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Phasing in Large-scale Expansion of Wind Power in the Nordic Countries

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Phasing in large-scale expansion of wind power in the Nordic countries*

by

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Abstract: There are plans of a substantial increase in the construction of renewable power in Scandinavia in the coming 10 years. The Nordic countries operate a common wholesale market, Nord Pool. Intermittent power (wind power, solar and small-scale hydro power) is stochastic and therefore needs other generating technologies to undertake the necessary adjustment of supply in order to keep up the continuous balance between demand and supply. There are several generating technologies in use in the Nordic countries; hydropower, conventional thermal, nuclear power, combined heat and power based on oil, coal and biofuel, and intermittent power, mainly wind power and run-of-the-river small scale hydro. Interesting questions are which technologies will counter the swings in intermittent power, and the consequences for the price of electricity as to variability. The purpose of the paper is to investigate these questions by using a theoretic dynamic model covering the main technologies used for generating electricity in the Nordic area in order to give qualitative conclusions about the interactions between the technologies and price impacts. A certain amount of intermittent power will be assumed, and then consequences of changes in intermittent power will be studied.

Keywords: Electricity; Intermittent power; Hydropower; Thermal power

JEL classification: C61, Q40, Q42

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1. Introduction

There are plans of a substantial increase in the construction of renewable power in Scandinavia in the coming 10 years. Norway and Sweden signed an agreement in 2010 to expand renewable power consisting of wind power, small-scale hydro power without reservoirs and generators using biofuel. In order to stimulate investment a common green certificate system is introduced starting in 2012 making it compulsory for buyers of electricity to also hold a certain share of certificates bought from renewable producers in order to make a planned expansion profitable in the market. The planned expansion in Norway and Sweden in yearly growth terms is about the double of the yearly increase in electricity consumption the last years, having implications both for the general price level and its variability.

The Nordic countries Norway, Sweden, Denmark and Finland, operate a common wholesale market for electricity with Nord Pool as the market place.[1] There are several generating technologies in use; hydropower in Sweden and Norway, nuclear power in Sweden and Finland, and coal-fired generation in Finland and Denmark, and the latter country also has a substantial share of wind power (15 - 20 %). There is also a significant capacity for combined heat and power production in Sweden, Finland and Denmark.

Intermittent power - wind power, solar and small-scale hydro power - is stochastic and uncontrollable (except wasting) and therefore needs other generating technologies to undertake the necessary adjustment of supply in order to keep the continuous balance between demand and supply. As mentioned Denmark has already a substantial share of wind power and has benefitted from participating in the common Nordic electricity market using its hydropower in Norway and Sweden as a back-up for its wind generation, thus not having to invest in that much back-up of coal-fired generators within Denmark itself. The new wind capacity in the other countries will compete with Danish wind power in using the Nordic system as back-up.

[1] Estonia joined the spot market in 2010.

It has been conjectured that storable hydro power in Norway and Sweden will increase in value because of the almost instant possibility of ramping up or down generation. Due to less than perfect forecasting of demand and supply even in the short term in the day-ahead wholesale market used in the Nordic area, there is a need for balancing power. It is a question if hydro power used for balancing purposes will be especially valuable. However, the investigation of the value of hydro as balancing power must necessarily be done within a model reflecting the stochastic nature of intermittent power, and this will not be pursued in the present paper using a deterministic model.

There are trading links between the Nordic countries and a country like Germany that has invested, and has expansion plans for much more investment, in wind power. An idea that has been floated in the media is that the hydropower of Norway and Sweden can serve as a battery for Europe (see e.g. The Economist, Where the wind blows (July 2006), http://www.economist.com/node/9539765). The idea is that surplus wind power can be absorbed by the hydro system simply by reducing the current use of stored water (even down to zero), and then exporting back when wind power is scarce.

The purpose of the paper is to prepare the ground for investigating these ideas by using a theoretical dynamic model covering the main technologies used for generating electricity in the Nordic area. We can then gain some qualitative insights into the effects on the electricity system in the Nordic electricity area that may be helpful for formulating energy policy.

The study will only consider utilisation of capacities and will not look into investment issues, like whether the investment in Renewables is socially profitable. We will also leave out the important issues of investment in transmission network to accommodate all the new renewable generation (Førsund, 2007b). The best wind resources are often found in remote areas or far from major consumption nodes so necessary transmission investments may be substantial. It may also be the case that the new lines may be environmentally controversial projects as such, spoiling and disfiguring pristine landscapes and creating problems for migratory birds and valuable species like eagles and owls along the coast of Norway. The study will try to derive qualitative insights into main consequences of increasing the share of intermittent power using Kuhn –

Tucker conditions[2]. A large-scale simulation model for the Nordic electricity system is the EMPS model developed by SINTEF Energy[3], Norway, over many years, originating in Hveding (1968), and it has been used to simulate consequences of wind power (Warland et al., 2011). In Førsund et al. (2008) the consequences for the use of hydro when expanding wind in a Northern region of Norway is explored using the same model.

An important simplification in the present paper is that uncertainty is not considered. Since a characteristic feature of intermittent energy is uncertainty about availability this is obviously a weakness of this study. The question of balancing power will then not be analysed. The approach taken is to assume availability of expected intermittent energy for each period, and then use the resulting utilisation of generation resources as a benchmark when exploring consequences of variability by assuming extreme values of intermittent energy as certain events. The exercise will then have the character of a sensitivity analysis.

A further simplification is to regard the countries involved as a single unit and not study trade flows, thus excluding the issue of hydropower functioning as a battery through international trade. However, key characteristics of the battery property will be revealed also within our single unit.

The model is cast in the tradition of a social planner having the objective of maximising social consumer- plus producer surplus. However, the characteristics of the solutions for prices and quantities are directly relevant for the outcomes in a competitive wholesale market with many independent electricity producers and consumers. Thus, our stylised aggregate model should throw light on how hydropower interacts with other generating technologies including intermittent power in an electricity market.

The paper is organised in the following way. The model will be presented in Section 2 together with conditions of optimality. In Section 3 the optimality conditions are interpreted and qualitative characteristics are derived. In Section 4 influences on price and mix of technologies

[2] In Baumol (1972, p. 165) it is stated that "the Kuhn – Tucker conditions may perhaps constitute the most powerful single weapon provided to economics theory by mathematical programming".

[3] According to Wolfgang et al. (2009) EMPS is the acronym for EFI's Multi-area Power-market Simulator. SINTEF Energy Research was created as a merger between EFI (Elektrisitetsforsyningens forskningsinstitutt) and SINTEF Energy in 1998.

are studied introducing an increase or a decrease in intermittent power in a period. Section 5 concludes and topics for further research are discussed.

2. The model framework

As mentioned in the introduction the electricity production in the Nordic area is based on several generating technologies. For simplicity we will lump together all thermal technologies into one sector (in Førsund and Hjalmarsson (2011) it is distinguished between conventional thermal and nuclear power), and the intermittent technologies into another, so there are three technologies in the model; hydropower with reservoir, thermal generators and intermittent generation (for ease intermittent will also be called wind)[4]. Individual hydro plants and storage capacities may be added together, under certain conditions, according to Hveding’s conjecture (Hveding, 1968; Førsund, 2007a). In the model the aggregated system for each group of technologies is then represented. The consumer sector is also aggregated into just one group and no transmission is specified. The model is partial, i.e. no interaction with the rest of the economy is modelled.

Time in the model is discrete, but length of the time period is open to choice, from e.g. one hour up to the most aggregate level of two periods (summer - winter seasons) within a year. The planning horizon used in practice may be one year, following a natural yearly cycle, or up to 3 - 4 years in order to account for fluctuations in inflows between years. The problem of optimising use of water over time is inherently dynamic when having reservoirs because water used today can alternatively be used tomorrow. Electricity is as a rule measured as energy, i.e. in MWh and not measured as power (MW) as is more the standard in engineering literature. The reason for this simplification is that for the typical length of period using energy suffices to bring out the qualitative characteristics we seek. One assumption, then, often used about power is that the use of power is constant over the period length considered. For longer periods, like a season, an

[4] In order to focus on the basic relationships between intermittent power on one hand and hydro and thermal on the other combined heat and power is not included due to the special structure of this generation.

assumption about an exogenous power distribution for the period may be introduced by using e.g. load curves.

The social planner maximises consumer plus producer surplus using demand functions (on price form) for electricity for each period and the cost function for the thermal sector. As mentioned in the Introduction an important simplification is that there is full certainty about the intermittent production profile, the inflow to the reservoirs and the period demand functions (temperature effects are also assumed perfectly predictable). The social optimisation planning problem is:

$$
\begin{aligned}
&\max \sum_{t=1}^{T} [\int_{z=0}^{x_t} p_t(z)dz - c(e_t^{Th})] \\
&\text{subject to} \\
&x_t = e_t^H + e_t^{Th} + e_t^I \\
&R_t \le R_{t-1} + w_t - e_t^H \\
&R_t \le \overline{R} \\
&e_t^{Th} \le \overline{e}^{Th} \\
&e_t^I \le a_t \overline{e}^I ,\ a_t \in [0, \overline{a}] \\
&x_t, e_t^H, R_t, e_t^{Th} \ge 0,\ t = 1,..,T \\
&T, R_o, \overline{R},\ \overline{e}^{Th}, \overline{e}^I, w_t, a_t (t = 1,...,T) \text{ given,} \\
&R_T \text{ free}
\end{aligned}
\tag{1}
$$

The symbols of the model are:

x_t : the demand for electricity during period t
$p_t(x_t)$: the demand function for period t on price form
e_t^H : the production of electricity from hydro power during period t
e_t^I : the production of electricity from intermittent technology during period t
$\overline{e}^I$: the capacity limit on intermittent generation in MW
a_t : the intermittent factor converting capacity to electricity in period t
$\overline{a}$: upper limit on the conversion factor
e_t^{Th} : the production of electricity from thermal generators during period t
$\overline{e}^{Th}$: the capacity limit on thermal generation
$c(e_t^{Th})$: the variable cost function for thermal generation during a period
R_t : the amount of water in the reservoir at the end of period t
$\overline{R}$: the capacity of the reservoir
w_t : the inflow to the reservoir during period t

Standard non-linear programming is used to find the first-order Kuhn – Tucker conditions. It is not possible to find an explicit analytical solution, but because of the special recursive structure of the dynamic water accumulation equation, it is easy to apply the Bellman (1957) backward induction for dynamic programming problems to get qualitative insight into the nature of the optimal solution. No discounting is specified because the time horizon is usually too short for this to make an impact, but discounting is straightforward to introduce (Førsund, 2007a).

Hydropower and intermittent generation are assumed quite realistically to have zero current cost that varies with output; e.g., labour overseeing the operations and maintenance costs are assumed to be dimensioned to given capacities and do not vary with fluctuations in output. Such fixed costs and capital costs are neglected in the analysis since we are only looking at the problem of optimal management of existing capacities, assuming that it is profitable to supply electricity when neglecting sunk capital costs and other costs not varying with output.

The thermal cost function comprises all thermal technologies including nuclear. There are no changes in primary energy prices between the periods and no technical change. The variable current costs constitute primary fuel costs that depend on the output level. The fixed cost part is not included in the cost functions. The aggregate cost function is constructed as a merit-order function according to marginal cost and is assumed that we have a unique ranking of capacities. This represents a simplification. Start-up costs and close-down costs are not specified. It is straightforward to make a step function over different technologies if a unique merit order holds. The total output of the thermal sector is constrained as seen in (1).

The accumulation of water is represented by the second constraint: the reservoir level at the end of period t must be equal to or less than the water in the reservoir at the end of the previous period t -1 plus the inflow during period t and the consumption of water represented by the hydro power production during period t. All the water variables are expressed in energy units. A production function for electricity is behind the conversion of water into electricity (Førsund, 2007a). Strict inequality means that there is overflow. Then the relation between the current level of the reservoir and the capacity (third constraint in (1)) must hold with equality.

We only include a constraint on the size of the water reservoir, but not on production or power, implying that all available water in the reservoir may be produced within a single period. We also leave out environmentally-based constraints on water flow and on ramping up and down over periods. (Such constraints are more relevant using a fine time resolution and multiple producers, see Førsund, 2007a.)

The intermittent generation of electricity is represented by a time-dependent coefficient converting wind, sunshine or run-of-the-river water into energy based on the installed power capacity. The coefficient, reflecting the average availability of the primary energy source for intermittent energy (e.g. wind conditions) may take the value between zero and a maximal value based on full utilisation of the power capacity for the period.[5] Wind mills usually need a wind blowing over 4 m/s to produce, and then production picks up until it levels off at about 12-13 m/s with standard gears, and finally the wind mill has to stop production if the wind blows too hard, above about 25 m/s.

The terminal condition is the simplest one having the reservoir level free. It is straightforward to introduce a scrap value or minimum level for the terminal period (Førsund, 2007a).

The Lagrangian function, substituting for the energy balance, is

$$
\begin{aligned}
L = {} & \sum_{t=1}^{T}\Big[\int_{z=0}^{e_t^H + e_t^{Th} + e_t^I} p_t(z)\,dz - c(e_t^{Th})\Big] \\
& -\sum_{t=1}^{T}\theta_t (e_t^{Th} - \overline{e}^{Th}) \\
& -\sum_{t=1}^{T}\lambda_t (R_t - R_{t-1} - w_t + e_t^H) \\
& -\sum_{t=1}^{T}\gamma_t (R_t - \overline{R})
\end{aligned}
\tag{2}
$$

Intermittent generation is assumed not to be subject to optimisation, but to be utilised within the feasible capacity, i.e. equality is assumed in the fifth condition in (1). (In principle potential output may be curtailed (using pitch control of the rotor blades or shutting down some turbines of a wind farm), but, e.g., in Germany this is not permitted.)

The necessary first-order conditions are

[5] In a disaggregated framework the distribution of the coefficient is site –specific.

$$
\begin{aligned}
&\frac{\partial L}{\partial e_t^H} = p_t(e_t^H + e_t^{Th} + e_t^I) - \lambda_t \le 0 \;(=0 \text{ for } e_t^H > 0) \\
&\frac{\partial L}{\partial e_t^{Th}} = p_t(e_t^H + e_t^{Th} + e_t^I) - c'(e_t^{Th}) - \theta_t \le 0 \;(=0 \text{ for } e_t^{Th} > 0) \\
&\frac{\partial L}{\partial R_t} = -\lambda_t + \lambda_{t+1} - \gamma_t \le 0 \;(=0 \text{ for } R_t > 0) \\
&\lambda_t \ge 0 \;(=0 \text{ for } R_t < R_{t-1} + w_t - e_t^H) \\
&\gamma_t \ge 0 \;(=0 \text{ for } R_t < \overline{R}) \\
&\theta_t \ge 0 \;(=0 \text{ for } e_t^{Th} < \overline{e}^{Th}), \qquad t = 1,..,T
\end{aligned} \tag{3}
$$

Qualitative insights are based on interpreting these first-order conditions. The shadow price λ_t expresses the increase in the objective function of a marginal increase in the amount of stored water at the end of period t. It therefore seems appropriate within this model to call this shadow price for the water value. The complementary slackness condition for this shadow price yields a value of zero if overflow occurs. The third first-order condition is the explicit dynamic relation in the model solution relating the water value in one period to the water value in the next period.

The shadow price on intermittent energy is simply the period price, applying the envelope theorem. An increase of the intermittent coefficient a_t increases the production by $de_t^I = da_t e^I$. An increase in the intermittent power capacity increases the objective function with $\sum_{t=1}^{T} p_t a_t$. The measurement unit of the price is money per MWh, while the coefficient a_t is measuring MWh per MW, so the sum expression is measured as money per MW, i.e. the total revenue on an incremental increase in the intermittent power capacity over all the periods.

3. Qualitative results

We start out from the basic assumption that there is a unique optimal solution to problem (1) characterised by the first-order conditions (3). Furthermore, we adopt the reasonable assumptions that electricity delivered to the consumers is positive in every period, and that demand for electricity is never satiated. The last assumption implies a positive optimal price for all periods. Intermittent energy is assumed to be used when available to zero production-dependent cost. Thus, intermittent energy will influence the solutions for how to use all the other types of technologies through appearing in the demand functions only. It is straightforward to split intermittent energy into wind power, solar power and run-of-the-river hydropower. In order to study the influence of intermittent energy it will be of special interest to discuss extreme periods when intermittent energy is zero and at the maximal level.

Interior solutions

An interior solution means that the first three conditions in (3) hold with equality. The third first-order condition reads

$$-\lambda_t + \lambda_{t+1} = 0 \qquad (4)$$

This means that as long as the reservoir level stays in between full and empty, then the water value remains constant. A set of consecutive periods with interior solutions having the same price p_j is termed $T_j(i)$, and we have J such sub-periods within the planning horizon T. The shadow price on the reservoir capacity is then zero according to the complementary slackness conditions. The connection between the social price, water value and marginal cost of thermal is then:

$$p_t(e_t^H + e_t^{Th} + e_t^I) = \lambda_t = c'(e_t^{Th}) = p_j\ ,\ t \in T_j(i)\,,\ j = 1,...,J \qquad (5)$$

The optimal price equals water value equals marginal thermal cost and all the technologies typically supply positive amounts.[6]

This implies that the optimal prices for the two periods are equal, and furthermore that the marginal thermal cost is equal to the common price implying an equal utilisation of thermal generation in the two periods. Notice that the result of a common price holds for as many consecutive periods as Eq. (4) holds.

[6] It is often said that in a mixed system with hydro and thermal that marginal thermal cost determines the price. However, equality between water value and marginal cost is a condition for optimality in a simultaneous optimal interior solution.

We may note that for consecutive periods within a sub period $T_j(i)$ of a common price if thermal capacity is constrained it has to be constrained for all the consecutive periods. From the two first conditions of (3) we have

$$p_j(e_t^H + e_t^{Th} + e_t^I) = \lambda_t = c'(e_t^{Th}) + \theta_t, t \in T_j(i) \quad (6)$$

As long as the price stays constant the shadow price on the thermal capacity is typically positive, and the maximal amount of energy is produced in each period.

It is not optimal to use thermal at all if

$$p_t(e_t^H + e_t^I) - c'(0) \leq 0 \quad (7)$$

As a general property we may well have $c'(0) > 0$. (This is not the same as start-up costs.[7]) The condition (7) can then be fulfilled with inequality at the same time as we have a positive price of electricity.

We see from (5) that total optimal production x_t^* in each period within a sub period $T_j(i)$ with the same price p_j, $x_t^* = e_t^H + e_t^{Th} + e_t^I$, varies between periods if demand varies. Because thermal output is locked to the same level due to the common price p_j, then hydro power has to accommodate both the variation in the intermittent energy and the variation in demand between periods.

The size of the swing for two consecutive periods t and t +1 within a sub period $T_j(i)$ with equal price p_j is, using (5):

$$\underbrace{e_{t+1}^H - e_t^H}_{\text{hydro swing}} = \underbrace{(x_{t+1}^* - x_t^*)}_{\text{demand change}} - \underbrace{(e_{t+1}^I - e_t^I)}_{\text{wind change}}, t \in T_j(i), j = 1,...,J \quad (8)$$

The first term on the right-hand side is the demand change between the periods and the second term is the change in the intermittent power. If we look at a constant demand the maximal down-

[7] A more detailed modelling of thermal generation may be necessary to get a technology description more correct in an engineering sense. In addition to start-up costs the marginal cost may start at a high level and decrease in output up to maximal capacity. Such non-convexities may create problems for finding a unique solution.

swing in hydro occurs when wind is blowing maximally period t +1, $e^I_{t+1} = \overline{a} \bullet \overline{e}^I$, and with no wind in period t, $e^I_t = 0$, resulting in the negative adjustment $\overline{a} \bullet \overline{e}^I$. This reduction in the use of hydropower is only possible if the downswing can be accommodated within the reservoir capacity. The maximal upswing in hydropower occurs if the intermittent energy changes from the maximal level in period t to zero in period t +1, resulting in the positive adjustment $\overline{a} \bullet \overline{e}^I$. For this upswing to be realised it must be enough water in the reservoir. Changes in demand can either dampen or increase the swing in hydropower. Going from night to day demand normally increases, and vice versa from day to night.

A conclusion about the impact of variation in intermittent energy is that for periods when hydro power is used, but no hydro constraints are binding, then this variation has no explicit qualitative price implications. (Of course, the absolute price level in a sub period $T_j(i)$ is another matter, and this level will be influenced in principle in the simultaneous solution by the amount of intermittent energy.) But the number of consecutive periods with equal price may well be influenced by variations in intermittent energy.

The number of sub periods $T_j(i)$ the price stays constant and the number of periods within each sub period are endogenous in the model. A conjecture may be that both the number of sub periods and the length of sub periods will be reduced due to the variation in intermittent energy. The reason is that when hydropower acts as a swing producer the constraints, both upper and lower, of the reservoir may more often become binding.

Price changes

It was pointed out already in Hveding (1968) for a pure hydro system that price only changes if a reservoir constraint become binding (empty or full). In our case with several generating technologies this is still the case for the system price when hydro is used as is seen from the third condition giving the relation between water values over time in (3). In our aggregate model the reservoir can be emptied within a single period, implying that it can under our assumption of non-satiation of demand never be optimal to have overflow. If overflow threatens in a period, then the shadow price on the reservoir constraints will typically become positive (however, note that zero is a formal possibility[8]). This means that the water value for the period when overflow threatens will typically be smaller than for the next period:

$$\lambda_t = \lambda_{t+1} - \gamma_t \tag{9}$$

The period prices become equal to the period water values. Assuming positive prices implies that the shadow price on the reservoir constraint in period t must typically be smaller than the water value in period $t+1$.

If it should be optimal to empty the reservoir at the end of a period, then the shadow price on the upper reservoir constraint is zero and the water value in the period when the reservoir is emptied will typically be greater than the water value in the next period:

$$\lambda_t \geq \lambda_{t+1} \tag{10}$$

The same relation holds between the optimal prices. The reason the reservoir is emptied is that the water is worth more in the current period than the next.

Note that it is not optimal that reaching the upper constraint of the thermal capacity for a period generates a system price change by itself. We have from (6) that this is not optimal if hydro is in use.

A price collapse

An interesting situation arises if it is optimal not to use any water in a period. The condition for this to take place is:

$$p_t(e_t^{Th} + e_t^{I}) \leq \lambda_t \tag{11}$$

When hydro is not used in a period water value is typically greater than the price. For this situation to be optimal there has to be room for more water to be stored in the reservoir. The water value for the current period t will be equal to the water value for the first future period when water will be used again.

[8] In the following we will refer to typical results and suppress in the discussion the often arbitrary possibilities in this type of aggregate system-wide model.

Figure 1 provides an illustration[9]. As time resolution we may think of period t as night time and

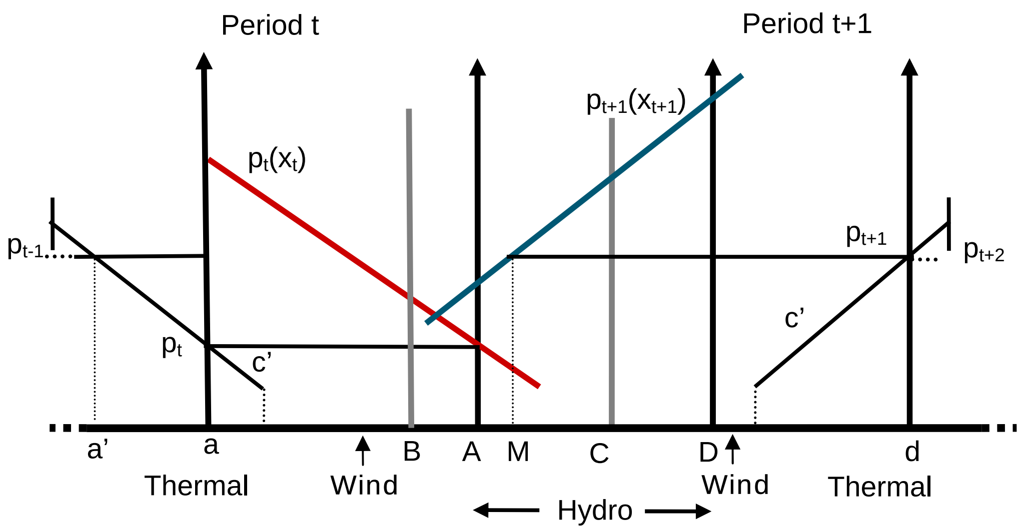

Figure 1. Energy bathtub for periods t and t +1

period t +1 as the following daytime. The hydro bathtub (Førsund, 2007a) for period t and t +1 is indicated by the bottom line from A to D, and by walls erected from these points. Period t price is measured along the left-hand wall of the bathtub, and period t +1 price along the right-hand wall. The water resource available for period t, made up of water inherited from the period before period t and the inflow during period 1, is AC, and the inflow in period 2 is CD.[10] The storage capacity for water is given by BC, and the walls erected from these two points illustrate the reservoir capacity. Note that the storage capacity is greater than the available water in period t, and the vertical line marking the left wall of the reservoir erected from B is therefore to the left of the hydro bathtub wall erected from A.

For period t the production possibilities are extended to the left of the wall of the hydro bathtub, due to the intermittent and thermal power, indicated with marginal cost curves for wind energy

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[9] Illustrations of price changes of the type (9) and (10) due to reaching the reservoir constraints are found in Førsund and Hjalmarsson (2011) in the case of two periods.

[10] Although we refer to the hydro resource as water, we measure the hydro bathtub in energy units, e.g. MWh.

following the floor of the extended bathtub since the variable cost is zero, starting from the left-hand hydro bathtub wall at A and following the horizontal axis to the left, and then comes the marginal cost curve for thermal capacities. The cost curve is for simplicity made linear in the figure (it could be made as a step curve, as is common in applied studies). The marginal cost curve has standard slope implying increasing marginal cost.

Now, the extension of the hydro bathtub including the two other technologies for period t +1 on the right-hand side is a mirror image of the marginal cost curves for period *t,* starting with the marginal cost curve for wind from D along the horizontal axis to the right and continuing with the thermal marginal cost curve. We have assumed that there is considerably more wind power available in period t than in period t +1.

The demand curve for electricity for period t is anchored on the left-hand energy wall erected from point a, and electricity consumption is measured from left to right. The demand curve for period t +1 is anchored on the right-hand energy wall erected from point d (the anchoring is not shown explicitly) and electricity consumption is measured from right to left. Both demand curves are drawn linear for ease of illustration. Period t is a low-demand period and period t +1 is a high-demand period.

The optimal solution to the management problem implies that the placement of the outer walls of the extended energy bathtub is *endogenously* determined (Førsund, 2007a). For ease of exposition, we erect the two walls such that we get illustrations consistent with the optimal underlying model solution (3) of a nature we want to discuss.

The two-period window in Fig. 1 is extended to a multi-period setting with one more period at each end by entering prices for period t -1 and t +2 assumed to be the optimal prices. The price in period t +2 is coming from the future (this is how Bellman's backward induction works) and is assumed to be part of a sub-period j with equal prices.

We assume water to be used in period t -1, t +1 and t +2, but not in period t. This may be part of an optimal solution because if a constant price level is to be realised including the period with the abundant wind this may not be feasible. The price level in the period with abundant wind will then be determined independently of the price level for the other periods within the sub-interval we are studying. From Eq. (5) we have the connection between the water values in period t -1 and t; $\lambda_{t-1} = \lambda_t$. Furthermore, we have $\lambda_t \geq p_t(e_t^{Th} + e_t^I)$ and $\lambda_t = \lambda_{t+1}$, implying that $p_{t-1} = p_{t+1} \geq p_t$. As a typical case the price with abundant wind is lower than the price in the period before and in the period after, and these latter prices are equal. The optimal price in period t must balance demand and available supply from wind and thermal, illustrated by the intersection of the period t demand curve and the hydro wall erected from point A.

The amount of thermal is shown by the intersection of the marginal cost curve for thermal and the energy bathtub wall up from point a. If thermal is in use in the wind-rich period the price will be equal to thermal marginal cost as seen from the second first-order condition in (3). A higher and same amount of thermal will be used in period t + 1 shown by the intersection of the marginal cost curve and the energy bathtub wall up from point d, and in period t – 1 as indicated by the vertical dotted line from point a’ in Figure 1. But due to the lower electricity price a smaller thermal capacity will be used in period t than in the periods before and after. Therefore the thermal capacity will typically not be constrained in such a situation. It may be the case that the price becomes so low that thermal is not used at all. This will happen if the price is lower than marginal cost at zero output. By assumption demand for electricity is not satiated so we have a positive price even without using thermal. (This assumption may not be empirically appropriate to make in a situation with maximal wind.)

The fall in the price in the wind-abundant period when it is not optimal to use water creates a “dip” in the common price, so in case this is the only occurrence the sub-period j is divided into two sub periods.

The multi-period nature of Figure 1 is also shown by the transfer of water between periods. All available water in period t is transferred to period t + 1, while the amount AM is transferred from period t + 1 to t + 2. We have a battery effect of saving water in the period with abundant wind, and then using this water to the benefit of reducing the price in the other periods of the two distinct sub-periods encompassing t – 1 and t + 1 with the same price. Notice that due to certainty the abundant wind makes it possible to increase the use of water and thereby reduce the price level both before and after the event. The situation is crucially different in the case of uncertainty of the available wind.

4. Sensitivity analysis of change in wind

Before carrying out a sensitivity study of the impact of variation in wind we conclude the discussion of a feasible optimal price path by an illustration set out in Figure 2. (The time

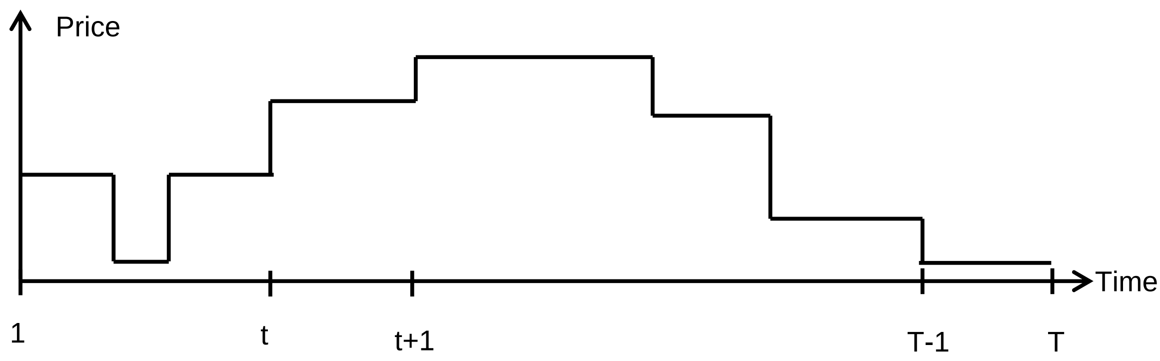

Figure 2. A feasible optimal price path for a full yearly cycle

intervals are just indications and are not spaced according to a common scale.) The time periods from t =1 to t have a common price level except for one period with a sharp price dip. Such a period is explained by Eq. (11) and illustrated and discussed in connection with Figure 1. The price profile will be a step curve in our type of model.

When studying possibilities of price changes due to variation in the wind power we should note that the optimal development of price is found by backward induction, so in any period we have to know the price "coming from the future". So the first price to determine is the price for the last period, T. We have assumed that demand is never satiated so the price in the last period will by assumption be positive. The reservoir is emptied in this period because we have not introduced any constraint on the reservoir level or any scrap value. Going backwards in time from the terminal period the two main price-changing situations for periods when water is used is that the reservoir is emptied in another period and that the reservoir constraint becomes binding in another period. To be more specific of the price-path profile illustrated in Figure 2 we have to give some structure to the development of the period demand functions. For simplicity we will look at a yearly cycle and place the terminal period in a period with empirically the lowest yearly reservoir levels and low demand. In Scandinavia this will be later spring – early summer. To end up with an empty reservoir in the terminal period would most likely imply a gradual reduction of the reservoir during some periods before the terminal period, but it is also possible that the reservoir becomes empty in a period some distance from the terminal period. It seems unreasonable that the reservoir can become full before the next to last period before the terminal period. We will therefore assume that the first event as to a corner solution of the reservoir level going backwards in time is another emptying of the reservoir. We then have from (10) that the price for this period typically will be greater than the price in the terminal period and then greater than all prices in the periods from the terminal period and backwards to the second emptying of the reservoir. If the reservoir stays empty in all periods in between we may have monotonic increase in the price backwards in time until we reach the time period in question. The hydro power plant then functions as a flow-of-the-river plant.

After finishing with periods emptying the reservoir going backwards in time we will assume that we come to a period with the upper constraint on the reservoir becoming active. After all, a normal situation in a hydro-dominated system is that the reservoir capacity is not sufficient for the prices becoming equal for all time periods. We will assume that from the period the upper reservoir constraint is active back to the first period the reservoir is in between full and empty, thus the price is constant for these periods. In Scandinavia the low-demand season is the spring/ summer season, and the high-demand season is the winter season. The main filling of the reservoirs following the general thawing and melting of snow during late spring and summer coincides with the low-demand season, while the periods with low inflows corresponds with the high-demand winter season due to heating of buildings and shorter daylight days. Because less water than needed to keep prices flat during the whole year can be transferred to the high-demand season a full reservoir should be realised in order for the subsequent high-price periods to have as low common price as possible. We have from (9) that the price in the periods before and including the period when the upper constraint on the reservoir becomes binding is typically lower than in the periods after. In the periods when the reservoir level is building up we obviously must have the inflow of water on the average being greater than the release of water on to the turbines. In the period when the reservoir constraint becomes binding this must especially be the case. For the other periods the reservoir level does not necessarily increase in a monotonic fashion. Entering a new higher price regime after the period with a binding reservoir constraint the release of water will on the average be greater than the inflows and must be that in the period when the reservoir becomes empty.

With the calibration mentioned above going forward in time the price will increase in the first period after reaching the upper limit. There may be several episodes of constraining the capacity leading to a gradual increase in price until the peak price is reached. But before that a possible episode of a dip in the price due to abundance of wind is illustrated. After the periods with a common peak price a more or less gradual reduction in the price will be encountered each time the lower constraint of the reservoir is reached, ending in the terminal period when the reservoir is always emptied.

Let us first consider the consequence of the wind power increasing in a period leading up to the first period when the reservoir constraint becoming binding, moving forward in time. One possible case is that the optimal period for the reservoir to become full does not change. A reason for this is that a full reservoir is realised in a specific period in order to use as much water as possible in subsequent periods with high demand, and that the price level or use of water in the periods leading up to the period with a full reservoir are completely disconnected from the pricing and water usage in the periods after. The common price of the periods leading up to a full reservoir must then necessarily go down in a situation with more wind power. Notice that more wind in just one period will have price consequences for all periods between the start period and the period with a full reservoir. Because thermal power capacity has the same capacity utilisation for all these periods leading up to a full reservoir determined by the price it follows that the capacity utilisation goes down, implying reduced profit for the thermal sector. Hydropower will also generate less profit, but because the variable current costs are zero it is not the question of withdrawing capacity as may be the case for thermal capacity. The total amount of water processed in the periods leading up to a full reservoir is not influenced by a variation in the wind power as long as the period with a full reservoir is the same. Thermal power absorbs the full impact of the shift in wind power due to the price change, but distributed over all periods leading up to a full reservoir and not only in the specific period when wind power is actually increased. (The role of hydro as a swing producer discussed in Section 3 was based on the assumption of a constant price.)

Increased wind power may influence the optimal choice of the period to have a full reservoir. Postponing the period may result in being able to keep a lower price during the high-price period after reaching a full dam. However, the distribution of inflow and demand functions must exhibit a special pattern to make this possible, so this is an empirical question.

Because more electricity is available it is physically possible to fill the reservoir earlier. But this is in general also possible without increased wind. The decisive point is the choice of the high-price periods determined by how to use a full reservoir optimally over the subsequent periods. More wind in the periods before the constraint becomes binding does not influence the size of the full reservoir. It may be difficult to see that the optimal choice of the period when reaching a full reservoir can change due to increased wind in a period leading up to a full reservoir.

A special situation may occur if the wind power becomes so abundant in a period that water will not be used at all, as illustrated in Fig. 1. For such a period Eq. (11) holds. This may be part of an optimal solution because if a constant price level is to be realised including the period with the abundant wind this may not be feasible, i.e. the common price level has to be so low that a full reservoir to meet high-price periods cannot be realised. Increasing wind generation in a period may lead to creating a dip in the price series as explained in connection with Fig. 1 in Section 3 and illustrated in Figure 2.

The abundance of wind is bad news for thermal generation because the price is low in that period; while no water is used so hydro generators do not suffer from this low-price period. However, hydro will also suffer lower prices in general. If water is not used in a period due to abundant wind then the common price level for the other periods leading up to the period with a full reservoir will become lower because there is more water to be used in these other periods.

It may be possible that we will have a smoother transition from the periods with accumulating reservoir and the high-demand periods with a running-down of the reservoir. The first period with a full reservoir may be followed by another period of a full reservoir, either the consecutive period or some periods later. Several periods with a full reservoir may form a transition from a low-price period to a high-price period. For each time we have a period with a full reservoir going forward in time the price will typically increase according to Eq. (9).

Having a period with abundant wind may influence the sequence and number of periods with a full reservoir because with more wind the reservoir can become full more rapidly.

In the case of realising a lower wind power the conclusion about the influence on price will be in the opposite direction of what is described above. Of course, less wind cannot lead to a period without use of water if that did not happen in the reference scenario.

If more abundant wind happens in a high-price period after the last period with a full reservoir the price level will decrease in the case of the period with empty reservoir going forward in time remains the same. Reduced peak prices will reduce the profitability of peak-load thermal capacity. Due to these periods being high-demand periods we will not expect so abundant wind that a period with no use of water will occur, but this is an empirical question.

It may be optimal with several periods of empty reservoir going forward in time. We may have a development over time with falling demand (due to increase in temperature and more daytime light) and increasing inflows (due to melting of snow).[11] If there are several periods following each other with empty reservoir we have the case of a run-of-the-river generation. Each time the reservoir get emptied the price will fall going forward in time. If the horizon is just one year we will end up with an empty reservoir in the terminal period. The period in between the terminal period and the next period with empty reservoir going backwards in time will be the sub period with the lowest price.[12]

More wind in one of the periods with falling prices will in general have price-reducing effects if the sequence of periods with empty reservoir does not change, but may also influence this

[11] In a longer perspective than a yearly cycle it is also a question of providing enough room in the reservoir for all the snow melting to be captured.

[12] Having a longer horizon the price cannot increase again until after the first period with a full reservoir, going forward in time, because this price can at most be equal to the price before the last period with empty reservoir. But a second period with a full reservoir can give a higher price in the periods after.

sequence, leading to more incidents of lower constraint being reached and in this way decreasing the average price.

5. Conclusions and extensions

Concerning the predictable variations in intermittent power we see from (4) when (5) holds the crucial role played by hydro power when the price for a number of consecutive periods is the same. Hydro fills in all the swings in intermittent power and demand because thermal output is constant for a constant electricity price. But variations in intermittent power may cause both the sub-periods with the same price to become shorter and the price level to change. In the case of more power being available the price level will decrease in sub-periods with the same price. This will reduce the profitability of thermal. The opposite will occur if wind gets a reduction.

When price varies caused by hydro reservoirs hitting a constraint a reduction in the intermittent power will increase the period price if the hydro reservoir is at the lower constraint, and reduce the price if intermittent power increases, and opposite if the reservoir is at the upper constraint.

The profitability of thermal is most affected by the new price patterns following introduction of intermittent power because thermal may have to produce relatively more in low-price periods. If thermal capacity is withdrawn this will have the consequence of increasing the price in high-price periods, but not the price in low-price periods if close-down is within reasonable limits.

The possibility of no use of water in a period is caused by sufficient intermittent power and thermal producing at a price lower than the price realised in a future period, and assuming that water can be stored to be used in that future period. This possibility will increase relatively the profitability of hydro power, but reduce the profitability both of thermal and intermittent power. This is the battery effect.

There are several ways of extending the analysis based on the same type of model as presented in (1). We will discuss such ideas below. One interesting and important issue that should be explored is how the optimal solution should be implemented. A natural continuation of the study is to investigate if a competitive market in electricity may realise the optimal conditions. However, we will not expand on this issue here, but focus on issues making the type of model used more realistic.[13]

Scandinavian hydro power as a battery for Europe

Seen in a wider European perspective it has been suggested that the hydro power of Norway and Sweden can serve as a battery for Europe. The idea is that surplus wind power can be absorbed by the hydro system simply by importing the wind power and reducing current use of stored water, and then export back when wind power is scarce. The picture of hydro power as a battery in the electricity supply is not new with intermittent power. Norway has had night-time - day-time exchange with Denmark before Nord Pool was founded (von der Fehr and Sandsbråten, 1997). The new aspect of intermittent power is the scale of this source of electricity generation and also the consideration of both hydro power and intermittent power being green technologies in terms of emission of climate gasses. If hydro power can contribute to making intermittent power more profitable this will have a positive environmental impact.

To discuss this issue in more detail we need a model encompassing trade in electricity. For the unit having hydropower (Nord Pool) there are two basic situations for trade, i) Nord Pool is sufficiently small to regard trading prices as exogenously given within the markets of trading partners (e.g. Germany), or ii) prices are determined endogenously by the trade in electricity. The model concept (1) can be extended to cover both possibilities (Førsund, 2007a; 2011).

In order to be able to trade across national borders there must be interconnectors in place. An important factor for analysing effects on prices is the capacity of the interconnectors. When this capacity is constrained import and export prices will not be equal (disregarding losses on the lines), being lower on the export side than the import side. If Germany wants to export electricity in a situation with abundant wind power depressing the price Nord Pool can import up to capacity of the interconnector. In a situation when lack of wind power creates a shortage in

[13] Wolfgang et al. (2009) study the impact of deregulation of the Norwegian wholesale market on the pattern of degree of filling of reservoirs.

Germany and a high price, Nord Pool can export stored power from the import periods and receive a, may be, substantially higher price than was paid for the import. It may sound like hydropower is the commercial winner here. But the alternatives for Germany should be born in mind. The alternative to export in a period of abundant wind power would be to consume the electricity at home to even lower prices than obtained by exporting or by wasting electricity, and not receiving any export income. The alternative to not importing in a high-price scarcity period would be an even higher price. It should also be taken into account that Germany may save a lot in not building domestic reserve capacity as stand-by for shortage periods.

Pumped storage

In a situation illustrated in period *t* in Figure 1 with available reservoir capacity, it might be socially profitable to run pumped-storage capacity. Pumped storage increases the amount of stored water over a yearly period, and hence increases the flexibility of hydropower. The power capacity will also increase in the case of stand-alone pumped storage, and when a facility with reversible turbine has an additional feeding tunnel. The necessary condition for using such capacity is that the income on a unit of water in a later period than when pumped up is greater than the cost of pumping up the water, assuming that more electricity has to be used to pump up a unit of water than generated by the same amount in a later period. In addition, when considering an investment project there are fixed costs, especially capital costs. But in the case of trading electricity across national borders both for energy purposes and for providing balancing power pumped storage may become more profitable.

The role of uncertainty

For long-term management of hydro reservoirs uncertainty about inflows will play a distinct role for the price formation in the pure hydro case. Following a forward-looking strategy it will be optimal to process less water when inflows fall short of expectations held in the previous period, resulting in an optimal price increase, and vice versa if inflows are above expectations; see Førsund (2007a). When considering that also the wind resource is stochastic, the analysis becomes more involved. A conjecture is that the optimal strategy is to react to wind variability in the same way as to inflow variability. This means that a lower wind than predicted should lead to a reaction on the hydropower side similar to the reaction to less inflow than predicted; less water should be processed and hence the price should increase if less wind is realised than expected in the previous period, and vice versa for more wind than expected. The greater the share of wind power, the greater the necessary reaction on the water side. A question is whether there is any correlation between wind availability and water inflows. If not, then the rule above for how to react to wind variation is valid, but if there is a correlation, this must be taken into consideration and may either strengthen or weaken the price variation, depending on the sign of the correlation. This issue does not seem to have been researched empirically yet.

The treatment of uncertainty may be especially crucial for high-demand periods and low reservoir levels. If the wind resource disappears in such a situation, the price may become a price spike of considerable magnitude. To avoid or cushion such price episodes, if this should be part of the social preferences, it may be optimal to keep more water in the reservoirs to face such contingencies due to the stochastic nature of the wind. However, it is costly for the society to keep such reserves, and individual hydro generators cannot be expected to keep such reserves unless they are paid for this in excess of the current spot price. This is the same situation as paying for stand-by thermal capacity (capital uplift).

Wind may disappear quite suddenly, so it is also a question of having more power capacity in reserve. It does not help to have enough energy in the form of stored water if that water cannot be processed instantly in sufficient quantities. Thus, the reserve issue created by the stochastic wind concerns both energy and power capacity.

Balancing power

It has been conjectured that storable hydro power in Norway and Sweden will increase in value because of the almost instant possibility of ramping up and ramping down generation, and the higher need for balancing power caused by increased intermittent energy. As indicated above uncertainty about inflows and intermittent power leads to a gap between expected and realised quantities that has to be dealt with a one-period lag. The continuous optimal adjustment up or down is the balancing problem within our social planning model. Due to uncertainty the possibility is opened up of both shortfall and surplus in real time. The social cost of regulating hydropower up in real time is the foregone the opportunity of producing the water in a future period increasing the value of the social objective function. If hydropower is to be regulated down the social loss per unit of output has the current period price as an upper limit if water is spilled. The loss may be reduced if water can be transferred to a future period.

The operation of balancing in practice is related to the type of market design both for energy and for balancing power. The key to understand the value of regulating power is the arbitrage opportunities between the current (spot) energy market and the regulation market. For a single price-taking hydro power actor the alternative to offering his power to a balancing market will be to sell to the energy market. Up-regulation means that the generator will forego the opportunity to process his water at a later period with a predicted higher price in the energy market. His expected cost per unit electricity is therefore the difference between the expected best future energy market price and the price paid in the balancing market. Down-regulation means that he has to hold on to the water longer (if physically possible, otherwise the water is lost) and will forego the current energy price and receive an expected lower price in the future. To be an attractive option the balancing market must offer at least the difference (Førsund and Hjalmarsson, 2011).

Although coal-fired generators and nuclear power is costly to regulate down and up again this is no reason for regulating services from hydro to demand a price premium as long as the total hydro power capacity to serve the balancing market is there. It is an empirical question whether this is the case.

References

Baumol WJ (1972). *Economic theory and operations analysis*. London: Prentice/Hall International, Inc. Third edition.

Bellman R (1957). *Dynamic programming*. Princeton: Princeton University Press

von der Fehr N-H M and Sandsbråten L (1997). Water on fire: gains from electricity trade. *Scandinavian Journal of Economics* 99, 281-297

Førsund FR (2007a). *Hydropower Economics*. New York: Springer Science+Business Media

Førsund FR (2007b). Investment in transmission. Working Paper No. 34/07, SNF Project No. 3215, Institute for Research in Economics and Business Administration, Bergen

Førsund FR (2011). Energy in a bathtub: electricity trade between countries with different generation technologies. In P.-O. Johansson and B. Kriström (Eds): *Modern Cost-benefit Analysis of Hydropower Conflicts*. Edward Elgar Publishing, 76-96

Førsund FR and Hjalmarsson L (2011). Renewable energy expansion and the value of balance regulation power. In P.-O. Johansson and B. Kriström (Eds): *Modern Cost-benefit Analysis of Hydropower Conflicts*. Edward Elgar Publishing, 97-126

Førsund FR, Singh B, Jensen T and Larsen C (2008). Phasing in wind-power in Norway: network congestion and crowding-out of hydropower. *Energy Policy* 36(9), 3514-3520

Hveding V (1968). Digital simulation techniques in power system planning. *Economics of Planning* 8(2), 118-139

Warland G, Mo B and Helseth A (2011). Handling balancing power in a power market with a large share of hydropower. Paper accepted for presentation at the 2011 IEEE Trondheim Power Tech, 1-7

Wolfgang O, Haugstad A, Mo B, Gjelsvik, Wangensteen AI and Doorman G (2009). Hydro reservoir handling in Norway before and after deregulation. *Energy* 34(10), 1642-1651