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In the Shadow of the Labour Market

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In the Shadow of the Labour Market*

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Abstract

Why do not people evade more taxes when their gain from evasion is higher than the expected penalties? Why does only a small minority evade when a large majority is willing to? These tax evasion puzzles are explained in a labour market framework where employees may combine reported work in firms with self-employed shadow work. On the margin, time spent on self-employed work reduces labour productivity in reported work. This creates an equilibrium where small, low-productive firms offer jobs with low wage rates but time for self-employed shadow work, while larger, more efficient firms offer jobs with higher reported wage rates but no time for shadow work. Improving the tax morale may not reduce evasion but only sort the honest people into jobs with no time for shadow work. Shadow work leads to an inefficient allocation of employees since it has the effect of a subsidy to low productive firms. Both lower taxes and minimum wages reduce evasion and improve labour allocation but harms low productive firms

Keywords: Shadow work, Tax evasion, Labour market
JEL classifications: H26,J29,K24

Highlights:
I model a labour market with opportunities for self-employed shadow work
The model explains the puzzle of why people do not evade more
Shadow work acts like a subsidy to low productive firms
Improving peoples tax morale may not reduce total tax evasion
Minimum wages may reduce evasion and improve labour allocation

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1 Introduction

The debt crisis in EU reminds us of the importance of a fair and efficient tax system. With a large shadow sector, as in Greece and Italy, it may simply be difficult to finance a modern welfare state. Shadow production reduces the tax base, and the need to hide may reduce total production. Yet, many aspects of the shadow economy is still poorly understood, and this makes it difficult to evaluate its effects and to know what policies that will reduce it. Paradoxically, the main puzzle in the tax evasion litterature has been that people seem to evade too little. In particular, a large majority does not evade at all. Follwing the standard portfolio-choice model of tax evasion, pioneered by Allingham and Sandmo (1972), the individual should choose the evasion that maximizes his expected utility. However, for reasonable values of the detection probability, penalty tax and risk aversion, peoples gain from evasion seems to exceed their expected penalties (Dhami and al-Nowaihi, 2007).

Many attempts to solve the tax evasion puzzle have focused on non-economic costs of tax evasion, such as costs of breaking social norms (Myles and Naylor, 1995 and Fortin et al, 2007) or religious norms (Torgler, 2006), fairness considerations (Spicer and Becker, 1980 and Bordignon, 1993) and guilt, shame and stigma (Erard and Feinstein (1994)). In their study of differences in tax morale across countries, Alm and Torgler (2006) conclude that tax morale, or "the intrinsic motivation to pay taxes", might help explain the puzzle of why so many individuals pay their taxes". While non-economic considerations may be important for peoples willingness to evade, several studies indicate that opportunities for evasion may be what determines actual evasion: While norms against tax evasion do not vary systematically with firm size and firm productivity, workers in large, productive firms tend to evade less than workers in small, less productive firms (Barth and Ognedal , 2010).

Employees typically have no opportunities to evade the tax on their wage incomes, since these are reported by their employers. Taxes on incomes from self-employment, however, can be evaded without being detected wth certainty. Estimates based on data from the National Research Program in US find only 1 percent underreporting for wage incomes, but 57 percent for self-employment business income (Slemrod, 2007). The same pattern was found in a large randomized tax enforcement experiment from Denmark, analysed in Kleven et al (2010). With the low tax evasion rates of wage incomes, overall tax evasion rate is low despite a high tax evasion rate for self-reported incomes. Thus, tax evasion seems to be low because most people choose jobs where they are unable to cheat on taxes, not because they are unwilling to. The next question is then why people choose jobs where they are unable to cheat: If expected gains from evasion exceed the costs, and people are willing to evade, why do they not choose more shadow work as self-employed?

I address this question in a framework where all individuals have access to self-employed shadow work, but need to be employed in firms where incomes are third party reported. In a crude way this captures the premise that a job in a regular firm improves the opportunities for shadow work, for example by providing access to potential customers. Typcally, the employee may be allowed to do self-employed shadow work in between his reported work for the firm. The firm determines how much shadow work the employee can do by determining his reported tasks. For example, a carpenter that rebuilds a house for a customer of the firm, may do maintenance on the customer's garage as shadow work. The time left for shadow work is determined by how fast his employer expects him to finish the rebuilding and get on to new customers.

An employee may be willing to accept a lower reported wage in a job can be combined with self-employed shadow work, which means the job must not require too many reported work hours.[1]. When work hours per employee matters for his labour productivity in the firm, the employer faces a trade-off between low wages and high labour productivity: By offering jobs with time for shadow work, the firm pays lower reported wages but its employees are less productive. The higher the firm-productivity, the larger is the loss from inefficiently short work days. As a result, high-productive firms offer jobs with less time for shadow work and higher reported wages that smaller, less productive firms. In equilibrium, employees are indifferent between the high-wage jobs with little or no oppportunities for evasion in high productive firms and the low-wage jobs with more opportunities for evasion in low productive firms.

A key result in the paper is that workers may differ in how much they evade although they do not differ in tax morale and other characteristics. In particular, a large majority may not evade at all even though they are as willing to evade as those who do it. Employees in large productive firms evade less than those in smaller, less productive ones. These results are in line with the empirical finding in Barth and Ognedal (2010) that employees evade less the larger and more productive the firm, other things equal. An important policy implication is that campaigns to improve peoples tax morale may have no effect on tax evasion even if they succeed in improving peoples tax morale.

Another key result is that shadow work distorts the allocation of employees because it acts like a subsidy to low-productive firms. The reason is that shadow work lowers the labour costs of the least productive firms relative to the more productive firms. As a result, too many low-productive firms survive and they attract too many employees. I discuss how lower taxes or minimum wages may reduce shadow work. Paradoxically, low productive firms lose from lower taxes and all employees lose from minimum wages.

In my framework, a worker's net expected revenue from self-employed shadow work exceeds his net reported wage. This implies that his gain from tax evasion exceeds his expected penalty, the socalled tax evasion puzzle. Thus, there seem to be a violaton of the arbtrage principle, since workers do not choose jobs with more opportuntes for shadow work. The explanaton is that the employer captures part of the gain from the self-employed shadow work of their employees by lowering the reported wages. They can do this because workers need a job in a regular firm. Moving to a job with more opportunties for shadow work means lower reported wages, with net expected income unchanged.

[1]The official reported work hours may not vary between firms even though the effective hours spent on reported work does

2 Explaining the tax evasion puzzle

Consider an economy where output in produced either in firms that report all their incomes or by self-employed shadow work.

2.1 The model

Each individual supplies one unit of labour, which he may divide between reported work and shadow work. He is risk neutral and have no moral cost of doing shadow work, but for the same net expected income, he prefers reported work. A crucial assumption is that everyone needs a job in a regular firm. This captures the idea that working in a regular firm makes it easier to find shadow work, and also lowers the probability of a tax audit. The number of reported work hours can be arbitrarily close to zero, however, which means that there is no exogenous constraint on the number of shadow work hours.

Each individual also decides whether or not he will become an entrepreneur, i.e. a firm owner. While all individuals have the same labour productivity, they differ with respect to entrepreneurial talent. Entrepreneurial talent increases the revenue in a regular firm, but not the revenue from self-employed shadow work. The decision to become a firm owner or not is then a simple one: Those who have entrepreneurial talent enough to run a firm with positive profit become owners, the others only workers. For convenience, I assume that being an entrepreneur requires no labour effort, such that firm owners also sell one unit of labour. This implies that there is a fixed supply of employees, and that one can separate the individual's employment decision from his decision to become an owner or not.

Self-employed shadow work is independent of the worker's entrepreneurial talent and the number of work hours. In a crude way this captures the fact that self-employed shadow work is often less organized than work in firms and uses less capital since it needs to be hidden (De Paula, A. and J.A.Sheinkman, 2008). Let s^g be the gross revenue from shadow work per unit of labour. With probability q there is an audit, and all evaded revenue is detected and penalized with a penalty tax rate τ which is higher than the regular tax rate t. The interesting case is where the regular tax rate exceeds the expected penalty, i.e. $t > q\tau$, such that evading is profitable

The revenue of a firm is increasing in the entrepreneurial talent of the owner, the number of employees and work hours per employee. I use the simple revenue function $\theta y(h)n$, the revenue of an employee that works h hours times the number of employees n. The parameter θ is the entrepreneurial talent of the owner. Its cumulative distribution function is $F(\theta)$. Revenue per work hour, $\theta y(h)/h$, is first increasing and thereafter decreasing in h. Marginal revenue of work hours, $\theta y'(h)$ is positive and decreasing in h. I assume decreasing returns to employment and model this as an increasing, convex cost of employment, $b(n)$.

Let w_i be the reported wage rate offered by firm i. With a profit tax rate t, the same as for labour income, the net profit of a firm i is

$$\pi_i = (1-t)\{[\theta_i y(h_i) - w_i)]n_i - b(n_i)\} \tag{1}$$

The term in the square bracket is revenue per employee. Since work hours per employee matters for revenue per employee, the firms offer jobs with a specified number of reported work hours, h . This leaves the employee with opportunity to do self-employed shadow work for $1-h$ hours if he accepts a job in firm i. The job offers may differ between firms. Thus, the employees' opportunities for shadow work also differ between firms.

Consider now the employment decision of an individual: A job in firm i gives him a net reported income $(1-t)w_i h_i$. In the hours he does not work in firm i he can either take a job in another firm or do shadow work at net expected revenue s. Let v denote the highest net expected income he can get per unit of labour in alternative jobs. The maximum net expected labour income if he accepts a job in firm i is then $h_i(1-t)w_i + (1-h_i)max[s,v]$. He compares job offers and chooses the one that gives him the highest net expected labour income.

Since all workers have the same labour productivity, all jobs must give the same net expected labour income v in equilibrium, also referred to as the value of a job. Firm i must therefore offer a combination of reported wage rate and work hours such that the value of a job in the firm equals v, i.e.

$$v = h_i(1-t)w_i + (1-h_i)max[s,v] \tag{2}$$

This no-arbitrage condition is crucial for the results of the model. It implies that there can only be shadow work in an equilibrium where the expected shadow revenue exceeds the value of a job, i.e. $s > v$. To see why, consider the case where $s \leq v$. Since shadow work do not pay more than reported work, employees prefer reported work. Equation (2) then implies that all firms pay the same wage, $w = v/(1-t)$. In the rest of the paper, I focus on the equilibriums with shadow work, i.e. where $s > v$.

When $s > v$, the no-arbitrage condition (2) can be rewritten as

$$w_i = \frac{1}{1-t}[s - \frac{s-v}{h_i}] \tag{3}$$

Since the reported wage rate is an increasing function of reported work hours, h, a firm may lower the reported wage rate by reducing the number of reported work hours. With fewer reported work hours, the employee has more time for shadow work, which compensates him for the lower wage rate. The firm gains from this as long as the wage reduction exceeds the lost revenue from fewer work hours per employee.

To see the firm's tradeoff between wages and work hours, insert w from (3) into the profit function (1). This gives

$$\pi = [(1-t)\theta y(h) + s(1-h) - v]n - (1-t)b(n) \tag{4}$$

The first two terms in the square bracket is the total revenue from one unit of labour: The net revenue from h hours of reported work plus net expected revenue from $1-h$ hours of shadow work. Thus, it is *as if* the firm receives the entire revenue, reported and unreported, and pays each employee his alternative income v. Since the production function is separable in h and n, the optimal h is independent of n and vice versa.The firm chooses the number of reported work hours (h) that maximizes the total revenue per employee, reported plus unreported. Next, it chooses the number of employees(n) such that the maximized revenue per employee equals the marginal cost of employment.

Figure 1 below illustrates the optimal choice of reported work hours. The three decreasing curves depicts the net marginal revenue from work hours for firms with productivity parameters θ_A, θ_B and $\bar{\theta}$ The horisontal line is the expected shadow revenue s. For the firm with productivity $\bar{\theta}$, the marginal gain from reported work equals the shadow revenue s for full time reported work, i.e. for $h = 1$. Thus, the firm offers jobs with full-time reported work, and therefore no time for shadow work. For firms with productivity above $\bar{\theta}$, such as the one with θ_A , the marginal gain from from full time reported work is higher than the shadow revenue s. Thus, firms with $\theta > \bar{\theta}$ offer jobs with full-time reported work. For firms with productivity below $\overline{\theta}$, such as the one with θ_B , the marginal gain from full time reported work is lower that the shadow revenue s. Thus, firms with $\theta < \bar{\theta}$, such as firm B, offer jobs with less than full time reported work, i.e. $h < 1$. The optimal number of work hours in a firm with $\theta < \bar{\theta}$ is determined by equality between the marginal revenue from reported work and shadow work, i.e. by $(1-t)\theta y'(h) = s$. With less than full time reported work, these jobs give time for shadow work. The lower θ is, the lower is the optimal number of reported work hours, h, and the higher the number of shadow work hours.

Figure 1: The choice of reported work hours (h) in firms with different productivity parameters (θ).

It follows from(3) that when firms' reported work hours differ, their reported wage rates also differ. All firms with full time reported work offer the same wage rate $v/(1-t)$. Firms that offer jobs with less than full time reported work, offer lower wage rates. The reported wage rate is decreasing in their productivity parameter.

2.2 Equilibrium

Maximizing the profit, as given by (4), with respect to work hours (h) and employment (n), gives us the optimal number of work hours and demand for employees from each firm i. It is easily verified that a firm's demand for employees is a function of v,s and t . Also, v,s and t determines the critical value of entrepreneurial talent necessary to be a firm owner, θ_0, and thereby the number of firms. Thus, the total demand for employees is a function of v,s and t . Total labour supply is fixed, since all individuals, including firm owners, supply one unit of labour. Equality between the total demand for employees and the fixed supply determines the value of a job, v. Consequently, the equilibrium values of n_i, h_i, w_i, θ_0 and v are functions of s and t. In appendix A I derive the equilibrium and show that it can be characterized as follows:

Proposition 1 *In an equilibrium where the expected revenue from shadow work (s) exceeds the net reported wage rate for all employees*
(i) individuals with entrepreneurial talent above a critical level θ_0 *organize firms. Those with* $\theta < \theta_0$ *become employees only.* θ_0 *is increasing in* t *and decreasing in* s.
(ii) firms can be separated into two groups, A and B. A-firms, with productivity of at least $\overline{\theta}$, *offer jobs with full time reported work, and therefore no time for shadow work. B-firms, with productivity below the critical level, offer jobs with time for shadow work. Time for shadow work in a B-firm is decreasing in firm productivity, and increasing in shadow revenue (s) and tax rate (t). The critical level* $\overline{\theta}$ *is increasing in* s *and* t.
(iii) the total net expected labour income,v, is increasing in s *and decreasing in* t.

The proof is given in Appendix A.

2.3 The tax evasion puzzles

Proposition 1 suggests a pattern of tax evasion in line with observed patterns of tax evasion implying that the so-called tax-evasion puzzle is no puzzle at all:

1. In an equilibrium with shadow work, the individual's expected shadow revenue exceeds his net reported wage rate.

As shown, an equilibrium with shadow work requires the expected shadow revenue must exceed the value of a job, $s > v$. From (2), it then follows that $s > (1-t)w$, i.e. that the expected shadow revenue exceeds the net reported wage. This is the socalled tax evasion puzzle, as it seems to imply that there is a gain from arbitrage: If the expected shadow revenue exceeds the net reported wage, employees should replace reported work with shadow work. The reason why this does not happen is that an employee cannot choose the number of reported work hours in a given job, and since all employees need a job in a firm, the employers determine the work hours as long as they match the outside option, v. The employee may switch to a job with more opportunities for shadow work, but as long as all jobs yield the same net income v, he is indifferent between them. A job which offers more shadow work pays correspondingly lower reported wages.

One may then ask why it does not pay for the firm to let their employees do more shadow work when the employees would gain. The explanation is that the firm chooses the reported work hours that maximizes total revenue from labour, reported plus unreported. Thus, there is no total gain from replacing reported work with shadow work for employees and employer taken together. The reason why an employee would gain if he could replace reported work with shadow work for given w, is that he collects the full revenue s from shadow work but shares the reported revenue with his employer. Thus, the gap between the employees expected gain from shadow work and his expected penalties is the employers' gain from the shadow work, in the form of lower reported wages.

Two assumptions are crucial for this result: First, that all workers need to be employed in a regular firm to get access to self-employed shadow work. Second, that work hours per employee matters for revenue per worker. To see this, consider what would happen if one of these conditions did not hold: If workers did not need to be employed in a regular firm, they would not accept a net reported wage below the expected shadow revenue. Thus, $s > v$ could not be an equilibrium outcome, since all individuals would then choose full-time self-employed shadow work. To attract employees, the firms would have to raise their reported wages to match the shadow revenue. Next, if hours per employee did not matter for revenue per worker, the employee could choose as many or few work hours he wanted at the going market wage. If the shadow revenue exceeded the net reported wage, the employees would ask for less reported work hours. Again, to attract workers the firm would have to raise their reported wages to match the shadow revenue.

2. Tax evasion differs between people who are equal in tax morale and productivity and who face the same expected penalties.

Proposition 1 implies that some workers evade, those in A-firms, and some do not, those in B-firms. However, the workers in A- and B-firms do not differ in either tax morale, productivity or expected penalties. The explanation for this outcome is the no-arbitrage condition, equation (2): Although the jobs differ with respect to reported wages and time for shadow work, the net expected income (v) is the same in all jobs. Workers in firms who pay high wages have little or no time for shadow work, while those in low-wage jobs have more time for shadow work. The time for shadow work compensates for low reported wage rate.

3. Only a small minority may do shadow work although a large majority wants to do it.

The employees in A-firms do no shadow work since it is optimal for these firms to require full time reported work. Thus, if A- firms employ a majority of the workers, prediction 3 follows directly: All workers want to do shadow work, since the expected shadow revenue exceeds their reported wage, but only the minority employed in B-firms actually does it. Lower shadow revenue (s) increases the fraction that does no shadow work and decreases the fraction that does. First, as s goes down some B-firms become A-firms since shadow work becomes relatively less profitable. Formally, the productivity parameter $\bar{\theta}$ that divides between A- and B-firms , determined by $\bar{\theta}y'(1) = s$, goes down with s. Second, a lower s reduces the equilibrium value of a job, v , and thereby increases employment in A-firms.

4. Workers in large, efficient firms evade less than those in small, inefficient firms even if the shadow work is not organized by the firms.

This prediction follows directly from Proposition 1 (iii): Since firms require more work hours per employee the higher the firm productivity θ , an employee has less time for shadow work the higher the firm productivity. The intuitive explanation why jobs in large, efficient firms leave less time for shadow work is the following: More shadow work means fewer reported work hours per employee, which in turn lowers the productivity per work hour. The drop in hourly productivity is larger the more productive the firm, i.e. the higher is. Large, efficient firms therefore offer jobs with little or no time for shadow work, while smaller, less efficient firms offer jobs with time for shadow work.

Points 2, 3 and 4 are in line with the empirical findings in Barth and Ognedal (2010). Using Norwegian survey data, they find that evasion per worker is increasing in firm size- and productivity, controlling for norms, probability of detection and a host of other factors. Also while a large majority are willing to evade, only a minority actually does.

5. Improved tax morale of a group of individuals may not reduce tax evasion.

Several studies indicate that tax morale matters for peoples willingness to evade or not (Cummings et al, 2009). However, even if tax moral affects peoples willingness to evade, it may not affect the total tax evasion in the economy. To see why, consider a campaign that succeeds in making a group of workers law obedient in the sense that they will not do shadow work. However, as long as the number of law abiding workers are lower than the total number of employees in A-firms, their improved tax morale does not affect the equilibrium outcome: With a reshuffling of workers between jobs, all the law abiding workers will be employed in the A-firms, while workers who are not law abiding will be employed in B-firms. Since A-firms offer jobs with no time for shadow work even if no workers were law abiding, this reshuffling of workers take place with no changes in wages. The equilibrium labour income v stays the same, and so does then the other endogenous variables. Consequently, the amount of shadow work does not change.

3 The harm from shadow work

Shadow work acts like a subsidy to the least productive firms. The reason is that a firm's gain from shadow work is higher the lower its firm productivity parameter, θ. This leads to an inefficient allocation of labour. To see how, consider an increase in the shadow revenue s caused by lower expected penalties, such that the social value of shadow work is not affected. First, since higher shadow revenue lowers the labour costs of B-firms, the demand for labour from B-firms goes up. This raises the net expected labour income, v, to equal supply and demand for labour. Since A-firms pay higher wages but do not gain from the shadow work, their demand for labour is reduced. As a result, employees are reallocated from A-firms to the less productive B-firms. Also, the higher shadow revenue makes it optimal for B-firms to offer contracts with a lower reported wage but more opportunities for shadow work. Finally, since a higher s makes the labour contracts of B-firms more profitable, more A- firms become B-firms and more B-firms survive. Both changes lead to more shadow work at the expense of the more productive work in regular firms and we have:

Proposition 2 *A lower expected penalty for shadow work leads to a less efficient al-*

location of labour. First, employees are reallocated to firms with lower marginal labour productivity. Second, work hours are reallocated from reported work to the less efficient shadow work. Third, more socially inefficient firms survive.

The proof is in appendix B.

Since higher shadow revenue reduces the labour cost of B-firms, it increases the total demand for employees and therefore the net expected labour income v. Thus, all workers benefit, including those who do no shadow work. Whether or not an owner gains, depends on his entrepreneurial talent θ. Owners of A-firms lose, since the only effect of higher shadow revenue on A-firms is that they pay higher wages, since v goes up. Owners of B-firms lose from higher v but gain from the higher shadow revenue of their employees. The gain from higher s is larger the lower the firm productivity , since employees do more shadow work the lower is. The gain from higher s exceeds the loss from higher v for B-firms with productivity below a critical value. Thus, we have:

Proposition 3 *A lower expected penalty for shadow work increases the net expected labour income (v) for all workers. Profit goes up in B-firms with productivity below a critical value and down in all firms with productivity above.*

The proof is in appendix B.

4 Policy implications

Since a lower tax rate reduces the gain from evasion, it is no surprise that it reduces tax evasion and improves the allocation of employees between firms. Paradoxically, a lower tax rate also reduces the profit for some firms, the least productive ones. Other policies to reduce tax evasion, may also have paradoxical effects when we take the labour market effects into account: Minimum wages,which is often accused of inducing more shadow work, may in fact lead to less shadow work and also improve the labour allocation. The effects of lower tax rate and of minimum wages are demonstrated below.

4.1 Taxes

A lower tax rate improves allocation by making shadow work relatively less profitable. First, lower taxes leads to less shadow work, which by assumption is less productive than reported work: Some B-firms become A-firms, and B-firms offer contracts with less opportunity for shadow work. Also, lower tax rate reduces labour costs in high-productive firms relative to low productive ones. The reason is that employees in high productive firms pay more taxes. A reduction in labour costs of high productive firms relative to the low productive ones, leads to a reallocation of workers from low productive to high productive firms. A lower tax rate increases total demand for labour since labour costs go down. In the new equilibrium, the workers net labour income v is therefore higher.

A paradoxical result is that low productive firms lose from lower taxes. The reason is that their loss from lower market wages (v) exceeds their gain from lower taxes. There are two reason why taxes harm low productive firms less than high productive ones: First, their employees only pay taxes on a small fraction of their income. Second, their profit tax is low because their profit is low. To sum up:

Proposition 4 *A lower tax rate (t) leads to a more efficient allocation of labour. Net labour income goes up. Profit goes down in B-firms with productivity below a critical level, and up in other firms.*

The proof is in appendix C.

4.2 Minimum wages

A well known effect of minimum wages, explored by Cuff et al (2011) ,Tonin (2007) and others, is that they may increase tax evasion because they make it more attractive for firms to go underground. My framework captures an effect that goes in the opposite direction: Minimum wages make it less attractive for firms that are not underground to offer jobs that can be combined with shadow work.

Consider a minimum wage that binds for at least some B-firms. Since a firm's reported wage is lower the lower its productivity, the minimum wage binds for the least productive firms. These B-firms can no longer offer jobs with lower wage rates in exchange for more time for shadow work. As a result, they cannot gain from from the shadow work of their employees, and will therefore demand more reported work hours. Moreover, with less profitable labour contracts, they reduce their demand for labour. Reduced demand for labour leads to a lower v, which in turn leads to a reallocation of labour from the low productive firms where the minimum wage binds, to the more productive firms where it does not. Thus, by reducing the ability to profit from shadow work , a minimum wage may in fact improve labour allocation. Paradoxically, all workers are worse off with a minimum wage, since it lowers the equilibrium net labour income v. Hence, we have:

Proposition 5 *Minimum wages reduces shadow work and improve labour allocation, but makes all workers worse off.*

The proof is in appendix D.

If firms can go underground, minimum wages then have two opposing effects on tax evasion: It makes it more attractive for firms to go underground but reduces shadow work among the firms that do not. The net effect depends on how easily firms can establish themselves underground and how much going underground reduce their productivity.

5 Concluding remarks

Most people evade little or no taxes although the expected gain seem to exceed the cost. This so-called tax evasion puzzle is explained in a framework where individuals may combine reported work in regular firms with self-employed shadow work outside the firms. The individuals trade off the gain from tax evasion against the expected penalties, in line with the standard portfolio choice model, pionered by Allingham and Sandmo (1972). The novelty lies in modelling their opportunities for tax evasion in the labour market by doing self-employed shadow work.

Two assumptions are crucial in my framework: First, individuals need a job in a regular firm. Second, labour productivity increases with work hours per employee. Together, these assumption imply that employees face a trade-off between reported wages and opportunities to combine reported work with self-employed shadow work: Large, high productive firms offer high wage jobs with long work hours and little or no opportunities for shadow work. Small, low productive firms offer low wage jobs with short work hours and therefore opportunities for shadow work. In equilibrium, workers are indifferent between the jobs.

The predictions of the model are in line with empirical observations related to the tax evasion puzzle: First, the expected revenue from shadow work exceeds the net reported wage in equilibrium. Second, workers may differ in how much they evade although they do not differ in tax morale and other characteristics In particular, a large majority may not evade at all even though they are as willing to evade as those who do it. Finally, employees in large high productive firms evade more than those in small, low productive firms even though the firms are not involved in the evasion.

There are several studies of how differences in individual characteristics may lead to differences in tax evasion. For example, Boeri and Garibaldi (2005) discuss the sorting of low-skilled workers into shadow jobs. A core result of my model is that differences in tax evasion between people may also arise even if individuals where equal in such characteristics as labour productivity and tax morale. In my model, the differences in tax evasion between the individuals are caused by different opportunities to combine their regular job with self-employed shadow work. An important implication is that differences in characteristics, such as tax morale, may have no effect on the level of evasion in the economy even though it determines whether or not an individual is willing to evade. For example, I demonstrate how improving the tax morale of a group of individuals may have no effect on evasion. The reason is that their preference for honesty may be accomodated by employing them in firms that do not offer jobs with time for shadow work.

The shadow work acts like a subsidy to the lowest productive firms. This leads to an inefficient labour allocation. Too many low-productive firms (B-firms) survive and too many employees are employed in low productive firms. Also, the employees in these firms spend time on self-employed shadow work, which by assumption is less productive that their work in regular firms. Both lower taxes and minimum wages reduce tax evasion and improves labour allocation, since they reduce the value of shadow work.

The reason why an individual's expected gain from tax evasion exceeds his expected penalty is that the firm captures part of the gain from his evasion by paying lower wages. The reason why the firms can capture part of the gain from the self-employed shadow work is that all individuals need a job in a regular firm. A job in a regular firm gives access to potential shadow customers and makes an audit less likely, which means it has a value above the reported wage. The workers "pay" for the access to shadow work in the form of lower reported wages. If a job in a regular firm had no value above the reported wage, firms could not offer jobs with reported wages below the expetced shadow revenue, and the gain from evasion would be equal to the cost at the margin.

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Appendix A: Proof of proposition 1

To determine the optimal work hours in firms with different productivities, we define a critical productivity level $\bar{\theta}$ by $(1-t)\bar{\theta}y'(1) = s$. It is immediate that $\bar{\theta}$ is increasing in s and t. In firms with $\theta \geq \bar{\theta}$, called A-firms, $(1-t)\theta y'(1) \geq s$. It is therefore optimal for A-firms to offer jobs with $h_A = 1$, i.e. full time reported work. In firms with $\theta < \bar{\theta}$, called B-firms, $(1-t)\theta y'(1) < s$. It is therefore optimal for B-firms to offer jobs with $h_B < 1$, where h_B is determined by

$$(1-t)\theta y'(h_B) = s \tag{A.1}$$

Since $y''(h) < 0$, h is increasing in θ and decreasing in s and t, i.e. $h_B = h_B(s, t; \theta)$.

The profit for A- and B-firms, given by (4), can be written as

$$\pi_A = [(1-t)\theta y(1) - v]n_A - (1-t)b(n_A) \tag{A.2}$$

$$\pi_B = [(1-t)\theta y(h_B) + s(1-h_B) - v]n_B - (1-t)b(n_B) \tag{A.3}$$

The demand for employees from A- and B-firms are found by differentiating the profit with respect to n. The first order condition are

$$\frac{d\pi_A}{dn_A} = (1-t)[\theta y(1) - \frac{v}{1-t} - b'(n_A)] = 0 \tag{A.4}$$

for A-firms

$$\frac{d\pi_B}{dn_B} = (1-t)[\theta y(h_B) - \frac{v - s(1-h_B)}{1-t} - b'(n_B)] = 0 \qquad \text{(A.5)}$$

for B-firms, where $h_B = h_B(s, t; \theta)$. From (A.4) and (A.5), the demand for employees from both A-firms and B-firms is decreasing in v and t and increasing in θ. The demand for employees from B-firms is also increasing in s.

With free entry, the number of firms in the market is determined by zero profit for the marginal firm, i.e.

$$\pi_0 = [(1-t)\theta_0 y(h_0) + s(1-h_0) - v]n_0 - (1-t)b(n_0) = 0 \qquad \text{(A.6)}$$

Since the marginal firm is a B-firm, h_0 is determined by $(1-t)\theta y'(h_0) = s$. Equation (A.6) then determines the productivity θ_0 of the marginal firm as a function of v, s and t. θ_0 is increasing in v. For given v, θ_0 is increasing in t and decreasing in s.

Together (A4)-(A.6) gives us the total demand for employees as a function of v, t and s. With a constant total supply of employees, M, the labour market equilibrium condition is then

$$\int_{\theta_0(v,s,t)}^{\bar{\theta}(s,t)} n_B(v, s, t; \theta) dG(\theta) + \int_{\bar{\theta}(s,t)}^{\theta^{max}} n_A(v, t; \theta) dG(\theta) = M \qquad \text{(A.7)}$$

The left hand side is the total demand for employees from all firms in the market. (A.7) determines the value of a job, v, that makes the demand equal to the supply .Inserting v into (A.4) and (A.5) determines n_A and n_B.

To find the effects of changes in s and t on v, we write (A.7) as $N(v; s, t) = M$ and differentiate with respect to factor j, where $j = s, t, v$. This gives

$$\frac{\partial N}{\partial v}\frac{dv}{dj} + \frac{\partial N}{\partial j} = 0 \qquad \text{(A.8)}$$

Partial differentiation of $N(v; s, t)$ yields

$$\frac{\partial N}{\partial s} = -n_0 g(\theta_0)\frac{\partial \theta_0}{\partial s} + \int_{\theta_0}^{\bar{\theta}} \frac{\partial n_B}{\partial s} dG(\theta) \qquad \text{(A.9)}$$

$$\frac{\partial N}{\partial t} = -n_0 g(\theta_0)\frac{\partial \theta_0}{\partial t} + \int_{\theta_0}^{\bar{\theta}} \frac{\partial n_B}{\partial t} dG(\theta) \int_{\bar{\theta}}^{\theta^{max}} \frac{\partial n_A}{\partial t} dG(\theta) \qquad \text{(A.10)}$$

$$\frac{\partial N}{\partial v} = -n_0 g(\theta_0)\frac{\partial \theta_0}{\partial v} + \int_{\theta_0}^{\bar{\theta}} \frac{\partial n_B}{\partial v} dG(\theta) \int_{\bar{\theta}}^{\theta^{max}} \frac{\partial n_A}{\partial v} dG(\theta) \qquad \text{(A.11)}$$

From (A.4), (A.5) and (A.6) we find the sign of the partial derivatives for θ_0 ,n_A and n_B: $\partial\theta_0/\partial v > 0$,$\partial\theta_0/\partial s < 0$,$\partial\theta_0/\partial t > 0$,$\partial n_A/\partial s = 0$,$\partial n_B/\partial s > 0$,$\partial n_i/\partial v < 0$ and $\partial n_i/\partial t < 0$ for $i = A, B$. This implies that $\partial N/\partial s > 0$, $\partial N/\partial t < 0$ and $\partial N/\partial v < 0$.

Appendix B: Proof of proposition2 and 3

Proposition 2: The effects of lower expected penalties on total social revenue

We study the effect of lower expected penalties, $q\tau$. Thus, the private value of shadow work s goes down while the social value s^g is unchanged. To prove the effect of a change in s on social revenue, it is useful to first derive the effect on v,n_A,n_B and θ_0.

The labour market equilibrium condition $N(v; s, t) = N$ implies that

$$\frac{\mathrm{dv}}{\mathrm{d}s} = -\frac{\partial N/\partial s}{\partial N/\partial v} = -\frac{-n_0 g(\theta_0)(\frac{\partial\theta_0}{\partial s}) + \int_{\theta_0}^{\bar{\theta}} \frac{\partial n_B}{\partial s} dG(\theta)}{-n_0 g(\theta_0)(\frac{\partial\theta_0}{\partial v}) + \int_{\theta_0}^{\bar{\theta}} \frac{\partial n_B}{\partial v} dG(\theta) + \int_{\bar{\theta}_0}^{\theta^{max}} \frac{\partial n_A}{\partial v} dG(\theta)} \tag{B.1}$$

From (A.4) and (A.5) we find the partial derrivatives $\partial n_B/\partial s = -\left[1 - h_B(\theta)\right]\partial n_B/\partial v$ and $\partial n_B/\partial s = -\left[(1 - h_0(\theta)\right]\partial\theta_0/\partial v$. Equation (B.1) can then be rewritten as

$$\frac{\mathrm{d}v}{\mathrm{d}s} = -(1 - h_0)\lambda \tag{B.2}$$

where

$$\lambda = -\frac{n_0 g(\theta_0)\frac{\partial\theta_0}{\partial v} - \int_{\theta_0}^{\bar{\theta}} \frac{1-h_B}{1-h_0}\frac{\partial n_b}{\partial v}}{\partial N/\partial v} \tag{B.3}$$

Since the fraction $(1 - h_B)/(1 - h_0)$ goes from 1 to 0 as θ goes from θ_0 to $\hat{\theta}$, and $\partial n_A/\partial v > 0$,the numerator is lower than the denominator in absolute value, and so $\lambda < 1$.From [B.2), this implies that $dv/ds < 1 - h_0$. Thus, we can conclude that $1 - h_0 < \lambda < 1$

From (A.4), n_A is decreasing in v and independent of s. Since $dv/ds > 0$, it follows that n_A goes up when s goes up. The effect on n_B is less thraightfdorward: From (A.5) we get $sgn(dn_B/ds) = sgn[(1 - h_B) - dv/ds]$. Let $\tilde{h}$ be determined by

$$(1 - \tilde{h_B}) - \frac{\mathrm{d}v}{\mathrm{d}s} = 0 \tag{B.4}$$

and let $\tilde{\theta}$ be the productivity level that makes $\tilde{h}$ the optimal choice of reported wprk hours, i.e. $\tilde{h}$ is determined by $\tilde{\theta}f'(\tilde{h}) = s$. Since $0 < dv/ds < 1 - h_0$, it follows that $h_0 < \tilde{h} < 1$, and consequently that $\theta_0 < \tilde{\theta} < \bar{\theta}$ .It follows from (B.2) and (B.3) that firms with $h_0 < h < \tilde{h}$ increase their employment as s goes up,while firms with $h > \tilde{h}$ reduce their employment. This implies that $dn_B/ds > 0$ for firms with $\theta_0 < \theta < \tilde{\theta}$ and $dn_B/ds < 0$ for firms with $\theta > \tilde{\theta}$. To sum up, we have proven that dn_A/ds, $dn_B/ds < 0$ for $\theta > \tilde{\theta}$ and $dn_B/ds > 0$ for $\theta < \tilde{\theta}$.

To find the sign of $d\theta_0/ds$ we differentiate (A.6) with respect to s. This gives us $sgn(d\theta_0/ds) = sgn[dv/ds - (1 - h_0)]$. Since$1 - h_0 < \lambda < 1$, $d\theta_0/ds < 0$.

To prove that a higher s leads to a less efficient labour allocation, I prove that the social revenue goes down as s goes up. Let R_B denote the social revenue from a B-firms and R_A the social revenue from an A-firm, i.e.

$$R_A = \theta y(1)n_A - b(n_A) \tag{B.5}$$

$$R_B = [\theta y(h_B) + s^g(1-h_b)]n_B - b(n_B) \tag{B.6}$$

where s^g is the exogenous gross shadow revenue. Total social revenue is then

$$R = \int_{\theta_0(v,s)}^{\bar{\theta}(s)} R_B dG(\theta) + \int_{\bar{\theta}}^{\theta^{max}} R_A dG(\theta) \tag{B.7}$$

Differentiating R with respect to s gives

$$dR/ds = -R_0 g(\theta_0)d\theta_0/ds + \int_{\theta_0(v,s)}^{\bar{\theta}(s)} (dR_B/ds)dG(\theta) + \int_{\bar{\theta}}^{\theta^{max}} (dR_A/ds)dG(\theta) \tag{B.8}$$

dR_A/ds and dR_B/ds are found by differentiating (B.5) and (B.6):

$$dR_A/ds = [\theta y(1) - b'(n_A)](dn_A/ds) \tag{B.9}$$

$$dR_B/ds = n_B[\theta y'(h_B) - s^g](dh_B/ds) + [\theta y(h_B) + s^g(1-h_B) - b'(n_A)](dn_B/ds) \tag{B.10}$$

where $dn_B/ds = \partial n_B/\partial s + (\partial n_B/\partial v)dv/ds$ and similar for n_A and θ_0. Using this together with the first order conditions (A.1), (A.4) and (A.5) and the zero-profit condition (A.6), we can rewrite (B.6) as

$$sgn\frac{dR}{ds} = -sgn\left(-n_0 g(\theta_0)(1-h_0)\frac{d\theta_0}{ds} - \int_{\theta_0}^{\bar{\theta}} n_B \frac{dh_B}{ds} dG(\theta) + \int_{\theta_0}^{\bar{\theta}} (1-h_B)\frac{dn_B}{ds} dG(\theta)\right) \tag{B.11}$$

The bracket is the change in total shadow work, i.e. it can also be written as

$$sgn\frac{dR}{ds} = -sgn\frac{d}{ds}\int_{\theta_0}^{\bar{\theta}} (1-h_B)n_B dG(\theta)$$

The first term in (B.11) is the increase in B-firms. The second term is the increased time for shadow work in B-firms. The third term is the change in employment in B-firms. The first two terms are positive, since $d\theta_0/ds < 0$ and $dh_B/ds < 0$. The third term is also positive, although some of the changes in n_B are negative. To see this, note that since employment goes down in all A-firms,

$$\int_{\theta_0}^{\bar{\theta}} (dn_B/ds)dG = -\int_{\bar{\theta}}^{\theta^{max}} (dn_A/ds)dG > 0$$

In the third term of (B.11), each of the changes in n_B is weighted with $1-h_B$. Since the weights go from $1-h_0$ to 0, the increases in n_B gets larger weights than the reductions. Thus, the intergral must remain positive with the weighting, and so the third term in (B.11) is also positive. Since all three terms in (B.11) are positive, we can conclude that

$dR/ds < 0$.

Proposition 3: The effect of lower expected penalties on profits

The profit of A-frms obviously go down as s goes up. To fnd the effect on π_B , we differentiate the maximized value of π_B with respect to s , taking into account the effect of s and v. This gives

$$sign(d\pi_B/ds) = sgn[(1-h_B) - dv/ds] = sgn[(1-h_B) - \lambda(1-h_0)] \tag{B.12}$$

where λ is given by (B.3). Let $\hat{h}$ be the h-value that gives $d\pi_B/ds = 0$, i.e.

$$\hat{h} = 1 - \lambda(1-h_0) \tag{B.13}$$

Since $0 < \lambda < 1$, $h_0 < \hat{h} = 1 - \lambda(1-h_0) < 1$. Let $\hat{\theta}$ be the productivity parameter that makes it optimal to choose $\tilde{h}$, given s and t. Since h_B is increasing in θ it follows that $d\pi_B/ds > 0$ in firms where $\theta < \hat{\theta}$ and $d\pi_B/ds < 0$ in firms where $\theta > \hat{\theta}$

Appendix C: Proof of proposition 4

The effects of a higher tax rate on profits and social revenue

From (A.3) the maximized profit of a B-firm can be written as

$$\pi_i = (1-t)\left(\theta y(h_i)n_i - \frac{[v - s(1-h_i))]n_i}{1-t} - b(n_i)\right) \tag{C.1}$$

Differentiating π_i with respect to t, using the envelope theorem, gives

$$\frac{d\pi_i}{dt} = \left(\frac{dv}{dt} + \frac{v - s(1-h_i)}{1-t}\right) - \frac{\pi_i}{1-t} \tag{C.2}$$

To find an expression for dv/dt we differentiate the labour market equilibrium condition $N(v; s, t) = M$ with respect to t and get

$$\frac{dv}{dt} = -\frac{\partial N/\partial t}{\partial N/\partial t} = \frac{-n_0 g(\theta_0)\frac{\partial \theta_0}{\partial t} + \int_{\theta_0}^{\bar{\theta}} \frac{\partial n_B}{\partial t} dG(\theta) + \int_{\bar{\theta}}^{\theta^{max}} \frac{\partial n_A}{\partial t} dG(\theta)}{-n_0 g(\theta_0)\frac{\partial \theta_0}{\partial v} + \int_{\theta_0}^{\bar{\theta}} \frac{\partial n_B}{\partial v} dG(\theta) + \int_{\bar{\theta}}^{\theta^{max}} \frac{\partial n_A}{\partial v} dG(\theta)} \tag{C.3}$$

From (A.4)-(A.6) we get

$$\frac{\partial x_i}{\partial t} = \frac{\partial x_i}{\partial v}\frac{v - s(1-h_i)}{1-t} \tag{C.4}$$

for $x_i = \theta_0, n_A, n_B$. Using (C.4) to substitute for $\partial n_A/\partial t$,$\partial n_B/\partial t$ and $\partial\theta_0/\partial t$ in (C.3) yields

$$\frac{dv}{dt} = -\frac{[v - \lambda s(1-h_0)]}{1-t} \tag{C.5}$$

Where λ is given by (B.3). We can then rewrite (C.2) as

$$\frac{d\pi_i}{dt} = \frac{s}{1-t}[(1-h_i) - \lambda(1-h_0)] - \frac{\pi_i}{1-t} \tag{C.6}$$

It is immediate that $d\pi/dt > 0$ for the marginal firm, the one with h_0 work hours, and that $d\pi/dt < 0$ for firms with $h = 1$, the A-firms. Thus, profit goes up for firms with productivity parameter θ_0 and down for firms with productivity parameter $\bar{\theta}$ and above. From (C.6), $d\pi/dt$ is decreasing in θ since h_i and π_i are increasing in θ. This implies that there is a critical value θ' between θ_0 and $\bar{\theta}$ such that the profit goes up in all firms with $\theta < \theta'$ when t goes up.This proves the second part of proposition 4, that B-firms with productivity parameter below a critical level lose from lower taxes.

Next, I prove that social revenue goes down when t goes up. To simplify the exposition, there is no loss from setting the expected penalty equal to zero, such that $s^g = s$. Differentiating R, given by (B.6), and using similar derivations as in Appendix B to simplify the expression we find

$$sgn\frac{dR}{dt} = -sgn\frac{d}{dt}\int_{\theta_0}^{\bar{\theta}}(1-h_B)n_B dG(\theta) \tag{C.7}$$

Thus, as for dR/ds the change in R as t goes up depends on the effect on the total amount of shadow work. In more detail, we can write

$$\frac{dR}{dt} = -\frac{st}{1-t}\left\{-n_0(1-h_0)g(\theta_0)\frac{d\theta_0}{dt} - \int_{\theta_0(v,t)}^{\bar{\theta}(t))} n_b\frac{dh_B}{dt}dG(\theta) + \int_{\theta_0}^{\bar{\theta}(t)}(1-h_B)\frac{dn_N}{dt}dG(\theta)\right\} \tag{C.8}$$

The first term is the reduction in surviving firms as t goes up. The secon term is the increase in reported work hours. The third term is the change in employment in B-firms, which is positive for low-productive firms and negative for high productive. The change in employment in firm i as t changes is

$$\frac{dn_i}{dt} = \frac{\partial n_i}{\partial t} + \frac{\partial n_i}{\partial v}\frac{dv}{dt} \tag{C.9}$$

This can be written as

$$\frac{dn_i}{dt} = \frac{\partial n_i}{\partial v}\left[\frac{v - s(1-h_i)}{1-t} + \frac{dv}{dt}\right] = -\frac{\partial n_i}{\partial v}\left[(1-h_i) - \lambda(1-h_0)\right]\frac{s}{1-t} \tag{C.10}$$

Since $\partial n_i/\partial v < 0$, it is immediate that n_i goes up for $h = h_0$ and down for $h = 1$, and $\partial n_i/\partial t$ is decreasing in h_i. Let h'' be the value of h that gives $(1-h_i) - \lambda(1-h_0) = 0$, and θ'' the productivity parameter of the firm that chooses $\tilde{h}$. It then follows from (C.10) that firms with $\theta < \theta''$ increase their employment as t goes up and firms with $\theta > \theta''$ decrease their employment as t goes up.

Appendix D: Proof of proposition 5

We assume that a minimum wage m is imposed, where $w_0 < m < w_A$. Before a minimum wage was imposed, firms would offer a reported wage given by

$$w = \frac{1}{1-t}\left[s - \frac{(s-v)}{h(\theta)}\right] \tag{D.1}$$

,where h is determined by $(1-t)\theta y'(h) = s$. Since the optimal work hours is an increasing function of θ, the reported wage can be written as $w(v;\theta)$, where $dw/dv > 0$ and $dw/d\theta > 0$. A minimum wage m binds for firms where $w(v;\theta) \leq m$. For given m and v, this gives us a critical productivity level θ^m such that the minimum wage binds for firms with $\theta \leq \theta^m$. θ^m is decreasing in m and v.

Let v_0 be the equilibrium value of v before a minimum wage m is introduced. Given v_0, a minimum wage m binds for firms with $\theta \leq \theta^m(m; v_0)$. However, in firms where the minimum wage binds, the marginal gain from employment is lowered. As a result, demand for labour goes down in these firms. Since labour demand is unchanged in firms where $\theta > \theta^m(m; v_0)$, total labour demand goes down, which means that v must go down. At the lower equilibrium value of v, the minimum wage binds for firms with $\theta \leq \theta^m(v_m; m)$, where $\theta^m(v_m; m) > \theta^m(v_0, m)$. Employment is now lower in firms with $\theta < \theta^m(v_m; m)$ and higher in firms with $\theta > \theta^m(v_m; m)$

In firms where the minimum wage binds, the right hand side of (D.1) must equal m. This determined the number of work hours as $[m(1-t) + s - v]/s = l(v; m)$. It is immediate that l is increasing im m and decreasing in v. Thus, introducing a minimum wage increases reported work hours in firms where the minimum wage binds.

The effects of minimum wages on social revenue follows easily from the effects on employment and work hours: Employment goes down in B-firms with $\theta \leq \theta^m(v_m; m)$ and up in firms with $\theta \geq \theta^m(v_m; m)$. This increases social revenue, since employees are reallocated to firms with higher productivity. In addition, social revenue is also increased because work hours goes up in firms with $\theta \leq \theta^m(v_m; m)$. Thus, social revenue goes up when a minimum wage m is imposed where $w_0 < m < w_A$.