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Ragnar Nymoen
Anders Rygh Swensen
Eivind Tveter

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Department of Economics
University of Oslo

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P. O.Box 1095 Blindern
N-0317 OSLO Norway
Telephone: + 47 22855127
Fax: + 47 22855035
Internet: http://www.sv.uio.no/econ
e-mail: econdep@econ.uio.no

In co-operation with
The Frisch Centre for Economic Research

Gaustadalleén 21
N-0371 OSLO Norway
Telephone: +47 22 95 88 20
Fax: +47 22 95 88 25
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Interpreting the evidence for New Keynesian models of inflation dynamics*

Ragnar Nymoen†
Department of Economics, University of Oslo,
Anders Rygh Swensen
Department of Mathematics, University of Oslo
Eivind Tveter
Statistics Norway

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Abstract

We present a framework for interpretation of the empirical results of New Keynesian models of inflation dynamics. Both the rational expectations solution of the structural New Keynesian Phillips curve, NKPC, and the reduced form VAR analysis of the multivariate time series properties give insight about the joint implications of the evidence in the NKPC literature. For example, we show that the unit-root form of non-stationary may be implied for inflation even though the econometric model initally assumed stationarity. The uniqueness and form of a rational expectations solution may depend on whether dynamic (in)homogeneity is present, and on the size of the forward-coefficient in the NKPC.

Keywords: *New Keynesian Phillips Curve, forward-looking price setting, rational expectations, VAR model.*

JEL classification: *B41,C22, E31, E52*

1 Introduction

In this paper we present a framework for assessing the implications of the estimation results for New Keynesian Phillips curve models (NKPC hereafter) of inflation dynamics. Early in the new century, the NKPC model became the new standard for modelling of inflations dynamics, in particular in the macro models used for monetary policy analysis, see e.g., Smets and Wouters (2003).

We present the main hypotheses that researchers have tested for the parameters of the NKPC model, and the associated empirical evidence (section 2). The evidence is mainly from studies that estimate the hybrid NKPC model as a single

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† Corresponding author, University of Oslo, Department of Economics P.O.B. 1095 Blindern, 0316 Oslo, Norway. ragnar.nymoen@econ.uio.no.

structural equation, but also from the growing literature that embed the NKPC in the vector autoregressive model, VAR. The main references are Galí and Gertler (1999, henceforth GG) on US data, and Galí, Gertler and López-Salido (2001, henceforth GGL (2001)) on euro-area data, and the more critical assessments in Bårdsen et al. (2004) and Rudd and Whelan (2007). A recent paper that uses the VAR approach to testing the NKPC is Fanelli (2008).

Several aspects of the evidence are relevant for the rational expectations solution for inflation, as section 3 shows. In particular, the feature called forward-dominance, which in isolation is interpreted as supportive of the NKPC model, becomes problematic when it appears together with another typical result, namely dynamic homogeneity. In this case, the rational expectations solution may imply non-stationary inflation, which most users of the NKPC model would not assume from the outset. Another possibility is that a rational expectations solution does not exist. In this regard, we correct an error in the NKPC literature about the existence of a rational solution in the case of dynamic homogeneity, forward-dominance, and a forcing variable which is Granger non-caused by inflation (an assumption made by e.g., GG and GGL).

In section 4, we embed the NKPC in the reduced form VAR, which provides the natural statistical model for linear dynamic relationships in economics. Examples of VAR based empirical tests of the NKPC are Fanelli (2008) and Boug et al. (2010). We show that the logical implications of the homogeneity restrictions are well defined in the VAR analysis. The VAR analysis and the rational expectations solution give internally consistent results, but are nevertheless complementary since they highlight different aspects of the dynamics of the NKPC model.

In section 5 we conclude the paper with a short discussion. In terms of scope and relevance, our analysis applies to the constant parameter and rational expectations version of the NKPC which has become the workhorse of operational DSGE models of monetary policy, see e.g., Gali (2008) and Wickens (2008, Ch 13).[1] The model with time varying coefficients and subjective expectations formation by Cogley and Sbordone (2008) represents a new development that raises new possibilities and issues. One relevant special case is a NKPC with a time varying intercept, e.g., to accommodate the transition to an inflation target, but with constant derivative coefficients. Our discussion below is relevant also for this extension of the standard NKPC.

2 The NKPC and the evidence

The hybrid NKPC is given as

$$\pi_t = \underset{\geq 0}{a^f} E_t[\pi_{t+1}] + \underset{\geq 0}{a^b} \pi_{t-1} + \underset{>0}{b} s_t + \varepsilon_{\pi t}, \tag{1}$$

where π_t is the rate of inflation, $E_t[\pi_{t+1}]$ is the expected rate of inflation in period $t+1$, given the information available for forecasting at the end of period t. The intercept of the equation has been omitted for simplicity. The variable s_t denotes the

[1] Hammond (2010) Table C shows that 20 out of 27 inflation targeting central banks either use or are developing a models with NKPC equations for forecasting and policy analysis.

logarithm of firms' real marginal costs and $\varepsilon_{\pi t}$ is a disturbance term with zero mean. In many applications, notably GG and GGL, the disturbance term is omitted, which suggests a stronger interpretation which is often referred to as the NKPC holding in "exact form". We will also consider the exact form, in section 4 below, but for the time being we keep the more traditional econometric formulation with a disturbance term.

The theory consistent signs are given below the parameters. The 'pure' NKPC is specified without the lagged inflation term ($a^b = 0$). In the case of the pure NKPC, Roberts (1995) has shown that several New Keynesian models with rational expectations have (1) as a common representation, including the models of staggered contracts developed by Taylor (1979, 1980) and Calvo (1983), and the quadratic price adjustment cost model of Rotemberg (1982), see also Pesaran (1987, Ch 4.3).[2]

The rationale for allowing $a^b > 0$ is that the theory applies to a significant portion of optimal price adjustments in period t, but not to all. Hence, in each period, a share of the overall rate of inflation is determined by last period's rate of inflation, for example because of backward-looking expectations.

Regarding the sum of the inflation coefficients, it is custom to specify $a^f + a^b \leq 1$ as a restriction, which rules out an explosive solution in the purely backward-looking case.

In the following, the third variable in (1), s_t, is the logarithm of the wage-share, which is the common operational definition of firms' marginal cost of production. The coefficient b is expected to be strictly positive, and there are no other economic explanatory variables in this model of inflation dynamics for the closed economy case.

The most influential papers supporting the empirical relevance and generality of the NKPC are the mentioned papers by GG and GGL (2001, 2005). A distillation of the typical empirical results can be made in the following numbered points:

1. **Forcing variable** When real marginal costs are measured by the log of the wage-share, the coefficient b is positive and significantly different from zero at conventional levels of significance.

2. **Forward-looking** The two null hypotheses of $a^f = 0$ and $a^b = 0$ are rejected both individually and jointly in the hybrid NKPC. Hence, forward-looking inflation dynamics is a feature of the hybrid NKPC. Quite often, the evidence also support forward-dominance, defined as $a^f > a^b$.

3. **Homogeneity** The hypothesis of $a^f + a^b = 1$ is typically not rejected at conventional levels of significance.

Even a cursory look at the literature shows that all three results have been debated. Rudd and Whelan (2005,2007) in particular refuted that the wage-share was a strong forcing variable on US data. It goes without saying that this was an important conclusion, because without a forcing variable there is no Phillips curve.[3] Neither

[2] The overlapping wage contract model of sticky prices is also attributed to Phelps (1978).

[3] Bårdsen et al. (2004) showed that the significance of the wage-share in the GGL (2001) euro model is fragile, as it depends on the exact implementation of the GMM estimation method used.

was it surprising that Rudd and Whelan's conclusions provoked replies. For example Gali et al. (2005) claimed that their results were robust to the critique.

Other researchers have pointed out that it may be too simple to ask whether it is the wage-share alone that drives inflation. It may be more fruitful to ask whether there are other variables that drive inflation together with the wage-share, and whether omission of such explanatory variables reduce or strengthen the results obtained for the wage-share in the hybrid NKPC. Bårdsen et al. (2004), using an encompassing approach, show that for small open economies in particular there is an omitted variables bias in NKPC models that omit the relative price of imports.

Regarding the prevalence of forward-lookingness, Bårdsen et al. (2004) and Bjørnstad and Nymoen (2008) test the encompassing capability of the NKPC with respect to competing models of inflation dynamics and find that forward-dominance may be a spurious finding. Castle et al. (2010) show theoretically how forward dominance may be a result of intermittent structural breaks. They also re-analyse the US (GG) and euro data (GGL) and show that the size of the estimated forward coefficient a^f is highly responsive to the inclusion or omission of indicator variables that capture breaks.

The homogeneity restriction in 3 seems to be robust across studies that estimate NKPCs on different data sets. In their review from 2007, Rudd and Whelan report nine estimates, from GG, GGL and their own research. The average value for $\hat{a}^f + \hat{a}^b$ is 0.98 with 1.002 as the highest estimate, and 0.958 as the smallest, see Rudd and Whelan (2007, Table 2). The standard errors of the point estimates are 0.04 or higher. Hence, and as an example, it would take a very high negative correlation in order to reject a null hypothesis of homogeneity based on any of these estimations, see also e.g., Chao and Swanson (2009).

It is also noteworthy that Rudd and Whelan, in their review of the wider NKPC literature, identify dynamic homogeneity as a typical case, for example, they write "...consider the case in which $\gamma_f + \gamma_b = 1$, a restriction that is directly imposed by several popular hybrid models and one that confirms closely to the estimates reported in Table 2."[4].

As we show below, the case with both dynamic homogeneity and forward-dominance may in fact be inconsistent with i) stationary inflation and, ii) the existence of a non-explosive rational expectations solution as defined in Blanchard and Kahn (1980).

The above summary is based on studies that estimate and test the NKPC as a single structural equation. There is now a literature where the NKPC is embedded in a VAR, see for example Fanelli and Palomba (2010) and the references in that paper. The main impression is that the results for the NKPC parameters in the VAR studies are consistent with the results from the earlier single equation studies. However, an important new result is that the NKPC is identified as a cointegrating relationship between variables that are non-stationary in the unit-root sense. This possibility is also contained in our discussion below.

Fanelli (2008) using a vector autoregressive regression model on the euro-area data set, finds that the NKPC is a poor explanatory model. On US data, Mavroeidis (2006) has shown that real marginal costs appear to be an irrelevant determinant of inflation, see also Fuhrer (2006).

[4]Rudd and Whelan (2007, p 167), $\gamma_f + \gamma_b = 1$ is the homogeneity restriction in their notation.

3 The rational expectations solution

The joint evidence for the NKPC has importance for the closed form rational expectations solution. In particular care must be taken if the homogeneity restriction (item 3) is imposed on the solution. It also seems relevant to consider both the case of $a^f > 0.5$ and $a^f < 0.5$, since forward-looking dominance in the form of $a^f > 0.5$ is a good deal stronger feature than 'mere' forward-looking relevance ($a^f > 0$) in models of inflation dynamics.

It is instructive to consider the closed form rational expectations solution when s_t follows an autoregressive process of order k :

(2) $$s_t = c_{s1}s_{t-1} + \cdots + c_{sk}s_{t-k} + \varepsilon_{s,t} \ .$$

Equations (1) and (2) define the NKPC model. For simplicity we assume that the two disturbances $\varepsilon_{\pi,t}$ and $\varepsilon_{s,t}$ are independently normally distributed variables. The two equations define a model of one-way Granger causality between s_t and π_t, hence the name *forcing variable* for s_t is well chosen.[5] The one-way causation represents a market break with earlier models of the wage-price spiral though, see e.g., Sargan (1980) and Blanchard (1987).

We obtain the solution for π_t as

(3) $$\pi_t = r_1\pi_{t-1} + \frac{b}{a^f r_2}\sum_{i=0}^{\infty}(\frac{1}{r_2})^i E_t s_{t+i} + \frac{1}{a^f r_2}\varepsilon_{\pi,t}$$

where r_1 and r_2 are the two roots of $r^2 - (1/a^f)r + (a^b/a^f) = 0$.[6] $E_t s_{t+i}$ denotes the rational expectation for s_{t+i}, conditional on (2) and information available in period t.

A stable solution of the pure NKPC, with $a^b = 0$, requires $r_2 = 1/a^f > 1$ ($a^f < 1$). For the hybrid NKPC we follow custom and assume $a^f + a^b \leq 1$. If $a^f + a^b < 1$, both roots are real. This means that any cyclical behaviour of inflation around a steady-state must be explained by expectations formation with regard to the forcing variable s_t.[7] This may be a separate argument for considering higher order processes for s_t with $k \geq 2$.[8] If $a^f + a^b = 1$, one of the roots r_1, r_2 is unity.

One important reason for considering the hybrid NKPC in the first place was to be able to explain the persistence of real world inflation rates. From (3) we see that a constellation with two positive roots is necessary for achieving this. Note that from *Viète's* rules

(4) $$r_1 r_2 = \frac{a^b}{a^f} \text{ and } r_1 + r_2 = \frac{1}{a^f}$$

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[5] Sbordone (2002) provides a solution for a different interpretation of the model, and for the non-hybrid version, where nominal unit-labour costs drive inflation.

[6] See Bårdsen et al. (2005, Appendix A) who build on Pesaran (1987, p. 108-109).

[7] Of course, another change in the specification that would open up for cyclical behavior of inflation is to include more than one lag of inflation in the hybrid NKPC.

[8] Mavroeidis (2005) shows that $k \geq 2$ is required for identification of the parameters of the NKPC.

we have that r_1,$r_2 > 0$ is implied by the sign assumptions for the parameters a^b and a^f of the NKPC.

It is usual to define r_1 as

$$r_1 = \frac{1 - \sqrt{1 - 4a^f a^b}}{2a^f} \tag{5}$$

which is $0 \leq r_1 < 1$ under the assumption of $a^f + a^b < 1$.

However care must be taken when we consider the homogeneity restriction $a^f + a^b = 1$. In that case, $r_1 = 1$ and r_2 is given by

$$r_2 = \frac{1 - a^f}{a^f} \tag{6}$$

If $0 < a^f < 0.5$, $r_2 > 1$, the solution becomes:

$$\pi_t = \pi_{t-1} + \frac{b}{a^f r_2} \sum_{i=0}^{\infty} (\frac{1}{r_2})^i E_t s_{t+i} + \frac{1}{a^f r_2} \varepsilon_{\pi,t} \tag{7}$$

which will have unit-root properties. The sum involving $E_t s_{t+i}$ can be expressed by $s_t, \ldots, s_{t-k+1}$. The s_t process can be causal (all roots inside the unit circle), or it can contain a unit-root. In a common notation we say that the inflation rate is integrated of degree 1, $\pi_t \sim I(1)$, even in the case when the forcing variable is $s_t \sim I(0)$. Details can be found in Appendix A.

If $a^f = 0.5$, $r_2 = 1$ is implied together with $r_1 = 1$. A closed form solution can still be derived by starting from equation (7), but we now need the extra assumption that the s_t process does not contain any unit roots, hence $s_t \sim I(1)$ is not allowed in this case.

In the case when the estimation results show both homogeneity $a^f + a^b = 1$, and forward-dominance $0.5 < a^f < 1$, the solution (7) can be replaced by

$$\pi_t = r_2 \pi_{t-1} + \frac{b}{a^f} \sum_{i=0}^{\infty} E_t s_{t+i} + \frac{1}{a^f} \varepsilon_{\pi,t} \tag{8}$$

since $r_2 < 1$ in this case. Indeed, this is stated in Rudd and Whelan (2007).[9] However, (8) does not represent a unique solution for π_t, as shown in Appendix B. If (8) is used as a solution, it is because we choose a stationary inflation rate, not that this stationarity is logically implied the a unique solution alone.

4 VAR implications for stationarity and cointegration

Above, we discussed the joint implications of the evidence for the dynamic solution of the structural NKPC model. In this section, we make use of the reduced form VAR to analyse the implications that poignant parameter constellations of the NKPC have for the degree of integration, and for the possibility of cointegration. Since integration and cointegration of π_t and s_t are system properties, we expect that there is a good correspondence between the structural form and the reduced form

[9] See equation (13) in section 3.3 in their paper.

analysis. However, for applied work, the VAR model is a well established framework to use.

Without loss of generality, we assume that any deterministic shifts as a cause of non-stationarity have been removed from the two variables. In line with the assumptions of the structural model, the forcing variable s_t is assumed to be not Granger caused by inflation, π_t. We consider the model

$$Y_t = A_1 Y_{t-1} + ... + A_p Y_{t-k} + \varepsilon_t \tag{9}$$

for fixed values of $Y_{-k+1}, ..., Y_0$, and a zero-mean disturbance vector ε_t.

Our premise is that (1) implies that the parameter vector $(a^f + a^b - 1, b)'$ is in the cointegration space defined by (9), which in turn is determined by the *rank* of the matrix Π:

$$\Pi = \sum_{i=1}^{k} A_i - I \tag{10}$$

where I denotes the two dimensional identity matrix.

1. $rank = 2$. In this case $(\pi_t, s_t)'$ is stationary, $I(0)$. There are two separate long-run means, which we denote m_π and m_s, corresponding to the steady-state solution of the system. In this case $(a^f + a^b - 1, b)'$ defines a linear combination of the two long-run means, i.e., by taking the unconditional mean on both sides of (1) we obtain:

$$\left(a^f + a^b - 1\right) m_\pi - b m_s = 0.$$

2. $rank = 1$. There is one cointegration vector, i.e., the cointegration space is spanned by $(a^f + a^b - 1, b)'$.
   (a) $a^f + a^b \neq 1$ and $b \neq 0$.
   (b) $a^f + a^b = 1$ and $b \neq 0$.
   (c) $a^f + a^b \neq 1$ and $b = 0$.
3. $rank = 0$. $(\Delta\pi_t, \Delta s_t)'$ is stationary, $I(0)$

Case 1, where both inflation and the forcing variable are stationary, $I(0)$, hence $rank = 2$, may be seen as the reference case. As noted by Fanelli (2008) this is how the variables are treated when (1) is estimated by GG and GGL(2001), and it is part of the rationale for inflation targeting regimes which takes the stationarity of inflation as a premise. This does not rule out that inflation can be highly persistent in many samples. (3) is the rational expectations solution corresponding to $a^f + a^b < 1$.

When $rank = 1$, Case 2, there are three possibilities. First, in Case 2a, with $a^f + a^b \neq 1$ and $b \neq 0$, the NKPC equation can be interpreted as a cointegration equation between the two $I(1)$ variables π_t and s_t. From an econometric perspective, identified cointegration relationships represent partial structure, because they are invariant to omitted stationary variables, and as such they are usually regarded as interesting entities.

The rational expectations solution is (3) also for this case, notably with a non-stationary forcing variable, $s_t \sim I(1)$. This solution requires $a^f < 1/2$ as noted, which does not conform well with the tendency to find forward-dominance in estimated NKPCs. However, one might doubt that an inflation targeting central bank would be comfortable with the implication that inflation has unit-root properties.

Case 2b is the constellation with dynamic homogeneity $a^f + a^b = 1$, and $b \neq 0$. From Lemmas 1 and 2 in Appendix C it follows that the determinant of the characteristic polynomial in general has a root at unity regardless of whether for the exact and inexact version of the NKPC formulation is assumed for the VAR model. The dynamic properties of the variables in the VAR can however differ according to the constellations of the parameters.

If $0 < a^f < 1/2$, the rational expectations solution is (7), i.e., with $r_1 = 1$. The implication is that inflation is $I(1)$, which refutes an assumption about a stationary inflation rate. The same feature can be seen in the VAR model, at least in the exact formulation. In this case, the VAR does not have a causal representation.

It is interesting to note that if (8) is chosen as the rational expectations solution for the case of forward-dominance and homogeneity, $\pi_t \sim I(0)$ is implied by the choice of solution. It is as if the rational expectations model avoids the consequence of the unit-root that $a^f + a^b = 1$ represents in the VAR representation.

Case 2c is a constellation which is again consistent with stationary inflation. However, Case 2c cannot be reconciled with the idea that marginal cost is the explanatory variable of inflation, since $b = 0$ in the cointegrating vector. There is no NKPC that can support monetary policy.

In Case 3, with $rank = 0$, the vector hypothesized by the NKPC takes the form $(a^f + a^b - 1, b)' = (0, 0)'$, hence the economic content/interpretation of the NKPC has no counterpart in the properties of the VAR since $b = 0$, even though dynamic homogeneity in itself allows both forward-relevance and forward-dominance.

As examples of VAR based studies we first have Fanelli (2008), who analyses euro-area data. He shows that the stationarity assumption is difficult to maintain for euro-area. Boug et al. (2010) analyse both euro-area and US data. They obtain sum of $\hat{a}^f$ and $\hat{a}^b$ is 1.03 for the US data and 1.05 for the euro-area data. The estimates of b are numerically low and statistically insignificant.[10] Finally Fanelli and Palomba (2010) analyse euro area inflation with a model with learning. They also find that the cointegrated VAR is the most credible econometric model for inference, but the main results for the NKPC parameters do not depend critically on setting reduced rank rather than full rank. In both cases, forward-dominance is found empirically. Juselius (2011) analyses a larger VAR that also embeds the variables that typically appear in an New Keynesian IS curve. He also finds that unit roots are dominant in the euro data set, and he interprets the New Keynesian Phillips curve in particular as a cointegrating relationship.

5 Summary and discussion

The above analysis shows that care must be taken when we assess the implications of the evidence for structural econometric relationships that include leads in variables.

[10] Barkbu and Batini (2005) use the same method, due to Johansen and Swensen (1999), for Canadian data. Their full sample results give a single cointegration relationship with $a^f + a^b = 1$, which fits into category 2b in our typology.

In particular, the near homogeneity often found by investigators of empirical NKPC may indicate a contradiction of the initial assumption about stationarity of the rate of inflation. This illustrates the principle of assessing the joint evidence and not just each single pieces of evidence sequentially, see Ericsson and Hendry (1999). In particular we find that the occurrence of forward-dominance, which is invariably seen as a result which is supportive of the NKPC, lead to an internal inconsistency if it is joined up with dynamic homogeneity. In this case, the implication of the joint evidence is that a rational expectations solution does not exist.

The result hold for the hybrid NKPC model with an exogenous forcing variable, which is the model in Galí and Gertler (1999) and later journal papers. As pointed out by Bårdsen et al. (2004), different dynamics, also for the case of homogeneity, may be implied if the wage-share is modelled as an endogenous variable. This parallels the "old" Phillips curve system, which is dynamically unstable in the case of a vertical long-run Phillips curve and exogenous unemployment rate, but stable if the rate of unemployment provides the right equilibrium correction mechanism. In the same way, the NKPC with forward dominance and homogeneity may very well have a stable rational expectations solution if there are equilibrium correction mechanism "elsewhere" in the DSGE macro model. Discussion of these mechanism goes beyond the scope of this paper though. We note however that the state of the art operational DSGE models represent inflation and real marginal costs as stationary from the outset since they are defined as deviations from their respective unconditional means, see Del Negro et al. (2006), thus implying that a stationary rational expectations solution always exists.

It is also an interesting issue whether the results above carry over to the second generation of NKPC model in Blanchard and Galí (2007). This model augments the Phillips curve with "new" explanatory variables, e.g., the rate of unemployment, and are also used together with bargaining models of wage setting, e.g., Rossi and Fabrizio (2008). Assessment of these developments within in a common econometric framework is an interesting area for future research. In contrast, Cogley and Sbordone (2008) represent a change of statistical framework to the stochastic parameter framework which is a very flexible way of modelling inflation dynamics within sample.

Another, more econometrically oriented research strategy is to adopt a more general framework that allows inflation to be non-stationary due to regime-shifts rather than unit-roots (so π_t may be $I(0)$ conditional on such breaks), see e.g., Bårdsen and Nymoen (2003) and Castle et al. (2010). This approach also allows the specification of testable hypotheses about lead-variables, in the light of pre-existing evidence from inflation dynamics modelling and empirical evidence about structural breaks.

A The closed form rational expectations solution

We use (2) to obtain the rational expectation solution for $E_t s_{t+i}$. If we define $ss_t = (s_t, \ldots, s_{t-k+1})'$, $\epsilon_t = (\varepsilon_{s,t}, 0, \ldots, 0)'$ and the companion matrix

$$C_s = \begin{pmatrix} c_{s1} & c_{s2} & \cdots & & c_{sk} \\ 1 & 0 & \cdots & 0 & 0 \\ \vdots & & & & \vdots \\ 0 & \cdots & & 1 & 0 \\ & & & & \end{pmatrix},$$

(2) can be expressed as

$$ss_t = C_s ss_{t-1} + \epsilon_t$$

Hence,

$$E_t(ss_{t+i}) = C_s^i ss_t \quad \text{and} \quad E_t(s_{t+i}) = e' C_s^i ss_t,$$

where $e = (1, 0, \ldots, 0)'$. If all the eigenvalues of $\frac{1}{r_2} C_s$ have modulus less than 1

$$\sum_{i=0}^{\infty} (\frac{1}{r_2})^i E_t s_{t+i} = \sum_{i=0}^{\infty} e'(\frac{1}{r_2} C_s)^i ss_t = e'[\sum_{i=0}^{\infty} (\frac{1}{r_2} C_s)^i] ss_t = e'(I - \frac{1}{r_2} C_s)^{-1} ss_t.$$

the full solution of (3) becomes:

$$\pi_t = r_1 \pi_{t-1} + \frac{b}{a^f r_2} K_{s1} s_t + \cdots + \frac{b}{a^f r_2} K_{sk} s_{t-k+1} + \frac{1}{a^f r_2} \varepsilon_{\pi,t}, \tag{11}$$

where $K_{s1} = 1/(1 - c_{s1}(\frac{1}{r_2}) - \cdots - c_{sk}(\frac{1}{r_2})^k)$ and $K_{si} = (c_{si}(\frac{1}{r_2}) + \cdots + c_{sk}(\frac{1}{r_2})^{k-i+1})/(1 - c_{s1}(\frac{1}{r_2}) - \cdots - c_{sk}(\frac{1}{r_2})^k)$, $i = 2, \ldots, k$.

For concreteness we consider the special case of $k = 2$, which is also sufficient for identification. In the case of $k = 2$, the constants K_{s1} and K_{s2} can be expressed as

$$\begin{aligned} K_{s1} &= -\frac{r_2}{r_{s2} - r_{s1}} \left\{ \frac{\frac{r_{s1}}{r_2}}{1 - \frac{r_{s1}}{r_2}} - \frac{\frac{r_{s2}}{r_2}}{1 - \frac{r_{s2}}{r_2}} \right\} \\ &= \frac{1}{1 - \frac{1}{r_2}(c_{s1} + \frac{1}{r_2} c_{s2})}, \end{aligned} \tag{12}$$

$$\begin{aligned} K_{s2} &= \frac{r_2}{r_{s2} - r_{s1}} \left\{ \frac{r_{s2}\frac{r_{s1}}{r_2}}{1 - \frac{r_{s1}}{r_2}} - \frac{r_{s1}\frac{r_{s2}}{r_2}}{1 - \frac{r_{s2}}{r_2}} \right\} \\ &= \frac{c_{s2}}{r_2} K_{s1}. \end{aligned} \tag{13}$$

r_{sj} ($j = 1, 2$) are the roots of the characteristic equation associated with (2). It is important to note that these expressions are based on the assumption

$$\left| \frac{r_{sj}}{r_2} \right| < 1 \text{ for } j = 1, 2 \tag{14}$$

Because $r_{s1}, \ldots, r_{sk}$ are the eigenvalues of C_s, the assumption (14) is in general the condition that the eigenvalues of $\frac{1}{r_2} C_s$ have modulus less than 1.

If we first assume $|r_{s1}| < 1$ and $|r_{s2}| < 1$ so that $s_t \sim I(0)$, then $r_2 \geqq 1$ is a sufficient condition for (14) to hold. Hence, the rational expectations solution for π_t exists and is given by (11) with $0 < r_1 < 1$, meaning that $\pi_t \sim I(0)$. In the special case of $r_2 = 1$, it follows from (4) that $a^f > a^b$ and $0 < a^f < 1$ jointly imply $0 < r_1 < 1$.

Next, continue with the assumption of $|r_{sj}| < 1$ for $i = 1, 2$, but consider $r_1 = 1$ which is equivalent with $a^f + a^b = 1$. If $a^f \leq 0.5$ we have $r_2 \geq 1$ which satisfies the requirement (14), and because of $r_1 = 1$, the rational expectations solution (11) predicts that $\pi_t \sim I(1)$ even though the forcing variable is stationary, $s_t \sim I(0)$. With forward-dominance, $a^f > 1/2$, this conclusion may first appear to be changed. If we follow Rudd and Whelan (2007) and write the partial solution as in (8), and (14) is replaced by the condition

$$|r_{sj}| < 1 \text{ for } j = 1, 2 \tag{15}$$

This condition is satisfied for $s_t \sim I(0)$, and therefore the rational expectations "solution" in (8) predicts $\pi_t \sim I(0)$, regardless of $r_1 = 1$. However, with reference to Blanchard and Kahn (1980) we can conclude that (8) is in fact not a solution of the NKPC model. This result is shown in appendix B below. Hence, it remains true that with $|r_{sj}| < 1$ for $j = 1, 2$ in the forcing process, homogeneity in the NKPC, $a^f + a^b = 1$, the rational expectations solution implies $\pi_t \sim I(1)$.

A special case of NKPC homogeneity is $a^f = 0.5$ and $a^b = 0.5$. This implies $r_1 = 1$, $r_2 = 1$, and although we retain that $|r_{sj}| < 1$, meaning that $s_t \sim I(0)$, the rational expectations solution gives $\pi_t \sim I(1)$.

Finally consider the case of $r_1 = 1$ and $s_t \sim I(1)$, which is equivalent to $|r_{sj}|^{\max} = 1$. In the light of (14), $|r_{sj}|^{\max} = 1$ may be allowed as long as

$$\left|a^f\right| < \frac{1}{|r_{sj}|^{\max} + 1}, \text{ when } a^f + a^b = 1, \tag{16}$$

i.e., $a^f < 1/2$. This result, that there may exist a (well defined) rational expectations solution also in the case that both π_t and s_t are non-stationary $I(1)$ variables, is consistent with Blanchard and Kahn (1980). In this sense we have rational expectation theory of a non-stationary rate of inflation. However, we need $a^f < 1/2$ as noted.[11]

B The rational expectations solution with forward-dominance and homogeneity

To verify that (8) is not a unique solution, consider the rational expectations model defined in (1) and (2) where in addition the homogeneity restriction $a^f + a^b = 1$ is imposed. The 2×2 matrix defining this structural model, denoted by A in Blanchard and Kahn (1980) is

$$\begin{pmatrix} 0 & 1 \\ (a^f - 1)/a^f & 1/a^f \end{pmatrix}$$

[11] The above discussion generalizes to the case of a *k'th* order s_t-process in (2). In the general case, $|r_{si}|^{\max}$ is defined as $|r_{si}|^{\max} = \max_{1 \leq i \leq k} |r_{si}|$.

and has eigenvalues given by the equation

$$(\lambda - 1)(\lambda - \frac{1-a^f}{a^f}) = 0.$$

Thus exactly one eigenvalue is outside the unit circle if $(1-a^f)/a^f > 1$ i.e., $a^f < 1/2$. If $a^f \geq 1/2$ all eigenvalues are on or inside the unit circle. In the system defined by (1) and (2) there is one non-predetermined variable and one predetermined variable. According to Propositions 1 and 2 in Blanchard and Kahn (1980), there is therefore a unique non-explosive solution if $a^f < 1/2$, and none if $a^f \geq 1/2$.

C The homogeneity restriction in the VAR

Lemma 1 *Consider the vector auto-regressive (VAR) model of the form* $Y_t = (\pi_t, s_t)'$

$$Y_t = A_1 Y_{t-1} + \cdots + A_k Y_{t-k} + \varepsilon_t$$

where $\varepsilon_{k+1}, \ldots, \varepsilon_T$ *are independent variables with covariance* Ω*. Let* $A(z) = \{A_{ij}(z)\}_{i,j=1}^2$ *be the associated characteristic polynomial.*

The two conditions

i)

(17) $$\pi_t = a^f E_t[\pi_{t+1}] + a^b \pi_{t-1} + b s_t, \text{ where } a^f + a^b = 1,\ a^f \neq 0$$

and

ii) π_t *does not Granger-cause* s_t*,*

imply that the determinant of the characteristic polynomial of the VAR-model is

$$\det[A(z)] = (z-1)(z - \frac{a^f}{1-a^f}) \det[A_{22}(z)].$$

For $1/2 \leq a^f < 1$ *the solutions of* $\det[A(z)] = 0$ *are equal to* $z = 1$ *or have modulus* $|z| > 1$ *provided* $\det[A_{22}(z)] = 0$ *also implies* $|z| > 1$*. Furthermore,* Y_t *is neither* $I(0)$ *nor* $I(1)$

For $0 < a^f < 1/2$ *some solutions of* $\det[A(z)] = 0$ *have modulus* $|z| < 1$*.*

Proof. Assume a remodelled bivariate VAR with k lags

(18) $$Y_t = A_1 Y_{t-1} + A_2 Y_{t-2} + \ldots A_k Y_{t-k} + \varepsilon_t,$$

so $\varepsilon_t,\ t = 1, \ldots$ are independently distributed errors, with mean zero. The associated characteristic polynomial is given by

(19) $$A(z) = I - A_1 z - A_2 z^2 - \ldots - A_k z^k.$$

For convenience let $k = 3$, without loss of generality, in the following. Granger non-causality for s_t implies

$$A(z) = I - \begin{pmatrix} a_{11}^{(1)} & a_{12}^{(1)} \\ 0 & a_{22}^{(1)} \end{pmatrix} z - \begin{pmatrix} a_{11}^{(2)} & a_{12}^{(2)} \\ 0 & a_{22}^{(2)} \end{pmatrix} z^2 - \begin{pmatrix} a_{11}^{(3)} & a_{12}^{(3)} \\ 0 & a_{22}^{(3)} \end{pmatrix} z^3.$$

By introducing the homogeneity restriction we can write the exact form of (17) as

$$\begin{aligned} \left(a^f + a^b\right)\pi_t &= a^f E_t \pi_{t+1} + a^b \pi_{t-1} + b s_t \\ a^f E_t \Delta\pi_{t+1} &= a^b \Delta\pi_t - b s_t, \end{aligned} \tag{20}$$

where $a^f \neq 0$. Note that we follow GG and GGL and use the "exact form" of the NKPC here, which is not a trivial simplification.

Leading (18) one period and taking expectation conditional on the information set in period t, we get

$$E_t Y_{t+1} = A_1 Y_t + A_2 Y_{t-1} + A_3 Y_{t-2}.$$

Subtracting by Y_t on both sides yields

$$E_t \Delta Y_{t+1} = (A_1 - I) Y_t + A_2 Y_{t-1} + A_3 Y_{t-2}. \tag{21}$$

The first line in (21) is given by

$$E_t \Delta\pi_{t+1} = \left(a_{11}^{(1)} - 1\right)\pi_t + a_{11}^{(2)}\pi_{t-1} + a_{11}^{(3)}\pi_{t-2} + ... \tag{22}$$

Comparing (20) and (21) gives the following parameter restrictions $-a_{11}^{(2)} = a_{11}^{(1)} - 1 = a^b/a^f$, $a_{11}^{(3)} = 0$, i.e.,$a_{11}^{(1)} = (a^b + a^f)/a^f = 1/a^f$.

This puts further restrictions on the characteristic polynomial

$$\begin{aligned} A(z) &= I - \begin{pmatrix} a_{11}^{(1)} & a_{12}^{(1)} \\ 0 & a_{22}^{(1)} \end{pmatrix} z - \begin{pmatrix} a_{11}^{(2)} & a_{12}^{(2)} \\ 0 & a_{22}^{(2)} \end{pmatrix} z^2 - \begin{pmatrix} 0 & a_{12}^{(3)} \\ 0 & a_{22}^{(3)} \end{pmatrix} z^3 \\ &= \begin{pmatrix} 1 - a_{11}^{(1)} z - a_{11}^{(2)} z^2 & -a_{12}^{(1)} z - a_{12}^{(2)} z^2 - a_{12}^{(3)} z^3 \\ 0 & 1 - a_{22}^{(1)} z - a_{22}^{(2)} z^2 - a_{22}^{(3)} z^3 \end{pmatrix} \\ &= \begin{pmatrix} 1 - a_{11}^{(1)} z - a_{11}^{(2)} z^2 & A_{12}(z) \\ 0 & A_{22}(z) \end{pmatrix} = \begin{pmatrix} 1 - a_{11}^{(1)} z - \left(1 - a_{11}^{(1)}\right) z^2 & A_{12}(z) \\ 0 & A_{22}(z) \end{pmatrix}. \end{aligned}$$

The determinant of $A(z)$ is given by

$$\det[A(z)] = \det\left[\left(1 - a_{11}^{(1)} z - \left(1 - a_{11}^{(1)}\right) z^2\right) A_{22}(z)\right].$$

Because $1 - a_{11}^{(1)} - 1 + a_{11}^{(1)} = 0$, it follows that $\det(A(1)) = 0$, which implies that $\det A[(z)] = 0$ has a unit root at $z = 1$ and hence Y_t is not $I(0)$. In fact, the solutions to $A_{11}(z) = 1 - a_{11}^{(1)} z - \left(1 - a_{11}^{(1)}\right) z^2 = 0$ is 1 and $1/(a_{11}^{(1)} - 1) = a^f/(1 - a^f)$, It remains to show that Y_t is not $I(1)$. From (20) it follows that the vector $\beta = (0, b)'$ belongs to the cointegration space. The error-correction coefficients are the elements of $\alpha = (1, 0)'$, due to the Granger non-causality. Therefore $\beta_\perp = (1, 0)'$ and $\alpha_\perp = (0, 1)'$, and $\alpha'_\perp \Gamma \beta_\perp = \Gamma_{21}$. The matrix Γ is given by $\Gamma = -\frac{d}{dz}A(z)|_{z=1} - \Pi$. Because π_t does not Granger cause s_t, $\Gamma_{21} = 0$, proving that Y_t is not $I(1)$. ■

Actually, under fairly general conditions some of the conclusions are valid for a restriction which is *non-exact* in the sense that an additional innovation term is allowed.

Lemma 2 *Under the same assumptions as in Lemma 1, the three conditions*

i)

$$\pi_t = a^f E_t[\pi_{t+1}] + a^b \pi_{t-1} + b s_t + u_t, \tag{23}$$

where u_t is a sequence of innovations, i.e., $E_{t-1}[u_t] = 0$, and where $a^f + a^b = 1$, $a^f \neq 0$,

ii) π_t does not Granger-cause s_t and

iii) $a_{11}^{(1)} \neq (1 - a^f)/a^f$ and $a_{21}^{(1)} \neq -b/a^f$

imply that the determinant of the characteristic polynomial of the VAR-model equals zero at $z = 1$.

Thus, $\{Y_t\}$ is not $I(0)$. If in addition, the roots of $\det[A(z)] = \det[A_{11}(z)]\det[A_{22}(z)] = 0$ *which do not equal 1, have modulus larger than 1, $\{Y_t\}$ is not $I(1)$ either.*

Proof. Leading (23) one lag and using iterated conditional expectations, the restriction from the NKPC takes the following form, because u_t is a sequence of innovations

$$E_t[\pi_{t+1}] = a^f E_t[\pi_{t+2}] + a^b \pi_t + b E_t[s_{t+1}]. \tag{24}$$

Similarly, the conditional expectation $E_t[Y_{t+2}]$ may be expressed as

$$E_t[Y_{t+2}] = (A_1^2 + A_2)Y_t + \cdots + (A_1 A_{k-1} + A_k)Y_{t-k+2} + A_1 A_k Y_{t+1-k}.$$

Combining the two, using the method of "reversed engineering" as in Kurmann (2007) or Fanelli (2008), yields the following restriction, equation (23) in Boug et al. (2010),

$$\sum_{i=1}^{k} a_{21}^{(i)} = \frac{(\sum_{i=1}^{k} a_{11}^{(i)})(1 - a^f a_{11}^{(1)}) - a^b - a^f(\sum_{i>1}^{k} a_{11}^{(i)})}{a^f a_{21}^{(1)} + b}.$$

By Granger non-causality this has to equal 0. Consider first the case where $1 - a^f a_{11}^1 = 0$. Then also $0 = a^b + a^f(\sum_{i>1}^{k} a_{11}^{(i)}) = 1 - a^f + a^f(\sum_{i>1}^{k} a_{11}^{(i)})$, so that $a^f(\sum_{i=1}^{k} a_{11}^{(i)} - 1) = 0$. Because $\det[A(1)] = (\sum_{i=1}^{k} a_{11}^{(i)} - 1)(\sum_{i=1}^{k} a_{22}^{(i)} - 1)$, $\det[A(1)] = 0$.

Consider then the case where $1-a^f a^1_{11} \neq 0$ so that $\sum_{i=1}^k a_{11}^{(i)} = [a^b + a^f(\sum_{i>1}^k a_{11}^{(i)})]/(1-a^f a_{11}^{(1)})$. Then, because $a^f + a^b = 1$,

$$\sum_{i=1}^k a_{11}^{(i)} - 1 = \frac{a^b + a^f(\sum_{i>1}^k a_{11}^{(i)}) - 1 + a^f a_{11}^{(1)}}{1 - a^f a_{11}^{(1)}} = \frac{a^f(\sum_{i=1}^k a_{11}^{(i)} - 1)}{1 - a^f a_{11}^{(1)}}.$$

Hence

$$(1 - a^f a_{11}^{(1)})(\sum_{i=1}^k a_{11}^{(i)} - 1) = a^f(\sum_{i=1}^k a_{11}^{(i)} - 1)$$

so

$$(\sum_{i=1}^k a_{11}^{(i)} - 1)(1 - a^f a_{11}^{(1)} - a^f) = 0,$$

which implies that $(\sum_{i=1}^k a_{11}^{(i)} - 1) = 0$ and $\det[A(1)] = (\sum_{i=1}^k a_{11}^{(i)} - 1)(\sum_{i=1}^k a_{22}^{(i)} - 1) = 0$. Thus in both cases Y_t is not $I(0)$.

The homogeneity assumption $a^f + a^b = 1$ implies that (24) may be written

$$a^f E_t[\Delta\pi_{t+2}] - a^b E_t[\Delta\pi_{t+1}] + bE_t[s_{t+1}] = 0.$$

Hence $(0, b)$ is also now a cointegration vector, and that Y_t is not $I(1)$ follows as in Lemma 1. ■

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