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From Macro Growth to Disaggregated Production Studies

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From Macro Growth to

Disaggregated Production Studies

by

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May 11, 2011

Abstract: Professor Leif Johansen (1930 – 1982) made significant contributions to a large number of fields in economics. A short survey of his contributions is presented. The main focus in the paper is on his growth – production program constituting an important part of his research. The key concepts are embodied technical change, irreversibility, sunk cost, rigidities and heterogeneity. The impact of these factors on the nature of economic growth at the macro level is the point of departure of gradually disaggragating the level of analysis right down to the individual firm. An important tool for the analysis for dynamic structural change at the industry level is the short-run function capturing the underlying heterogeneity of the technologies of the firms within an industry. Technical rigidities represent constraints on how an economy develops from the level of a single industry up to the aggregated macro level of an economy.

Keywords: Dynamic structural change; Heterogeneity; Macroeconomic growth; Short-run industry production function; Technical rigidities; Vintage

JEL classification: C61, D24, D92, E22, E23

1. The research contributions of Leif Johansen

Professor Leif Johansen (1930 – 1982) was an extraordinary economist and a rare generalist with significant contributions to a large number of fields in economics. His research career, extending over a period of 30 years, revealed a highly productive mind, always with a strong attitude for the betterment of mankind, and a deep desire for justice. Except for a one-year stay abroad in the late 50's, he spent all his time at the University of Oslo, offering brilliant and research-based lectures, while producing a large number of books and articles published in top journals. Whatever field he embarked upon, whatever problem had caught his mind, he produced self-contained papers of high originality and deep insight. At the time of his death he was still very active, always orienting towards new fields and problems to be solved. Today it is difficult to imagine any economist being capable, as Johansen was, to penetrate almost any kind of problem, and at the same time come up with interesting conclusions – of high social value – without being lost in insignificant and empty technicalities.

Based on his profound knowledge of the themes Leif Johansen had a special gift for creating clarity of various contributions in the literature. He unified different approaches, and in the process made a series of novel contributions, both theoretical and empirical. His main fields of research were aggregate and disaggregate growth theory, dynamic input – output models, public economics, production theory, macroeconomic planning, socialist economies, and game theory and bargaining. It is also typical of his interest in economic policy and the dissemination of economic theory that he contributed numerous papers to popular journals interpreting current economic events on the basis of economic theory. Typical of Leif johansen's interest in what was going on is his numerous and insightful book reviews. A good example of his capability to create clarity based on a profound understanding and then making main points accessible is the explanation of Arrow's Impossibility Theorem (Johansen 1969).

The number of citations of his works may not seem that impressive, but his research papers were more typical of being the professors' professor. His research was quickly noticed and quoted by a group of influential economist, among them several future Nobel Prize winners. To show the extent of his research Table 1 shows a few of his most well-known articles chosen on the basis of the number of citations.

Table 1. Key articles by Leif Johansen and their citations

Article	**Journal**	**No. of citations**
Substitution versus fixed production coefficients in the theory of economic growth - a synthesis	Econometrica, 1959 27(2), 157-176	141*
Some notes on the Lindahl theory of determination of public-expenditures	International Economic Review, 1963 4(3), 346-358	46
The theory of public goods - misplaced emphasis	Journal of Public Economics 1977 7(1), 147-152	26
On the status of the Nash type of noncooperative equilibrium in economic theory	Scandinavian Journal of Economics 1982 84(3), 421-441	24
The bargaining society and the inefficiency of bargaining	Kyklos, 1979 32 (3) , 497-522	23

*ISI web of Science as of May 2010. Three self-citations are removed, but two citations not in ISI are added.

The five most cited papers might be a good representation of his wide interest in research themes The citation numbers from ISI Web of Knowledge may be rather modest, but the first article in the table was quoted by no less than ten future Nobel Prize winners.

Johansen spread his papers consciously around on different journals based on the interest of the journals in specific research topics. His 40 journal papers are published in 22 different international journals. In addition he had 30 papers in 16 national economic journals representing 10 countries.

Leif Johansen coined the name Public Economics for a field that used to be called public finance. The second paper in Table 1 is an example of how Leif Johansen systematically goes through the literature of a research filed and produces novel insights in the process. It was natural that he was one of the founders of Journal of Public Economics. The third paper in Table 1 is a good example of his interest in the relevance of theoretical contributions to the field. Another paper from his period of interest in public economics showing his gift for creating understanding is the paper by Johansen (1958) on the role of the banking system in macro-economic models. The insights provided there are still useful in the light of the international financial crisis the last years.

The last two papers show Leif Johansen's interest in applying game theory. His distinct interpretation of a Nash equilibrium was an eye-opener for people not thinking deeply enough about the economics interpretation of the concept. The last paper in Table 1 clearly points to his interest in applying game theoretical concepts in a wider context of understanding decision-making at various levels in a society.

Leif Johansen also published books based on his research. His books and their citations are set out in Table 2 in chronologically order.

Table 2. The books of Leif Johansen and their citations

Name of book	**Citations**
A Multi-Sectoral Study of Economic Growth (1960; 1964)	130
Public Economics (1965)	63
A Multi-Sectoral Study of Economic Growth (1974)	31
Production Functions. An integration of micro and macro, short run and long run aspects (1972)	162
Lectures on Macroeconomic Planning (1977)	34
Lectures on Macroeconomic Planning (1978)	30

The first book is his doctoral dissertation from 1960 (second issue 1974), and the third item in Table 2 is an edition enlarged with the experience of using the multi-sectoral growth model (shortened MSG) for indicative planning purposes in

Norway. This path-breaking contribution is the theme of a special forthcoming issue of Journal of Economic Policy Modeling, and will therefore not be dealt with here. We will just point out as an example of the importance of the research themes Leif Johansen took up that in his book on the multi-sectoral growth model Leif Johansen not only cited the seven first Nobel Prize winners, but also cited six more winners within a 20 year span after his book was published (Bjerkholt 2009).

The book on public economics is a textbook thatoffers insights about policy issues at a research level. The two volumes on macroeconomic planning were also intended primarily as textbooks, but represented an integration of many of Leif Johansen's research contribution and represented fresh research ideas.

His book on production functions represents a natural final stop on Leif Johansen's research on the importance of rigidities in an economy. Such rigidities follow from his vintage approach, starting at the macro level in is 1959 paper, and ending at the micro level in this book. His works from macro to micro, based on elements like embodiment of technology, irreversibility, sunk cost and rigidities, is the story we want to tell in the present paper based on some of Leif Johansen's research contribution.

To avoid making a superficial survey of his entire research output,[1] we want to highlight one research path that we find to be a very important part of his research agenda, what we call Johansen's "Growth - Production research program". This program starts with his seminal "Synthesis"– paper in Econometrica 1959 on "Ex ante substitution and ex post fixed coefficients" – later coined "Putty Clay" by Phelps – and ending with his book "Production Functions" with the well-chosen subtitle "An integration of micro and macro, short run and long run aspects" from 1972. We feel there is a distinct line of thought between these publications, and in

[1] See Førsund (1987) for a collection of his works, and Solow (1983) and Bjerkholt (2009) for excellent expositions.

our opinion we still find a lot of very stimulating and highly relevant issues that should be conveyed to the profession of today. The common thread we want to highlight here is the role played by technical rigidities, irreversibility, and heterogeneity in an economy's production structure. In our opinion it is rather difficult to discuss issues like structural unemployment, intra-industry structural changes and changes in an industry's capacity distribution, and obsolescence or economic lifetime of capital equipment or plants, without introducing technical rigidities. The main line of this research sequence is surveyed in Sections 2 – 5, emphasizing the impact of technical rigidities in production on growth and structural changes. In section 6 we offer concluding remarks on various issues stemming from technical rigidities, and point out some research inspired by the Growth - Production program.

2. Technical rigidities and macro growth

The production structure in standard growth theory at the time Johansen published his "Synthesis" in Econometrica 1959, was either based on fixed production coefficients or substitutability between total labour and capital stock; the latter assumption was adopted in his MSG-study, as well. The Johansen-synthesis offered a compromise between these two approaches by distinguishing between an ex ante-stage (prior to the point in time of installment of new capital equipment) and an ex post-stage (once the installment had taken place). At the planning or ex ante-stage, the firm has an infinite number of technical options regarding the choice of factor proportion – or substitution possibilities at the margin. Or as he expressed it: "Any gross *increment* in the rate of production can be obtained by different combinations of *increments* in capital and labour input" (Johansen 1959, 158). However, within this framework, once equipment is completed and installed – the ex post stage – the plant is operated with a fixed technique or with a fixed amount of labor (fixed proportion ex post); or in his own words: "Once a piece of capital is produced and has been put into operation, it will continue to operate through all its life span in cooperation with a constant amount of labour input" (Johansen 1959, 158). At this stage there are no further substitution possibilities between total labor and existing capital stock. In Phelps's terminology from 1963, we have a putty choice-of-technique function ex ante; ex post we have a clay production structure.[2]

Johansen kept his approach as close as possible to the standard neoclassical growth theory, with an exogenous time path for total labour force, full employment and a fixed fraction of net income being saved. However, due to the distinction between substitution ex ante and fixed proportion ex post, the analysis required a more sophisticated framework for how capital depreciated over time. In his model, consisting of six equations between production, capital and labour, as well as their rates of change, also expected lifetime of a newly produced unit of capital was introduced by imposing different assumptions about the survival function of a unit of capital.

Although we will later present another version of this set-up, the original Johansen model is:

$$y(t) = \varphi(n(t), k(t)) \tag{1}$$

This is the ex ante function, indicating that new production techniques can be utilized only by installing new capital equipment, so-called embodied technical progress. As opposed to disembodied technical progress, that comes as "manna from heaven", with a malleable capital stock benefitting uniformly from the technological progress, one can only take advantage of embodied technical improvements by investing in new or modern capital equipment, embodying the

[2] It is surprising to note that we do not find any reference to Johansen or to Phelps, in "modern" textbooks like "Endogenous Growth Theory", by Aghion and Howitt, from 1998, or in "Modern Economic Growth", by Acemoglu from 2009. The profession's retrospective shortening or memory loss is, to be honest, disappointing.

new and superior knowledge about production technology.[3] As formulated by Hahn and Matthews in their survey from 1964 (p. 837): "The difference from the orthodox approach is merely that now the manna of technical progress falls only on the latest machines."

If an amount $k(t)dt$ of newly produced equipment is put into operation during the time interval $\left[t, t + dt\right]$, and combined with an amount of labour $n(t)dt$, the rate of gross increase in production is $y(t)dt$, according to (1). The depreciation, shrinkage or survival of capital is expressed according to some given "death rate table": At some point in time $\tau > t$, a fraction $f(\tau - t)$ of the amount $k(t)dt$ of capital installed during $\left[t, t + dt\right]$ will still be operating at time τ, with $f(0) = 1$ and f, in general, non-increasing. Hence, because the factor proportion is assumed to be fixed throughout the life span, capital shrinkage according to the survival function will imply that required labour input will shrink in the same proportion; i.e. at τ an amount $f(\tau - t)n(t)dt$ of labour is operating the capital equipment $f(\tau - t)k(t)dt$, with a corresponding rate of production $f(\tau - t)y(t)dt$. Hence, total rate of production or "gross macro output", $x(t)$, is then obtained by integrating the rate of output over all vintages, when taking into account the shrinkage according to the chosen survival function $f(t - \tau)$ showing the fraction of capacity installed at τ operating at t:

$$x(t) = \int_{-\infty}^{t} f(t - \tau)y(\tau)d\tau \tag{2}$$

Total labour force at t is given by

$$N(t) \text{ (an exogenous function).} \tag{3}$$

[3] Even though there have been some critics against the embodiment hypothesis, like Denison claiming it to be unimportant, we find the distinction between new and old capital equipment with different qualities and efficiencies reflecting the technological know-how at the date of construction an empirically reasonable approach, and very hard to reject, at least for materials-processing activities.

Full employment requires that the available labour force is distributed over plants of different ages or vintages, according to

$$\int_{-\infty}^{t} f(t-\tau)n(\tau)d\tau = N(t) \tag{4}$$

The assumption of full employment will of course impose restrictions on the development of wages and rate of interest. The choice of technique ex ante must, along with prices, move in such a way as to get entrepreneurs to "choose to absorb both the flow of savings and the flow of disposable labour at all times" (Johansen 1959, 164).

Total stock of capital available at t in the economy will therefore be

$$\int_{-\infty}^{t} f(t-\tau)k(\tau)d\tau = K(t) \tag{5}$$

To derive a concept of net income, we need a rule for valuing capital. Johansen uses a prospective method by valuing a unit of capital according to its remaining life span. The expected lifetime of newly produced unit of capital equipment can be expressed as $T(0) = \int_{0}^{\infty} \tau \frac{d}{d\tau}(1 - f(\tau))d\tau = \int_{0}^{\infty} f(\tau)d\tau$, whereas a capital unit of age η will have an expected remaining lifetime given by $T(\eta) = \frac{1}{f(\eta)}\int_{\eta}^{\infty} f(\tau)d\tau$. A capital unit of age η relative to a new one, will then be valued according to $\frac{T(\eta)}{T(0)}$. The value of the total capital stock at t, $V(t)$ can then be found as $V(t) = \frac{1}{T(0)}\int_{-\infty}^{t} f(t-\tau)T(t-\tau)k(\tau)d\tau = \frac{1}{T(0)}\int_{\tau=-\infty}^{t}\left[\int_{\omega=t-\tau}^{\infty} f(\omega)d\omega\right]k(\tau)d\tau$, from which we can derive the rate of net investment at t as $\dot{V}(t) := I(t) = k(t) - \frac{K(t)}{T(0)} = k(t) - D(t)$, where $D(t)$ is depreciation at t.

The last equation of the model, the one that relates savings and net income, follows from assuming that a constant fraction of net income, $x - D$ being saved. With an exogenous savings rate α, we have $I(t) = \alpha \cdot \left[x(t) - D(t)\right] = k(t) - D(t)$ when using that $I = k - D$ from above. Then it follows that:

$$k(t) = \alpha x(t) + (1-\alpha)\frac{K(0)}{T(0)} \tag{6}$$

With $T(0)$ defined above, the model consists of the six equations $(1)-(6)$ between the six time functions $y(t), n(t), k(t), N(t), K(t), x(t)$.

Some special features of the model will be captured by making specific assumptions about the "death rate table", the decrement series function or the survival function. In his article, Johansen analyzes the asymptotic growth properties of the model, under three types of decrement series: One is with infinite lifetime, $f(\tau) = 1 \; \forall \tau \geq 0$, a Cobb-Douglas ex ante production function and exponential growth of the labour force. In that case the asymptotic growth rate of total output is shown to be independent of the saving ratio and identical to what would have been derived from a standard neoclassical growth model. As should be obvious, growth itself will be increasing in the savings rate, because the higher is the savings rate, the more and more modern equipment will be installed. However, even though the asymptotic growth properties are identical, the asymptotic growth rate is expected to be reached sooner in a standard neoclassical world than in a world with putty-clay technologies. In the latter world, capital equipment cannot be changed momentarily, so history will prevent us from adopting the most efficient and modern equipment instantaneously.

In another version, capital shrinkage follows an exponential form, $f(\tau) = e^{-\delta\tau}$ where δdt is the flow probability that a unit of capital is destroyed during $\left[t, t+dt\right]$, with δ, a constant and age-independent parameter, being the inverse of the expected lifetime of a newly produced capital unit. Again with a similar structure as above, one can derive a Bernoullian differential equation for $k(t)$, whose asymptotic solution exhibits a constant growth rate, with an asymptotic growth rate for total output as above. The role played by the "rate of destruction", δ is a bit complicated. Not only is δ affecting the rate of shrinkage, but its magnitude will also have some influence on the accumulation of new equipment and "the speed with which new techniques can be introduced", (Johansen, 1959; 170). Another feature of that model is the relationship between the rate of technical progress (ε) in the ex ante production function and the rate of destruction, δ. The asymptotic level of total output will, for a low rate of technical progress be higher the smaller is δ, as we should expect. However, for $\varepsilon > 0$, it might be the case that the asymptotic level is higher the higher is δ, conveying a relationship between the rate of destruction, or scrapping, and the speed with which old equipment is replaced by new and better equipment.

The third case analyzed by Johansen as to the form of the shrinkage function, is one with fixed technical life time for each unit of capital, or a "sudden death" after some given time span. Here it is assumed that $f(\tau) = 1$ for $\tau \leq \theta$ and $f(\tau) = 0 \; \forall \tau > \theta$, saying that each unit of capital has a given life time equal to the exogenous parameter θ and that each unit will retain its original productive characteristics throughout a period of length $T(0) = \theta$. Even this "simple" structure will turn the model into a rather complicated one, even for a linear ex ante production function, with a mixed difference-differential equation for the rate of gross investment. In this case we get a large number of solutions, with real or complex growth rates, depending on a measure of roundaboutness (the product of expected lifetime and marginal productivity of new capital). If the growth rate in total labor force exceeds (falls below) the real positive solution for the growth rate in gross investment, we get an asymptotic value (long run growth) of income per capita, if the saving rate is not too low. The model also contains an infinity of complex solutions, which under some reasonable assumptions will exhibit damped cycles of length depending, among other things, on θ.

In the above version of the model, capital equipment is worn out solely for technical reasons. Johansen was fully aware of obsolescence or the fact that capital units of certain vintages may become economically unprofitable before its technical life time has elapsed. (Obsolescence was discussed in more detail in a paper from 1967 – see the next section.) Hence, in the present version of the model, an issue like "structural" unemployment cannot play any role, as it will in real life.

Phelps (1963) studied a similar topic as Johansen did, but he relaxed one technical assumption by considering "the longevity of machinery as a dependent variable rather than a parameter", Phelps (1963, 267).[4]

To fully recognize the role of prices for the scrapping-of-equipment decision, we can extend the Johansen-model with the remarks offered by Kurz and Phelps. To capture the essential features of obsolescence, it seems appropriate to introduce the Marshallian concept "quasi rent", within a more disaggregated setting.[5] An investment decision during the time interval $[\tau, \tau + d\tau]$, $k(\tau)d\tau$, will require an amount of labour $n(\tau)d\tau$ and will produce output $y(\tau)d\tau$. With constant productivity over time and with fixed or frozen factor proportion, this vintage will yield a quasi rent equal to $y(\tau) - w(t)n(\tau)$ at $t > \tau$, when the output price is set equal to one. Only current real wage w will be relevant for whether the capital unit is operated at t or not.[6] Suppose that real wage is expected to increase over time. Then there exist some point in time, t^* at which the vintage τ becomes unprofitable and will be taken out of operation. This critical moment of time is determined from the

[4] See also the comment by M. Kurz, who claimed that Johansen failed to explain "the time at which a "machine" will be withdrawn from service due to economic reasons", Kurz (1963, 210).

[5] The subsequent story is to a large extent material taken from Johansen's unpublished lectures on economic growth in 1975.

[6] We ignore, so far, any type of disembodied technical progress; hence we have only embodied technical progress in the ex ante production function $\varphi(n(\tau), k(\tau), \tau)$.

condition $w(t^*) \geq y(\tau) / n(\tau)$. If we define $\theta := t^* - \tau$ as the *age* of the oldest equipment in use, then the economic age is determined from $y(t^* - \theta) = w(t^*) \cdot n(t^* - \theta)$, saying that the average productivity of labour operating vintage of age θ is equal to the wage at t^*, when it will be scrapped.[7] Hence, we can write this age as $\theta(w)$, making the economic life time of equipment an endogenous variable. Then actual life time of any equipment should be determined as the smaller of technical and economic life time. The speed with which capital units are scrapped will then depend on the wage increase and the rate of embodied technical progress in the choice-of-technique function. The higher is the rate of embodied technical progress, the earlier will equipment be scrapped, so as to have more labour being released to be combined with new and more effective capital equipment.

To see more clearly what factors may affect the investment decision, on the one hand, and the operating decision on the other; let us extend the present version of the model along lines as suggested by Phelps. The ex ante decision at τ is characterized by a choice of equipment and an associated labour requirement, according to the choice-of-technique function φ, so as to maximize the difference between the expected present value of future quasi-rents over the expected operating life of the equipment (taken here to be the economic life), and the current expenditure on new equipment at τ. Let this present value be

$$\Pi(\tau;\theta) = \int_{\tau}^{\tau+\theta} \left[\varphi(n(\tau), k(\tau), \tau) - w(t)n(\tau)\right] e^{-r(t-\tau)} dt - k(\tau) \qquad (7)$$

where $w(t)$ is the expected real wage rate at t, and r is a constant discount rate. (The model is a one-sector model; hence capital equipment is measured in units of output, with a unit price.) This formulation captures embodied technical progress, obsolescence and the role played by expectations.[8] We suppose that the real wage

[7] We assume here that either all equipment of vintage τ , or nothing, is in use at some point in time *t*.
[8] See Kemp and Thãnh (1966), and Biørn and Frenger (1992) for more on expectations and putty clay.

schedule is increasing over time – hence once a vintage is scrapped it is taken out of operation permanently. Given some regularity conditions, the optimal choice of equipment, along with the associated number of man-hours (labour), at τ, will then obey:

$$\frac{\partial \varphi}{\partial k}\int_{\tau}^{\tau+\theta} e^{-r(t-\tau)}dt = 1 \qquad \text{(8-i)}$$

$$\frac{\partial \varphi}{\partial n}\int_{\tau}^{\tau+\theta} e^{-r(t-\tau)}dt = \int_{\tau}^{\tau+\theta} w(t)e^{-r(t-\tau)}dt \qquad \text{(8-ii)}$$

These two conditions determine one point at some isoquant – or factor intensity, which is the capacity output, among the prevailing techniques at τ. Equipment should be acquired up to a point where discounted present value of marginal productivity over the expected life time is equal to the unit price of new capital. The associated amount of labour is set so that the discounted present value of its marginal productivity is equal to the present value of expected wage bill for the marginal man-hour employed on the capital unit over the expected life time, over which quasi rents are non-negative. Note that if the wage schedule is overall increasing, then we have that labour hired on plant (vintage) installed at τ, will have a marginal productivity in the range $\left(w(\tau), w(\tau+\theta)\right)$. Because capital intensity cannot be changed ex post, the optimal choice at τ will then involve a more capital intensive technique than what would have otherwise been chosen. Lack of ex post substitution possibilities is therefore discounted by the entrepreneur in such a way that a higher capital intensity will be chosen, producing higher quasi-rents and a longer expected life span of the plant than if a less capital intensive technique should be chosen.

From this setting we can consider an extended macro-version of the ’59-paper, as given by the eight equations below, between the eight time functions $\{x, N, y, n, k, w, r, \theta\}$:

$$y(t) = \varphi(n(t), k(t), t) \tag{1)'$$

$$x(t) = \int_{t-\theta}^{t} y(\tau)d\tau \tag{2)'$$

$$N(t) = N_0 e^{\nu t} \tag{3)'$$

$$N(t) = \int_{t-\theta}^{t} n(\tau)d\tau \tag{4)'$$

$$k(t) = \alpha x(t) \tag{5)'$$

$$\frac{\partial \varphi(n,k,t)}{\partial k} = \frac{r}{1-e^{-r\theta}} \tag{8-i)'$$

$$\frac{\partial \varphi(n,k,t)}{\partial n} = \frac{\int_{t}^{t+\theta} w(\tau)e^{-r(\tau-t)}d\tau}{\int_{t}^{t+\theta} e^{-r(\tau-t)}d\tau} \tag{8-ii)'$$

$$y(t-\theta) = w(t)\cdot n(t-\theta) \tag{9}$$

We could have discussed whether a balanced growth path will exist in this model. However, we want to consider some other aspects. There are (at least) three points within this setting we can highlight. The first one is that as capital equipment is scrapped, labour that was combined with this equipment will now be released, and made available for new projects. Along with “new” labour, we have an upper limit on the amount of labour that can be used for new investments. From (4)’ and with $\theta = \theta(t)$, we have:

$$\dot{N}(t) = n(t) - n(t-\theta)\cdot(1-\frac{d\theta}{dt}) \Rightarrow n(t) = \dot{N}(t) + n(t-\theta)\cdot(1-\frac{d\theta}{dt}) \tag{10}$$

In addition to natural increase in the labour force, we have an additional inflow due to labour being released from equipment of age θ that is scrapped at t, adjusted for the possibility of a change in the age of oldest equipment over time.

Another point that might be of some interest is that we in this aggregated version are able to derive a short-run macro production function or a relationship between total employment and total output. From the definition of the marginal capacity in use at t, cf. (9), we can express the age of this capacity as a declining function of w at t. The date of installation of this marginal capacity is $t-\theta(w)$ at t; hence (2)′ and (4)′ will show that total output (or supply) as well as total employment (or total demand for labour) at t be decreasing functions of the real wage at t. We have:

$$x(t) = \int_{t-\theta(w)}^{t} y(\tau)d\tau := F(w) \qquad (2)''$$

$$N(t) = \int_{t-\theta(w)}^{t} n(\tau)d\tau := G(w) \qquad (4)''$$

(Here $y(\tau)d\tau$ is the ex post output from vintage τ, characterized by fixed coefficients, which matter for all operating plants of age above the one that is installed at t.)

Given sufficient differentiability, we have $F'(w)$ and $G'(w)$ both negative. From (4)″ we then have the inverse relationship $w = G^{-1}(N)$, which can be inserted into the supply function (2)″, to obtain: $x = F(G^{-1}(N)) := \Phi(N)$, with $\Phi'(N) = \dfrac{F'(w)}{G'(w)}$ as the marginal productivity of labour in the short-run macro production function $\Phi(N)$. At some given point in time t, we have $F'(w) = y(t-\theta)\cdot\theta'(w)$ and

$G'(w) = n(t-\theta)\cdot\theta'(w)$, and so $\Phi'(N) = \dfrac{y(t-\theta)}{n(t-\theta)}$. The marginal productivity of the short-run macro production function is equal to the average productivity of labour of the oldest vintage in use at t. The explanation for this result is that if employment should increase marginally at t, real wage has to decline in a competitive economy so as to get firms to hire the additional unit of labour. The lower wage will affect the quasi rents on all vintages that were operated originally. However a marginal reduction in real wage at t will turn the vintage that was the first to leave "just before" the wage decline into the new marginal vintage, which will be of age $t-\theta$, at t, and will therefore obey (9). Hence the marginal increase in output is just the average productivity $\dfrac{y(t-\theta)}{n(t-\theta)}$. If the real wage rate is increasing over time, $w(t) = \dfrac{y(t-\theta)}{n(t-\theta)} = \Phi'(N) < \dfrac{\partial\varphi(n(t),k(t),t)}{\partial n}$, due to embodied technical progress. This inequality, representing the difference between average and marginal productivity, indicates the gains to be reaped from structural changes or modernization of capital equipment within sectors. Similar features are also derived from more disaggregated versions of a Johansen-economy, as demonstrated in his "Production Functions".

The last point we want to emphasize at this stage is a rather aggregated version of what was termed "a complete growth equation" in Johansen (1972, 171-175). To achieve a very crude representation at the macro level of this growth equation (which disentangles growth in output per unit of time into input increases, various types of disembodied technological progress – like capacity-increasing and input-saving disembodied technological progress – in addition to embodied technical progress as manifested through new capital equipment), we reformulate (2)′′ to capture both disembodied and embodied technological progress in the ex ante relation (1)′. Rewrite the ex ante function so that $y(\tau,t) = \varphi^{*}(n(\tau),k(\tau),\tau,t)$, where disembodied technological progress is captured by the argument t. Let this type of technological progress be neutral and represented by a multiplicative term $A(t)$. Then the relationship conveying aggregate output can be expressed as:

$$x(t) = A(t) \int_{t-\theta(t)}^{t} y(\tau) d\tau \qquad (2)'''$$

where the output ex post of any operating plant or firm is characterized by fixed coefficients. Differentiating (2)''' with respect to t, gives Johansen's aggregate version of the complete growth equation:

$$dx(t) = \frac{\dot{A}(t)}{A(t)} \cdot x(t)dt + \frac{y(t) - w(t)n(t)}{k(t)} \cdot k(t)dt + w(t) \cdot \dot{N}(t)dt \qquad (11)$$

Interpreted as a relation that will hold on a macro level of the economy, we observe that this growth equation shows the various factor behind growth in GDP. The first term shows the neutral capacity-or-output-increasing disembodied part of technological progress. The second term captures the growth due to embodied technological progress caused by installing new equipment at t. (This term is reflected by the quasi rent per unit new equipment scaled up by the total amount of new capital.) The last term shows the value-added due to increased labour force at t. Even within this simple setting, the "Johansen growth equation" offers a more detailed and, perhaps, more convincing specification of the factors used in standard growth accounting, where beyond growth in inputs (capital and labour), the residual growth component (TFP) is interpreted as the rate of increase in technological progress.

The 1959-article inspired a lot of highly prominent economists to elaborate on further issues raised by the putty-clay vintage-approach. We have already mentioned Phelps and Kurz, who introduced economic lifetime and obsolescence, instead of technical life span as in Johansen's original paper. Existence of long-run equilibrium (balanced) growth and properties of such equilibria in a vintage framework were analyzed further by Inada (1964), Sheshinski (1967), Bliss (1968) and Bardhan (1969). In his seminal paper on "Learning by Doing" from 1962, Arrow refers especially to the novel way of introducing embodied technical progress in the capital goods sector, as proposed by Johansen. Kemp and Thãnh (1966) discussed more explicitly the role of expectations, and they even formulated a multi-sector version of the putty-clay model. A two-sector putty-clay model, with one capital goods sector and one consumption goods sector, was formulated by Adachi (1974), where durability of capital goods, along with economic lifetime, was made part of the choice set. This added a new dimension: If a capital unit or "machine is expected to be used for a shorter time, it will be made less durable than if it is expected to be used for a longer time", Adachi (1974, 773).

If we go back to equation (10), with full employment all available labour at some point in time would be hired to operate new machinery. Such a smooth development will of course not go on automatically. Because buying new machinery will under normal circumstances depend on investors' expectations of future profitability, there might be times when not all available labour will be hired. We might enter states of structural unemployment with clay production structure, as discussed by Inada (op.cit.), Sheshinski (op.cit), Akerlof (1969), and Akerlof and Stiglitz (1969).

The introduction of production functions with substitutable inputs ex ante, but frozen factor proportions ex post, offers a theoretical framework or structure that in a rather nice way makes it possible to explain and convey dynamic and persistent social phenomena like cycles, unemployment or labor "scrappage", when older machines are taken out of operation. The framework provides a natural way of incorporating heterogeneity among the operating units in an industry or an economy. Also the role of expectations seems to have a much better resonance in this type of model, as compared to "smooth" neoclassical structures, where history has a minor effect. The modelling approach offers ass wll some more realism as to the impact of future uncertainty about input prices on current investment decisions, because of the persistent role of such decisions on the supply side of the economy; see Moene (1985). Albrecht and Hart (1983) have studied the role of future demand risk on fixed investment decisions within a similar context, whereas Gilchrist and Williams (2000, 2005) have combined the putty-clay structure with general equilibrium business cycle-effects due to productivity shocks. The putty clay framework has recently also been used to explain stock market volatility caused by irreversible investments; see e.g. Wei (2003) and Gourio (2011). At last we want to point at Moene and Wallerstein (1997) who have discussed the role of centralized resp. decentralized (or local) wage bargaining on wage compression and the impact on the dynamics of entry and exit of firms resulting from a heterogeneous firm structure derived from the putty clay or vintage framework.

3. Technical rigidities and decisions at the micro level

As shown in Section 2 the distinction between substitution ex ante and fixed proportions ex post was applied to a macro production function in the context of aggregate growth. In Johansen (1972) the attention was turned to modelling at the micro level. The dynamics of aggregate economic growth was abandoned. Instead attention was focused on the implication for the industry structure of investing in new pieces of equipment embodying the latest technology. We now turn to the modelling at the most disaggregated level of, for example, a piece of equipment, a plant, or a firm.

3.1 The Johansen production function concepts

The fundamental observation made by Johansen in "Production Functions" is that the nature of production-function concepts to be estimated is often not stated clearly enough in the empirical literature. As he puts it:

> Econometric research on production functions is growing ever more sophisticated as far as functional forms and statistical methodology are concerned.

And he continues:

> …the basic notion of production functions has remained almost untouched. The crudeness of the concept of the production function, as it is being used in most econometric research, is accordingly out of proportion with the sophistication of the theories and methods by which it is surrounded (Johansen 1972, 1).

In order to build more fruitful notions of the production function he introduced four concepts of production functions:

(1) The ex ante function at the micro level

(2) The ex post function at the micro level

(3) The short-run function at the macro level

(4) The long-run function at the macro level

The micro and macro levels may be interpreted in several ways. Concerning micro one may start with a piece of capital equipment, or have a plant in mind, and also a firm with several plants. The empirical content of the micro function concept will then, of course, vary. The macro level follows naturally from the definition of the micro level because the macro level is the level of the aggregated micro units. In Johansen (1972) the term short-run industry function was also suggested; we find this name most appealing in our context.

The dynamics of production was then to study the consequences of the change of the short-run industry function of entry of new pieces of equipment with different embodied technologies. The dynamics is revealed by the sequence of short-run functions over time.

In Johansen (1972) a taxonomy of the forces acting on the short-run function by expanding the capacity on the industry was presented in the form of a *complete growth equation* incorporating not only the effect of embodied technical change, but also the effect of disembodied technical change of the output-increasing or input-saving type (for the latter type; cf. "the Horndal effect", introduced by Lundberg (1961, 129-133) based on productivity development at a steel plant.) The complete growth equation was applied at the macro level in Section 2, but was actually introduced exploring the industry level. The decomposition applies equally to the macro level as to the industry level. The short-run industry production function and its use in a dynamic analysis will be explored in Section 5.

The last production function concept covers the situation in steady state with no technical change, and connects to his former analysis of long-term growth studied in Section 2. We will not be concerned with this concept below.

3.1.1 The ex ante micro production function

The ex ante micro production function is of the same type as he introduced at the aggregated economy-wide level in Johansen (1959) (see (1) in Section 2). This function for a micro unit exhibits the standard neoclassical properties of substitutability between inputs, including capital as an input.[9] But once the investment is made the ex post micro production function is characterised by fixed variable input coefficients, and there are no longer any substitution possibilities. The choice of factor proportions made on the ex ante function is "frozen" into fixed variable input coefficients ex post, and capital only serves the role of defining the capacity limit. This is the extreme version of a vintage production function (or putty-clay, as Phelps (1963) termed it).

[9] Notice that we assume that the ex ante function is valid for a new unit in isolation. We do not consider the unit as an integral part of a larger unit, e.g., a piece of machinery within a plant integrated with other different pieces of equipment.

For simplicity we will only specify a single output, but use multiple inputs. The ex ante production function for a micro unit considered at time t = 0 can formally be expressed as

$$y = \varphi_o(x_1,...,x_n,k),\ \ \varphi'_{oi},\varphi'_{ok} > 0, i = 1,...,n \tag{12}$$

where y is output, x_i (i = 1,…,n) are current inputs[10] and k is a measure of capital equipment. The same symbols are used as introduced in Equation (1) in Section 2, but the single variable labour input used there is expanded into multiple variable inputs. Since we are dealing with vintages the variables and the production function must in principle be dated both with current time and the time of the construction of the vintage. In order to simplify, only the ex ante production function is dated with the current time t = 0 in (12). The production function is assumed to have standard neoclassical properties as to marginal productivities and exhibit variable returns to scale.

3.1.2 From ex ante to ex post at the micro level

When a choice is made from the ex ante function, i.e., a point on an isoquant is selected, as illustrated in the left part in Figure 1 with capital (k) and labour (n) as inputs, the volume of capital k_0 is fixed ex post, and we assume that there are no longer any substitution possibilities between variable inputs. The ratios of input per unit of output are frozen and reflecting the point picked on the isoquant of the ex ante function. Correspondingly, there is an upper limit on the output capacity given by the output label y_0 on the chosen isoquant.

[10] The number of current inputs was in Johansen (1972) limited to two.

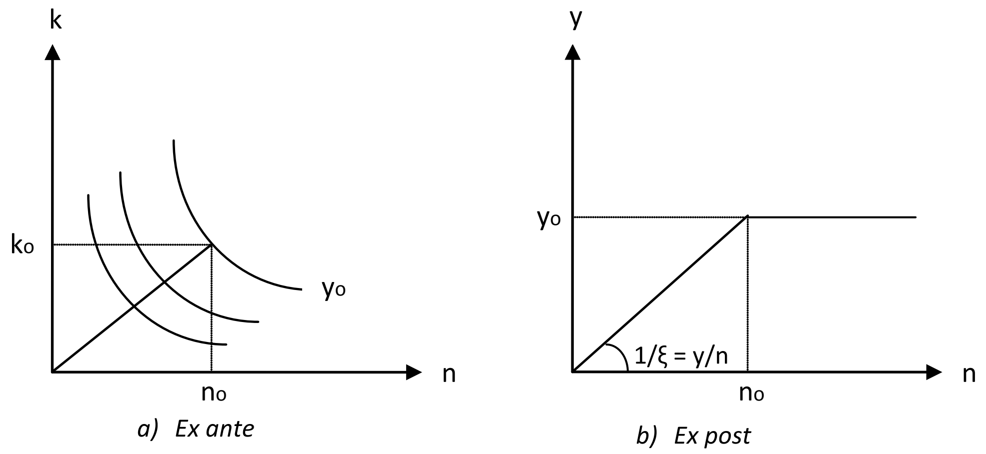

Figure 1. The ex ante and the ex post micro production functions

In contrast to the literature on quasi-fixed factors the nature of the production function changes fundamentally between the ex ante and the ex post stage. The latter can be expressed as a limitational law, or as a Leontief production function, when using the notion as given in (12):

$$y(t,\tau) = \min\left[\frac{x_1(t,\tau)}{\xi_1(\nu)},\ldots,\frac{x_n(t,\tau)}{\xi_n(\nu)},\overline{y}(\nu)\right], \tau \geq \nu \tag{13}$$

where *t* is current time and τ the date of investment. The output capacity is $\overline{y}(\tau)$. The "frozen" input coefficients are defined by

$$\xi_i(\tau) = \frac{\overline{x}_i(\tau)}{\overline{y}(\tau)}\ ,\ i = 1,..,n \tag{14}$$

where $\overline{x}_i(t,\tau)$ is the full capacity use of variable input *i*. The right-hand part of Figure 1 illustrates the ex post micro function and the given input coefficient ξ.

There is no explicit capital variable in the short-run function, only capacity of the unit measured by output. Capital is indirectly represented by the production capacity. Assuming that there is no waste in the short run implies that all input coefficients can be measured by observed inputs and output. (Waste of labour in the ex post function may be illustrated in Figure 1 with a horizontal line continuing from the maximal output point with $y = y_0 \ \forall n \geq n_0$.) Then no sophisticated estimation method is needed for the coefficients. However, it may be more realistic to assume that the input coefficients vary with the capacity utilisation, e.g., becoming larger the smaller the capacity utilisation, requiring more sophisticated methods than calculating ratios.

One may try to test this extreme version of the vintage model econometrically (Belifante 1978; Fuss 1978). One problem will then be the nature of the production unit. A firm may consist of many plants, and a plant may again comprise several distinct types of equipment, and it may be that the extreme vintage assumption is most suitable for the most disaggregated level of a piece of equipment. However, concerning testing the vintage hypothesis it may also be a good idea to follow the recommendation provided by Leif Johansen: "In fact, I think the best way to test the putty-clay hypothesis is simply to inspect production equipment and talk with engineers and technicians" (Johansen 1972, 226).

3.2 The investment decision at the micro level

The introduction of new capacity embodying the technology of the time of construction is crucial for understanding the change in the short-run function. It is therefore of importance to study the investment decision and choice of factor proportions. We will assume that the investment applies to a micro unit, and not to some replacement of equipment within an existing unit.

In Section 2 the investment decision was cast within an aggregated macro setting. In order to bring out key characteristics of investment decisions at the micro level we will make the following simplifying assumptions:

- No uncertainty about future prices
- Only one unit consisting of a single vintage
- Economic lifetime shorter than physical lifetime
- Investment at time $t = 0$; hence we disregard timing of investment

- Disregarding that carrying out investment takes time
- Full capacity utilisation until scrapping
- No scrap value of capital
- No disembodied technical change
- No maintenance costs of capital

Under the assumptions above the present value of net profit over the economic lifetime, *T*, for the firm at time *t* = 0 may be written

$$\pi(0) = \int_{t=0}^{T} e^{-rt}[p(t)y(t,0) - \sum_{i=1}^{n} q_i(t)x_i(t,0)]dt - q_k(0)k(0) \tag{15}$$

The current input prices are q_i (i = 1,..,*n*) and the price per unit of capital is q_k, while *r* is the fixed discount rate. The integral is the present value of the Marshallian quasi-rent as in the aggregated case treated in Section 2. The firm has to make an initial choice as to output level and levels of variable inputs and capital, resulting in fixed input coefficients for the operation of the firm. By assumption there is full capacity utilisation over the economic lifetime, *T*, which has to be determined at time *t* = 0. The optimization problem can be stated as

$$\begin{aligned} &Max\left\{\pi(0) = \int_{t=0}^{T} e^{-rt}[p(t)y(t,0) - \sum_{i=1}^{n} q_i(t)x_i(t,0)]dt - q_K(0)k(0)\right\} \\ &\text{subject to} \\ &y(0,0) = \varphi_o(x_1(0,0),\dots,x_n(0,0),k(0)) \\ &y(t,0) = \overline{y}(0) \;;\; x_i(t,0) = \xi_i(0)\overline{y}(0) \text{ for } t \in [0,T] \end{aligned} \tag{16}$$

where the first constraint is the ex ante function dated with current time and period of construction, and the second constraint is the ex post production function. After determining implicitly the input coefficients for variable inputs, capital investment and maximal output based on the ex ante function, as expressed by the first constraint, the limitational law (13) holds for the actual production, as expressed by the second constraint.

The necessary first-order conditions for an interior solution for the variable inputs are

$$\begin{aligned} &\frac{\partial \pi(0)}{\partial x_i(0,0)} = \int_{t=0}^{T} e^{-rt}[p(t)\frac{\partial \varphi_o}{\partial x_i(0,0)} - q_i(t)]dt = 0 \Rightarrow \\ &\frac{\partial \varphi_o}{\partial x_i(0,0)} \int_0^T e^{-rt} p(t)dt = \int_0^T e^{-rt} q_i(t)dt \;, i = 1,\ldots,n \end{aligned} \tag{17}$$

The first term on the left-hand side of the last equation is the present value of the marginal productivity of variable input *i*. This value should be set equal to the present value of the outlay on a unit of the input. Current prices in the static textbook case are replaced with the present value of prices. The current value of the marginal productivity may now never be equal to the current value of the input price. This relationship will only hold in an average sense, forming the average of prices dividing the integrals by the economic lifetime, *T*.

The choice of factor ratios will be directly influenced by the forecasted input prices:

$$\frac{\dfrac{\partial \varphi_o}{\partial x_i(0,0)}}{\dfrac{\partial \varphi_o}{\partial x_j(0,0)}} = \frac{\int_{t=0}^{T} e^{-rt} q_i(t)dt}{\int_{t=0}^{T} e^{-rt} q_j(t)dt} \;,\; i,j = 1,\ldots,n \tag{18}$$

A factor with a relative low average price will be substituted for a factor with a relatively high average price. For example, if the wage rate is expected to increase more than the energy price, the initial choice will be to use relatively more energy for a given output level.

The necessary first-order condition for optimal capital choice is:

$$\frac{\partial \pi(0)}{\partial k(0)} = \int_0^T e^{-rt} p(t) \frac{\partial \varphi_o}{\partial k(0)} dt - q_k(0) = 0 \Rightarrow$$
$$\frac{\partial \varphi_o}{\partial k(0)} \int_0^T e^{-rt} p(t) dt = q_k(0) \tag{19}$$

The present value of the marginal productivity is set equal to the capital price. Combining (17) with (19) we have that a project with a given output capacity will be more capital intensive the higher the present value of input prices are relatively to the initial capital price.

Inserting the first-order conditions (17) and (19) into the profit expression in (16) yields

$$\pi(0) = \int_{t=0}^T e^{-rt} [p(t)y(t,0) - \sum_{i=1}^n q_i(t)x_i(t,0)]dt - q_k(0)k(0) =$$
$$\int_{t=0}^T e^{-rt} p(t)\overline{y}(0)dt - \sum_{i=1}^n \int_{t=0}^T e^{-rt} p(t) \frac{\partial \varphi_o}{\partial x_i(0,0)} x_i(0,0)dt - \int_{t=0}^T e^{-rt} p(t) \frac{\partial \varphi_o}{\partial k(0)} k(0)dt = \tag{20}$$
$$\int_{t=0}^T e^{-rt} p(t)\overline{y}(0)(1 - \varepsilon_o)dt$$

The last expression is obtained employing the "passus equation" (Frisch, 1965), where ε_o is the "passus coefficient" or scale elasticity of the ex ante function (12). If it is optimal to choose an output level equal to the optimal scale output, then the present value of profit is zero, and the rate of return on the investment is equal to the rate of discount, *r*. However, it may be optimal to have a level of output greater than optimal scale, and then the rate of return on the capital investment will exceed the rate of discount.

The economic lifetime is determined from the condition

$$\frac{\partial \pi(0)}{\partial T} = e^{-rT} [p(T)y(T,0) - \sum_{i=1}^n q_i(T)x_i(T,0)] = 0, \tag{21}$$

implying

$$p(T)y(T,0)=\sum_{i=1}^{n}q_i(T)x_i(T,0)\Rightarrow p(T)=\sum_{i=1}^{n}q_i(T)\frac{x_i(T,0)}{y(T,0)}=\sum_{i=1}^{n}q_i(T)\xi_i(0), \tag{22}$$

inserting the input coefficients (14). The last right-hand expression in (22) is the variable unit cost of production at time *T*. Production will be terminated when the current variable unit cost becomes higher than the current output price. The difference between the output price and the variable unit cost is the unit quasi-rent. Thus, the economic lifetime under our assumption is determined as a quasi-rent criterion: production is terminated when the quasi-rent becomes negative.

3.3 Efficiency considerations

Economic theory offers strong conclusions about the role prices play as means of achieving efficient allocation of resources. However, most of these conclusions are derived from static models with substitution possibilities or a putty production structure, so that profit-maximizing firms can respond to changes in factor prices by altering the factor proportions smoothly and instantaneously. When we take into account the production structure Johansen outlined in his 1959-article, the situation is significantly changed. With substitution possibilities ex ante, and fixed factor proportions ex post, not only current prices will affect a firm's decisions, but also future prices, at the time when equipment is installed, as was shown above. When current decisions about factor proportions are taken, future opportunities are constrained because these factor proportions are frozen for a long period of time.

A relevant question is then: What consequences will such technical rigidities have for the role prices may play as means of achieving efficiency?

This issue was raised by Johansen in an interesting article from 1967; see Johansen (1967a). Within a disaggregated and planning-inspired model, he asked whether there exist a price system that will induce profit-maximizing firms with putty-clay technologies to choose factor proportions that are socially efficient.

In standard neoclassical models with variable factor proportions, price changes will motivate profit maximizing firms to instantaneous and continuous changes in factor proportions, and under such circumstances, current changes in equilibrium prices will convey sufficient information to agents to take optimal decisions, as given by a sequence of single-year efficiency conditions. Not so when the choice of technique is determined at the time of installment of equipment, leaving factor proportions fixed for a long period of time as long as the plant earns non-negative quasi rent. When such dynamic constraints are prevailing in the economy, it seems important to see whether a market economy with decentralized profit-maximizing firms is able to realize a dynamically efficient allocation. For an economy with plants characterized operating fixed factor proportions, we will, according to Johansen, have to demand more from the price system if efficiency should be achieved, than what would be the case if factor proportions could be changed instantaneously and continuously.

To support his view, Johansen formulated a disaggregated version of his 1959-paper. The putty-clay machinery was rolled out in a model with one final goods sector (called the secondary sector) and two intermediate goods sectors (the primary sectors producing fuel, power or raw materials), where pricing issues along with fixed factor proportions ex post, were at the forefront. The secondary sector uses labour, new capital equipment (or gross investments) and the two intermediate goods as inputs to produce a final good (a consumption good), whereas each intermediate goods sector uses only new capital equipment and labour.[11] For some

[11] As a benchmark, Johansen derives standards conditions for production efficiency within a traditional setting, with variable factor proportions, both ex ante and ex post, with malleable capital that is transferable between sectors, and supplied in a fixed amount. Within such a setting, there exist equilibrium prices so that producers will take choices compatible with production efficiency. With

given output program for the final good, and a given supply of capital or new investments to the sectors, efficiency will require a specific distribution of labour and capital between the sectors, and a certain development of factor proportions.

In this economy each production unit within all three sectors is characterized by a discrete "vintage", indicated by τ, and with choice-of-technique functions, as given by:

$$\left.\begin{aligned} \tilde{v}_i^\tau &= f_i^\tau(n_i^\tau, k_i^\tau) \quad for\ i = 1,2 \\ y^\tau &= g^\tau(n_y^\tau, k_y^\tau, v_1^\tau, v_2^\tau) \end{aligned}\right\} \tag{23}$$

Here output from a primary sector i of vintage τ, denoted $\tilde{v}_i^\tau$, is generated from a choice in the input space (n_i^τ, k_i^τ), describing all available techniques of production at τ, the time of setting up the plant. On the other hand, output from the secondary or final good sector of vintage τ follows from the production function g^τ, where n_x^τ (k_x^τ) is labour input (gross investments) in plant of vintage τ in the secondary sector, while v_i^τ is total amount of intermediate goods used as input in the secondary sector and produced by all operating plants in primary sector no. i.[12] From the outset it is assumed that there is a physically-determined lifetime, θ_j, for capital equipment installed in sector $j = 1,2,y$. Any plant is assumed to be set up at the beginning of a year, and if installed at the beginning of year τ, with a technically-determined lifetime θ_i, will operate during the years $\tau, \tau+1, ..., \tau+\theta_i-1$, for $i = 1,2,y$. Hence, for each year in the planning period, $t = 1,2,...,T$, and for each sector, we have a balancing condition, when vintages installed prior to year 1, are ignored:

malleable capital, price changes would be translated into immediate changes in factor proportions. Under such circumstances price expectations have no merit.

[12] The vintage-time superscript imposed on the production functions indicates that there might be some kind of technical improvements over time.

$$\sum_{\tau=\max\{1,t+\theta_i-1\}}^{t} f_i^{\tau}(n_i^{\tau},k_i^{\tau}) = \sum_{\tau=\max\{1,t+\theta_x-1\}}^{t} v_i^{\tau} \quad for\ i=1,2;\ t=1,2,...,T \tag{24}$$

In this planning-inspired model, Johansen also assumed that total supply of investment goods in year t to the three sectors, as given by k^t, is fixed, and supplied to sector i at an opportunity cost, z_i^t. Hence, for each year and because capital equipment is no longer putty and transferable between different users, there is a resource constraint as given by:

$$k^t = \sum_{i\in\{1,2,y\}} z_i^t k_i^t \quad for\ t=1,2,.....,T \tag{25}$$

To demonstrate his claim, Johansen formulated a model without bringing in too many elements, by simply focusing on dynamic production efficiency alone, while leaving aside any issue of finding an optimal consumption profile. (In a subsequent paragraph we outline how technical rigidities will matter for this question.) This means that we have a *given* output program for the secondary sector, with a fixed sequence of outputs, one for each year t, as given by

$$Y^t = \sum_{\tau=\max\{1,t-\theta_y+1\}}^{t} y^{\tau} \quad for\ t=1,2,...,T \tag{26}$$

The given supply of the final good in year t is provided by all operating plants in the secondary sector in year t.

In this economy, the goal is to achieve production efficiency, which is derived from minimizing a weighted or discounted sum of total labour inputs over the planning period from $t=1$ to $t=T$, subject (24), (25) and (26), with an objective function as:

$$L = \sum_{t=1}^{T} \alpha^t \left[\sum_{\tau=\max\{1,t-\theta_1+1\}}^{t} n_1^\tau + \sum_{\tau=\max\{1,t-\theta_2+1\}}^{t} n_2^\tau + \sum_{\tau=\max\{1,t-\theta_y+1\}}^{t} n_y^\tau \right]$$

Here α^t is the weight or discount factor for year t.[13]

The solution to this problem – the efficient program – as given by the vector $\left\{ n_1^t, n_2^t, n_y^t, k_1^t, k_2^t, k_y^t, v_1^t, v_2^t \right\}_{t=1}^{t=T}$, is a derived distribution of labour and gross investments among the three sectors, a certain production program for the two primary sectors, and a specific development of the factor proportions in all sectors. The conditions characterizing an efficient solution convey a strong dynamic structure due to frozen factor proportions ex post. Without going into too many details, it might be sufficient, as Johansen did, just to describe the structure of these efficiency conditions. We know that Lagrangian multipliers related to the constraints above can be given a price-like interpretation. On organizing the optimality conditions, we can present them as follows, where the Lagrangian multipliers are positive, with γ_i^t the one for the t^{th} constraint in (24) for primary sector *i*, with μ^t as the multiplier related to the supply of total gross investment to be allocated on plants installed in period t in (25), and with λ^t as the multiplier for the t^{th} constraint in (26):

$$\frac{\sum_{t=s}^{\min\{s+\theta_i-1,T\}} \alpha^t}{\frac{\partial f_i^s(n_i^s,k_i^s)}{\partial n_i^s}} = \frac{\mu^s z_i^s}{\frac{\partial f_i^s(n_i^s,k_i^s)}{\partial k_i^s}} = \sum_{t=s}^{\min\{s+\theta_i-1,T\}} \gamma_i^t \quad for\ i = 1,2 \tag{27}$$

$$\frac{\sum_{t=s}^{\min\{s+\theta_y-1,T\}} \alpha^t}{\frac{\partial g^s(n_y^s,k_y^s,v_1^s,v_2^s)}{\partial n_y^s}} = \frac{\mu^s z_y^s}{\frac{\partial g^s(n_y^s,k_y^s,v_1^s,v_2^s)}{\partial k_y^s}} = \frac{\sum_{t=s}^{\min\{s+\theta_y-1,T\}} \gamma_i^t}{\frac{\partial g^s(n_y^s,k_y^s,v_1^s,v_2^s)}{\partial v_i^s}} = \sum_{t=s}^{\min\{s+\theta_y-1,T\}} \lambda^t \quad for\ i = 1,2 \tag{28}$$

[13] None of the intermediate goods can be stored.

(Note that when considering a plant installed in period s, it will be operating over a period from $t = s$ and no longer than the lower of the lifetime of the equipment and the remaining part of the planning period. The partial derivatives are marginal productivities.) These conditions convey a familiar optimality structure, even though the Lagrangian multipliers enter in a rather unfamiliar way.

The first equality in (27) shows that marginal cost of any input used in primary sector *i*, in terms of labour, should be equalized, where the numerators can be given a price interpretation. The second equality in (27) gives the common value of the marginal cost of output from primary sector i in units of labour, and can be interpreted as a shadow price, measured in units of labour, per unit of output delivered by a vintage s from primary sector i, and given by $\sum_{t=s}^{\min\{s+\theta_i-1,T\}} \gamma_i^t$. (The multiplier γ_i^t is "the marginal value in terms of decreases in L of a given increase in the supply of goods of the type produced by primary sector No. *i*" (Johansen 1967a, 139). The first equality shows a cost minimizing input combination in primary sector i, or cost efficiency, whereas the second one can be seen as a condition for profit maximization, given the specified prices. Except for gross investment with an initial outlay, the remaining inputs, as well as output, are priced according to some average price taken over the period of operation of the plant. (The multiplier μ^t is the marginal value in terms of lower *L* of a higher gross investment in period *t* allocated to our segment of the economy.)

The conditions in (28) have similar interpretations, but for the secondary sector. The first three, with $i = 1,2$, are conditions for cost minimization or cost efficiency, equalizing marginal cost of production, in terms of labour, for the final good sector, whereas the last one, can be interpreted as a condition for the profit maximizing scale of operation, stating that marginal cost is equal to a price term, $\sum_{t=s}^{\min\{s+\theta_y-1,T\}} \lambda^t$. (Here the multiplier λ^t shows the marginal cost in terms of increases in L of a unit increase of the final good in period t. The eight conditions in (27) and (28), for any $s = 1,2,\ldots,T$, along with the side constraints, will give us the full set of conditions for production efficiency in our segment of the economy.

Before the general case is analyzed in more detail, it is a special case that warrants some attention. Let capital equipment in all sectors have the same technically-determined lifetime; i.e., $\theta_j = \theta$ for $j = 1,2,y$. In that case, for any period $s = 1,2,\ldots,T$, the conditions in (27) and (28) are reduced to:

$$\frac{\sum_{t=s} \alpha^t}{\sum_{t=s} \gamma_i^t} = \frac{\dfrac{\partial g^s}{\partial n_y^s}}{\dfrac{\partial g^s}{\partial v_i^s}} = \frac{\partial f_i^s}{\partial n_i^s} \Leftrightarrow f_{1n}^s \cdot g_{v_1}^s = f_{2n}^s \cdot g_{v_2}^s = g_{n_y}^s \text{ for } s = 1,2,\ldots,T \qquad \text{(29-i)}$$

$$\frac{\mu^s}{\sum_{t=s} \gamma_i^t} = \frac{\dfrac{\partial f_i^s}{\partial k_i^s}}{z_i^s} = \frac{1}{\dfrac{\partial g^s}{\partial v_i^s}} \cdot \frac{\dfrac{\partial g^s}{\partial k_y^s}}{z_y^s} \Leftrightarrow \frac{f_{1k}^s}{z_1^s} \cdot g_{v_1}^s = \frac{f_{2k}^s}{z_2^s} \cdot g_{v_2}^s = \frac{g_{k_y}^s}{z_y^s} \text{ for } s = 1,2,\ldots,T \qquad \text{(29-ii)}$$

When we "ignore" the price terms of new capital equipment to the various sectors in the denominators of (29-ii), these conditions are (almost) identical or parallel to conditions for production efficiency for the case with variable factor proportions and malleable and transferable capital. On the margin, any primary input (labour and capital), should have, either directly or indirectly, the same return. In this special case, Johansen concludes, "there exist such prices that adaption to current prices in each period would lead to fulfillment of the optimum conditions", Johansen (1967a, 139). Optimal decisions are implemented "without taking account of future prices", Johansen (1967a, 140).

To "prove" his general point of view, when technical lifetimes differ among the sectors, that we have to demand more from the price system when factor proportions are fixed for a considerable period of time than what would have been the case if factor proportions can be changed instantaneously, Johansen considers the following case: For year s, the primary sector i has a continuum of investment projects to choose among, as given by the all feasible input combinations $\left(n_i^s, k_i^s\right)$, leading to output given by $\tilde{v}_i^s = f_i^s(n_i^s, k_i^s)$. If price per unit output from primary sector *i* in period s is set at q_i^s, wage rate at w^s, and the accounting price of capital is Z_i^s, then the investment project can be evaluated according to the profit, where the future sequence of all prices is used in the valuation:

$$\pi_i^s = \tilde{v}_i^s \cdot \sum_{t=s}^{s+\theta_i-1} q_i^t - n_i^s \cdot \sum_{t=s}^{s+\theta_i-1} w^t - Z_i^s k_i^s = f_i^s(n_i^s, k_i^s) \cdot \sum_{t=s}^{s+\theta_i-1} q_i^t - n_i^s \cdot \sum_{t=s}^{s+\theta_i-1} w^t - Z_i^s k_i^s \qquad (30)$$

A profit-maximizing firm in primary sector *i*, when taking prices as given, will choose inputs and output so that:

$$\frac{\sum_{t=s}^{s+\theta_i-1} w^t}{\frac{\partial f_i^s}{\partial n_i^s}} = \frac{Z_i^s}{\frac{\partial f_i^s}{\partial k_i^s}} = \sum_{t=s}^{s+\theta_i-1} q_i^t \qquad (31)$$

A similar set of conditions can be given for the secondary sector as well, with an investment project in period s evaluated according to profits, with $\left\{p^t\right\}$ as a given (expected) price sequence, one price for each period per unit final good:

$$\Pi^s = g^s(n_y^s, k_y^s, v_1^s, v_2^s) \cdot \sum_{t=s}^{s+\theta_y-1} p^t - n_y^s \cdot \sum_{t=s}^{s+\theta_y-1} w^t - \sum_{i=1}^{2} v_i^s \cdot \sum_{t=s}^{s+\theta_y-1} q_i^t - Z_y^s \cdot k_y^s \qquad (32)$$

With profit-maximizing firms acting as price-takers, their choice will be characterized by:

$$\frac{\sum_{t=s}^{s+\theta_y-1} w^t}{\dfrac{\partial g^s}{\partial n_y^s}} = \frac{\sum_{t=s}^{s+\theta_y-1} q_i^t}{\dfrac{\partial g^s}{\partial v_i^s}} = \frac{Z_y^s}{\dfrac{\partial g^s}{\partial k_y^s}} = \sum_{t=s}^{s+\theta_y-1} p^t \qquad \text{for } i = 1,2 \tag{33}$$

When undertaking an investment project in some period *t*, it is necessary for the entrepreneur to choose an efficient factor combination and the associated scale of production to take into account the entire sequence of expected prices from *t* towards the expected final period of operation. Price expectations play therefore a significant role. However, the problem is not what is required, but how to get these prices. Within such a dynamic framework there might be market-clearing but with incorrect prices! As Johansen puts it (op.cit., 142): ”…. it is necessary to have consistent and correct expectations about future prices in order that decentralized decisions with profit maximization shall lead to fulfillment of the optimality requirements”. He doubted very much whether a system of free markets or an unguided competitive economy without any help from a planning body would be able to implement efficient allocations. Even a full set of futures markets will not be sufficient! This pessimistic proposition is supported by: “Even if managers of existing enterprises were able to adapt as they should in such a complicated system, one would still have to face the fact that *much of what is to be produced by means of vintages installed in year t is to be delivered to enterprises not yet established at that time”* (our italics). This point of view was also expressed in his strong belief in indicative planning – with a planning agency providing necessary information to economic agents so as to induce them to take socially desirable decisions in a coordinated way.

Above it was assumed that the lifetime of new capital equipment was technologically determined, and it was supposed that a plant was operating at full capacity during this lifetime. However, within the framework of putty-clay, there are two reasons why a plant might be operated at less than full capacity. Efficiency will require that equipment is taken out of operation before it is physically worn out. Because factor proportions are fixed ex post, we showed in the previous paragraph that changes in relative input prices may make the quasi rent of equipment at some vintage negative. Another reason why capital equipment can be taken out of operation is due to embodied technical progress that can only be exploited by investing in new equipment and scrapping old plants. If such technical progress takes place at a high rate, it seems worthwhile to close down older equipment "early" so as to have essential inputs available that might have higher rate of return in combination with modern equipment.

To see how prices enter the problem under these circumstances, as it also involves obsolescence; cf. the remarks by Phelps and Kurz, Johansen modified the model above, by making explicit assumptions as to how labour and other inputs are released and outputs are contracted when a plant of some vintage is operated at less than full capacity. He therefore introduced another variable $h_j^{\tau,s} \in [0,1]$ as the fraction of equipment of vintage τ in sector $j = 1,2,y$; still operating in year $t \in [\tau, \tau + \theta_j - 1]$. (This formulation might be justified by assuming that each vintage consists of a large number of plants of equal quality. Also, it is assumed that the economic lifetime is shorter than the technical lifetime.) As a fraction of equipment of a certain vintage is not operated, output and other inputs used on this equipment will shrink in the same proportion. Without going into technical details as to the formulation of the modified optimization problem, except mentioning that both sides of the constraints in (24) and (26) are modified properly by the relevant $h_j^{\tau,s}$, while the objective function is turned into

$$L^{*}=\sum_{t=1}^{T}\alpha^{t}\left[\sum_{\tau=\max\{1,t-\theta_1+1\}}^{t}h_{1}^{\tau,t}n_{1}^{\tau}+\sum_{\tau=\max\{1,t-\theta_2+1\}}^{t}h_{2}^{\tau,t}n_{2}^{\tau}+\sum_{\tau=\max\{1,t-\theta_y+1\}}^{t}h_{y}^{\tau,t}n_{y}^{\tau}\right], \text{with } h_{j}^{\tau,s}\in[0,1]$$

for $j=1,2,y$, and for any relevant t, as additional decision variables, and when replacing the Lagrangian multipliers with prices, we have for primary sector $i=1,2$, that optimality now requires $h_i^{\tau,t}=1$ if $q_i^t\tilde{v}_i^\tau-w^t n_i^\tau\geq 0$ at t, and $h_i^{\tau,t}=0$ if the quasi rent is negative. (When the quasi rent equals zero, the firm is indifferent between closing and operating.) In this case the operating decision is taken on the basis of current prices only, as has been outlined earlier. For the final good sector we have now that if the quasi rent, $p^t-w^t n_y^\tau/y^\tau-\sum_j q_j^t v_j^\tau/y^\tau$ is positive, capital equipment of vintage τ in this sector will be operated at full capacity at t; i.e., $h_y^{\tau,t}=1$. (If negative quasi rent, the plant is closed.) All factor proportions are fixed and determined at τ, the date of installment, where modified versions of (27) and (28) now will characterize an efficient allocation, with expected prices as determinants for the optimal factor proportions. However, decisions about what plants to operate will be taken on the basis of current prices alone. Because expectations normally will differ from realized prices, plants will operate either for a longer or a shorter period of time than what was expected at the time of choosing equipment, even if the decision-maker takes obsolescence into account ex ante. (If relative factor prices are increasing over time, then plants will be fully operated over some years, before permanently taken out of operation; cf. our discussion in the previous paragraph.)

In the one-good macro growth-model studied earlier we have, with static expectations about the wage rate, that marginal productivity of labour on new equipment is equal to the wage rate, which, in equilibrium, is equal to the average labour productivity of the oldest plant in use. When we turn to the present model, we have that the oldest plant in use at t is of vintage t', or of age $t-t'$, and characterized by $\frac{\tilde{v}_j^{t'}}{n_j^{t'}} = \frac{w^t}{q_j^t}$. On the other hand, the vintage of new equipment installed in year t, and expected to operate over θ^* periods, being the minimum of the technical and the economic lifetime, will have a marginal productivity of labour determined by $\frac{\partial f_j^t}{\partial n_j^t} = \frac{\sum_{s=t}^{t+\theta^*-1} w^s}{\sum_{s=t}^{t+\theta^*-1} q_j^s}$, which in general will differ from $\frac{w^t}{q_j^t}$, being equal to the average productivity of labour on the oldest plant being operated at t.

What are the lessons from this model? Even though the model is strongly planning-motivated, conditions for production efficiency will be the same whether the economy is competitive or not. The main question is what information is required among the entrepreneurs for achieving efficiency. In the standard neoclassical framework, with capital and labour being substitutable ex post, and capital equipment fully transferable between sectors, current equilibrium prices are sufficient for entrepreneurs to choose efficient or cost-minimizing factor proportions. However, within a dynamic framework where factor proportions are fixed for a long period of time, and determined by entrepreneurs through a sequence of expected prices (not necessarily identical and correct ex post), at the date of installment, production efficiency will require more information than current prices. Ex post, such factor proportions will in general turn out to be incorrect, but despite this, it will not be worthwhile to scrap the equipment as long as current quasi rents are positive. Due to the technical rigidity stemming from substitution ex ante – fixed proportions ex post, we will experience a strong sluggishness in the economy causing the structure of plants not to change as fast as would be predicted by neoclassical models. The entrepreneurs have therefore to live with incorrect factor proportions, viewed from ex post, for a long period of time, before old vintages of capital are replaced by new equipment. As noted above, Johansen had some beliefs in indicative planning to overcome this kind of market failure, with a planning agency providing information to entrepreneurs so as to induce them to choose factor proportions that will ex post turn out to be "more correct" than under a system of uncoordinated and decentralized decisions. It is, however, not obvious how the planner should be able to get access to that kind of price information.

4. Technical rigidities and optimal two-sector growth

Johansen's famous MSG-model, which is a dynamic general equilibrium model, will determine the structural sectoral changes over time; i.e. how labour moves between sectors during a growth process (see Johansen, 1960, 1964, 1974). The MSG-model does not comprise any planning authority, equipped with instruments so as to maximize some preference function, which was playing a crucial role in the planning models Johansen developed; see Johansen (1977,1978a). In this planning literature, the authorities are normally equipped with a large set of instruments and some preference function that is maximized carefully subject to the relevant constraints. In the survey so far, no planning authority has been explicitly introduced. In the previous section efficiency was discussed within a setting where the sequence of final consumption goods was exogenously given. Hence it seemed very sensible that Johansen wanted to combine the multi-sectoral framework he had developed with both planning and technical rigidities. This was done in two demanding papers, Johansen (1964, 1967b). In these papers he analyzed how the sequence of consumption goods could be determined within the context of optimal saving in a two-sector framework, with a planner for an economy characterized by technical rigidities. The technical rigidity differed from the one studied above; now he imposed sector-specific vintage equipment with some technically fixed life time.[14]

[14] Calvo (1976) studied in detail optimal growth in a pure pure putty-clay model. See also Cass and Stiglitz (1969).

His point of departure for analyzing optimal growth in this two-sector setting was that most problems related to optimal savings were formulated in models with all production being aggregated into one production sector. Such aggregation would, according to Johansen, conceal important rigidities of technical nature. As stated in Johansen (1967b, 125): "The main feature which distinguishes the model from other models of optimal growth, is that it assumes technological rigidity as regards the division of total production into production of investment goods and production of consumption goods. That means, there are two production sectors, one for investment goods and one for consumption goods, and capital which is invested in one of the sectors cannot be transferred to the other sector and employed in production there." (This framework is close to dynamic input-output models or planning models adopted in developing countries in the period after the Second World War.)

The model in Johansen (1967b) is identical to the one presented in 1964, which paid more attention to the structure of the solution based on programming, whereas he in the 67-paper devoted some space and time to the existence of an optimal plan for an infinite horizon.

The model highlights accumulation of capital and roundaboutness, with some technically fixed life-time, in both sectors, under embodied technical progress, with the resulting sequence of consumption goods being determined from maximizing an intertemporal preference function or a social welfare function. Time is divided into periods, with consumption in (the beginning of) period t, as given by C_t, whereas total gross investment in period t is I_t, being allocated to both sectors, according to

$$x_t + z_t = I_t \tag{34}$$

where x_t $(resp.\ z_t)$, non-negative, is investment allocated to the investment (resp. consumption) good sector in period t. (The investment sector is denoted *I*-sector.) The production structure is simple, but embody technical rigidity in the sense that output from the consumption sector (*C*-sector) in period *t* is determined from investments undertaken during the last θ periods, when equipment invested in period τ, has productivity that is reflected by the technological know-how in that period. If the time-dependent capital productivity in the *C*-sector is given by the constant α_τ, for period τ, we then have the following linear production function in the *C*-sector:

$$C_t = \sum_{\tau=t-\theta}^{t-1} \alpha_\tau z_\tau \qquad (35)$$

A unit of equipment installed in this sector in period τ, will be fully operated during the periods $\tau+1, \tau+2, ..., \tau+\theta$. (Note that no attention is paid to obsolescence here.) In the same manner, β_τ is the productivity of capital equipment of vintage τ, installed in the *I*-sector in that period. Capital equipment has identical life-time, θ, irrespective of where it is used. In that case output from the *I*-sector (or total gross investment available to the economy) is given by:

$$I_t = \sum_{\tau=t-\theta}^{t-1} \beta_\tau x_\tau \qquad (36)$$

The planning period starts in $t=0$ and extends over T periods. One type of preference function studied by Johansen is the linear one; $\Omega = \sum_{t=1}^{T+1} \Omega_t(C_t)$, where

$\Omega_t(C_t)$ is an increasing and concave, with marginal utility $\frac{d\Omega_t(C_t)}{dC_t} = \omega(C_t)$ assumed to be very high as consumption becomes sufficiently low.[15]

If consumption and investment are measured in units so that they can be added, income in period t (in the closed economy) is $C_t + I_t$, and the gross savings rate in period t will be $\frac{I_t}{C_t + I_t} = \frac{\sum_{\tau=t-\theta}^{t-1} \beta_\tau x_\tau}{\sum_{\tau=t-\theta}^{t-1} (\alpha_\tau z_\tau + \beta_\tau x_\tau)}$. We observe that this savings rate is determined solely by history and capital productivities of the various vintages in use. The savings rate can therefore only be altered gradually over time by the future allocation of the output from the *I*-sector. Therefore the present model will be characterized by a lot of inertia, sluggishness or path-dependence as compared to models where the savings rate can be freely chosen by the planner.

One might now ask how this model would behave under balanced or proportional growth. Suppose that there is no embodied technological progress, i.e., with $\alpha_t = \alpha$ and $\beta_t = \beta$, and let the proportional growth rate be denoted by ρ, obeying:

$$C_t = (1+\rho)^t C_0, \quad I_t = (1+\rho)^t I_0, \quad z_t = (1+\rho)^t z_0, \quad x_t = (1+\rho)^t x_0 \tag{37}$$

Now, under proportional growth, the savings rate is constant and equal to:

$$s = \frac{I_t}{C_t + I_t} = \frac{\beta x_0}{\alpha z_0 + \beta x_0} = \frac{x_0}{I_0} \tag{38}$$

[15] He discussed various properties of the preference function; like one with a "scrap or bequest value" encompassing utility after the end of the planning period. Also he discussed various versions of the periodic utility function itself. But the one provided in the text seems to be the one that captures the main idea.

if units of measurement are chosen so that $\alpha = \beta$. To find this growth rate, use (37) for I in (36), to give: $I_t = (1+\rho)^t I_0 = \beta \sum_{\tau=t-\theta}^{t-1} (1+\rho)^\tau x_0$. From this we get:

$$1 = \frac{\beta x_0}{I_0} \sum_{\tau=t-\theta}^{t-1} (1+\rho)^{\tau-t} = \frac{\beta x_0}{I_0} \left[(1+\rho)^{-\theta} + \ldots + (1+\rho)^{-1} \right] \tag{39}$$

If $\theta \to \infty$, featuring a case with infinitely-lived capital equipment, the proportional growth rate turns out to be equal to the one that can be derived in Domar-like growth models:

$$1 = \frac{\beta x_0}{I_0} \sum_{\tau=t-\theta}^{t-1} (1+\rho)^{\tau-t} \to_{\theta \to \infty} \frac{\beta x_0}{I_0} \frac{\frac{1}{1+\rho}}{1 - \frac{1}{1+\rho}} = \frac{\beta x_0}{\rho I_0} = 1 \Rightarrow \rho(\theta = \infty) = \frac{\beta x_0}{I_0} \tag{39'}$$

The growth rate is proportional to the initial investment that is allocated to the investment sector.

Returning to our original set-up we get a rather complex and interesting dynamic structure of the model, if we use (34) and (36) in (35), to get an explicit expression for consumption in period *t*:

$$\begin{aligned} C_t &= \sum_{\tau=t-\theta}^{t-1} \alpha_\tau z_\tau = \sum_{\tau=t-\theta}^{t-1} \alpha_\tau (I_\tau - x_\tau) = \sum_{\tau=t-\theta}^{t-1} \alpha_\tau (\sum_{i=\tau-\theta}^{\tau-1} \beta_i x_i - x_\tau) \\ &= \sum_{\tau=t-2\theta}^{t-2} \left(\beta_\tau \sum_{i=Max\{t-\theta,\tau+1\}}^{\min\{t-1,\tau+\theta\}} \alpha_i \right) x_\tau - \sum_{\tau=t-\theta}^{t-1} \alpha_\tau x_\tau \end{aligned} \tag{40}$$

Output in the *C*-sector in period *t* is fully determined by the sequence of historical investment allocations in the *I*-sector, $\left\{ x_{t-2\theta}, x_{t-2\theta+1}, \ldots\ldots, x_{t-2}, x_{t-1} \right\}$. From this expression we can derive explicitly $\frac{\partial C_t}{\partial x_{t-j}}$ for different values of j. The coefficient in front of x_τ in the first sum in the second line of (40), shows the positive effect on consumption in period t of a unit increase in investment allocated to the *I*-sector in period τ, under the assumption that the entire increase in output from the investment sector, implied by the marginal increase in x_τ, is allocated to the *C*-sector. The negative term in the second line of (40) shows the direct loss in consumption in period *t* caused by investing more in the *I*-sector in period τ. We can summarize these effects in the following way:

$$\frac{\partial C_t}{\partial x_\tau} = \begin{cases} 0 \quad for \ \ t > \tau + 2\theta \ or \ t < \tau + 1 \\ \beta_\tau \sum_{i=t-\theta}^{\tau+\theta} \alpha_i \quad for \ \ \tau + \theta + 1 \le t \le \tau + 2\theta \\ \beta_\tau \sum_{i=\tau+1}^{t-1} \alpha_i - \alpha_\tau \quad for \ \tau + 1 < t \le \tau + \theta \\ -\alpha_\tau \quad for \ t = \tau + 1 \end{cases} \tag{41}$$

From this set-up we can calculate the rate of return on investment in period τ. Suppose one unit of investment is allocated to the *C*-sector in period τ. The increment in capacity output from this sector is then α_τ. Alternatively, the unit of gross investment can be allocated to the *I*-sector, while the resulting output is subsequently invested in the *C*-sector. In this latter case, we get an additional capacity in the *C*-sector equal to $\beta_\tau \alpha_{\tau+1}$ in the first period, then, two period later, $\beta_\tau \alpha_{\tau+2}$, and so on, up to $\beta_\tau \alpha_{\tau+\theta}$ after θ periods. The rate of return, denoted g_τ, is the rate of discount that will then solve

$$\beta_\tau \sum_{i=\tau}^{\theta} \alpha_{i+1} (1 + g_\tau)^{-i} = \alpha_\tau \tag{42}$$

We can now make use of these relationships in the overall preference function which can be written as: $\Omega = \sum_{t=1}^{T+1} \Omega_t(C_t) = \sum_{t=1}^{T+1} \Omega_t(\sum_{\tau=t-\theta}^{t-1} \alpha_\tau(\sum_{i=\tau-\theta}^{\tau-1} \beta_i x_i - x_\tau))$.

The optimal growth path is then found by choosing the investment allocation to the *I*-sector so as to maximize this preference function subject to the constraints on the control variables; $x_t \in \left[0, \sum_{\tau=t-\theta}^{t-1} \beta_\tau x_\tau\right]$, for any $t = 1,2,....,T$.

As reported in a number of similar multi-sector planning models of this type, there are different phases of development over time. In an initial phase, extending not longer than θ periods, all investment is allocated to the investment sector, whereas the consumption level is kept unchanged, at a low level. (Such a period might not take place if marginal utility of consumption becomes very high if consumption falls below some low level.) In a second or some "unconstrained phase of the programme", both sectors are developed, with some investment allocated to the consumption sector, as well. In a "terminal" phase, all investment is allocated to the consumption sector and nothing to the investment sector.[16] In the second phase where investment is allocated to both sectors in some period τ, the following optimality condition must hold: $\frac{\partial \Omega}{\partial x_\tau} = \sum_{t=1}^{T+1} \omega(C_t) \frac{\partial C_t}{\partial x_\tau} = 0$. On taking advantage of (41), and when writing the marginal utility as $\omega(C_j) = \omega_j$, this condition turns out to be:

$$\frac{\partial \Omega}{\partial x_\tau} = -\alpha_\tau \sum_{i=1}^{\theta} \omega_{\tau+i} + \beta_\tau \left\{ \sum_{s=\tau+1}^{\tau+\theta} \alpha_s \sum_{j=s+1}^{s+\theta} \omega_j \right\} = 0 \tag{43}$$

showing the direct and indirect (roundabout) impact on welfare of a marginal increase in investment allocated to the *I*-sector in period τ, when each item of

[16] The first phase might vanish, but whether phase three will vanish, depends on the planning period. If the horizon is finite, this phase will appear as part of the solution, which is not the case under infinite horizon.

investment has some productive power for θ periods. This difference equation, of order $2\theta - 1$, can under some simplifying assumptions, with constant productivities, be analyzed by looking at the characteristic equation corresponding to (42). This will reveal that the development can, depending on whether θ is odd or even, have damped or non-damped oscillations, or permanent growth. In this latter case, the solution is $(1+g)^{-1}$ if $\alpha_\tau = \alpha, \beta_\tau = \beta$ for all periods, and $\beta\theta > 1$, which can be seen as a condition for the gains from roundaboutness, or in the words of Johansen: "...a unit of investment goods is able to do more through its lifetime than reproduce itself", Johansen (1967b, 129), with g as the constant rate of return from (42). In case of infinite horizon, the solution with a trend component seems most relevant. This trend component will along an optimal path exhibit the property $-\frac{\omega_\tau - \omega_{\tau-1}}{\omega_\tau} = g$, which is in accordance with results from standard optimal growth. If the preference function is expressed as $\Omega_t(C_t) = (1+r)^{-t}\Omega_0(C_t)$, where r is a constant utility discount rate (or pure rate of time preference), then along an optimal path we get a modified Böhm-Bawerk-Ramsey-Keynes condition as given by

$$\frac{g}{1+r} = r - \frac{\omega_0(C_t) - \omega_0(C_{t-1})}{\omega(C_t)} \approx r + (-\hat{\omega}_0(C_t)) \cdot \frac{C_t - C_{t-1}}{C_t} \tag{44}$$

(The term $(1+r)$ in the denominator is caused by the lag structure of the model, whereas $\hat{\omega}_0(C_t)$ is the elasticity of the marginal utility function ω_0.[17])

Hence in the unconstrained phase along an optimal path in this two-sector model, classical results are reproduced, when taking account of lags and the technically given lifetime of capital equipment. This is neatly reported in Johansen (1964, 168-170) for the case $\theta = 1$. Also, the relationship to standard one-sector models is considered. This is done by relaxing the assumption that capital is sector-specific and

[17] In modern literature this term is related to a measure of intertemporal substitution; see Blanchard and Fischer (1993, p.40).

non-transferable. If capital can freely be transferred between sectors, when assuming equality between α_t and β_t, then the optimal program is solved by maximizing $\Omega = \sum_{t=1}^{T+1} \Omega_t(C_t)$ subject to the feasibility constraints: $C_t + I_t = \sum_{\tau=t-\theta}^{t-1} \beta_\tau I_\tau$. Hence, an optimal solution must obey:

$$\frac{\partial \Omega}{\partial I_t} = -\omega_t + \beta_t \sum_{\tau=t+1}^{t+\theta} \omega_\tau \leq 0 \quad \text{for } t = 1,2,\ldots,T \tag{45}$$

with strict equality if $I_t > 0$.

Even though there are similarities with the standard, neoclassical one-sector approach, the two-sector planning model of Johansen reveals, at least with the technical assumptions being imposed, that it might be harder to achieve sustainable economic growth in the two-sector setting. Again history matters and the saving rate is determined by past investment allocations. The development might be less smooth and might exhibit cycles, as well. Hence the model integrates business cycles and economic growth. (An interesting task is to see how the model will behave if we let the capital productivities undergo some shocks.)

In a medium-term perspective, with capital equipment being more sector-specific, with less flexibility than in the real long run, it might be more appropriate to model an economy along the lines suggested by Johansen, conveying more sluggishness than what we find in standard neoclassical models, with rather smooth adjustments. The present two-sector model, with its technically fixed lifetime of capital equipment, has some nice reference points to the role of "durability" of capital.[18] This was an issue Leif Johansen had been analyzed elsewhere; especially the relation between economic growth and durability of capital; as in Johansen (1961). Later he discussed, within a dynamic input-output structure, the role of different construction profiles for the completion of capital equipment with finite life-time on

[18] Cf. the above-mentioned paper by Adachi.

growth; see Johansen (1978b). Although this paper falls outside our constructed common thread, covering "growth-production studies" from 1959 to 1972, we feel that it should be mentioned as relevant for this survey because this paper is concerned with technical rigidities as well, now related to time-consuming construction periods of capital equipment. (This was a topic that Leif Johansen's teacher, Professor Trygve Haavelmo, had been working on for several years; see Haavelmo (1960).)

5. Technical rigidities and industry production structure

The short-run industry production function is a new concept in the production function literature. In Johansen (1972) both a discrete approach and a continuous capacity approach are used. The focus of the analysis was the connection between the distribution of capacity of micro units in the input coefficient space and the form of the short-run function, related to the approach in Houthakker (1955-56).

The short-run industry production function is a construct of the analyst. The normative question behind it is how the given capacities of the micro units should be utilised. An aggregate production function is introduced utilizing existing capacities in a certain way, and it is possible that the actual utilisation of units may deviate from this reference utilisation.

5.1 The short-run industry production function

In the short run the total output and use of current inputs in an industry is determined by the utilisation of individual firm output capacities and the short-run micro productions function (13) with the input coefficients determined by (14). In Johansen (1972) a production function covering the industry as a unit was defined, using the classical definition of a production function. The industry consists of N units with homogeneous output and inputs. This procedure can be regarded as a special kind of aggregation: the question asked is how given current inputs and the given micro-unit capacities should be utilised in order for the aggregated industry output to be maximized:

$$\begin{aligned}&\text{Max } Y = \sum_{j=1}^{N} y_j \\ &\text{subject to} \\ &\sum_{j=1}^{N} \xi_{ij} y_j \leq \bar{X}_i , i = 1,\ldots,n \\ &y_j \leq \bar{y}_j , j = 1,\ldots,N\end{aligned} \tag{46}$$

Total output and total inputs are denoted by uppercase letters. The formulation is built on the short-run micro production functions (13). The discrete distribution of capacity is given by $\left\{\xi_{ij}, \bar{y}_j\right\}$ *for* $i = 1,\ldots,n , j = 1,\ldots,N$. The solution to (46) yields an optimal way to utilize resources, and no market prices, only shadow prices, are involved. The observed way of utilizing resources and capacities may deviate from the solution of (46), so what is introduced is a benchmark for optimal utilisation of the micro units given available total resources.

The Lagrangian for the problem is

$$L = \sum_{j=1}^{N} y_j - \sum_{i=1}^{n} \lambda_i (\sum_{j=1}^{N} \xi_{ij} y_j - \bar{X}_i) - \sum_{j=1}^{N} \gamma_j (y_j - \bar{y}_j) \tag{47}$$

The functional forms have properties so that we need consider only the necessary first-order conditions, as given by:

$$\begin{aligned}&\frac{\partial L}{\partial y_j} = 1 - \sum_{i=1}^{n} \lambda_i \xi_{ij} - \gamma_j \leq 0 (= 0 \text{ for } y_j > 0) \\ &\lambda_i \geq 0 \ \ (= 0 \ \ if \ \sum_{j=1}^{N} \xi_{ij} y_j < \bar{X}_i) \\ &\gamma_j \geq 0 \ \ (= 0 \ \ if \ \ y_j < \bar{y}_j)\end{aligned} \tag{48}$$

If the solution is unique, as assumed, the endogenous variables can be written as functions of the exogenous variables:

$$y_j = F_j(\bar{X}_1,...,\bar{X}_n,\bar{y}_1,..,\bar{y}_N);(j=1,...,N),\sum_{j=1}^{N} y_j = Y \Rightarrow$$
$$Y = F^*(\bar{X}_1,...,\bar{X}_n,\bar{y}_1,..,\bar{y}_N) = F(X_1,...,X_n) \quad (49)$$

In the last equation we have suppressed capacities as arguments, as well as considering a parametric variation in the given total inputs. The mapping F is the short-run industry production function. Notice that capital has no role as an argument in this function. The output capacities of the micro units are arguments in this function, but these capacities are fixed in the short run, and therefore for notational convenience, being included in the functional form. The standard characterization of a production function by substitution- and scale properties also apply to the short-run function. However, with the micro functions having fixed input coefficients the isoquants will be piecewise linear.

An optimal solution will imply that a micro unit may be in one of three states: fully utilised, partly utilised, or not used at all. The shadow price, λ_i, on the input constraint no. *i* has the interpretation, in an optimal solution, of the change in the objective function of a change in the resource availability, i.e., the shadow price shows directly the marginal productivity of the resource in question. From the Envelope Theorem we have:

$$\frac{\partial Y}{\partial \bar{X}_i} = \frac{\partial L}{\partial \bar{X}_i} = \lambda_i \Rightarrow \frac{\partial F}{\partial X_i} = \lambda_i \quad (50)$$

The conditions in (48) give the characterization of the three states mentioned above:

1) Fully utilised units: $1 - \sum_{i=1}^{n} \lambda_i \xi_{ij} = \gamma_j \geq 0$

2) Partly utilised units: $1 - \sum_{i=1}^{n} \lambda_i \xi_{ij} = 0$ (51)

3) Units not in use: $1 - \sum_{i=1}^{n} \lambda_i \xi_{ij} \leq 0$

The common expression on the left-hand sides above can be interpreted as the unit Marshallian *quasi-rent*, as defined earlier. We have that $\sum_i \lambda_i \xi_{ij}$ is the variable cost per unit of output of unit *j* when λ_i is interpreted as a price. Now the measuring unit for the shadow price is output per unit of input *i*. The sum is therefore dimensionless, and subtracted from 1 is then the quasi-rent in the case of the output price (*p*) being normalized to unity, or the shadow price defined as $\lambda_i = q_i / p$, where q_i is the price of input *i*.

In the case of a fully utilised unit the quasi-rent will typically be positive with γ_j being interpreted as the shadow capacity cost of production unit *j*. A partly utilised unit will have zero quasi-rent, whereas an inactive unit will typically face a negative quasi-rent if positive production is undertaken.

A remark on the continuous distribution approach

Johansen (1972) also introduced a continuous representation of production capacity, in the spirit of Houthakker (1955-56). He argued that (p. 28): "However, many of the theoretical properties of this type of function can be more efficiently revealed by shifting over to considering a continuum of such production units at the micro level instead of keeping track of a finite number of individual units." We will not go in detail here, but just give a taste of his approach, but refer the interested reader to Hildenbrand (1981), Muysken (1985), and Seierstad (1981, 1982, 1985).

A distribution of capacity over a two-dimensional[19] region in the $(\xi_1, \xi_2)-$plane was introduced, termed the capacity distribution $f(\xi_1, \xi_2)$. The meaning of this function is

[19] This limitation is not as serious as it may seem. Let one current input be labour and the other energy and a third materials. Then it is often the case that raw materials are used in fixed proportion with output that is the *same* for all micro units due to some basic physical law governing the process in question. An example is the use of aluminium oxide in the production of aluminium. Then this input does not play any role for the utilisation pattern of micro units within the short-run function.

that $f(\xi_1^0, \xi_2^0)\Delta\xi_1\Delta\xi_2$ indicates approximately the total capacity of production for units with input coefficients in $\left[(\xi_1^0, \xi_1^0 + \Delta\xi_1), (\xi_2^0, \xi_2^0 + \Delta\xi_2)\right]$.

The total capacity of production with input coefficients in this region is obtained by integrating $f(\xi_1, \xi_2)$ over this region. The function is thus a continuous analogue to the discrete capacity distribution which could be indicated by the collection of numbers $\left\{\overline{y}_j; (\xi_{1j}, \xi_{2j})\right\}$ for $j = 1, 2, \ldots, N$, as above.

Define the (closed and compact) set called the utilisation region as:

$$G(q_1, q_2) = \{\xi_1, \xi_2 \,|\, \xi_1 \geq 0, \xi_2 \geq 0, q_1\xi_1 + q_2\xi_2 \leq 1\} \tag{52}$$

Here (q_1, q_2) is a vector of input prices deflated by the output price. The quasi-rent line is defined by the boundary $q_1\xi_1 + q_2\xi_2 = 1$. Total output and total use of inputs are then found by integrating over the set $G(q_1, q_2)$:

$$\begin{aligned} Y &= \int\int_{G(q_1, q_2)} f(\xi_1, \xi_2) d\xi_1 d\xi_2 \\ X_i &= \int\int_{G(q_1, q_2)} \xi_i f(\xi_1, \xi_2) d\xi_1 d\xi_2 \end{aligned} \tag{53}$$

Eliminating the prices, we get the short-run industry production function, defined simply as:

$$Y = F(X_1, X_2) \tag{54}$$

As in standard neoclassical theory, we can derive scale, as sell as, substitution properties of this function, the shape of the substitution region and properties of the isoquants. Here we confine ourselves to a rather brief survey; for a detailed derivation; see Chapter 4 in Johansen (1972).

Scale and substitution properties

The scale properties of the short-run function can be characterised by the scale elasticity. Since the build-up of capacity is based on a merit-order concept it is rather obvious that the scale elasticity can maximally be 1 and will in general decrease as more and more inefficient units have to be included in order to increase industry output. Applying the scale elasticity to the industry production function with two inputs, derived either from the discrete case or the continuous case, we have:

$$\varepsilon Y = X_1 \frac{\partial F}{\partial X_1} + X_2 \frac{\partial F}{\partial X_2} \Rightarrow \varepsilon = q_1 \frac{X_1}{Y} + q_2 \frac{X_2}{Y} \tag{55}$$

To facilitate treating the discrete case together with the continuous capacity case, q_i is also used for the shadow price λ_i. (The expressions for the marginal productivities in (50) have been used to derive the last equation.) While the marginal productivity interpretation in the discrete case follows directly from the optimization problem (46), the derivation is rather more involved in the continuous case; see Chapter 4 in Johansen (1972). The ratio $X_i / Y \quad (i = 1,2)$ is the input coefficient at specific corresponding values for X_i and Y in the discrete case and the input coefficients at the centre of gravity in input coefficient space for the continuous case, using the zero quasi-rent line: $q_1\xi_1 + q_2\xi_2 = 1 \Rightarrow \xi_2 = \frac{1}{q_2} - \frac{q_1}{q_2}\xi_1$. The following illustration is found in Johansen (1972, p. 65) for the continuous case[20]:

[20] The original notation uses V_i for industry inputs and X for industry output.

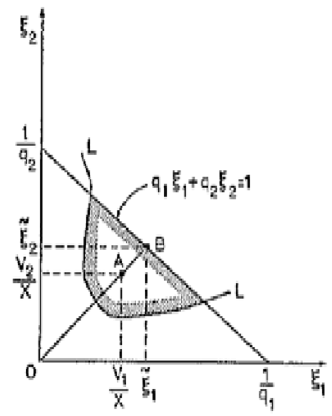

Figure 2. Determination of scale elasticity in the continuous case
Source: Johansen (1972, Fig. 4.2, p. 65)

The shaded region below the boundary line is the region of positive capacity for the given input prices. Point A is the centre of gravity for the utilised capacity. The scale elasticity is defined as the proportion between the input coefficients of the average unit and the input coefficients of the marginal unit with the same factor proportions. The scale elasticity is then simply OA / OB.

In the case of a discrete capacity distribution we have derived the production function $Y = F(X_1, X_2)$. This function can be portrayed in the input-coefficient space by transforming the isoquants and the borders of the substitution region. The isoquants and the borders are piecewise linear, and all corner points can be transformed to the input coefficient space simply by dividing by the output in question. This is done in Fig. 3.[21] Together with this capacity region the raw observations are entered (20 units in all) as squares with input coefficients determined by the mid-point of a square and the size of the square being proportional with output capacity (i.e. the information $(\xi_{ij}, \overline{y}_j)$ for $i = 1,2, j = 1,\ldots,20$).

[21] The notation in the figure is *X* for output, and *L*, *E* for labour and energy inputs, respectively.

In the discrete case the scale elasticity can be found in a very similar way as illustrated in Figure 3 by using real data (Swedish cement industry in 1974 with

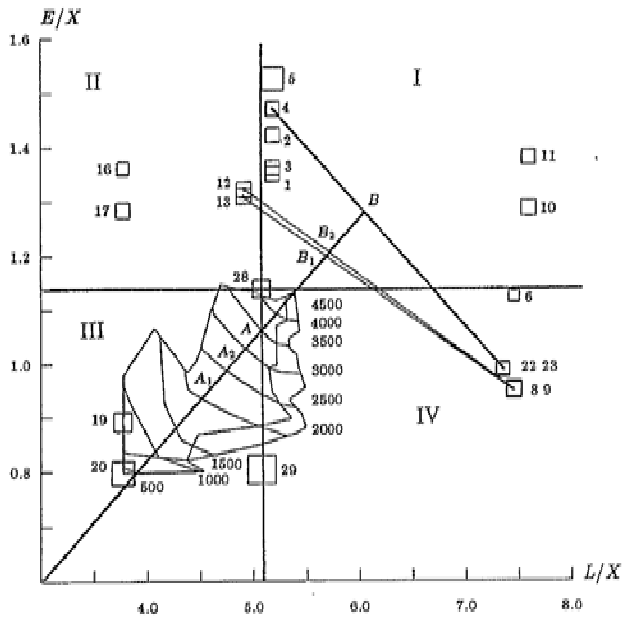

Figure 3. Determination of scale elasticity in the discrete case
Source: Førsund and Hjalmarsson (1987, Fig. 5.2, p.148

labour and energy as current inputs). A point like A is chosen on the corresponding isoquant (marked 3500) and a factor ray is drawn. The two units being partially utilised at the linear isoquant segment of point A are found to be units 4 and 23. For all the capacity supporting the isoquant level at A to be used, the zero quasi-rent line must be the line connecting units 4 and 23 as drawn in the figure. The factor ray through A ends up in point B on the zero quasi-rent boundary. We now have the average input coefficients in point A and the marginal input coefficients with the same factor ratio as A in point B. The scale elasticity is therefore OA / OB.

An expression for the elasticity of substitution can be derived analytically in the case of a continuous capacity distribution, but must be calculated numerically, and approximated because the isoquants are piecewise linear, and in the discrete case based on the numerically derived isoquants. We will not develop these aspects further, but refer to Johansen (1972), and Førsund and Hjalmarsson (1987).

5.2 The dynamics of structural change

In a dynamic perspective the short-run function reflects the history of the ex ante function over time, and past choices of factor proportions. Each of the short-run functions is static, but as Baumol (1962, 1078) remarks, "the static theory of a firm [here the industry] is a helpful snapshot description of a system in motion… ". As Johansen (1972, 26) expresses it: "A study of the dynamics of production of a sector requires a study of how the short-run production function changes through time." The short-run production function is dependent on the technical characteristics and capacities of existing production units. We may say that the short-run macro production function reflects both the history of ex ante functions over time and past choices made from these ex ante functions.

In Johansen (1972) the short-run function is also called the "Transient production function", because the "short-run function is itself shifting through time in such a way that time series observations do not provide a basis for estimating *one* such function (Johansen (1972, 27). Many factors, like technical change, etc, influence the short-run function over time, and one might therefore expect the changes in the short-run function to be more complicated and less accessible to a representation in terms of a limited number of parameters.

The relation to Salter dynamic analysis

A significant extension of the traditional structural analysis was done in Salter (1960), and he studied the dynamic impact of structural change on productivity change, as well. The production function concepts introduced in Johansen (1972) fit nicely in with the type of analysis of Salter (1960). (In fact, Salter (1960) was one of the inspirations of Johansen's approach.)

A first step towards an empirical analysis of firms is to sort all firms in an industry according to increasing variable unit cost, thus creating a *merit-order* sorting of the firms. The difference between the current market price and the unit costs shows what is available for remuneration of fixed capital. This is the quasi-rent. If a firm does not have a positive quasi-rent, then this firm is a candidate for being closed down. Even with a positive quasi-rent a firm may be unable to repay debt and ownership rights may change hands, but it is in general profitable, from a social point of view, to keep producing as long as the quasi-rent is positive. (Another issue is that new owners might be able to increase efficiency of the firm.)

Heckscher (1918) used a simple sorting of firms according to unit costs as the point of departure of his analysis of consequences for domestic firms of reducing the duty on imports from foreign competing industry's product and hence a lowering of the market price. A diagram showing sorted unit costs and the market price line was therefore termed a *Heckscher diagram* in Førsund and Hjalmarsson (1987).

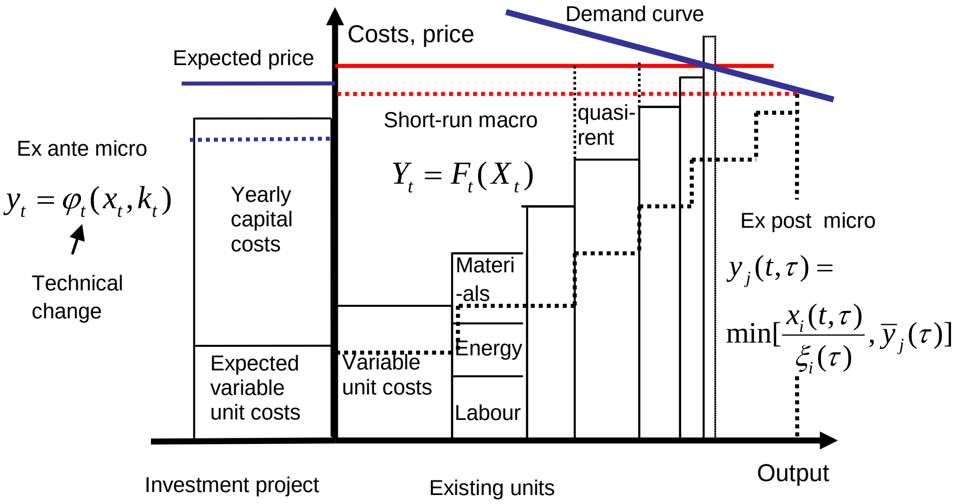

Figure 4. Salter's dynamic analysis and the Johansen production function concepts

This diagram is illustrated in the right-hand part of Figure 4 marked "Existing units"on the horizontal axis.

The sorting according to unit costs makes the distribution into a merit-order curve. The output capacity of firms is proportional to the length of the unit cost histograms. The quasi-rent, i.e. the difference between revenue and variable costs, is indicated for unit no. 4 (from the left). We see that there is one small unit at the right-hand end of the distribution earning negative quasi-rent at the prevailing market price (the solid horizontal line at the top). This firm is assumed not to be operating, indicated by the dashed right-hand line of the histogram. Firms with different cost characteristics and representing different technologies, reflecting different more or less outdated blueprint technologies, co-exist and earn positive quasi-rents.

On disentangling the variable unit cost into components such as labour, energy and materials, as done for the second unit in the Heckscher diagram in Figure 4, the impact on total unit costs of different increases in the costs of types of inputs can be traced. The consequences for, e.g., the aluminium industry of an increase of the electricity price can be analyzed using a Heckscher diagram with a subdivision of costs (cf. Bye and Holmøy (2010)).

Salter (1960, 59) extended the Heckscher diagram by introducing potential investment in production capacity, exhibiting best practice technique. The new technology may imply lower unit variable cost than the most efficient existing unit, but in addition to variable costs, capital cost must now be considered when making an investment decision. The situation is set out in the left part of Figure 4 (marked "Investment project" on the horizontal axis). An investment with the capacity represented by the width of the histogram is profitable if the output price the investor expects to prevail is sufficiently high so as to cover both variable and capital costs, when the latter is expressed in the relevant unit of time. The solid price line indicates a profitable investment, while the dotted one represents an expected price that makes the investment unprofitable.

The Johansen production function concepts are illustrated in Figure 4. The ex ante micro function is the blueprint function valid when the investment is considered (here at time *t*), taking into account the substitution possibilities between inputs including capital. The investment decision is based on considerations outlined in Section 3.2. As we have seen earlier, the role of future prices will be crucial for choice of factor proportions as well as capacity.

Once the investment is undertaken we move to the right-hand part of Figure 4. The ex post micro functions existing at time t will consist of all operating vintages installed from time τ and onwards. Without any waste of inputs the unit variable costs shown in the diagram reflect the input coefficients ξ_{ij} for input *i* employed by unit *j*. The width of a histogram reflects the choice of capacity.

The short-run function $Y = F(X)$ will then express the production opportunities for the industry as a unit, based on the existing operating micro units with their ex post micro technologies. If the units face the same input prices the merit order shown in the figure shows the utilisation of capacity along the isocline reflecting the price structure of the current inputs (the expansion path). Only one unit will be left idle and the other units will be utilised to full capacity. The co-existence of micro units of different vintages with different unit costs is a typical situation when the technical change is embodied in the vintage capital, emphasising the importance of sunk cost for the industry structure. New investment must cover capital costs, but existing capacity has to cover variable costs only, i.e., existing units need only to have positive quasi-rents. We have illustrated the impact of technical change as lowering operating costs compared with the most efficient existing capacity.

If we assume that the indicated investment is undertaken, the new unit-cost distribution shifts to the right and is shown by the dotted lines of the step curve in the Heckscher diagram of Figure 4. The dotted lines represent the new merit-order cost curve. The reaction in the market to the entry of new capacity is shown by the new, lower short-run equilibrium price (horizontal dotted line) as the intersection of the supply curve of firms earning positive quasi-rent and the demand curve. As a result of new capacity one more firm runs into negative quasi-rent, and is removed from the supply curve. The reduction in the market price (maybe unexpected for the price-taking investors) due to investment in capacity may be typical for a capital-intensive industry that is uncoordinated as to investment decisions (see Section 3). The naïve prospects for the investments looked better ex ante than the ex post reality.

The short-run industry function for the period after the investment has been carried out is now based on the ex post micro production functions supporting the new distribution shown by the dotted lines. The exit of one unit due to economic obsolescence at the right-hand side of the distribution (caused by the entry of new capacity leading to a downward shift in the market price) and the entry of the new unit to the left of the distribution with the most modern technology will imply a lowering of unit variable cost, and will represent or constitute the dynamic forces reshaping the short-run function. (Adverse current input price changes may also cause exit of units due to economic obsolescence.) The short-run function is a snapshot of past technologies and the choices made about input proportions. Structural change and productivity development can be studied by having a series of short-run functions over time.

The last production function concept introduced in Johansen (1972) – the long-run macro function – provides a link to growth theory as surveyed in Section 2 and asymptotic results obtained there. In steady state with no technical change all units will be alike, and in a competitive market they will all have the same positive quasi-rent covering the same annual capital cost.

6. Concluding remarks

Our survey has concentrated on a single, but rather significant part of Leif Johansen's impressive research portfolio; namely the one we have coined his "Growth – Production program", with emphasis on material published in the period 1959 – 1972. Without any exaggeration it seems fair to say that Johansen's ideas on putty - clay from 1959 have shown long longevity and persistence. It offers a logical way of introducing heterogeneity and path dependence. The approach invented by Leif Johansen – with substitution opportunities ex ante, but fixed coefficients ex post – has shown to have a lot of explanatory power, at least in two areas: Technical rigidities associated with vintage capital seem to be the most appropriate way of modeling *heterogeneity* among the units on the production side of the economy. Secondly, the suggested production structure will also give rise to *sluggishness*, inertia or path dependence that can easily be explained without making artificial assumptions. Our view is far from controversial and is confirmed by the voluminous literature that have followed after Johansen's '59-paper.

Our objective has been to demonstrate what role *technical rigidities* in the production structure of a firm, industry or a whole economy will play. The lock-in of technology to vintages of capital does not give a different asymptotic growth rate in macro models compared with smooth substitution possibilities, but it may take longer time to reach the growth rate.

When two sectors - one sector producing capital goods, and another one producing consumption goods - are introduced the savings rate is determined solely by history and capital productivities of the various vintages in use. The savings rate can therefore only be altered gradually over time by the future allocation of the output from the *I*-sector. Such a two-sector model with vintages will be characterized by a lot of inertia, sluggishness or path-dependence as compared to models where the savings rate can be freely chosen by the planner.

Another consequence of technical rigidities addressed in a three-sector model of one final goods sector and two intermediate sectors providing raw materials including energy, is the inherent lack of efficiency due to the role of price expectations. Factor proportions are fixed for a long period of time based on price expectations at the date of installment, but price expectations will typically not be realised. The entrepreneurs therefore have to live with incorrect factor proportions, viewed from ex post, for a long period of time, before old vintages of capital are replaced by new equipment. Production efficiency will now require more information than current prices can provide. A competitive economy may be in equilibrium at any instant of time, but still be inefficient in a longer-term perspective.

The new key concept of a short-run industry function introduced in Johansen (1972) provides a way of keeping the heterogeneity of the underlying micro units and thus enables us to trace the dynamics of structural change over time caused by entry and exit of micro units and embodied technical change.

Heterogeneity is a key word in understanding real-life dynamics involving capital. However, the vintage idea may also be applied to labour. Leif Johansen utilized and suggested some very interesting ideas about the dynamics of heterogeneous labour quality during a business cycle in a very fascinating paper, "Some Notes on Employment and Unemployment with Heterogeneous Labour", from 1982. Dynamics created by workers' heterogeneity generates dynamics analogous to the dynamics of structural change portrayed in Section 5 based on technology embodied in the capital equipment. The main idea stems from an assumption that each member of the labour force has its own skill or quality (grade), not known by the employer ex ante. The labour input is made up of different workers, with an average grade as a weighted average of the group members' grades. Each worker is paid the same wage rate, and due to some learning process, the employer may obtain more precise information about the labour force. If marginal value of productivity of labour, properly measured, exceeds (falls below) the wage rate, recruiting new workers (laying off workers) will be profitable. During a hiring period (a boom), the quality of the newly hired workers, from the pool of unemployed, will have skill equal to the average skill of those being unemployed. Under a period of layoffs or recession, the employer, after having obtained better information about each worker, will be able to lay off workers with lower quality than the average skill of the unemployed. During this period of hiring and laying off, the difference between the average quality of those working and those being unemployed will increase. Depending upon wage rate and distribution of skills among those working and not working, we might get different equilibrium configurations. Simultaneous hiring and laying off might occur because the average quality among unemployed might exceed the average among those being laid off. There is an ongoing filtering process through cyclical firing and hiring. The difference in skills between those inside and those outside might therefore increase over time, and might create, in the words of Solow (1983) "a strong class barrier between high-grade and low grade workers".

The short-run industry function appraoch pioneered by Johansen has been extended and proveed to be valuable for attacking and understanding environmental regulation of an industry. To cope with a large number of environmental problems or hazards, a regulatory body has to intervene into the production sector of an industry, while taking into account that enterprises that constitute the industry might differ according to technological characteristics, like the input coefficients and/or the relationship between emissions and output. Then, as demonstrated by Hochman and Zilberman (1978) and Zilberman (2010), within a framework of heterogeneous firms in an industry of the Johansen-type, some kind of environmental regulation, say standards or upper bounds on emissions per unit output, will affect a different subset of firms than those being affected if, say, an emission tax should be imposed. Different firms are induced to exit from the industry under different policy regimes. On using standards or upper bounds on discharges per unit output, the government might induce highly output-efficient, but pollution-intensive firms to leave the industry. On the other hand, a pollution tax might have the opposite effect. Therefore, taking account of heterogeneity on the production side of an industry, the choice of instruments might have very different implications for the various production units and also for efficiency considerations. Acknowledging this variety of implications for the various production units due to the choice of policy instrument, one might also have a building block for issues related to "a political economy of environmental regulation"; or in general "a political economy of industrial policy". Benefits and costs are not uniformly shared among firms; hence some firms might have strong incentives to form coalitions so as to prevent the government from imposing, say, emissions standards. If other firms, with different objectives, do the same, we might end up with a complex policy-game, with a government, acknowledging the strategic motives of the players, while the various coalitions put in effort to affect the choice of policy instruments set by the government. Starting out with heterogeneous firms as we have focused on in this paper, and introduce a planner or government with some environmental goal in a game-like situation with different coalitions of producers, will generate a story with features very much in accordance with what Leif Johansen was concerned about; see Johansen (1977b, 1978a), and later revealed in his strong interest in bargaining and game theory; see e.g., Johansen (1979, 1982b, 1982c).

Regarding Leif Johansen's own ideas for further research his overall perspective on economics was that economic theory should be seen as a tool for economic planning (Sandmo, 1983). In a paper (Johansen, 1982d) read at the last conference he attended (the 25Th anniversary of the Econometric Institute at Erasmus University, Rotterdam), he stressed the need for microeconomic data for econometric estimation. He emphasised the importance of the supply side to look at different types of disequilibria, and the use of putty-clay in modelling production. It is quite interesting on the backdrop of the recent financial crisis that he stated that the theory of rational expectations gives an incomplete or inadequate formulation of the expectations mechanism when considering that governments can pursue discretionary policies.

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