 0$. In this case the total extraction period is *shortened*, which tends to increase climate costs if the present value of the social cost of carbon declines over time. To illustrate this case further, it is useful to consider the limiting case of completely inelastic demand, i.e. $D' = 0$.

If $D' = 0$, $A = B_1 = B_2 = 0$, and $\frac{dT_2}{dq_2} = -\frac{dT_1}{dq_2} = \frac{e^{rT_1}D}{H} < 0$, where

D is the demand for the resource or substitute in each country. The effect on resource extraction of an increase in q_2 is illustrated in Figure 2. Until T_1^0, the resource is used in both countries. At T_1^0 country 1 (which has the highest tax) switches to the substitute, while country 2 continues to use the resource until it is exhausted at T_2^0. If country 2 increases its tax the date of resource depletion is reduced to T_2^*, while the period of resource use in country 1 is extended till T_1^*. Since total resource extraction is given, the squares A and B in Figure 2 are of equal size. Resource extraction of this size is moved from a later to an earlier period, clearly increasing climate costs if the present value of the social cost of carbon declines over time.

With a constant social cost of carbon, denoted v, total climate costs are (when $x_1 + x_2 \equiv x$)

$$\Omega = v \int_0^\infty e^{-rt} x(t) dt \tag{13}$$

The change in emissions described by Figure 2 clearly increases Ω since $r > 0$. While a completely inelastic demand is unrealistic, continuity implies that Ω will increase as the carbon tax in the low-tax country also for a sufficiently small positive value of $-D'$. Hence, the following Proposition follows:

Proposition 1 *If the demand for the resource plus substitute is sufficiently price inelastic, total climate costs will increase if the carbon tax is increased in the country that initially has the lowest tax.*

Notice that this proposition will hold even if the social cost of carbon is not constant as assumed in (13). As long as the present value of the social cost of carbon is declining over time, early emissions are worse for the climate than later emissions. Therefore total climate costs will increase also under this less restrictive assumption if the conditions in Proposition 1 hold. The same is true for the subsequent propositions.

How relevant is Proposition 1 in a more realistic setting of many countries instead of only two? To answer this, it is again useful to consider the limiting case of completely inelastic demand. For this case each country's resource use is its demand over the time period it takes for the country's consumer price to go from $p_0 + q_j$ to b. As one country i reduces its carbon tax, p_0 declines, implying that T_j increases for all other countries. The reduced resource use by the country that increases its tax is thus moved to all other countries. Some of these countries may have higher taxes than the country increasing its tax, and the move in resource use to them will increase climate costs defined by Ω. However, there may also be countries having lower carbon taxes than the country increasing its tax. The move in resource use to these countries will *reduce* climate costs defined by Ω. The net affect on Ω is ambiguous. The possibility of climate costs increasing seems more likely the lower the initial tax is in the country increasing its tax, since this means that more of the resource use is moved to higher tax countries and hence used earlier.

Climate costs are only one component of a country's total welfare. To study the effects of a tax increase on a country's total welfare, we return to the simple two-country case. Welfare in country j is

$$W_j = \int_0^\infty e^{-rt} \left\{ u(x_j(t) + y_j(t)) - b y_j(t) + e^{rt} p_0 \left[\alpha_j x(t) - x_i(t) \right] \right\} dt - \frac{\Omega}{2}$$

The first two terms in curly brackets give utility from resource and substitute use minus the costs of substitute production. The third term is the value of country j's net export of the resource, if country j owns a share α_j of the resource. Each country is assumed to bear half of the total climate costs, giving the green welfare term $-\frac{\Omega}{2}$ for both countries.

Differentiating W_j w.r.t. q_i and using the fact that $y_j = 0$ for $u' = e^{rt} p_0 + q_j < b$ and $x_j = 0$ for $e^{rt} p_0 + q_j \geq b = u'$ gives

$$\frac{dW_j}{dq_i} = \frac{dp_0}{dq_i} \int_0^\infty \left[\alpha_j x(t) - x_i(t) \right] dt + q_j \int_0^\infty e^{-rt} \frac{dx_j(t)}{dq_i} dt - \frac{1}{2} \frac{d\Omega}{dq_i} \qquad (14)$$

The first term is a pure terms of trade effect; if $\alpha_1 + \alpha_2 = 1$ these terms vanish when we sum over the two countries. The second term reflects that a carbon tax gives a distortion in the economy if climate effects are ignored. As the time path of resource use is changed due to a change in a carbon tax, we get a negative or positive welfare effect, depending on how this time path is changed. Finally, the last term is the change in the country's climate costs.

The total welfare change for the two countries is (assuming $\alpha_1 + \alpha_2 = 1$):

$$\frac{d(W_1 + W_2)}{dq_i} = \int_0^\infty e^{-rt} \left[q_1 \frac{dx_1(t)}{dq_i} + q_2 \frac{dx_2(t)}{dq_i} \right] dt - \frac{d\Omega}{dq_i} \tag{15}$$

Consider first the case of increasing an common tax rate $q = q_1 = q_2$. Using (13) we find

$$\frac{d(W_1 + W_2)}{dq_1} + \frac{d(W_1 + W_2)}{dq_2} = (q - v) \int_0^\infty e^{-rt} \left[\left(\frac{dx(t)}{dq_1} + \frac{dx(t)}{dq_2} \right) \right] dt \tag{16}$$

From the discussion above we know that an increase in a common tax delays extraction, implying $\frac{dx(t)}{dq_1} + \frac{dx(t)}{dq_2} < 0$ for small t and $\frac{dx(t)}{dq_1} + \frac{dx(t)}{dq_2} > 0$ for large t. Since $r > 0$ the integral is therefore negative, implying that the whole expression is positive for $q < v$. The welfare maximizing carbon tax is of course the Pigovian rate $q = v$. We can summarize this (rather obvious) result as follows:

Proposition 2 *Increasing a common carbon tax will reduce climate costs, and will also increase social welfare in both countries if the common tax rate initially is below the Pigovian tax rate.*

When tax rates differ, there is not much in general one can say about welfare effects of changing one tax rate. However, if $\frac{d\Omega}{dq_i} > 0$, the last term in (14) and (15) may dominate all other terms if v is sufficiently large, since Ω is proportional to v. From Proposition 1 it therefore follows

Proposition 3 *If the social cost of carbon is sufficiently high, and demand for the resource plus substitute is sufficiently price inelastic, total welfare for both countries will decline if the carbon tax is increased in the country that initially has the lowest tax.*

From the discussion after proposition 1 it is clear that welfare may decline for all countries in a multi-country setting if the carbon tax raised in a country that initially has a relatively low tax rate.

5 Lower costs of producing the substitute

In the Appendix it is shown that differentiation of (3)-(6) w.r.t. b gives

$$\frac{dp_0}{db} = \frac{rp_0\left(e^{rT_1} + e^{rT_2}\right)D(b)}{-H} > 0 \tag{17}$$

$$\frac{dT_1}{db} = \frac{\left(e^{rT_1} - e^{rT_2}\right)D(b) + e^{rT_2}rp_0A}{H} > 0 \tag{18}$$

$$\frac{dT_2}{db} = \frac{\left(e^{rT_2} - e^{rT_1}\right)D(b) + e^{rT_1}rp_0A}{H} \text{ ambiguous sign} \tag{19}$$

Notice that the initial consumer price in both countries goes down as b is reduced. Hence, near-term emissions increase, so we get a weak green paradox. As we shall see below, total climate costs may nevertheless decline, in which case there is no strong green paradox.

It is useful to first consider the case in which both countries initially have the same tax, implying $T_1 = T_2 = T$. For this case it follows that

$$\frac{dT}{db} = \frac{e^{rT}rp_0A}{H} > 0$$

Hence, a reduction in b will for sure shorten the period of extraction. This result is well-known from the theory of non-renewable resources: A lower cost of a substitute will reduce the price path of the resource, and hence speed up resource use. It is straightforward to see that this result remains valid for positive but small tax differences, implying a small value for $T_2 - T_1$. The change in the extraction path implied by the reduction in b will increase climate costs when the present value of the social cost of carbon declines over time. As we shall see below, total welfare may nevertheless increase.

When tax rates differ, a lower renewable cost will for sure speed up extraction in the high-tax country. However, resource extraction lasts until T_2, and the direction in which T_2 moves as b is reduced is ambiguous. If $\frac{dT_2}{db} < 0$, the total period of extraction will increase as b is reduced. In this case the reduction in b will give increased early emissions (since p_0 goes down), and increased late emissions (since T_2 increases), and hence lower medium term emissions (from the resource constraint). It is thus not obvious how climate costs are affected by the reduction in b.

To illustrate the possibility of $\frac{dT_2}{db} < 0$, it is useful to again consider the limiting case of completely inelastic demand, i.e. $D' = 0$, implying $A = B_1 = B_1 = 0$. This gives $\frac{dT_2}{db} = -\frac{dT_1}{db} = \frac{\left(e^{rT_2} - e^{rT_1}\right)D}{H} > 0$, where D is the demand for the resource or substitute in each country. The effect on resource extraction of a reduction in b is similar to what we found in Figure 2, except now the initial switch dates are given by T_1^* and T_2^*, while the switch dates after the reduction in b are given by T_1^0 and T_2^0. Some of the resource extraction is thus moved from A to B, reducing climate costs if the present value of the social cost of carbon declines over time.

From the analysis above the following proposition immediately follows:

Proposition 4 *If the differences in carbon tax rates are sufficiently small, total climate costs increase if the cost of the substitute declines. For larger differences in carbon tax rates, this need not be true. In particular, if demand for the resource plus substitute is sufficiently price inelastic, total climate costs will decline if the cost of the substitute declines.*

The effect of reduced b on total welfare is found by proceeding as we did for the case of a tax change. We now find (ignoring the terms of trade term)

$$\frac{dW_j}{db} = \int_0^\infty -e^{-rt} y_j(t)dt + q_j \int_0^\infty e^{-rt} \frac{dx_j(t)}{db} dt - \frac{1}{2}\frac{d\Omega}{db} \qquad (20)$$

The first term, which is negative, is the direct effect of a change in b. This term tends to make welfare increase as b is reduced. The second and third term have exactly the same interpretation as in (14).

For the case of a common tax rate $q = q_1 = q_2$ the total welfare change for the two countries is, using (13)

$$\frac{d\left(W_1 + W_2\right)}{db} = \int_0^\infty -e^{-rt}\left[y_1(t) + y_2(t)\right] dt + (q-v) \int_0^\infty e^{-rt} \left[\frac{dx(t)}{db}\right] dt \qquad (21)$$

From the reasoning above it is clear that both integrals are negative. If $q = v$, it follows that welfare increases as b declines. However, if $q < v$, we cannot rule out the possibility that the increased climate costs that are implied by a reduction in b dominate the positive direct effect.

Proposition 5 *If carbon taxes in both countries are at the Pigovian rate, a lower cost of the substitute is welfare enhancing for both countries. With a lower common tax rate welfare for both countries will decline if the cost of the substitute is reduced, provided the social cost of carbon is sufficiently high.*

Turning next to the case different tax rates, there are not many general conclusions regarding overall social welfare. Perhaps the most interesting case is the case described in proposition 4, implying that lower b may reduce climate costs. In particular, if $\frac{d\Omega}{db} > 0$ and q_2 is sufficiently low, it is clear from (20) that $\frac{dW_2}{db} < 0$, while the sign of $\frac{dW_1}{db}$ will depend on q_1. This gives the following proposition:

Proposition 6 *If the tax rate in the low-tax country is sufficiently low and demand for the resource plus substitute is sufficiently price inelastic, a reduced cost of the substitute will increase welfare in the low-tax country. The sign of the welfare change in the high-tax country will depend on the tax rate in this country and on the social cost of carbon.*

6 Subsidizing the renewable substitute

In the previous section, the cost reduction of the renewable substitute was a real cost reduction. Alternatively, one could consider a cost reduction to the users of the substitute caused by a subsidy σ reducing the private cost of the substitute from b to $b - \sigma$. Notice that such a subsidy in this model is a promise or commitment from the government to hold the future price of the substitute at $b - \sigma$. There may be good reason to believe that such a promise or commitment is not credible, as it may be in the interests of the governments to terminate the subsidy once the carbon resource is depleted. It is nevertheless useful to consider how such a subsidy works if it were possible to convince resource owners that the subsidy would continue "for ever", or at least sufficiently beyond the date of resource depletion.

Consider first a common subsidy σ. The effects on resource extraction of increasing σ are identical to the effects of a reduction in b. The results in the previous section up to and including proposition 4 therefore remain valid. However, the welfare effects of increasing σ differ from the welfare effects of a reduction in b. The difference is in the first term in (20). Clearly, this term vanishes when the reduction in $b - \sigma$ is not caused by a reduction in b, but instead an increase in σ. Instead, if σ initially is positive, we get a term similar to the second term in (20). The exact expression is (ignoring as before the terms of trade term)

$$\frac{dW_j}{d\sigma} = -\sigma \int_0^\infty e^{-rt} \frac{dy_j(t)}{d\sigma} dt + q_j \int_0^\infty e^{-rt} \frac{dx_j(t)}{d\sigma} dt - \frac{1}{2} \frac{d\Omega}{d\sigma} \tag{22}$$

For the case of a common tax rate $q = q_1 = q_2$ the total welfare change for the two countries is, using (13)

$$\frac{d(W_1 + W_2)}{d\sigma} = -\sigma \int_0^\infty e^{-rt} \left[\frac{dy_1(t)}{d\sigma} + \frac{dy_2(t)}{d\sigma} \right] dt + (q - v) \int_0^\infty e^{-rt} \left[\frac{dx(t)}{d\sigma} \right] dt \tag{23}$$

The term in square brackets in the first integral is positive: An increased subsidy implies an earlier start of the use of the subsidy, and once it is used a larger subsidy implies larger use. If $q = v$, it follows that welfare decreases as the subsidy is increased. The second integral is positive, since an increase in the subsidy will speed up carbon extraction. For any $q \leq v$, the whole expression is hence negative:

Proposition 7 *If carbon taxes in the two countries are equal and do not exceed the Pigovian tax rate, subsidizing the renewable substitute will for sure lower social welfare in both countries.*

Notice that this proposition implies that introducing a small positive tax on the substitute in this case would increase social welfare, as also pointed out by van der Ploeg and Withagen (2010).

Turning next to the case of different tax rates, there are not many general conclusions regarding overall social welfare. Perhaps the most interesting case is the case described in proposition 4, implying that higher σ may reduce climate costs. In particular, if $\frac{d\Omega}{d\sigma} < 0$, and σ and q_2 are sufficiently small, it is clear from (20) that $\frac{dW_2}{d\sigma} > 0$, while the sign of $\frac{dW_1}{db}$ will depend on q_1. This gives the following proposition:

Proposition 8 *If the tax rate in the low-tax country is sufficiently low and demand for the resource plus substitute is sufficiently price inelastic, the introduction of a small subsidy for the renewable substitute will increase welfare in the low-tax country. The sign of the welfare change in the high-tax country will depend on the tax rate in this country and on the social cost of carbon.*

Finally, it is useful to consider the case where carbon taxes are equal in the two countries, but subsidies may differ. For this case equations (3)-(4) are changed to (for $i = 1, 2$)

$$x_i(t) = D\left(p_0 e^{rt} + q\right) \text{ and } y_i(t) = 0 \text{ for } t < T_i \quad (24)$$

$$x_i(t) = 0 \text{ and } y_i(t) = D\left(b - \sigma_i\right) \text{ for } t \geq T_i \quad (25)$$

with T_i determined by

$$p_0 e^{rT_i} + q = b - \sigma_i \quad (26)$$

We assume that $\sigma_1 > \sigma_2$, implying that $T_1 < T_2$. In the Appendix it is shown that

$$\frac{dp_0}{d\sigma_i} = \frac{rp_0 e^{rT_j} D(b - \sigma_i)}{H} < 0 \quad (27)$$

$$\frac{dT_i}{d\sigma_j} = \frac{-e^{rT_i} D(b - \sigma_j)}{H} > 0 \quad (28)$$

$$\frac{dT_i}{d\sigma_i} = \frac{e^{rT_j}\left[D(b - \sigma_j) - rp_0 A\right]}{H} < 0 \quad (29)$$

An increased subsidy in any country will reduce the initial consumer price in both countries. Just like for a reduction in b, near-term emissions therefore increase, giving a weak green paradox. If the subsidy in country 1 is increased, the total period of extraction is increased (since $\frac{dT_2}{d\sigma_1} > 0$). In this case the increased subsidy will give increased early emissions (since p_0 goes down), and increased late emissions (since T_2 increases), and hence lower medium term emissions (from the resource constraint). It is thus not obvious how climate costs are affected by the increased subsidy.

If instead the subsidy in country 2 is increased, the total period of extraction is shortened. Moreover, total extraction at any point of time up to the exhaustion date must go up, since the consumer price path becomes lower in both countries. This immediately gives the following proposition:

Proposition 9 *If the subsidy is increased in the country that initially has the lowest subsidy, total climate costs increase.*

For the same reason as for a carbon tax increase in a single county, this result is relevant also in a more realistic setting of many countries instead of only two: Each country's resource use is its demand over the time period it takes for the country's consumer price to go from $p_0 + q$ to $b - \sigma_j$. As one country i increases its subsidy, p_0 declines, implying that T_j increases for all other countries. The reduced resource use by the country that increases its subsidy is thus moved to all other countries. Some of these countries may have higher subsidies than the country increasing its tax, and the move in resource use to them will increase climate costs defined by Ω. However, there may also be countries having lower subsidies than the country increasing its subsidy. The move in resource use to these countries will *reduce* climate costs defined by Ω. The net affect on Ω is ambiguous. The possibility of climate costs increasing seems more likely the lower the initial subsidy is in the country increasing its subsidy, as this means that more of the resource use is moved to higher subsidy countries and hence used earlier.

To simplify the discussion of total welfare, assume that the common carbon tax rate is zero. Then we find an expression similar to (22):

$$\frac{dW_j}{d\sigma_i} = -\sigma_j \int_0^\infty e^{-rt} \frac{dy_j(t)}{d\sigma_i} dt - \frac{1}{2} \frac{d\Omega}{d\sigma_i} \tag{30}$$

The integral in (30) is positive for the country that increases its subsidy (since it switches to the substitute earlier and uses more of the substitute as a consequence of the increased subsidy) but negative for the other county (since it delays its switch to the substitute as a consequence of the increased subsidy). These results can be summarized in the following proposition:

Proposition 10 *Assume that both countries have zero carbon taxes but positive subsidies. An increased subsidy will reduce social welfare excluding climate costs in the county that increases its subsidy, but increase social welfare excluding climate costs in the other county. Since climate costs increase if the low-subsidy country increase its subsidy, total social welfare for this country will decline as it increases its subsidy.*

7 Conclusions

The analysis above is done with an extremely simple model. Perhaps the most drastic simplification is that carbon resources are homogeneous, and have constant unit costs up to a physical upper limit on total extraction. A much more realistic assumption would be to let extraction costs be rising in cumulative extraction. van der Ploeg and Withagen (2010), Gerlagh (2011) and Hoel (2010a,b) have shown that the effects on emission paths of changes in carbon taxes and costs of a renewable substitute may depend significantly on the properties of the extraction cost function. Moreover, the extraction cost function may differ between different types of fossil fuels.

A second drastic simplification is that the substitute for the carbon resource was assumed to be a perfect substitute, and that it had a constant unit cost of production. Relaxing these assumptions may change the conclusion that a lower cost of the substitute will speed up extraction in a world of homogeneous countries, see e.g. van der Ploeg and Withagen (2010), Grafton et al. (2010), and Gerlagh (2011).

The focus of the present paper has been to show that the degree of country heterogeneity may significantly affect the relationship between carbon taxes, costs and subsidies on the one hand and emission paths on the other hand.

To focus on this issue it has been useful to keep the model as simple as possible in all other dimensions. The analysis has shown that the effects on emission paths of changes in taxes, costs and subsidies may be very different in a world of heterogeneous countries than one would find in a hypothetical world of identical countries. Although details will differ, it seems reasonable to expect similar differences also in more general models of carbon resources with a substitute.

Appendix: Changes in taxes, costs, and subsidies

Inserting (3) into (6) gives

$$\int_0^{T_1} D\left(p_0 e^{rt} + q_1\right) dt + \int_0^{T_2} D\left(p_0 e^{rt} + q_2\right) = G_0$$

which together with the two equations (26) give three equations determining p_0, T_1 and T_2 as functions of q_1, q_2, σ_1, σ_2 and b. Differentiation of this equation system gives

$$M \cdot \begin{pmatrix} dT_1 \\ dT_2 \\ dp_0 \end{pmatrix} = \begin{pmatrix} -B_1 \\ -1 \\ 0 \end{pmatrix} dq_1 + \begin{pmatrix} -B_2 \\ 0 \\ -1 \end{pmatrix} dq_2 + \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} d\sigma_1 + \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} d\sigma_2$$

where

$$\mathbf{M} = \begin{pmatrix} D(b-\sigma_1) & D(b-\sigma_2) & A \\ rp_0 e^{rT_1} & 0 & e^{rT_1} \\ 0 & rp_0 e^{rT_2} & e^{rT_2} \end{pmatrix}$$

and A and B_i are as defined by (10) and (11) but with the subsidy rates included in the demand function.

These equations give (7)-(9) and (27)-(29), where H , defined by (12) with the subsidy rates included, is the determinant of the matrix M.

For $\sigma_1 = \sigma_2 = 0$ the equations above imply

$$\begin{aligned}
\frac{dp_0}{db} &= -\frac{dp_0}{d\sigma_1} - \frac{dp_0}{d\sigma_2} = \frac{-rp_0\left(e^{rT_1} + e^{rT_2}\right) D(b)}{H} \\
\frac{dT_1}{db} &= -\frac{dT_1}{d\sigma_1} - \frac{dT_1}{d\sigma_2} = \frac{\left(e^{rT_2} - e^{rT_1}\right) D\left(b\right) - rp_0 e^{rT_2} A}{H} \\
\frac{dT_2}{db} &= -\frac{dT_2}{d\sigma_1} - \frac{dT_2}{d\sigma_2} = \frac{\left(e^{rT_1} - e^{rT_2}\right) D\left(b\right) - rp_0 e^{rT_1} A}{H}
\end{aligned}$$

which is identical to (17)-(19).

References

Amigues, J.-P., G. Lafforgue and M. Moreaux (2010). Optimal capture and sequestration from the carbon emission flow and from the atmospheric carbon stock with heterogeneous energy consuming sectors. Toulouse School of Economics Working Paper 10-163.

Archer, D. (2005). Fate of fossil fuel CO2 in geologic time. *Journal of Geophysical Research*, *110*, C09S05.

Bohm, P. (1993). Incomplete international cooperation to reduce CO2 emissions: alternative policies. *Journal of Environmental Economics and Management*, *24*, 258–271.

Chakravorty, U., A. Leach and M. Moreaux (2011). Hotelling prices with pollution and learning-by-doing. Unpublished.

Chakravorty, U., B. Magne and M. Moreaux (2006). A Hotelling model with a ceiling on the stock of pollution. *Journal of Economic Dynamics & Control*, *30*(12), 2875–2904.

Chakravorty, U., M. Moreaux and M. Tidball (2008). Ordering the Extraction of Polluting Nonrenewable Resources. *American Economic Review*, *98*(3), 1128–44.

Eichner, T. and R. Pethig (2009). Carbon Leakage, the Green Paradox and Perfect Future Markets. *CESifo Working Paper*, *2542*.

Farzin, Y. and O. Tahvonen (1996). Global carbon cycle and the optimal path of a carbon tax. *Oxford Economic Papers, New Series*, *48*, 515–536.

Gerlagh, R. (2011). Too Much Oil. *CESifo Economic Studies*, *forthcoming*.

Grafton, R., T. Kompas and N. Long (2010). Biofuels subsidies and the green paradox. CESifo Working Paper no. 2960.

Hoel, M. (2010a). Climate change and carbon tax expectations. CESifo Working Paper 2966.

Hoel, M. (2010b). Is There a Green Paradox? CESIfo Working Paper no. 3168.

Hoel, M. and I. Isaksen (1995). The environmental costs of greenhouse gas emissions. In Carraro, C. and J. A. Filar (eds.), *Control and Game-Theoretical Models of the Environment*, vol. Control and Game-Theoretical Models of the Environment, 89–105. Boston: Birkhauser.

Hoel, M. and S. Jensen (2010). Cutting Costs of Catching Carbon - Intertemporal effects under different climate policies. CESifo Working Paper no. 3284.

Hoel, M. and S. Kverndokk (1996). Depletion of fossil fuels and the impacts of global warming. *Resource and Energy Economics*, *18*(2), 115–136.

Le Kama, A., M. Fodha and G. Lafforgue (2009). Optimal carbon capture and storage policies. Toulouse School of Economics Working Paper 09-095.

Long, N. and H.-W. Sinn (1985). Surprise Price Shifts, Tax Changes and the Supply Behavior of Resource Extracting Firms. *Australian Economic Papers*, *24*(45), 278–289.

Sinclair, P. (1992). High does nothing and rising and worse: carbon taxes should be kept declining to cut harmful emissions. *Manchester School of Economic and Social Studies*, *60*, 41–52.

Sinn, H. (2008a). *Das Grüne Paradoxon. Plädoyer für eine Illusionsfreie Klimapolitik*. Econ.

Sinn, H. (2008b). Public policies against global warming: a supply side approach. *International Tax and Public Finance*, *15*, 360–394.

Strand, J. (2007). Technology Treaties and Fossil Fuels Extraction. *The Energy Journal*, *28 (4)*, 129–142.

Tahvonen, O. (1995). Dynamics of pollution control when damage is sensitive to the rate of pollution accumulation. *Environmental and Resource Economics*, *5*, 9–27.

Ulph, A. and D. Ulph (1994). The optimal time path of a carbon tax. *Oxford Economic Papers*, *46*, 857–868.

van der Ploeg, F. and C. A. Withagen (2010). Is There Really a Green Paradox? CESifo Working Paper no. 2963.

Withagen, C. (1994). Pollution and exhaustibility of fossil fuels. *Resource and Energy Economics*, *16*, 235–242.

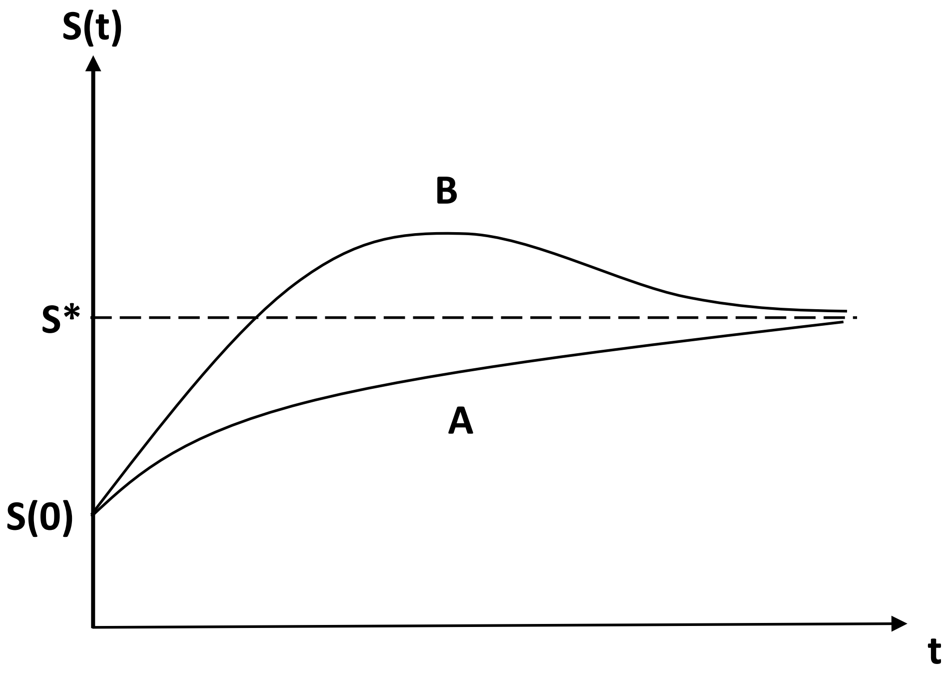

Figure 1: Carbon in the atmosphere for alternative emission paths

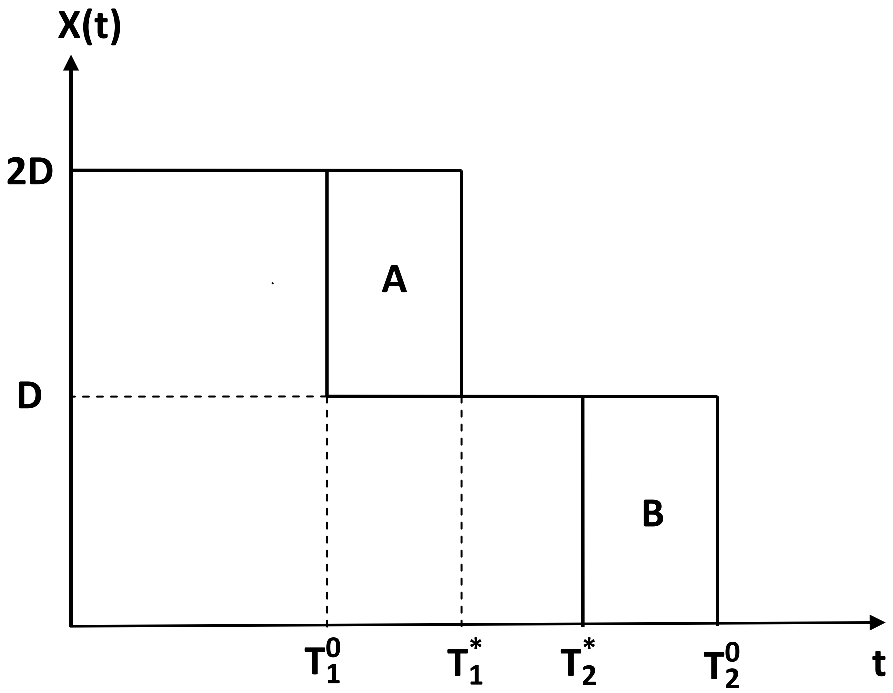

Figure 2: The effect on the extraction path of reduced cost of the substitute