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Motivating over Time:
Dynamic Win Effects in Sequential Contests[*]

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Abstract

We look at motivation over time by setting up a dynamic contest model where winning the first contest yields an advantage in the second contest. The win advantage introduces an asymmetry into the competition that we find reduces the expected value to the contestants of being in the game, whilst it increases the efforts exerted. Hence, a win advantage is advantageous for an effort-maximizing contest designer, whereas in expectation it is not beneficial for the players. We also show that the principal should distribute all the prize mass to the second contest. With *ex ante* asymmetry, the effect of the win advantage on the effort in the second contest depends on how disadvantaged the laggard is. A large disadvantage at the outset implies that, as the win advantage increases, total effort for the disadvantaged firm is reduced as the discouragement effect dominates the catching-up effect. If the inital disadvantage is small, then the catching-up effect dominates and the laggard increases its total effort, seeking to overturn the initial disadvantage. When there are more than two players, we find that introducing the win advantage is an effective mechanism for shifting effort to the early contest.

Keywords: dynamic contest, win advantage, prize division

JEL codes: D74, D72

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1 Introduction

Many contest situations have the features that (i) contestants meet more than once, (ii) winning in early rounds gives an advantage in later rounds, and (iii) the prize structure is such that the prize value in each stage may differ. In this paper, we set up a model to study such a contest situation. In this model, there are two contests run in sequence among the same set of players, and the winner of the first contest has a lower cost of effort than the other player(s) in the second one.

The win advantage from the early round introduces an asymmetry into the subsequent competition. A crucial insight from our analysis is that this reduces the expected value to the contestants of being in the game, whilst it increases the efforts exerted. Hence, a win advantage is advantageous for an effort-maximizing contest designer, whereas, in expectation, it will not be beneficial for the players. We also show that the principal should distribute all of the prize mass, if possible, to the second contest in order to maximize total effort across the two contests.

Losing the first-round contest with a win advantage may have one of two potential effects on a player before the second round: he may be discouraged by his earlier loss and the entailing disadvantage in effort costs and thus reduce his effort before the second contest relative to a case of no win advantage; or he may be encouraged to increase his effort in order to compensate for this disadvantage in the fight for the second-round prize. In our analysis, we find that, as long as the players are symmetric to start with, the former effect dominates, so that the win advantage discourages the early loser.

When we introduce *ex-ante* asymmetry, so that one player has an advantage already at the outset, the effect of the win advantage on the effort in the second contest depends on how disadvantaged the *ex-ante* laggard is. A large disadvantage at the outset implies that, as the win advantage from the first round increases, total effort for the disadvantaged player is reduced as compensating the accumulated disadvantage gets too costly. If the initial disadvantage is small, on the other hand, then the laggard increases his total effort, to try to overturn the initial disadvantage.

In the analysis below, we compare our basic setting, a sequence of two contests where an advantage in the second contest is obtained by the winner of the first contest, with an alternative, grand contest, where play is done once and the advantage is assigned one of the players at random. Whether the win advantage in this sense leads to more effort among the players depends on factors such as the number of players, the distribution of the prize mass across the contests, and the heterogeneity among the players at the outset. We show for example that introducing a win advantage is an effective method for shifting effort to the early contest when there are more than two players, as this encourages effort in the early round in order to be the single player with an advantage in the second contest.

There are numerous real-life situations, in areas such as business, politics, and sport, that have features resembling our set-up. Consider for example a major research programme organized by a national research council, where funds are awarded through a sequence of contests over the lifetime of the programme. It is reasonable to consider early winners having an advantage over other applicants before later rounds of contests. Sequences of contests are also in frequent use in sales-force management, with seller-of-the-month awards, *etc.*, in order to provide motivation for the sales force. In many organizations, promotion games may have the same multi-stage structure, and in a number of sports, teams meet repeatedly throughout the season. The winner of a pre-election TV debate may be seen as obtaining a win advantage in the ensuing election (Schrott,

1990). In a quite different setting, students are subject to a number of tests throughout the year, with the final ranking being based on an exam in the end.

Evidence pointing to the presence of a win advantage in sequential competition is found in experimental studies carried out by Reeve, *et al.* (1985) and Vansteenkiste and Deci (2003). These studies show that winners feel more competent than losers, and that winning facilitates competitive performance and contributes positively to an individual's intrinsic motivation.

The study closest to ours is that of Möller (2012). Like us, he posits a sequence of contests in which the principal can choose how to distribute the prize fund across the contests. His win advantage differs from ours, though. In his case, the first-round prize is restricted to being invested in improving the winner's technology before the second round, so that there is no direct benefit from the prize for the early winner. In our case, the first-round prize does have such a direct benefit to the early winner, in addition to the advantageous effect winning has on the winner's second-round technology.

Also related is the study of Beviá and Corchón (2013). Like us and Möller (2012), they study a sequence of contests where there is an asymmetry between players in the second contest that stems from the outcome of the first. However, their model is special in that all players receive prizes in each period, so that the "winner" in a round does not take it all but rather merely gets more than the other players. Moreover, their set-up requires *ex-ante* asymmetry to get out results and has not much to say on the symmetric case.

More generally, we are related to studies of dynamic battles; see the survey by Konrad (2009, ch 8). One such battle is the race, in which there is a grand prize to the player who first scores a sufficient number of wins. A related notion is the tug-of-war, where the winner of the grand prize is the one who first gets a sufficiently high lead. Early formal analyses of the race and the tug-of-war were done by Harris and Vickers (1987). A study of races where there, as in our paper, also are intermediate prizes in each round, in addition to the grand prize, is done by Konrad and Kovenock (2009).

Another variation of a dynamic battle is the elimination tournament, where the best players in an early round are the only players proceeding to the next round (Rosen, 1986). Thus, in an elimination tournament, the number of players decreases over time. A particularly interesting study of an elimination tournament is done by Delfgaauw, *et al.* (2012). They allow for intermediate prizes at each round of elimination, and they find, in line with our results, that the principal would like to have the early prizes small and the expected late prizes large and confirm in a field experiment that this leads to higher total effort.

Still another variation of dynamic battles is the incumbency contest, where the winner of one round of the contest is the "king of the hill" in the next round and thus has an advantage that resembles our win advantage; see the analyses by Ofek and Sarvary (2003) and Mehlum and Moene (2006, 2008).

While most of our analysis is carried out in a setting where the win advantage is exogenous while the prize distribution across contests can be decided by the principal, there is clearly a link to the work of Meyer (1992), who discusses a situation where the win advantage is endogenous. This link is spelled out in the analysis below.

Whereas we in the present analysis emphasize dynamic *win* effects, whereby an early win creates a later advantage, Casas-Arce and Martínez-Jerez (2009), Grossmann and Dietl (2009), and Clark and Nilssen (2013) discuss dynamic *effort* effects, whereby early efforts create later advantages. In Clark, *et al.* (2013), we carry out an analysis related

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to this one, where each stage contains an all-pay auction with a win advantage, whereas the present analysis is based on a Tullock contest at each stage.

The paper is organized as follows: In section 2, we present the model. In section 3, we discuss the case when the players are not identical at the outset. In section 4, we analyze the optimal prize structure. In section 5, we consider several extensions: we introduce decreasing or increasing returns to effort, we allow a loss disadvantage in addition to the win advantage, and we allow the principal to choose the win advantage. In section 6, we look at large contests, i.e., contests with more than 2 players. Finally, in section 7, we present some concluding remarks. Some of our proofs are relegated to an Appendix.

2   A simple model

Consider two identical players, 1 and 2, who compete with each other in two interlinked sequential contests. In each contest, a prize of size 1 is on offer, and the players compete by making non-refundable outlays. In the first contest, the players have a symmetric marginal cost of effort $x_{i,1}, i = 1, 2$, and the winner is determined by a Tullock contest success function:[1]

$$p_{1,1}(x_{1,1}, x_{2,1}) = \frac{x_{1,1}}{x_{1,1} + x_{2,1}} \tag{1}$$

where $p_{1,1}(x_{1,1}, x_{2,1})$ is the probability that player 1 wins the prize in the first contest, making $p_{2,1}(x_{1,1}, x_{2,1}) = 1 - p_{1,1}(x_{1,1}, x_{2,1})$ the probability that player 2 wins it. The linkage between the two contests occurs via a win advantage that reduces the marginal cost of effort of the first-contest winner in the second contest to $a \in (0, 1]$; the smaller is a, the larger is the win advantage. The loser of the first contest continues to the second contest with a marginal cost of effort of 1.

Efforts in the second contest determine the winner of the second-contest prize, according to the same rule as in (1). Denote by $x_{1,2}(i)$ and $x_{2,2}(i)$ the efforts of player 1 and 2 in the second period given that player i has won the first contest. Based on these efforts, the probability that player 1 wins the second contest is

$$p_{1,2}(x_{1,2}(i), x_{2,2}(i)) = \frac{x_{1,2}(i)}{x_{1,2}(i) + x_{2,2}(i)} \tag{2}$$

The players determine their efforts in each contest as part of a subgame perfect Nash equilibrium in which their aim is to maximize own expected payoff. The model is solved by backward induction starting with contest 2 in which the expected payoffs are given by

$$\pi_{i,2}(i) = \frac{x_{i,2}(i)}{x_{i,2}(i) + x_{j,2}(i)} - ax_{i,2}(i), \text{ and}$$

$$\pi_{j,2}(i) = \frac{x_{j,2}(i)}{x_{i,2}(i) + x_{j,2}(i)} - x_{j,2}(i), \ i = 1, 2, \ j \neq i$$

An internal second-contest equilibrium is characterized by the following first-order conditions for the first-contest winner and loser:

$$\frac{\partial \pi_{i,2}(i)}{\partial x_{i,2}(i)} = \frac{x_{j,2}(i)}{(x_{i,2}(i) + x_{j,2}(i))^2} - a = 0,$$

$$\frac{\partial \pi_{j,2}(i)}{\partial x_{j,2}(i)} = \frac{x_{i,2}(i)}{(x_{i,2}(i) + x_{j,2}(i))^2} - 1 = 0,$$

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[1] As axiomatized by Skaperdas (1996) and used in numerous contest applications; see, for example, Konrad (2009).

yielding equilibrium efforts in the second contest of

$$x_{i,2}(i) = \frac{1}{(1+a)^2} \tag{3}$$

$$x_{j,2}(i) = \frac{a}{(1+a)^2} \tag{4}$$

Hence, the winner of the first contest becomes more efficient at exerting effort in the second contest, and exerts more effort than the rival. This leads to a larger than one-half chance of winning the second contest – $\frac{1}{1+a}$, to be precise – and the more efficient player also has a larger expected payoff:

$$\pi_{i,2}(i) = \left(\frac{1}{1+a}\right)^2 \tag{5}$$

$$\pi_{j,2}(i) = \left(\frac{a}{1+a}\right)^2 \tag{6}$$

At the beginning of the first contest, each player has an expected payoff of

$$\pi_{1,1} = p_{1,1}(1 + \pi_{1,2}(1)) + (1 - p_{1,1})\pi_{1,2}(2) - x_{1,1}, \tag{7}$$

$$\pi_{2,1} = (1 - p_{1,1})(1 + \pi_{2,2}(2)) + p_{1,1}\pi_{2,2}(1) - x_{2,1}. \tag{8}$$

The expected payoff from the first contest consists of three elements: (i) the probability that a player wins the first contest multiplied by the prize for the first contest plus the expected profit from the second contest having won the first; (ii) the probability of losing the first contest multiplied by the expected payoff from the second contest having lost the first; and (iii) the first-period cost of effort.

Seen from the first period, the players solve identical maximization problems. Writing out the expected payoff for player 1 in full, using (1),(3),(4),(5), and (6), gives

$$\pi_{1,1} = \frac{x_{1,1}}{x_{1,1} + x_{2,1}} \left[1 + \left(\frac{1}{1+a}\right)^2\right] + \left(1 - \frac{x_{1,1}}{x_{1,1} + x_{2,1}}\right)\left(\frac{a}{1+a}\right)^2 - x_{1,1}$$

$$= \left(\frac{a}{1+a}\right)^2 + \frac{x_{1,1}}{x_{1,1} + x_{2,1}}\left(\frac{2}{1+a}\right) - x_{1,1}. \tag{9}$$

Differentiating this expression with respect to the choice variable of player 1, $x_{1,1}$, gives

$$\frac{\partial \pi_{1,1}}{\partial x_{1,1}} = \frac{x_{2,1}}{(x_{1,1} + x_{2,1})^2}\left(\frac{2}{1+a}\right) - 1.$$

At an interior symmetric equilibrium, we have $x_{1,1} = x_{2,1}$, giving rise to the solution

$$x_{1,1} = x_{2,1} = \frac{1}{2(1+a)}. \tag{10}$$

Denote total expected efforts in each contest as X_1 and X_2. Using (10) in (9), and adding (3) and (4), we find the following

Proposition 1 *In equilibrium,*

$$X_1 = X_2 = \frac{1}{1+a}.$$

The total expected value of the two-contest game to each player is

$$\pi_{1,1} = \pi_{2,1} = \frac{1}{2}\left(1 - \frac{a(1-a)}{(1+a)^2}\right).\tag{11}$$

Thus, expected efforts in each of the two contests are equal, and total expected effort over the two contests is

$$X = X_1 + X_2 = \frac{2}{1+a}\tag{12}$$

This is decreasing in a: the larger the win advantage (*i.e.*, the smaller a) the more effort can be expected by the players. When $a = 1$, the model reverts to being of two independent Tullock contests each over a prize of size one, with marginal cost of effort equal to one for each contestant in each period. In this case, expected effort for each player is $\frac{1}{4}$ in each period. When a decreases below 1, an asymmetry is introduced into the second period contest, since one player will be advantaged in relation to the other. This encourages the advantaged player to increase effort in the second contest, whilst the disadvantaged player slackens off to save effort cost. In the first contest, the players compete not only for the prize at that contest stage, but also the right to be the advantaged player in contest two. Hence efforts in contest one are increased compared to the level that would occur with two independent symmetric contests.

To see the role that asymmetry plays here, suppose that the contest designer can at the outset choose one of the players to have cost a, whilst the other has 1. In a grand contest with prize 2, the advantaged player will have an effort of $\frac{2}{(1+a)^2}$, and the rival $\frac{2a}{(1+a)^2}$. In sum this is the same as in (12). The effort equivalence of our simple model with that of a single asymmetric contest is shown in later sections to be a facet of i) the two-player model, ii) the equal distribution of prizes across contests and iii) the initial symmetry of players.

Whilst the relationship between the win advantage a and total expected effort in equilibrium is monotonic, the same is not the case for the total expected value of the two contest games given in (11) and graphed in Figure 1.

With two independent contests ($a = 1$), each contest would have a value of $\frac{1}{4}$ to each player, and hence the total value would be $\frac{1}{2}$ per player. It is apparent that introducing a win effect in the first contest will initially cause the expected value of the contest to fall for high values of a (*i.e.*, a small win advantage), and that this value will increase as the win advantage becomes larger (smaller values of a). From (11) it is easy to verify that $\pi_{1,1}$ and $\pi_{2,1}$ reach a minimum value of $\frac{7}{16}$ at $a = \frac{1}{3}$. As the win advantage becomes bigger, total expected effort increases as discussed above; why then does the expected value initially decrease and then increase as the win advantage gets larger? The cases $a \to 0$ and $a = 1$ both collapse to a single contest; the former since the winner of the first contest has almost zero cost of effort in the second contest, making the opponent give up, and the latter since the contests are no longer related, and the prize can be distributed in one go. When a is increased from zero, the winner of the first contest has a lower effort level, but at a higher cost. The loser increases effort. Taking into account the probability of winning and losing the first contest, and the fact that higher values of a lead the winner to do less but at a higher cost leads to a concave total expected cost of effort function given as

$$\begin{aligned}EC_1 &= EC_2 = x_{1,1} + p_{1,1}\left(x_{1,1}, x_{2,1}\right)ax_{1,2}(1) + (1 - p_{1,1}\left(x_{1,1}, x_{2,1}\right))x_{1,2}(2)\\&= \frac{1}{2}\left(1 + \frac{a(1-a)}{(1+a)^2}\right).\end{aligned}$$
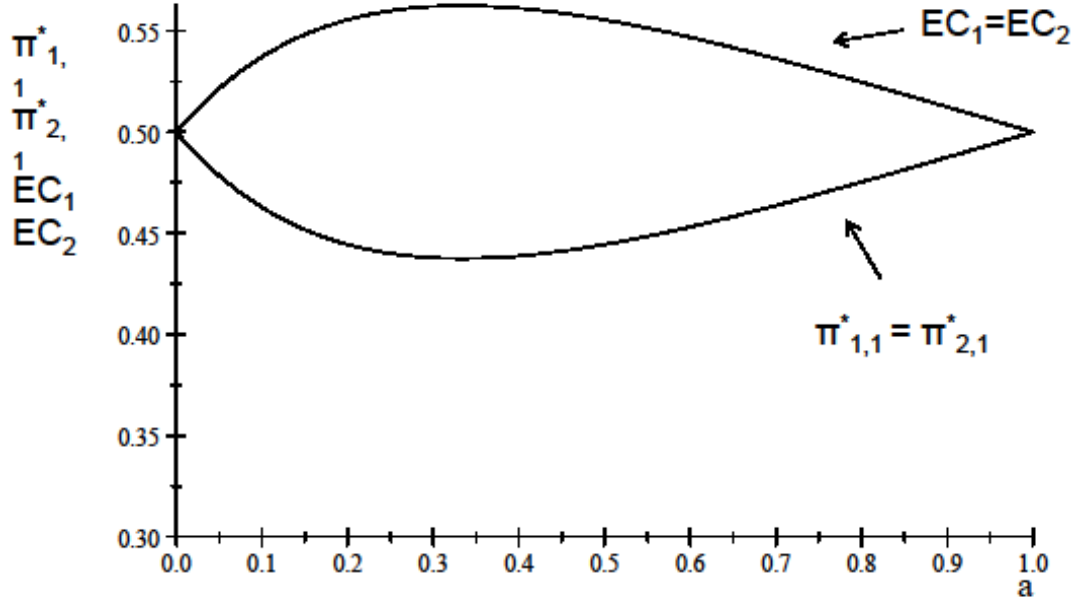
Figure 1: Profits and expected costs in equilibrium

This is also depicted in Figure 1. This expression reaches a maximum at $a = \frac{1}{3}$; as a is initially reduced from 1, the extra effort induced by the first contest winner occurs at quite a high cost, and as a falls further, the extra effort costs less and less at the margin, until the cost effect dominates and more effort actually costs less.

The win advantage introduces an asymmetry into the competition that reduces the expected value to the contestants of being in the game, whilst it increases the efforts exerted. Hence it may seem that a win advantage is advantageous for an effort maximizing contest designer, whereas in expectation it will not be beneficial for the players. A winning experience might in this sense be thought of as negative, although the player that actually ends up winning the first round will have an increase in expected payoff in the second contest.

3   *Ex-ante* asymmetry

Suppose now that player 1 has an initial cost advantage over player 2 at the beginning of contest 1. Specifically, let the initial marginal cost of effort for player 1 be $y < 1$, falling to ay in the second contest if he wins the first. Player 2 has initial marginal cost of 1. As above, a prize of 1 is available in each contest. The equilibrium efforts in the first contest are naturally no longer symmetric:[2]

$$x_{1,1} = \frac{\left(1 + 2ay + y^2\right)\left[2\left(1 + ay\right) - a^2\left(1 - y^2\right)\right]^2}{\left(1 + ay\right)^2\left(1 + y\right)^2\left[2\left(1 + ay\right) - \left(y + a^2\right)\left(1 - y\right)\right]^2}; \tag{13}$$

$$x_{2,1} = \frac{y\left(1 + 2ay + y^2\right)\left[2\left(1 + ay\right) - a^2\left(1 - y^2\right)\right]^2}{\left(1 + ay\right)^2\left(1 + y\right)^2\left[2\left(1 + ay\right) - \left(y + a^2\right)\left(1 - y\right)\right]^2}. \tag{14}$$

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[2]Calculations are to be found in the Appendix.

The leader (player 1) exploits his initial advantage by having more effort in the first contest: $x_{1,1} > x_{2,1}$. Moreover, $\frac{dx_{1,1}}{da} < 0$, $\frac{dx_{1,1}}{dy} < 0$, and $\frac{dx_{2,1}}{dy} > 0$, while the sign of $\frac{dx_{2,1}}{da}$ is ambiguous (positive for small y, negative for large).

Expected efforts in the second contest for the two players are given by

$$Ex_{1,2} = \frac{2 - a^2 + 3ay + 3a^2y^2 + ay^3}{(1 + ay)^2 (1 + y) [2(1 + ay) - (y + a^2)(1 - y)]}; \tag{15}$$

$$Ex_{2,2} = \frac{a(2 - a^2) + 2a^2y + (1 + a^3)y^2 + 2ay^3 + y^4}{(1 + ay)^2 (1 + y) [2(1 + ay) - (y + a^2)(1 - y)]}. \tag{16}$$

From this we have that $\frac{dEx_{1,2}}{da} < 0$, and $\frac{dEx_{1,2}}{dy} < 0$, whereas the effects of a and y on $Ex_{2,2}$ are ambiguous. The effects of a and y on the total effort of player 1 are thus monotonic, whereas the relationship for player 2, the *ex-ante* laggard, is more complicated. Let the sum of the expected efforts in the two periods for player i be given by Z_i, where

$$Z_i = x_{i,1} + Ex_{i,2},$$

and $Z = Z_1 + Z_2$ is total expected effort. Clearly, $\frac{\partial Z_1}{\partial a} < 0$, so that a smaller win advantage decreases the effort of the favoured player. Figure 2 depicts the areas in (y, a) space in which $\frac{\partial Z}{\partial a}$ and $\frac{\partial Z_2}{\partial a}$, respectively, are positive and negative.
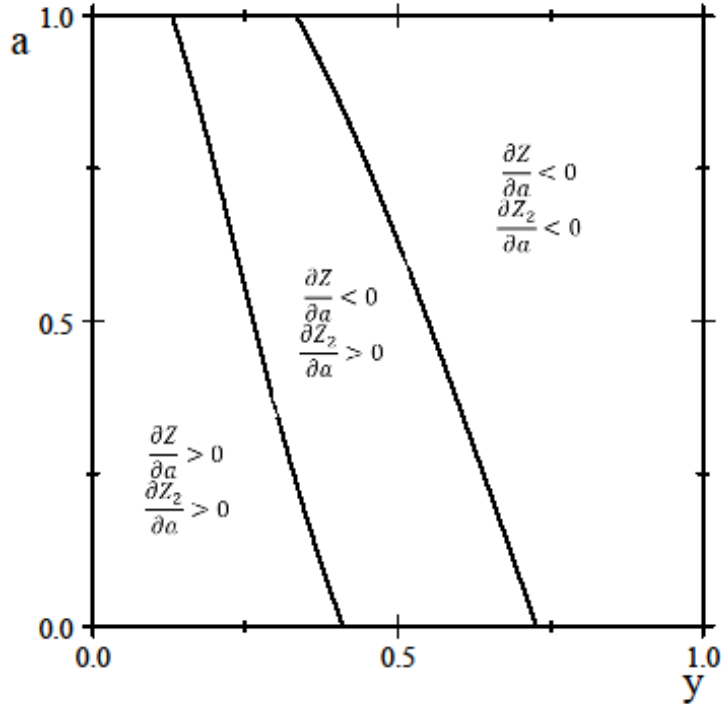
Figure 2: Effect of the win advantage on effort under asymmetry

The locus to the right in the figure delineates areas in which $\frac{\partial Z_2}{\partial a} > 0$ (to the left) and $\frac{\partial Z_2}{\partial a} < 0$ (to the right). For relatively low values of y, player 2 is at a large disadvantage at the outset; in this area, when a falls, meaning that the winner of the first contest gains an even larger advantage in the second contest, player 2 reduces effort. The chances are great that player 1 will win the first contest, but player 2 might win leading to him

evening out some of the initial disadvantage; this encourages effort. On the other hand, there is a great chance that player 1 will win the first contest and, as a falls, gain an even larger advantage in the second; this discourages effort. The latter effect dominates to the left of the right-hand locus in Figure 2. For quite large values of y, player 2 is not so disadvantaged initially; if a falls in this area, this player will react by increasing total effort, enticed by the possibility of catching up player 1's initial, relatively small advantage. The locus to the left in the Figure delineates areas in which $\frac{\partial Z}{\partial a} > 0$ (left side) and $\frac{\partial Z}{\partial a} < 0$ (right side). On the left, the effect of a on Z_2 dominates the opposing effect on Z_1. In the middle region of the Figure, the a exerts opposing effects on Z_1 and Z_2, with the effect on Z_1 now dominating.

We can see the effect of our contest on effort by considering a grand contest with asymmetry. Now there are two types of asymmetry that a principal can consider in including the win advantage parameter in a grand contest: i) it can be used to make the advantaged player even stronger at the outset, so that player 1 has a marginal cost of effort of ay, ii) it can be used to neutralize or overturn the initial asymmetry by making the marginal costs of player 1 and 2 equal to y and a, respectively. In the first case, with a very uneven contest, efforts by each player are $\frac{2}{(1+ay)^2}$ and $\frac{2ay}{(1+ay)^2}$, totalling $Z_{un} = \frac{2}{1+ay}$, and the more even contest yields $\frac{2y}{(y+a)^2}$ and $\frac{2a}{(y+a)^2}$, and $Z_{ev} = \frac{2}{y+a}$ in sum. The total effort in this case is higher in the second contest, so that the principal would increase effort by evening up the contest. These total efforts are compared to our mechanism in Figure 3.
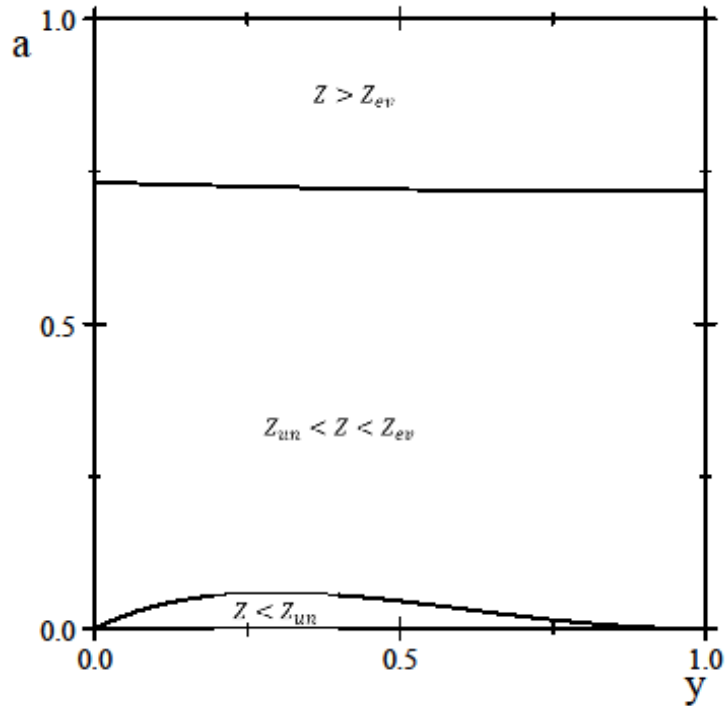
Figure 3: Effort comparison with win advantage and asymmetric grand contest

Our mechanism gives more total effort than the uneven grand contest unless a is very small. This is due to the large efficiency gain being introduced earlier in the grand contest. The even grand contest performs better than our mechanism for a larger set of values of a than the uneven one. But when the win advantage is relatively small, in the top region of the Figure, it is better for total effort if asymmetric players must compete

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to gain an advantage in the second contest.

4 Optimal prize distribution

Given that the contests are interconnected by a win-advantage effect, a contest designer might wish to exploit this in order to maximize total expected effort. Whilst the amount of the win advantage will probably be out of the designer's control,[3] another variable is at his disposal, namely the prize distribution between the two contests. Above, the prize was assumed to be distributed in two equal amounts. Here, we consider a designer (principal) who wishes to maximize expected total effort in the contest by choosing a distribution of the total prize mass across the two contests. We return to the case of *ex-ante* symmetry of Section 2. To maintain comparability with that section, we assume that the principal distributes a total prize mass of 2, saving an amount M for the second contest and awarding a prize of $2 - M$ in the first.

With this prize distribution, the expected payoffs in the second contest will be

$$\pi_{i,2}(i) = \frac{x_{i,2}(i)}{x_{i,2}(i) + x_{j,2}(i)}M - ax_{i,2}(i), i = 1, 2, j \neq i$$

$$\pi_{j,2}(i) = \frac{x_{j,2}(i)}{x_{i,2}(i) + x_{j,2}(i)}M - x_{j,2}(i)$$

Straightforward calculations give the following equilibrium values for efforts and payoffs in the second contest:

$$x_{i,2}(i) = \frac{1}{(1+a)^2}M$$

$$x_{j,2}(i) = \frac{a}{(1+a)^2}M$$

$$\pi_{i,2}(i) = \left(\frac{1}{1+a}\right)^2 M$$

$$\pi_{j,2}(i) = \left(\frac{a}{1+a}\right)^2 M$$

At the first contest, each player maximizes:

$$\begin{aligned}
\pi_{1,1} &= \frac{x_{1,1}}{x_{1,1} + x_{2,1}}\left(2 - M + \frac{1}{(1+a)^2}M\right) \\
&+ \left(1 - \frac{x_{1,1}}{x_{1,1} + x_{2,1}}\right)\left(\frac{a}{1+a}\right)^2 M - x_{1,1} \\
&= \left(\frac{a}{1+a}\right)^2 M + 2\frac{x_{1,1}}{x_{1,1} + x_{2,1}}\left(1 - \frac{a}{1+a}M\right) - x_{1,1}
\end{aligned}$$

Equilibrium values for the first contest can easily be found to be[4]

$$x_{1,1} = x_{2,1} = \frac{1}{2}\left(1 - \frac{a}{1+a}M\right), \text{ and}$$

$$\pi_{1,1} = \pi_{2,1} = \frac{1}{2}\left(1 - \frac{a(1-a)}{(1+a)^2}M\right). \tag{17}$$

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[3]But see Section 5.3 below.

[4]Note that equilibrium efforts are non-negative, since $a \leq 1$, and $M \leq 2$.

Total effort is the sum of the efforts expended in contest one, $X_1 = x_{1,1} + x_{2,1}$, and contest two, $X_2 = x_{i,2}(i) + x_{j,2}(i)$, so that

$$X_1 = 1 - \frac{a}{1+a}M, \; X_2 = \frac{1}{1+a}M, \text{ and}$$

$$X = X_1 + X_2 = 1 + \frac{1-a}{1+a}M. \tag{18}$$

Note that total effort in contest one decreases in M (the second-round prize), while total effort in contest two increases in it. In fact, we have

Proposition 2 $X_1 < X_2$ *if and only if* $M > 1$.

Hence, with two contestants, total effort in the second contest is larger than the first if and only if the second contest has the larger prize. The effect on second-round effort is the stronger, as can be seen from the expression for the sum of efforts, which is always increasing in M. Hence,

Proposition 3 *To maximize total efforts, the principal should set* $M = 2$.

To maximize total effort in the two contests, the principal should distribute all of the prize mass in the second contest. In the first contest, participants then compete to be the advantaged player in the second contest with no instantaneous prize. In the second contest, an asymmetry is introduced which would tend to reduce total effort compared to a symmetric contest, and to mitigate this effect the principal should award a large prize here. This prize distribution, however, minimizes the expected value of the two-contest game for the players as is seen from (17).

At the optimum $M = 2$, the loser's effort in the second contest is $\frac{2a}{(1+a)^2}$, while each participant's effort in the first contest is $\frac{1}{2}\left(1 - \frac{2a}{1+a}\right) = \frac{1}{2} - \frac{a}{1+a}$. Thus, if $a \in \left(\sqrt{5} - 2, 1\right]$ $\approx (0.236, 1]$, then the loser increases his effort from contest one to contest two. It is only when the win advantage effect is very large – $a < \sqrt{5} - 2$ – that the loser decreases effort in contest 2 compared to contest 1 when the principal's optimum distribution of the prize mass is instituted.

In Section 2, we showed that total efforts in our model with an equal prize split are the same as in a grand contest in which the principal gives the win advantage to one of the players at the outset. With the possibility of dividing the prize mass over two contests this equivalence breaks down, and comparing (18) with (12) shows that our mechanism yields most total effort for $M > 1$, *i.e.*, when the bulk of the prize is distributed in contest 2.

5   Extensions

In this section, we consider extensions to the basic model, where we first analyze the effect of returns to effort in the contests, then the introduction of a loss disadvantage, and finally the possibility of biased contests.

5.1 Returns to effort

Let us now, in the two-contestant model, allow for decreasing or increasing returns to effort in each contest, so that the probability that player 1 wins contest $t = 1, 2$ is given by

$$p_{1,t} = \frac{x_{1,t}^r}{x_{1,t}^r + x_{2,t}^r},$$

where $r > 0$ represents the elasticity of the odds of winning. When $r \in (0, 1)$, there are *decreasing* returns to effort in each contest, and when $r > 1$, there are *increasing* returns to effort; the analysis above posited constant returns, with $r = 1$. There is a prize of $2 - M$ in the first contest and one of M in the second. We can use the results of Nti (1999) to find the equilibrium outcome. Suppose that player 1 wins the first contest so that, in the second contest, expected profits of the two players are

$$\pi_{1,2} = \frac{x_{1,2}^r}{x_{1,2}^r + x_{2,2}^r} M - a x_{1,2} = a \left(\frac{x_{1,2}^r}{x_{1,2}^r + x_{2,2}^r} \frac{M}{a} - x_{1,2} \right), \text{ and}$$

$$\pi_{2,2} = \frac{x_{2,2}^r}{x_{1,2}^r + x_{2,2}^r} M - x_{1,2}.$$

In order for us to ensure a pure-strategy equilibrium in the second contest, we impose the condition

$$r - a^r < 1,$$

which follows from Nti (1999, Proposition 3).[5] Without any win advantage, so that $a = 1$, this condition amounts to $r < 2$. But the larger is the win advantage, *i.e.*, the lower is a, the stricter the condition becomes. Still, for the case of decreasing returns to efforts, $r < 1$, a pure-strategy equilibrium exists in the second contest for any $a \in [0, 1]$.

By Nti's (1999) equations (7) and (9), we obtain equilibrium efforts in the second contest equal to

$$x_{1,2} = \frac{r a^{r-1}}{(1 + a^r)^2} M, \text{ and}$$

$$x_{2,2} = \frac{r a^r}{(1 + a^r)^2} M.$$

This gives rise to second-contest payoffs equal to

$$\pi_{1,2} = \frac{(1-r) a^r + 1}{(1 + a^r)^2} M, \text{ and}$$

$$\pi_{2,2} = \frac{a^r (1 - r + a^r)}{(1 + a^r)^2} M.$$

Turning to the first contest, each player's expected profit is

$$\pi_{1,1} = \frac{x_{1,1}^r}{x_{1,1}^r + x_{2,1}^r} \left(2 - M + \frac{(1-r) a^r + 1}{(1 + a^r)^2} M \right)$$
$$+ \left(1 - \frac{x_{1,1}^r}{x_{1,1}^r + x_{2,1}^r} \right) \frac{a^r (1 - r + a^r)}{(1 + a^r)^2} M - x_{1,1}$$
$$= \frac{a^r (1 - r + a^r)}{(1 + a^r)^2} M + 2 \frac{x_{1,1}^r}{x_{1,1}^r + x_{2,1}^r} \left(1 - \frac{a^r}{1 + a^r} M \right) - x_{1,1}.$$

---

[5]Nti (1999) considers a contest in which players has the same marginal cost of effort and different prize valuations. Our framework can be transformed to his with prize values $V_1 = \frac{M}{a}$, and $V_2 = M$.

As above, the first term is a constant and has no effect on the players' choices of effort. Since the two players are symmetric, we can impose symmetry to obtain first-contest efforts equal to

$$x_{1,1} = x_{2,1} = \frac{r}{2}\left(1 - \frac{a^r}{1+a^r}M\right)$$

Thus, total effort in the first contest is

$$X_1 = r\left(1 - \frac{a^r}{1+a^r}M\right),$$

while total effort in the second contest is

$$X_2 = \frac{ra^{r-1}}{(1+a^r)^2}M + \frac{ra^r}{(1+a^r)^2}M = ra^{r-1}\frac{1+a}{(1+a^r)^2}M$$

Total effort over the two contests becomes

$$
\begin{aligned}
X_1 + X_2 &= r\left(1 - \frac{a^r}{1+a^r}M\right) + ra^{r-1}\frac{1+a}{(1+a^r)^2}M \\
&= r\left(1 + \frac{a^{r-1}(1-a^{r+1})}{(1+a^r)^2}M\right).
\end{aligned}
$$

Since the multiplier of M in this expression is always positive for $a \in (0,1]$, $r > 0$, and $r - a^r < 1$, total effort increases in M, and the result in Proposition 3, that the principal would like to have the full prize mass in the second contest, is robust to the introduction of returns to effort.

5.2 Loss disadvantage

In some applications, the implication of a first-round contest may just as well be a disadvantage in future contests suffered by the loser as an advantage gained by the winner. In order to discuss the notion of a loss disadvantage, we introduce a second-round effort cost bx for the loser of the first-round contest, where $b \geq 1$. The higher b, the larger the effect on a player's future costs from losing today. In addition, we retain the advantage accruing to the winner, so that the second-round effort cost is ax for the first period winner, with $a \in (0,1]$.

With prizes $2 - M$ in the first period and M in the second, players' expected second-period payoffs are

$$
\begin{aligned}
\pi_{i,2}(i) &= \frac{x_{i,2}(i)}{x_{i,2}(i) + x_{j,2}(i)}M - ax_{i,2}(i), i = 1,2, j \neq i \\
\pi_{j,2}(i) &= \frac{x_{j,2}(i)}{x_{i,2}(i) + x_{j,2}(i)}M - bx_{j,2}(i).
\end{aligned}
$$

These give rise to equilibrium second-round efforts and payoffs for the first-round winner and loser:

$$
\begin{aligned}
x_{i,2}(i) &= \frac{b}{(a+b)^2}M \\
x_{j,2}(i) &= \frac{a}{(a+b)^2}M
\end{aligned}
$$

$$\pi_{i,2}(i) = \left(\frac{b}{a+b}\right)^2 M$$

$$\pi_{j,2}(i) = \left(\frac{a}{a+b}\right)^2 M$$

In the first contest, each player thus maximizes

$$\pi_{1,1} = \frac{x_{1,1}}{x_{1,1}+x_{2,1}}\left(2 - M + \left(\frac{b}{a+b}\right)^2 M\right)$$

$$+ \left(1 - \frac{x_{1,1}}{x_{1,1}+x_{2,1}}\right)\left(\frac{a}{a+b}\right)^2 M - x_{1,1}$$

$$= \left(\frac{a}{a+b}\right)^2 M + 2\frac{x_{1,1}}{x_{1,1}+x_{2,1}}\left(1 - \frac{a}{a+b}M\right) - x_{1,1}$$

This leads to the following equilibrium values in the first round:

$$x_{1,1} = x_{2,1} = \frac{1}{2}\left(1 - \frac{a}{a+b}M\right), \text{ and}$$

$$\pi_{1,1} = \pi_{2,1} = \frac{1}{2}\left(1 - \frac{a(b-a)}{(a+b)^2}M\right).$$

Total expected effort is now

$$\left(1 - \frac{a}{a+b}M\right) + \frac{b}{(a+b)^2}M + \frac{a}{(a+b)^2}M = 1 + \frac{1-a}{a+b}M$$

Thus, the introduction of a loss disadvantage decreases total expected effort, for any $b > 1$, but does not alter the conclusion from Proposition 3 that the principal should put the prize mass in the second contest.

A particularly interesting case of a loss disadvantage is the symmetric one, for which $\frac{a+b}{2} = 1$, or $b-1 = 1-a$, so that the loss disadvantage exactly matches the win advantage. At $M = 1$, so that prizes are equal across rounds, we now have total expected effort equal to

$$1 + \frac{1-a}{2} = \frac{3-a}{2} > 1.$$

Thus, for any $a \in (0,1)$, total effort is higher than if both contests were run either simultaneously or as one big contest, for which total effort equals 1.

5.3 Biased contests

We have so far taken for granted that the principal has no influence on the size of the win advantage. In some applications, however, one can easily envisage the principal being able to determine how big the benefit of an early win shall be. This idea is very much in the line of the work of Meyer (1992) who discusses the merits of biased contests and tournaments. In our model, the result follows directly from equation (18) above: if the principal were to choose, she would not only want to have a high second-round prize M but also a large win advantage (a low a).

6   Large contests

Extending the basic model to the case of $n \geq 2$ participants is straightforward. For the case of $M = 1$, the following equilibrium values can be determined (with calculations in the Appendix) for the expected value of the game to each player $(\pi_s(n), s = 1, ..., n)$ and the expected total cost of effort to each player $(EC_s(n))$:

$$\pi_s(n) = \frac{2}{n^2}\left[1 - a(1-a)\left(\frac{n-1}{n+a-1}\right)^2\right]; \qquad (19)$$

$$EC_s(n) = \frac{2}{n^2}\left[(n-1) + a(1-a)\left(\frac{n-1}{n+a-1}\right)^2\right]. \qquad (20)$$

A similar picture emerges as in the case of $n = 2$, discussed in Section 2, with respect to how a affects these magnitudes. It is easily verified that $\pi_s(n)$ is convex in a, reaching a minimum at $a = \frac{n-1}{2n-1} \in \left[\frac{1}{3}, \frac{1}{2}\right)$, while $EC_s(n)$ is concave in a, reaching a maximum at the same value. One can also verify that $\frac{\partial \pi_s(n)}{\partial n} < 0$, so that more contestants reduce the expected value of the game to each player.

Total effort by all n competitors in contests 1 and 2 can be determined from (A5) through (A7) in the Appendix as

$$X_1(n) = \frac{n-1}{n+a-1}\left\{1 + (1-a)\frac{n-2}{n}\left[1 - \frac{a(n-1)}{n+a-1}\right]\right\}, \qquad (21)$$

$$X_2(n) = \frac{n-1}{n+a-1}. \qquad (22)$$

We have that $\frac{\partial X_1(n)}{\partial a} < 0$, and $\frac{\partial X_2(n)}{\partial a} < 0$, as is the case for $n = 2$ above. Further, $\frac{\partial X_1(n)}{\partial n} > 0$, and $\frac{\partial X_2(n)}{\partial n} > 0$, so that total effort increases in the number of competitors. Even though effort per player decreases in each period as more rivals are added, total effort increases since there are more players.

Since the square-bracketed term in (21) is positive for any $a \in (0, 1]$, we have the following result:

Proposition 4 $X_1(n) > X_2(n)$ for $n > 2$.

The case of $n = 2$ in Proposition 2 is thus special in that the contestants even out their efforts in each contest with an equal prize; this occurs since the game, at $M = 1$, is completely symmetric and each player has a one-half chance in equilibrium of being the advantaged player in the second contest. Players exert more effort in the first contest when there are more than two competitors. The game is still symmetric, but it is rational to move effort to the first contest since, at a symmetric situation, the probability of winning the first contest is $\frac{1}{n}$, so that a unilaterally increased effort gives a larger chance of beating $n-1$ rivals. This way, an asymmetry in the incentives between contests arises, causing effort to shift to the early contest.

As before, we can isolate the effect of competing for a win advantage as in our mechanism to that of creating an asymmetric situation at the outset. Suppose that the designer creates a grand contest with prize 2 in which one player has cost a initially, whilst all the others have 1. From (22), which is valid for a prize of 1, total efforts in this case are

$2X_2(n) = \frac{2(n-1)}{n+a-1}$. By Proposition 4 it is immediate that $X_1(n) + X_2(n) \geq 2X_2(n)$, and hence our mechanism yields weakly more effort than a grand asymmetric contest.

Calculations in the Appendix show that, when the principal can divide the prize mass into $2 - M$ and M between the two contests, the resulting total expected effort from all n participants is linearly increasing in M, and hence a corner solution obtains in which $M = 2$ maximizes total expected effort. As in the case with $n = 2$, this minimizes the expected value of the game to the players.

In contest models, one is often interested in the proportion of the prize that is dissipated. In our model, it is interesting to look at the effort-to-prize ratio in each period, even though this is not a dissipation rate as such. Moving prize mass between contests affects the effort-to-prize ratio in two ways, since both the denominator in the expression (the prize) and the numerator (effort) will be influenced. Using the expressions in the Appendix, these ratios in each period can be calculated to be

$$
\begin{aligned}
D_1 &\equiv \frac{X_1}{2-M} = \frac{n-1}{n} \frac{2 - Ma\left(1 + \frac{(1-a)\left[(n-1)^2 + a\right]}{(n+a-1)^2}\right)}{2-M}; \\
D_2 &\equiv \frac{X_2}{M} = \frac{n-1}{n+a-1}.
\end{aligned}
$$

Both these ratios are decreasing in a and increasing in n, with D_1 converging to $\frac{2-Ma(2-a)}{2-M}$ as the number of players gets large, whilst D_2 converges to 1. Hence, the larger the win advantage in the second period (lower a), the higher the effort-to-prize ratio in both periods since efforts increase. More players increase total efforts in each period also, again causing these ratios to rise. Since the total effort in contest 2 is multiplicative in M, the effort-to-prize ratio at this stage is independent of the distribution of prize distribution. Moving prize mass to the second contest (increasing M) increases D_1; there is a lower prize to be dissipated in this case $(2-M)$, and efforts fall, but by less than the decrease in prize. It is straightforward to show that $D_1 > (<) D_2$ for $M > (<) \overline{M}(a,n)$, where

$$
\overline{M}(a,n) \equiv \frac{2(n+a-1)}{n^2(1-a) + 2a(2n-1) - n}
$$

Note that $\overline{M}(a,n)$ is decreasing in n, with $\overline{M}(a,2) = 1$, and $\lim_{n\to\infty} \overline{M} = 0$. This means that the effort-to-prize ratio in contest 1 is always larger than contest 2 when most of the prize mass is distributed in contest 2, and that, as n increases, the effort-to-prize ratio in contest 1 is larger also for lower levels of M.

The distribution of effort in the general case can also be of interest, and from the expressions in the Appendix, one can determine that $X_1 > (<) X_2$ for $M < (>) \widetilde{M}(a,n)$, where

$$
\widetilde{M}(a,n) \equiv \frac{2(n+a-1)^2}{a\left[(2-a)n^2 - (3-4a)n + 2(1-a)\right] + n^2}
$$

We see that $\widetilde{M}(1,n) = 1$, and that $\widetilde{M}(a,n)$ is increasing in n, converging to $\lim_{n\to\infty} \widetilde{M}(a,n) = \frac{2}{1+2a-a^2}$. Hence, for sufficiently large M (i.e., $M > \frac{2}{1+2a-a^2}$), total effort in contest 2 will be larger than that in contest 1, independently of the number of players. Also, for $M < 1$, total effort will be larger in contest 1 independently of the number of players participating. Other comparisons depend upon the number of participants, as illustrated in Figure 4, in which $\overline{M}(a,3)$ and $\widetilde{M}(a,3)$ are drawn in as an illustration.

The figure is divided into three areas. For large values of M, there is a greater effort in the second contest, but the low prize in contest 1 gives a higher effort-to-prize ratio

M 2.0

$\widetilde{M}(a,3)$

1.5

$D_1 > D_2$
$X_2 > X_1$

1.0

$D_1 > D_2$
$X_1 > X_2$

0.5

$\overline{M}(a,3)$

$D_2 > D_1$
$X_1 > X_2$

0.0

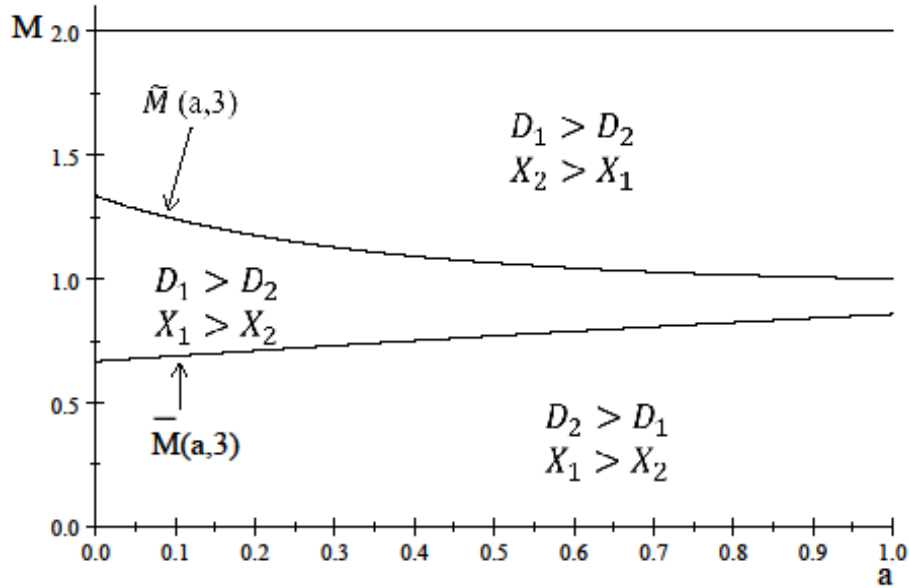0.0   0.1   0.2   0.3   0.4   0.5   0.6   0.7   0.8   0.9   1.0
a

Figure 4: Effort-to-prize ratios, and total effort

there. The result is reversed for low values of M, whilst between the two curves – for intermediate values of M – both the level of efforts and the level of dissipation are greatest in the first contest. This area gets larger as the number of players grows since $\widetilde{M}(a,n)$ pivots upwards around the point $M = 1, a = 1$, while $\overline{M}(a,n)$ falls downwards in this case.

7 Conclusions

We have analyzed a simple, dynamic contest, in which the winner of the first contest gains an advantage over the losing player in terms of reduced cost of effort in the second contest. The goal has been to shed light on an issue which is prevalent in a number of management, marketing, economics and political-science applications. Our results can add to the understanding of, among other things, how sales-force compensation schemes should be designed to increase sales effort incentives, and how R&D contests should be designed to maximize effort. The research is also related to research in psychology on (intrinsic and extrinsic) motivation in competitive environments.

Our results should be of particular interest to personnel managers. Personnel such as sales force and many others are involved in situations of intense internal competition, where employees are measured against each other. As we show here, any gains to early winners in such internal competitions are to the advantage of the personnel manager. Furthermore, if the manager can put the main prize mass at later stages, this would maximize her benefit from the situation. This calls, for example, for the use of promotions in sales-force management: it is when there is a sense among the sales force that a promotion of one of them is the climax of a sales season, or any other period of intense internal competition with dynamic win advantages, that the sales-force manager gets the most out of her employees.

A Appendix

A.1 *Ex-ante* asymmetry

Let the prize in each contest be 1. Player 1 has initial cost $y < 1$, and player 2 has marginal cost 1. A useful lemma for calculating efforts in this model is the following:

Lemma 5 *Consider a single-stage contest between two asymmetric contestants where, for contestant $i \in \{1, 2\}$, the value of winning the contest is W_i, the value of losing it is L_i, and unit effort cost is c_i. In the unique pure-strategy Nash equilibrium, contestant i's effort is*

$$x_i = \frac{c_j \left(W_i - L_i\right)^2 \left(W_j - L_j\right)}{\left[c_2 \left(W_1 - L_1\right) + c_1 \left(W_2 - L_2\right)\right]^2}, \ i, j \in \{1, 2\}, \ i \neq j,$$

while her expected payoff is

$$\pi_i = L_i + \frac{c_j^2 \left(W_i - L_i\right)^3}{\left[c_2 \left(W_1 - L_1\right) + c_1 \left(W_2 - L_2\right)\right]^2}, \ i, j \in \{1, 2\}, \ i \neq j.$$

Proof. Calculations are straightforward. Existence and uniqueness follow from Nti (1999, Prop. 3). ∎

In the second contest, player 1 has a marginal cost of ay if he has won the first. The prize for the winner of the second contest is $W_{1,2} = W_{2,2} = 1$. Lemma 5 yields efforts of

$$x_{1,2}(1) = \frac{1}{(1 + ay)^2} \tag{A1}$$

$$x_{2,2}(1) = \frac{a}{(1 + ay)^2} \tag{A2}$$

with payoffs

$$\pi_{1,2}(1) = \left(\frac{1}{1 + ay}\right)^2$$

$$\pi_{2,2}(1) = \left(\frac{ay}{1 + ay}\right)^2$$

Following a win by player 2 in contest 1, player 1 has cost y and player 2 cost per unit effort a. Efforts in the second contest are thus:

$$x_{1,2}(2) = \frac{a}{(1 + ay)^2} \tag{A3}$$

$$x_{2,2}(2) = \frac{y}{(1 + ay)^2} \tag{A4}$$

and payoffs

$$\pi_{1,2}(2) = \left(\frac{a}{1 + ay}\right)^2$$

$$\pi_{2,2}(2) = \left(\frac{y}{1 + ay}\right)^2$$

In contest 1, the contestants fight over two things: the stage prize 1 and the cost benefit a. The value for player 1 of winning the first contest is: $W_{1,1} = 1 + \pi_{1,2}(1)$, while his value of losing is $L_{1,1} = \pi_{1,2}(2)$, giving

$$W_{1,1} - L_{1,1} = 1 + \frac{1 - a^2}{(1 + ay)^2}.$$

Correspondingly for player 2, we have

$$W_{2,1} - L_{2,1} = 1 + \left(\frac{y}{1 + ay}\right)^2 - \left(\frac{ay}{1 + ay}\right)^2 = 1 + y^2 \frac{1 - a^2}{(1 + ay)^2}.$$

Initial costs are $c_1 = y$ and $c_2 = 1$. We have now, by Lemma 5, that contestants' first-period efforts are given in (13) and (14) in the text.

Total expected efforts per contestant are Z_1 and Z_2. These magnitudes are given by

$$
\begin{aligned}
Z_1 &= x_{1,1} + p_{1,1}x_{1,2}(1) + (1 - p_{1,1})x_{1,2}(2) = x_{1,1} + Ex_{1,2} \\
Z_2 &= x_{2,1} + p_{2,1}x_{2,2}(2) + (1 - p_{2,1})x_{2,2}(1) = x_{2,1} + Ex_{2,2}
\end{aligned}
$$

where $Ex_{i,2}$ is the expected effort of player i in contest 2. Using (A1)-(A4) in (2), we find (15) and (16) in the text.

A.2 The case of $n \geq 2$ players

Using the n-player equivalent of (1) and (2), with a prize of 1 in each round and player i as the winner of the first contest, efforts in contest 2 for the winner of contest 1 and the $n - 1$ losers are

$$x_{i,2}(i) = \frac{n - 1}{n + a - 1}\left(1 - \frac{a(n - 1)}{n + a - 1}\right); \tag{A5}$$

$$x_{j,2}(i) = \frac{a(n - 1)}{(n + a - 1)^2}, \ j \neq i. \tag{A6}$$

Expected profits for contest 2 are then

$$
\begin{aligned}
\pi_{i,2}(i) &= \left(1 - \frac{a(n - 1)}{n + a - 1}\right)^2 \\
\pi_{j,2}(i) &= \left(\frac{a}{n + a - 1}\right)^2, \ j \neq i.
\end{aligned}
$$

These values are then used in the expected profit for the first contest for player $s = 1, ..., n$:

$$\pi_{s,1} = \frac{x_{s,1}}{x_{s,1} + \sum_{v \neq s} x_{v,1}}(1 + \pi_{s,2}(s)) + \left(1 - \frac{x_{s,1}}{x_{s,1} + \sum_{v \neq s} x_{v,1}}\right)\pi_{j,2}(s) - x_{s,1}, \ j \neq s$$

Maximizing this expression with respect to $x_{s,1}$ and computing a symmetric equilibrium yield

$$x_{s,1} = \frac{n - 1}{n(n + a - 1)}\left[1 + (1 - a)\frac{n - 2}{n}\left(1 - \frac{a(n - 1)}{n + a - 1}\right)\right] \tag{A7}$$

From these equations, the expressions in (19) and (20) in the text can be derived.

Suppose now that there are prizes of total value 2, divided into $2 - M$ and M. Given that i wins the first contest, efforts in contest 2 are

$$x_{i,2}(i) = M\frac{n-1}{n+a-1}\left(1 - \frac{a(n-1)}{n+a-1}\right), \text{ and}$$

$$x_{j,2}(i) = M\frac{a(n-1)}{(a+n-1)^2}, \ j \neq i,$$

with total effort in contest 2 equal to

$$X_2 = M\frac{n-1}{n+a-1},$$

and expected payoffs in contest 2 equal to

$$\pi_{i,2}(i) = M\left(1 - \frac{a(n-1)}{n+a-1}\right)^2$$

$$\pi_{j,2}(i) = M\left(\frac{a}{n+a-1}\right)^2, \ j \neq i.$$

For player k, the expected profit at contest 1 is

$$\pi_{k,1} = \frac{x_{k,1}}{x_{k,1} + X_{-k,1}}\left[2 - M + \pi_{k,2}(k)\right] + \left(1 - \frac{x_{k,1}}{x_{k,1} + X_{-k,1}}\right)\pi_{k,2}(i) - x_{k,1}, \ k \neq i,$$

where $X_{-k,1}$ is the total effort in contest 1 of player k's rivals. Maximizing this expression with respect to k's effort, using the continuation payoffs above, yields a symmetric equilibrium effort in the first contest per player of[6]

$$x_{s,1} = \frac{n-1}{n^2}\left\{2 - Ma\left[1 + \frac{(1-a)\left((n-1)^2 + a\right)}{(n+a-1)^2}\right]\right\}, s = 1, 2, ..., n. \qquad (A8)$$

Total effort in contest 1 is hence $X_1 = nx_{s,1}$, whereas total effort in contest 2 is $X_2 = x_{i,2}(i) + (n-1)x_{j,2}(i)$. This leads to total efforts decreasing over time if M, the second-contest prize, is small enough, in particular if

$$M < \frac{\left(1 + a - a^2\right)n^3 - 4\left(1 - a^2\right)n^2 + \left(5 - 5a - 2a^2\right)n - 2\left(1 - a\right)^2}{a\left[(2 - a)n^3 + (5a - 6)n^2 + 6(1 - a)n - 2(1 - a)\right]},$$

which is strictly greater than 1 for any $n > 2$, equals $\frac{n^2-2}{n(n-1)}$ when $a = 1$, and approaches $\frac{1+a-a^2}{a(2-a)}$ as n goes to infinity.

Aggregate efforts over both contests are

$$X_1 + X_2 = \frac{n-1}{n}\left\{2 + M\frac{1-a}{(n+a-1)^2}\left[(1-a)n^2 - (1-4a)n - 2a\right]\right\}, \qquad (A9)$$

which is linear in M. It is easily checked that the square-bracketed term in (A9) is positive for feasible values of n and a. Hence, total effort increases in M, and the effort-maximizing choice of this variable is $M = 2$, as claimed in the text.

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[6]Note that the equilibrium effort in the first contest is non-negative, since $M \leq 2$ and the term in square brackets in (A8) is no greater than $\frac{1}{a}$ for feasible values of n and a.

Inserting equilibrium efforts into the payoff functions of the players yields the equilibrium expected payoff

$$\Pi(M, n) = \frac{2}{n^2} \left[1 - Ma\,(1 - a) \left(\frac{n - 1}{n + a - 1} \right)^2 \right],$$

which is clearly decreasing in M.

At $M = 2$, a loser's effort in the second contest is lower than his effort in the first contest if and only if

$$a < \frac{1}{n^2 - 4n + 3} \left(\frac{3}{2}n^2 - 3n + 2 - \frac{1}{2}\sqrt{5n^4 - 12n^3 + 12n^2 - 8n + 4} \right),$$

which increases in n and approaches $\frac{1}{2}\left(3 - \sqrt{5}\right) \approx 0.382$ as n goes to infinity.

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