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Energy Intensive Infrastructure Investments with Retrofits in Continuous Time: Effects of Uncertainty on Energy Use and Carbon Emissions[1]

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Abstract: Energy-intensive infrastructure may tie up fossil energy use and carbon emissions for a long time after investments, making the structure of such investments crucial for society. Much or most of the resulting carbon emissions can often be eliminated later, through a costly retrofit. This paper studies the decisions to invest in such infrastructure, and retrofit it later, given that future climate damages are uncertain and follow a geometric Brownian motion process with positive drift. It shows that greater uncertainty about climate cost (for given unconditional expected costs) then delays the retrofit decision by increasing the option value of waiting to invest. Higher energy intensity is also chosen for the initial infrastructure when uncertainty is greater. These decisions are efficient given that energy and carbon prices facing the decision maker are (globally) correct, but would be inefficient when they are lower, as typical in practice. Greater uncertainty about future climate costs will then further increase lifetime carbon emissions from the infrastructure, related both to initial investments, and to too infrequent retrofits when this emissions level is already too high. An initially excessive climate gas emissions level is then likely to be worsened when volatility increases.

Key words: Greenhouse gas emissions; long-term investments; retrofits; uncertainty; option value of waiting.

JEL classification: C61; Q54; R42.

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1 Introduction

Large and energy-intensive infrastructure poses a serious concern for climate policy. Such infrastructure is found on both the supply side (such as power plants and other energy supply infrastructure) and demand side of the energy-intensive sectors of the economy (such as urban structure and transport systems including the balance between public and private transport). It is the source of more than half of total greenhouse gas (GHG) emissions from fossil fuels in high-income countries, as well as a significant and growing share in emerging economies. Proper concern for such systems requires considering their impacts over long future periods (in some cases 100 years or more). Infrastructure with less permanent albeit still persistent effects on emissions includes motor vehicles (fossil-fuel versus electric or renewable-powered), household appliances, and home heating and cooling systems. Decisions for such long-lasting investments will always be subject to great uncertainty. Making "wrong" investments could tie up inefficiently high emissions levels for long future periods, and make it difficult to reach ambitious climate policy targets later. Many emerging economies today have currently, and are planning over the next 20–30 years, a high rate of such investments.

"Retrofits" can help alleviate this problem. For example, coal-fired power plants potentially (perhaps soon) could be retrofitted with carbon capture and sequestration technologies; or to instead utilize renewable, non-fossil, fuels. Urban areas which depend mainly on private transport, can likewise be "retrofitted" by adding public transport systems. Motor vehicle fleets can be "retrofitted" by replacing gasoline- or diesel-driven vehicles with electric vehicles (with renewable-based power production).

In this paper we consider an initial infrastructure investment carried out at a point of time that can be more or less energy intensive. Its emission intensity stays constant until it is retrofitted, which we define to mean that it is purged of all of its emissions, while the utility of its basic services is fully retained after the retrofit.

Two important issues related to energy use and GHG emissions are here 1) the optimal energy and emissions intensity for the initial infrastructure, and 2) the optimal retrofit policy (when, if ever, should the infrastructure be retrofitted). We first (in section 3) consider these issues under certainty, where combined energy and environmental costs follow a

deterministic, increasing, path over time. In sections 4–5 we assume that energy (including climate and other environmental) costs are uncertain and follow a geometric Brownian motion process with constant positive drift. The future retrofit cost is still considered as certain, and constant.

An important issue is efficiency. If policy makers face "globally correct" price signals (prices where all external effects are taken into consideration), the resulting (dynamic) allocation will tend to be socially optimal.[4] Most of our formal presentation focuses on cases of efficiency, focusing on questions (I)–(VI) below. It thus represents a starting point for a fuller analysis of cases, more plausible in emerging economies today, where energy prices are too low as decision makers are not, in their investment and retrofit decisions, properly accounting for global climate costs. Our model as presented however still has rather direct implications for cases of inefficiency. This is discussed in the final section below.

Some key questions to be discussed in this paper are:

(I)   When (if ever) will the infrastructure be "retrofitted"; and what principles govern a retrofit decision?

(II)   What is the probability distribution of the time to a "retrofit"?

(III)   What is the probability distribution of accumulated carbon emissions from the infrastructure?

(IV)   What is the optimal level of carbon emissions related to the initial infrastructure investment?

(V)   What are expected damages, and the distribution of damages, both in present discounted value terms, due to carbon emissions over the lifetime of the infrastructure investment?

(VI)   Is such infrastructure investment likely to be optimal in the real world; and if not, what types of distortions lead to non-optimality and in what way, and how can optimality, or an improved solution, be achieved?

Of particular concern is the impact of "volatility", by which we here mean the instantaneous relative variance on a continuous stochastic process by which the marginal GHG emission cost for society changes over time. We first (in section 2) study a "benchmark" case where the climate cost trajectory is assumed to be certain. Our main concern, however (in sections 3 – 4), is with the case of uncertain damages.

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[4] The main assumptions are that certain convexity conditions on choice and production sets are fulfilled; and that all economic actors behave competitively (are price takers).

Our paper is technically similar to Pindyck (2000, 2002); and most directly builds on Pindyck (2000).[5] Our solution for optimal timing of a technology retrofit, for given initially invested infrastructure (questions (I) and (II) above), reproduces Pindyck's result (albeit with a modification; see Framstad (2011)).[6] Our more important contribution is to extend Pindyck's analysis by deriving the initial infrastructure investment decision (question (IV)), simultaneously with the future retrofit decision. We are also the first to directly answer questions (III) and (V), on effects for accumulated carbon emissions. While we have less formally to say about point (VI), we discuss this issue and its relation to our model in the final section 6.

The combined infrastructure establishment and retrofit issue have been studied by Strand (2011a), Strand and Miller (2010) and Strand, Miller and Siddiqui (2011), albeit in simpler discrete (two-period) models. Shalizi and Lecocq (2009) provide a more descriptive (largely non-technical) framing. Anas and Timilsina (2009), simulating infrastructure investments in roads for Beijing, find that a higher level of road investments makes the chosen residential pattern more dispersed, and also makes later investments in mass transport ("retrofits" in this case) less valuable or more expensive. Vogt-Schilb, Meunier and Hallegatte (2012) discuss timing of sectoral abatement policies within a model of overall optimal climate policy, given that long-lasting effects of particular abatement investments can vary between sectors. They show that marginal sectoral abatement costs should differ by sector, with more "rigid" sectors investing relatively more in early abatement.

2 Overview of the Paper

Some of our analysis is analytically complicated, but the main results are simple. Given socially optimal pricing, increased volatility of climate damages from given emissions, when future retrofit is an option, implies that (a) retrofits will be executed later, and when marginal climate damage has reached a higher level; and (b) the initial infrastructure will be chosen with a higher energy intensity. These two factors together imply that both current

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[5] See also a follow-up paper by Pindyck (2002), which generalizes the analysis in some other directions than those pursued here (to simultaneously uncertain climate impact, and uncertain damage due to given climate impact. See also the review by Pindyck (2007). Balikcioglu, Fackler and Pindyck (2011) and Framstad (2011) rectify errors in the original Pindyck (2000, 2002) presentations.
[6] See also Framstad (2013) for a more general mathematical model, showing that the project in question can regardless of size be computed like if it had only infinitesial effect on the global emissions trajectory.

emissions while the infrastructure operates, and its aggregate lifetime emissions, will be greater when volatility is higher.

These results may be surprising to some, but have a simple intuitive explanation. A retrofit will be executed later because increased volatility leads to an increased option value of waiting. The result was shown in similar model context by Pindyck (2000, 2002). The principle is however long well known, as exposed e.g. in Dixit and Pindyck's (1994) wider-ranging analysis of investment decisions under uncertainty. It is due to the asymmetric effect of volatility when retrofit is an option. Since one can guard against (and thus avoid) "particularly bad" outcomes by retrofitting, the higher frequency of "particularly good" outcomes that also follows from higher volatility then takes on a greater importance, and leads to an overall benefit from "not yet" having carried out the retrofit. Higher volatility also increases the expected net utility of the infrastructure investment (not only the option value component), by reducing the expected value of actually realized damages, when (as here) "very bad" outcomes are avoided by either a retrofit, or (in a worst case) full closedown of the infrastructure system. For a wide class of parameters – we argue, those for which the model is otherwise reasonable – we show that the energy intensity of the initially chosen investment is higher with higher volatility, a result which in view of the above results is not surprising. In the same way as for the retrofit decision, higher volatility with respect to future environmental costs has an asymmetric effect on "good" versus "bad" outcomes: it gives the "good" outcomes (with low climate damage) relatively greater weight as the "worst" outcomes can always be avoided by retrofitting later. This draws in the direction of a more energy intensive infrastructure choice.

Such an outcome could clearly be problematic in a climate policy context: Greater uncertainty about the welfare consequences of GHG emissions leads to less and not more climate action in early problem stages. Greater unconditional uncertainty then makes it optimal to postpone mitigation. But such climate action is optimal *only given that decision makers face correct global costs*. More typically, decision makers do not face correct but instead too low GHG emissions costs, so that more and not less climate action is desirable. In such circumstances, greater uncertainty facing decision makers, together with "wrong" emissions prices, could together easily lead to a greater social loss, due to a more inadequate climate action.

Several policies can deal with GHG emissions *ex post*, related to an already established infrastructure. We consider the following[7]:

(1) *Energy use eliminated upon "retrofitting"*. This may represent a case where the initial fossil energy is replaced by renewable energy sources with very low ex post marginal production cost (which could include hydro, solar or wind); or some new energy source that is supplied in unlimited amount. In this case we need only be concerned with one set of prices or costs, namely the combined energy and emissions cost, from the start of the project and until a retrofit takes place.

(2) Energy is not eliminated, but *emissions are eliminated upon retrofitting*. This could be the case where CCS technology is adopted to existing power plants; or fossil fuels are replaced by renewable energy with no net emissions load. Energy costs could here be taken to follow a deterministic process independent of emissions costs.

(3) The infrastructure is *closed down* or abandoned. In this case all (energy and environmental) costs are removed, and the infrastructure provides no utility from then on. Should there be both a retrofit and a closedown option with given constant costs, we can disregard the more expensive of the two.

2.1   Assumptions

We shall below consider a set of exogenously given constraints or background assumptions put on the problem, *(A) through (D)*, assumed to hold in the main part of the analysis, but partly relaxed in Appendix A:

(A) The infrastructure is laid down initially, with no timing optimization;

(B) The infrastructure is operated forever, so that the only policy choices later would be when and how to retrofit (change the emission rates);

(C) A retrofit can be carried out at most once, in which case it eliminates emissions completely from the time of retrofit;[8]

(D) It is possible to retrofit at any future time.

In section 3 below we first study a "benchmark" case with full certainty, where the process for the marginal environmental cost of climate emissions, Θ, is non-stochastic and

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[7] A further type of policy to deal with the impacts of GHG emissions, not discussed further here, is to lower impacts directly (through "adaptation"); see Strand (2011b) for a related analysis.
[8] Note that removal of energy use is not a necessary assumption given that the cost q below only refers to environmental costs; see footnote 4.

has a constant relative growth ("drift") rate α. This leads to a deterministic stopping rule in this case, as the exact time of "stopping" can be decided already at the initial investment stage. We assume in section 3 that the retrofit cost is proportional to the reduction of emission level. In sections 4–5, we generalize this analysis by assuming that Θ follows a geometric Brownian motion process, and by assuming a more general retrofit cost structure. Net welfare to be maximized, over initial technology and retrofit implementation time, is (under assumptions (A)–(D)) assumed to be given by

$$V - \phi(E_0) - \mathsf{E}\left[\int_0^\infty e^{-rt}\Theta_t M_t\, dt + e^{-r\tau}K(E_0)\right] \tag{1}$$

where V is the discounted gross utility from the services of the infrastructure, assumed constant; the cost of the initial infrastructure investment, ϕ, depends on the level of energy consumption chosen, E_0; τ is the time of retrofit, to be chosen optimally (where $e^{-r\tau} = 0$ if retrofit never occurs); $K(E_0)$ is the cost of retrofit, assumed under (D) to eliminate all emissions; and M_t is the stock of greenhouse gases in the atmosphere. $\mathsf{E}[\cdot]$ is the expectation operator.

3 Deterministic Climate Damage

This section considers a simple "benchmark" case where the future path for climate costs is deterministic and known. This serves as a precursor to the later uncertainty analysis. It also provides a first analysis of optimal initial infrastructure investment with a retrofit option. Consider the simple current-value function for society:

$$U(E) = V - qE \tag{2}$$

where V is (gross) current utility flow to the public associated with the infrastructure once established, assumed given; at the initial time, the only choice variable is the initial emission (or energy consumption) rate E of the infrastructure. $q > 0$ incorporates the entire (expected discounted) damage caused by current energy use, for all future (in addition to the basic initial fuel cost). In the following we will typically assume that damage is of this proportional form also when a subsequent emission reduction is chosen optimally.

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3.1 Infrastructure Utility and Cost

The cost of establishing the infrastructure is $\phi(E)$, where $\phi'(E) < 0 < \phi''(E)$: it is less expensive to establish an infrastructure which requires a high ex post energy consumption level, but marginal investment cost savings are reduced as energy consumption increases (or opposite, when energy consumption is reduced, investment costs increase more steeply).[9] One simple example is the power function $\phi(E) = AE^{-a}$, where A, $a > 0$; then ϕ' is increasing from $-\infty$ to 0, so we have no corner solution; under (2), the optimal choice E_* for E would be given by the first-order condition $aA \cdot E_*^{-a-1} = q$, equivalent to $E_* = (q/Aa)^{-1/(a+1)}$. Net present value of the installation is then

$$V - (1+a)A^{\frac{1}{a+1}} \cdot (a/q)^{\frac{a}{a+1}}. \tag{3}$$

Assume that V is sufficiently large so that this expression is always positive; otherwise, the basic investment would not be worthwhile in the first place.

3.2 The Retrofit Decision

A "retrofit" will remove emissions permanently from the infrastructure thereafter. This cannot be done at the establishment time, but "later". Assumptions (A)–(D) are assumed to hold. The cost of permanently removing emissions E is $K(E)$. Assume that it is optimal to remove all emissions once a retrofit is carried out; this is always optimal when (as in Pindyck's (2000) main case) retrofit costs are proportional to removed emissions. The only choice variable once the investment is sunk is then the retrofit time, stopping emissions completely and permanently. In the following we suppress the energy cost dimension (considering basic energy costs as given and netted into the utility V and retrofit cost K), and focus on the (climate-related) environmental impact of carbon emissions. Following Pindyck (2000), current environmental costs are given by

$$\Theta_t M_t \tag{4}$$

M is the stock of GHGs in the atmosphere, assumed to obey the following law of motion:

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[9] The infrastructure cost function will need to take this form: considering the set of cost-minimizing infrastructure projects that all yield the same utility, it must be the case that economically relevant projects, that are less energy intensive (and thus has lower current energy cost E), must have a higher establishment cost ϕ.

$$dM_t / dt = E_t - \delta \cdot M_t \tag{5}$$

where δ (≥ 0) denotes a constant rate of decay of GHGs, assuming $0 < M_t < \max_t E_t / \delta$. Discounted damage caused by one unit of emissions at time t_1 equals $D(t_1)$ which in the non-stochastic case grows exponentially at rate α starting at $D(t_0)$, and is given by

$$D(t_1) = D(t_0)e^{\alpha t_1} = \Theta_0 e^{\alpha t_1} \int_0^\infty e^{-(r+\delta-\alpha)t} \, dt = \frac{e^{\alpha t_1}}{r+\delta-\alpha}\Theta_0 \tag{6}$$

The discounted value of damages due to one unit of emissions per time unit emitted forever, starting at $t = 0$, is $D_0 = \Theta_0 / (r - \alpha)(r + \delta - \alpha)$. Charging for emissions costs at a socially optimal rate along the optimal emissions path requires that the charge $q(t)$ equal $D(t)$ at all times (where q is interpreted as emissions cost alone).

Suppose E is emitted per time unit from time 0 on. Solving the differential equation for M_t yields

$$M_t = M_0 e^{-\delta t} + (1 - e^{-\delta t})E / \delta \tag{7}$$

Using that $\Theta_t = \Theta_0 e^{\alpha t}$, we can calculate accumulated emissions as

$$\begin{aligned}
&\Theta_0 \int_0^\infty e^{(\alpha-r)t} \left[\tfrac{E}{\delta} + (M_0 - \tfrac{E}{\delta})e^{-\delta t} \right] dt \\
&= \Theta_0 \cdot \left[\frac{E/\delta}{(r-\alpha)} + \frac{M_0 - E/\delta}{r+\delta-\alpha} \right] = \frac{\Theta_0 M_0}{r+\delta-\alpha} + \frac{\Theta_0 E}{(r-\alpha)(r+\delta-\alpha)}
\end{aligned} \tag{8}$$

Here the first term is a "fixed cost" unrelated to the project. Suppose that we retrofit the infrastructure at time $t*$, eliminating emissions from this project from that point on. We can then again insert for M_t (the dynamics of which are now split at $t*$ with $M_t = M_{t*}\exp(-\delta(t-t*))$ for $t > t*$), and write the net social value as $-\Theta_0 M_0 / (r+\delta-\alpha)$ (which is independent of chosen infrastructure), plus terms amounting to $W(E)$, that depend on infrastructure type:

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$$W(E) = -\phi(E) + v \int_0^\infty e^{-rt} \, dt - \frac{\Theta_0}{r+\delta-\alpha} \int_0^{t^*} e^{-(r-\alpha)t} E \, dt - e^{-rt^*} K(E)$$

$$= -\phi(E) + V - \frac{\Theta_0}{(r-\alpha)(r+\delta-\alpha)} \left(1 - e^{-(r-\alpha)t^*}\right) E - e^{-rt^*} K(E) \tag{9}$$

In (9), the first two terms comprise the ordinary net present value of the infrastructure. The third term constitutes present discounted emissions costs due to infrastructure operation; and the last term is the present value of the retrofit cost when done at time t^*; both the two last terms are discounted from the time of infrastructure investment.

A retrofit as noted removes E_t from all periods beyond the retrofit time. (9) is concave with respect to t^*. Taking the derivative yields the first-order condition[10]

$$\frac{dW(E)}{dt^*} = -\frac{\Theta_0 E}{(r+\delta-\alpha)} e^{-(r-\alpha)t^*} + r e^{-rt^*} K(E) \tag{10}$$

The solution for t^* is

$$\Theta_0 \cdot e^{\alpha t^*} = \Theta(t^*) = r(r+\delta-\alpha) \frac{K(E)}{E} \tag{11}$$

given a general retrofit function $K(E)$. Bearing in mind that $t^* \geq 0$, we need to have

$$\frac{K(E)}{E} \geq \frac{\Theta_0}{r(r+\delta-\alpha)} \tag{12}$$

If (12) did not hold, the "retrofit cost" K would be so low that a "retrofit" would be chosen already at $t = 0$ (the time of initial investment); and our model would not apply.

3.3   The Choice of Energy Intensity of Infrastructure

Choosing the energy intensity of the initial infrastructure investment amounts to maximizing (9) with respect to E. We assume in this section $K(E) = kE$: the retrofit cost is proportional to the emissions removed from the infrastructure. This is a particularly simple case since then

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[10] An alternative and more intuitive way to derive this condition in discrete time requires considering the discounted value of climate damages caused by one (infinitesimal) unit of installed energy capacity for one (small) discrete time unit at time t^*, found from (9). The current interest cost of this instalment is simply $rK'(E)$. The optimal solution is found at the point where these two quantities are equal, which yields (11).

the optimal retrofit time, from (11), is independent of E_*, the optimal E. A more general case is discussed in section 3. E_* is here found from:

$$-\phi'(E_*) - \frac{\Theta_0}{(r-\alpha)(r+\delta-\alpha)}\left(1 - e^{-(r-\alpha)t^*}\right) - e^{-rt^*}K'(E_*) = 0 \qquad (13)$$

where we assume $t^* > 0$. With $K'(E) = k$ = constant we then have

$$\phi'(E_*) = -\frac{\Theta_0}{(r-\alpha)(r+\delta-\alpha)} + \frac{k}{\gamma_0-1}\left[\frac{\Theta_0}{kr(r+\delta-\alpha)}\right]^{\gamma_0} \qquad (14)$$

where $\gamma_0 = r/\alpha > 1$.

The first main term on the right-hand side of (14) can be interpreted as the discounted cost associated with one unit of energy consumed forever. The last term modifies this calculation: it increases the value of energy use at the time of investment, due to the added option of a retrofit possibility, which is here always exercised.

We are here most interested how energy intensity and overall energy consumption over the infrastructure's lifetime, depend on the retrofit cost. Given $K(E) = kE$, with constant k (> 0), the effect of a small change in k on E_* is found, from (14), as:

$$\frac{dE_*}{dk} = \frac{-1}{\phi''(E_*)} \cdot \frac{\Theta_0}{r(r+\delta-\alpha)} k^{-\gamma_0} \qquad (15)$$

A higher retrofit cost leads, reasonably, to the choice of a lower energy intensity. This effect is weaker when k is larger. Intuitively, a retrofit is then done in the more distant future, and is less significant at the time of investment. Also, since E_* is bounded, by (14), also the elasticity $\mathcal{El}_k E_* = (k/E_*) \cdot (dE_*/dk)$ decreases in k, in the same way as $-k^{1-\gamma_0}$.

3.4 Implications of Changes in Cost Variables

The discounted value of damages caused by energy consumption resulting from the initial investment at $t = 0$ is given from (9) by

$$D = \frac{\Theta_0}{(r-\alpha)(r+\delta-\alpha)}\left(1-e^{-(r-\alpha)t^*}\right)E_* \tag{16}$$

Consider here first an upward shift in the path of damages, represented by a shift in Θ_0. Such a shift will shift D up for two reasons: first, directly as seen from (16); and secondly, as t^* is now reduced, from (11).

Consider next an increase in the unit retrofit cost, k. This affects the discounted damage from (16) in two separate, opposite, ways. First, the initial energy intensity of the infrastructure, E_*, is reduced, from (15). Secondly, t^* is increased, so that a retrofit occurs later (and energy consumption continues for longer), from (11). Using the latter expression and $-(r-\alpha)t^* = \alpha t^*(1-\gamma_0)$, discounted climate damages can be rewritten as a function of k:

$$D = \frac{\Theta_0}{(r-\alpha)(r+\delta-\alpha)}\left(1-\left[\frac{\Theta_0}{rk(r+\delta-\alpha)}\right]^{\gamma_0-1}\right)\cdot E_* \tag{17}$$

whose derivative with respect to k is a difference between positive terms:

$$\left[\frac{k}{\Theta_0}\cdot r(r+\delta-\alpha)\right]^{1-\gamma_0}\cdot\frac{(\gamma_0-1)\Theta_0 E_*}{k(r-\alpha)(r+\delta-\alpha)}+\frac{D}{E_*}\frac{dE_*}{dk}=\gamma_0\left(\frac{\Theta_0}{kr(r+\delta-\alpha)}\right)^{\gamma_0}E_*+\frac{D}{E_*}\frac{dE_*}{dk} \tag{18}$$

From the expression for dE_*/dk – namely, in particular, the ϕ'' in the denominator – we have the following:

Proposition 1: *There is no universal sign of the relation between the magnitude of (proportional) retrofit cost and expected discounted environmental damage.*

Cases with more general retrofit cost and uncertainty will yield the same negative result.

4 Infrastructure Investment and Retrofit when Environmental Damage is Stochastic

We now extend the above model to the more realistic and interesting but also more complicated case of stochastic environmental damages. Unless otherwise stated, we assume

that conditions Assumptions (A)–(D) hold[11]. Several alternatives exist for modelling energy and emissions costs over time under uncertainty. We have so far assumed that marginal emissions cost has a positive, constant and deterministic growth rate ("drift") α. We now assume that the environmental cost parameter Θ_t evolves according to a geometric Brownian motion process

$$\Theta_t = \Theta_0 \exp\{(\alpha - \frac{1}{2}\sigma^2)t + \sigma Z_t\} \mathrm{d}\Theta_t = \alpha\Theta_t \, \mathrm{d}t + \sigma\Theta_t \, \mathrm{d}Z_t \tag{19}$$

so that the change in Θ can be expressed via the stochastic differential equation

$$\mathrm{d}\Theta_t = \alpha\Theta_t \, \mathrm{d}t + \sigma\Theta_t \, \mathrm{d}Z_t \tag{20}$$

where Z_t denotes the standard Brownian motion. Marginal climate damages from one unit of emissions depend on time with constant growth rate α, and in addition on a purely random factor. A simplification is that, with this process, there is no direct relation between accumulated emissions and marginal climate damage. Considering the impacts of one single project, this may be reasonable as the project is likely to be "small" relative to global emissions (implementation of this project does not substantially alter the process for Θ). A similar stochastic model has already been studied by Pindyck (2000) for already committed infrastructure investment. We here reproduce some of his results (with a modification as set out also in Framstad (2011)), and extend them to the initial investment stage.

4.1   The Retrofit Decision for Given Investment Under Uncertainty

The optimal rule for timing of a retrofit (given that the infrastructure is not closed down) has been derived by Pindyck (2000), as:

$$\theta^* = \frac{\gamma}{\gamma - 1}(r - \alpha)(r + \delta - \alpha) \cdot \frac{K(E_*)}{E_*} \tag{21}$$

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[11] Part of the analysis could cover the case where assumption (B) is relaxed, and $K(E)$ capped at V. This case however turns out to be analytically intractable, at least with parsimonious investment cost; see Appendix A.

where E_* is the initially (and optimally) chosen energy intensity of the infrastructure, and where γ is given as the positive zero of $\frac{1}{2}\sigma^2\gamma(\gamma-1)+\alpha\gamma-r$:

$$\gamma = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{\alpha}{\sigma^2}-\frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} \qquad (\in(1,r/\alpha]) \qquad (22)$$

This expression characterizes the solution whenever $r > \alpha$ (otherwise, welfare is $-\infty$ for all positive $M_0 \cdot \Theta_0$). We here assume a general retrofit cost function $K(E)$, but note that, under the proportionality case treated in section 2, $K(E_*)/E_* = k$, a constant.

The key result proved by Pindyck (2000) can be stated as follows:

Proposition 2: *Consider the one-shot problem of maximizing (9) with respect to a retrofit-time for which E_t changes from E to 0 (where $E = E_*$ is given exogenously). Suppose that the laws of motion for M and Θ are given by (5) and (20), respectively, where $\alpha < r$, and with initial conditions $\Theta_0 = \theta > 0$ and $M_0 = m > 0$. Then the optimal retrofit-time τ^* is the first time – if ever – when Θ_t hits $[\theta^*,\infty)$, with θ^* given by (21), and the optimal net social value is the sum of $-\theta m/(r+\delta-\alpha)$ (which would incur even if the investment were never made), and $W(E)$ given by*

$$W(E) = V - \phi(E) - \begin{cases} K(E) & \text{if } \theta \geq \theta^*, \text{ and otherwise, if } \theta < \theta_*: \\ [\dfrac{\theta}{(r-\alpha)(r+\delta-\alpha)} - \dfrac{(\gamma-1)^{\gamma-1}}{\gamma^\gamma}(\dfrac{\theta}{(r-\alpha)(r+\delta-\alpha)})^\gamma \cdot (\dfrac{E}{K(E)})^{\gamma-1}]E \end{cases} \qquad (23)$$

As in the deterministic case, the value function can be decomposed as follows: $-\theta m/(r+\delta-\alpha)$ is the (negative-valued) climate damage incurred by inheriting the system in state $(\Theta_0, M_0) = (\theta, m)$, assuming no investment and no further emissions. $-\theta E/(r-\alpha)(r+\delta-\alpha)$ is the additional expected discounted social cost of the climate damage from E units of emissions emitted for all future periods[12]. This can at the discretion

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[12]This holds only when there are no effects of current emissions on future marginal costs θ; see comment in the text. If there are such effects, the social cost of emissions is higher.

of the social planner be exchanged at cost $K(E_*)$, and in optimum this will occur when Θ

hits $[\theta^*, \infty)$; below this, there is a value of the option to stop, equal to $(\theta / \theta^*)^\gamma \cdot K(E_*) / (\gamma - 1)$.

This solution converges to the deterministic case as $\sigma \rightarrow 0$. Then γ reduces to $\gamma_0 = r / \alpha$,

and θ^* reduces to the right-hand side of (11). From (21), θ^* is greater by a factor $\gamma / (\gamma - 1)$ (

> 1, as $\gamma > 1$). An increase in σ^2 implies that γ drops (moves closer to unity), so that

$\gamma / (\gamma - 1)$ increases, and θ^* thus increases.

Greater uncertainty then raises the level of damage required for mitigation action. The reason is that the option value of waiting with carrying out the retrofit (or, possibly, close the infrastructure down) then increases. Intuitively, greater uncertainty leads to more states of the world where, in the immediate future, damages will be reduced; this makes waiting more favorable. This can however be unfortunate from the point of view of mitigation: Once the infrastructure investment is sunk, a decision maker has an incentive to delay a costly retrofit that removes all emissions associated with the infrastructure, perhaps for a long time.

4.2   The Optimal Emission Intensity of Infrastructure

We now derive optimal emission intensity, E_0, of infrastructure at establishment and until it is retrofitted, assuming that the later retrofit decision is optimal. We then maximize welfare (9) with respect to E. Differentiating W (from Proposition 1) and defining

$$b = \frac{(\gamma - 1)\Theta_0}{\gamma(r - \alpha)(r + \delta - \alpha)} \quad \text{and} \quad \rho = \frac{\Theta_0}{\theta^*} = \frac{bE}{K(E)} \tag{24}$$

we have $W'(E) = -\phi'(E) - K'(E)$ for $\Theta_0 > \theta^*$, while for $\Theta_0 \leq \theta^*$ we find:

$$\begin{aligned} W'(E) &= -\phi'(E) - \frac{\gamma b}{\gamma - 1} + \frac{1}{\gamma - 1} b^\gamma \cdot \frac{E^\gamma}{K(E)^{\gamma - 1}} \cdot \frac{\mathrm{d}}{\mathrm{d}E} \big[\gamma \ln E - (\gamma - 1) \ln K(E) \big] \big| \\ &= -\phi'(E) - \frac{\gamma b}{\gamma - 1} + h(E). \end{aligned} \tag{25}$$

The effect of the "option element", denoted h, can be written in terms of the elasticity of K with respect to E, $\mathcal{E}\ell\, K$, given general differentiable function $K(E)$, as

$$h(E) = \left(\frac{bE}{K(E)}\right)^{\gamma} \cdot \frac{K(E)}{E}\left[\frac{\gamma}{\gamma-1} - \mathcal{E}\ell K\right] \qquad (26)$$

The first-order condition for optimal E $(= E_*)$ is straightforward. The second-order condition is more complicated in the general case of non-linear K function. We find

$$h'(E) = \left(\frac{bE}{K(E)}\right)^{\gamma} \cdot \left[\frac{\gamma K}{E^2}\{1 - \mathcal{E}\ell K\}^2 - K''\right] \qquad (27)$$

If we require W to be concave for any parsimonious convex ϕ, the appropriate condition $h' \leq 0$ requires K to be convex (see comment below Proposition 2). In particular, a convex power function is here sufficient as long as the exponent – i.e. the elasticity – is between 1 and $\gamma / (\gamma - 1)$.

Optimization with respect to E can in the proportional cost case be expressed by a sufficient first-order condition involving the function

$$g(\rho, \gamma) := \frac{1}{\gamma - 1}\left[\max\{1, \rho\}^{\gamma} - \gamma \cdot \max\{1, \rho\}\right] \qquad (\in [-1, 0]) \qquad (28)$$

The following key result can now be stated and proven.

Proposition 3: *Consider the optimal stopping problem from Proposition 2, with cost functions $\phi(E)$ and $K(E) = kE$, where $k > 0$ is a constant, ϕ is strictly positive and strictly convex, and $\phi(\infty) \geq 0$. Given that an optimal initial E_* exists, it depends only on parameters through b and γ. Then W is strictly concave, and E_* is either zero (iff $\phi'(0) \geq kg(\theta / \theta^*, \gamma)$) or uniquely characterized by the first-order condition*

$$\phi'(E_*) = kg(\theta / \theta^*, \gamma) \qquad (29)$$

which can be interpreted in the subgradient sense if ϕ is not C^1.

The proposition is proven for the case of proportional retrofit cost, with k constant. With strictly convex ϕ, as assumed, the option to retrofit will increase the initially chosen energy consumption level E_* for more general K function, as long as $\mathcal{E}\ell K < \gamma / (\gamma - 1)$. With larger

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elasticity the presence of the option will reduce E_*, the main reason being that it is then much cheaper (per unit of purged emissions) to retrofit a less polluting infrastructure. Pindyck (2000), section 4, shows how convex K could lead to gradual reduction in E through successive partial retrofits; this possibility is however ruled out here. Note that in our main case, we have proportional cost, implying $\mathcal{E}\ell\, K = 1$, and the above stated condition holds.

As in subsection 2.3 we can find an expression for the derivative of E_* with respect to the retrofit cost. To do so, we scale up the cost by κ into $\kappa \cdot K(E)$, and consider changes in κ. Instead of h, we get $h \cdot \kappa^{1-\gamma}$, and the first-order condition for E_* becomes

$$-\phi'(E_*) = b\gamma / (\gamma - 1) - h(E_*)\kappa^{1-\gamma} \tag{30}$$

Implicit differentiation with respect to κ yields comparative statics as below. Furthermore, the effect of the initial state Θ_0 is found by differentiating As the following results are straightforward, proof is omitted:

Proposition 4 (effect of (proportional) retrofit cost and state on the optimal E_*): *Suppose that Proposition 3 applies, with $W'(E_*) = 0 < W''(E_*)$. Then, at least piecewise,*

$$\frac{\mathrm{d}E_*}{\mathrm{d}\kappa} = \frac{(\gamma - 1)\kappa^{-\gamma} h(E_*)}{W''(E_*)} = -\frac{(\gamma - 1)h(E_*)}{\kappa^{\gamma}\phi''(E_*) - \kappa h'(E_*)} \tag{31}$$

and

$$\mathcal{E}\ell_\kappa E_* = -\frac{(\gamma - 1)h(E_*)E_*}{\kappa^{\gamma-1}\phi''(E_*) - h'(E_*)} \tag{32}$$

both of which have signs opposite to $h(E_*)$, i.e. the same signs as $\mathcal{E}\ell\, K - \gamma / (\gamma - 1)$. Also the elasticity tends to 0 monotonically as κ (and thus the denominator) increases.

As the retrofit cost increases, the value of the option to stop is reduced. Optimal emissions intensity in infrastructure is reduced and converges to the case with no retrofit

option. The characterization, in terms of elasticity of E_* with respect to κ, yields conditions for this convergence to be monotone and smooth, assuming that ϕ is strictly convex.

5 Distributional Properties of the Solution in the Stochastic Case

5.1 Probability Distributions

We now discuss distributional properties and comparative statics with respect to volatility, represented by the variance σ^2 on the stochastic process for climate damages. Assume that the initial value (at time $t = 0$ when the infrastructure investment is made) of Θ_t equals $\Theta_0 = \theta < \theta^*$. Since a retrofit intervention completely removes emissions from then on, aggregate lifetime emissions from this infrastructure is the optimized E_* multiplied by the (stochastic optimal stopping) time $\tau^* > 0$ to retrofit; it therefore suffices to give the distribution of τ for each choice of E. The solution for Θ_t can be written in log terms as

$$\ln(\Theta_t / \theta) = (\alpha - \frac{1}{2}\sigma^2)t + \sigma Z_t \tag{33}$$

τ^* is therefore the hitting time at the positive level $L^* = -\ln \rho$ of the right-hand side, which is a Brownian motion with drift α, and τ^* has probability density (Borodin and Salminen (2002) p. 295)

$$\frac{L^*}{|\sigma|\sqrt{2\pi}}\exp(-\frac{((\alpha - \frac{1}{2}\sigma^2)t - L^*)^2}{2\sigma^2 t})\, t^{-3/2} \tag{34}$$

We distinguish between the following parametric cases:

- For $\alpha < \sigma^2 / 2$ (where Θ_t tends to zero almost surely), this density does not integrate to 1, but to $\Pr[\tau^* < \infty] = \rho^{1-2\alpha/\sigma^2} < 1$. In this case the upward drift is too low to guarantee that the process will ever hit the trigger level for the infrastructure investment. This is however a case on which we here do not focus.

- For $\alpha = \sigma^2 / 2$, then (34) corresponds to the Lévy distribution (i.e. stable distribution with index of stability equal to $1/2$), which is finite with probability one but has infinite moments of order $1/2$ and above.

- For $\alpha > \sigma^2 / 2$, the process will hit the target value θ^* in finite time, with probability one. (34) is now the density of the inverse-Gaussian distribution with mean

$$\mathsf{E}[\tau^*] = L^* / (\alpha - \tfrac{1}{2}\sigma^2) \quad < \infty \tag{35}$$

and shape $(L^* / \sigma)^2$, i.e. variance

$$L^* \sigma^2 (\alpha - \tfrac{1}{2}\sigma^2)^{-3} < \infty \mid \mathrm{CDF}_{\hat{M}}(\hat{m}) = \mathrm{CDF}_{\tau^*}(\tfrac{1}{\delta} \ln \tfrac{m - E/\delta}{\hat{m} - E/\delta}) \qquad \text{on } \hat{M} \in (m, E/\delta) \tag{36}$$

Starting from $m < E/\delta$, the pollution stock M_t will keep increasing to a peak level $M_\tau = E/\delta + (m - E/\delta)\exp(-\delta\tau)$. For the optimal $\tau = \tau^*$, which does not depend on the initial pollution stock m, this peak level $\hat{M} = M_{\tau^*}$ will be distributed as

$$\mathrm{CDF}_{\hat{M}}(\hat{m}) = \mathrm{CDF}_{\tau^*}(\tfrac{1}{\delta} \ln \tfrac{m - E/\delta}{\hat{m} - E/\delta}) \qquad \text{on } \hat{M} \in (m, E/\delta), \tag{37}$$

where CDF_{τ^*} is the distribution of τ^* as above – notice that for $\alpha < \sigma^2 / 2$, the point mass at $\tau^* = \infty$ translates into an equal point mass for $\hat{M}$ at E/δ (a limit not attained in finite time).

We focus on our central case where $\alpha > \sigma^2 / 2$, and where τ^* is integrable[13]. From the moment-generating function evaluated at $-\delta$, it follows that the peak $\hat{M}$ has expected value

$$\mathsf{E}[\hat{M}] = \frac{E}{\delta} - [\frac{E}{\delta} - m]\exp\{\frac{\pi \cdot L^*}{|\sigma|^3}[1 - \sqrt{1 + 2\delta\sigma^4\pi^{-2}}\ ]\} = \frac{E}{\delta} - [\frac{E}{\delta} - m]\rho^\epsilon \tag{38}$$

where ϵ and π (both positive, the latter by assumption) are defined as:

$$\epsilon = \frac{\pi}{|\sigma|}[\sqrt{1 + 2\delta\pi^{-2}} - 1] = \frac{2\delta |\sigma|^{-1}}{\pi + \sqrt{\pi^2 + 2\delta}}, \qquad \text{and} \quad \pi = \frac{\alpha}{\sigma^2} - \frac{1}{2} \tag{39}$$

The peak *current marginal cost factor* is simply the non-random θ^*, so that the peak environmental *current damage rate* is simply $\theta^* \hat{M}$. The expected environmental total damage is the braced expression of (23) corrected for the expected discounted retrofit cost

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[13] Finite mean will hold in many plausible cases (Dixit and Pindyck (1994), page 81). To exemplify, consider "reasonable" values for α and σ, say, α (= the mean rate of increase in climate damage) = 2 percent per year, and σ (= the relative random change around trend, positive or negative, in impact of GHG accumulation) = 10 percent per year - this, we would claim, is a relatively high value). In this case, $\sigma^2 = 0.01 = \alpha / 2$.

$\mathsf{E}[e^{-r\tau^*}]K(E)$, which (Borodin and Salminen (2002) p. 295) equals $(\theta/\theta^*)^\gamma K(E)$. The following formula for expected environmental total damage – note the γ factor at the end due to the correction – is then (valid for non-optimized E_* and θ^* as well):

$$D = \frac{\gamma}{\gamma-1}\left\{\frac{\theta}{\theta^*} - \left(\frac{\theta}{\theta^*}\right)^\gamma\right\}K(E_*) \tag{40}$$

We can find an expression for the comparative static wrt a multiplicative κ on retrofit cost; however, as pointed out in subsection 2.4, we have no universal sign on this.

5.2 Effects of Increased Volatility

We now consider how the retrofit decision and initial infrastructure investment are affected by changes in σ^2. These together determine the time profile for carbon emissions, and thus also expected aggregate emissions resulting from the infrastructure. Note that $(\theta/\theta^*)^\gamma \cdot K(E)/(\gamma-1)$ is an option value which increases in σ^2 for fixed E. By the envelope theorem, this turns out to be the only first-order effect of increased volatility on both the expected environmental damage (decreasing in volatility) and optimal value of E (increasing in volatility) when optimized. At first glance counterintuitive, the reduced environmental damage will – as long as $\mathcal{E}\ell K(E_*) \in [1, \gamma/(\gamma-1)]$, as we shall see – be accompanied by an increased emissions rate, an increased (finite) expected time to retrofit, and increased expected total emissions.

It turns out to be convenient to give comparative statics results in terms of γ, from (22). From Proposition 2, for given retrofit cost function K, the optimal E_* depends on the parameters only via γ and b, where b depends on γ but not directly on volatility. The comparative static with respect to γ suffices to analyze the effect of volatility σ^2, and we note that γ is strictly decreasing in volatility: since $\sigma^2 = 2(r-\alpha\gamma)/(\gamma^2-\gamma)$, we have

$$\frac{\mathrm{d}\gamma}{\mathrm{d}(\sigma^2)} = -\frac{1}{2}\frac{\gamma^2(\gamma-1)^2}{(r-\alpha\gamma)\gamma + r(\gamma-1)} =: -\Delta \quad (<0) \tag{41}$$

Using Δ as shorthand notation, we arrive at the following:

Proposition 5 (Impact on value and decision rules): *Suppose the conditions of Proposition 3 hold, with $\Theta_0 / \theta^* = \rho < 1$. Then the derivatives with respect to volatility σ^2 of the optimal E_*, the optimal θ^*, and the value function $W^* = W(E_*)$, are found as, given sufficient differentiability:*

$$\frac{dE_*}{d(\sigma^2)} = \frac{\Delta}{-W''(E_*)} \cdot \frac{K(E_*)}{E_*} \rho^\gamma \left[\frac{\mathcal{E}\ell K(E_*) - 1}{\gamma} + \left(\frac{\gamma}{\gamma - 1} - \mathcal{E}\ell K(E_*) \right) \ln \frac{1}{\rho} \right] \tag{42}$$

$$\frac{d\theta^*}{d(\sigma^2)} = \theta^* \cdot \left[\frac{\mathcal{E}\ell K(E_*) - 1}{E_*} \cdot \frac{dE_*}{d(\sigma^2)} + \frac{\Delta}{\gamma(\gamma - 1)} \right] \tag{43}$$

$$\frac{d}{d(\sigma^2)} W^* = -\Delta \cdot \frac{d}{d\gamma} W^* = \Delta \cdot \frac{K(E_*)}{\gamma - 1} \rho^\gamma \ln \frac{1}{\rho} > 0 \tag{44}$$

W^* always increases in volatility, while E_* and θ^* always do so given that $\mathcal{E}\ell K(E_*) \in [1, \gamma / (\gamma - 1)]$. There is no universal sign of $dD / d(\sigma^2)$, i.e. no universal sign of the relation between volatility and optimized expected environmental damage – not even under proportional retrofit cost.

The optimal values of E_* and θ^* increase as expected in volatility under the stated conditions. The impact on the decision rules depend on the more precise shape of the retrofit cost function, where a proportionate retrofit cost function, $K(E) = kE$, is a special case (in which $\mathcal{E}\ell K(E_*) = 1$). We are also interested in effects of increased volatility on retrofit time and emissions. The discussion on the time to retrofit will be simplified if we merely consider its expectation, assumed finite, i.e. $\alpha > \sigma^2 / 2$. In that case, we have the following result:

Proposition 6 (Impact on time to retrofit, total emissions and peak pollution stock): *Assume the conditions for Proposition 5 hold, and in addition that $\alpha > \sigma^2 / 2$. Then*

$$\frac{dE[\tau^*]}{d(\sigma^2)} = \frac{1}{2(\alpha - \frac{1}{2}\sigma^2)^2} \ln \frac{1}{\rho} + \frac{1}{(\alpha - \frac{1}{2}\sigma^2)} \cdot \frac{1}{\theta^*} \frac{d\theta^*}{d(\sigma^2)} \tag{45}$$

which is always positive given $d\theta^* / d(\sigma^2) \geq 0$. Expected total emission, $E[\tau^*] \cdot E_*$, always increases with respect to volatility given that $\mathcal{E}\ell K(E_*) \in [1, \gamma / (\gamma - 1)]$. There is no universally valid sign on the

relationship between expected peak pollution stock $\mathsf{E}[\hat{M}] = \mathsf{E}[M_{\tau^*}]$ *and volatility, not even when* $K(E) = \kappa E$.

The proof (given in Appendix B) is straightforward, noting that $\mathcal{E}\ell K(E_*) \in [1, \gamma / (\gamma - 1)]$ grants that E_* and θ^*, and thus also τ^*, all increase in volatility. Expected emissions thus increase "faster" in volatility than either initial energy intensity E_*, or expected time to retrofit, θ^*, since each is increasing.

Appendix B gives examples of cases with either sign of the relationship for expected peak stock. It is unclear whether the slightly more complicated peak expected current damage rate $\theta^* \mathsf{E}[\hat{M}]$ has any universal sign of relation with volatility. It is also not clear whether expected total emissions increase with respect to volatility given that E_* does not – a case where the first-order condition for E_* does not guarantee uniqueness.

6   Conclusions

In this paper we have derived simultaneous solutions for the following two decisions:

(1) The (fossil-fuel) energy and carbon emissions intensity embedded in an infrastructure object, from the time of investment until the time it is (later, if ever) "retrofitted". As long as operated and not retrofitted, this infrastructure gives rise to a constant energy consumption and carbon emissions per unit of time.

(2) The retrofit time for the infrastructure. Fossil energy use and carbon emissions are then eliminated, while the infrastructure in other respects keeps operating with a given and constant utility stream to the public.

Both decisions have implications for aggregate fossil energy consumption and carbon emissions resulting from the infrastructure's lifetime operation. In section 3 of the paper, we assume that marginal emission costs evolve according to a geometric Brownian motion process with constant positive drift. We derive analytical results related to each of the two decisions; one of these is an already known result, while the other is new. The first result, previously shown by Pindyck (2000), is that increased variance of the stochastic process ("volatility"), facing the party making the investment, leads to postponement of the retrofit decision, due to the increased option value of waiting when volatility increases. Our second

(and new) result deals with optimal energy and emissions intensity of infrastructure when established, which is shown to also increase in volatility. This is not surprising: it follows from the first-order effect of an increase in the associated value function when the "upside" (or benign) risk increases. This effect is reinforced by the increased option value of waiting to retrofit, which works in the same direction. The retrofit decision here "takes care of" (eliminates) any major increases in "downside" risk; the principal effect of greater volatility is that "good" outcomes become better and more frequent, which increases the value of the project and of having a high established energy consumption level. A related and also new result is that the aggregate expected damage from carbon emissions, from the lifetime operation of the infrastructure (considering also the future retrofit), also increases in volatility. This result is less obvious: it happens despite the increased frequency of "good" outcomes which in isolation lowers emissions damage. The higher energy intensity in the initially established infrastructure, and the longer time to a retrofit, is here shown to always more than outweigh this benign effect.

For efficient investment decisions involving long-lasting and potentially energy-intensive infrastructure, both at the initial infrastructure investment stage, and at later stages when the infrastructure can (at least in principle) be retrofitted, decision makers need to face globally correct energy and emissions prices. Otherwise, two serious related problems, to be read directly out of our analysis, can result. First, infrastructure whose structure is difficult or expensive to change later, will be too energy intensive and established at a too large scale. Secondly, too low emissions and energy costs can lead to inadequate action also in the process of operating the infrastructure, as necessary retrofits are not implemented. This problem would be exacerbated if, at the same time, the cost of executing a retrofit is too high, which it may often be in less developed economies that do not have access to the most advanced retrofit technology.

We find that when the level of volatility or uncertainty facing policy makers increases, energy consumption due to both the energy intensity of infrastructure, and the unwillingness to retrofit this in the future, at the same time increase. This result indicates that great attention should be paid to correct pricing of emissions and energy, for all decision makers in both the short and long run. An indication is also that appropriate pricing becomes even more crucial under uncertainty, and in the long run, with the additional

pressure thereby created to raise the fossil energy consumption related to long-lasting infrastructure.

The issues discussed in this paper are of particular concern for emerging and expanding economies which are making, or plan to make, large volumes of infrastructure investments of a long-lasting nature. Given that decision makers in these economies face globally correct emissions and retrofit costs both today and in the future, our results show that they will choose to make their infrastructure investment more energy- and emissions-intensive, and to retrofit them later, when volatility of the process for emissions costs increases.

But policy and wider implications of the concrete results we derive are not entirely obvious at this stage. A major political complication today is that the relevant decision makers, most of whom are in emerging economies, in most cases do not face globally efficient emissions prices, but instead prices that are (often far) lower. In our model as it stands, since increased volatility of global emissions costs leads to a higher globally optimal emissions level, it might be reasonable to conjecture that when decision makers face lower than optimal global emissions costs, resulting emissions rates will be excessive. We know however less about the relationship between these two; it requires comparisons of discretely different systems, most likely necessitating simulations. Then the problem of excessive energy intensity and retrofits to be made too late, associated with such infrastructure will be exacerbated by greater volatility. A high perceived likelihood of "upside" (low-cost) risks, when such perceived effects are not warranted, might be particularly harmful. Such "upside" risks can also be interpreted as a low rate at which carbon emissions stemming from the infrastructure will actually be charged; and not necessarily as a low (true) climate cost to society (be it the local, national or global).

Note also that high "downside" (high-cost) risks play no effective role in decision makers' choices in our model, as such high costs can always be purged by retrofitting later (at a limited and "moderate" cost). This is of course unrealistic. If retrofits are impossible or very costly in some cases (such as completely altering urban structure or transport systems), and high "downside risk" (such as the possibility of a "climate catastrophe") is underrated, emissions, in particular those associated with the initial infrastructure investment, will also tend to be excessive. A further formal analysis of such cases must however await future research.

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Appendix A: Alternative Assumptions

A1. Multiple Retrofit Times (Relaxing Assumption (D))

Thus far we have assumed that emissions are reduced at most once, and if so to zero. We shall give conditions for the latter; if it is not optimal even in a one-shot model to remove all emissions, then it is neither optimal when gradual emissions are allowed.

Suppose the problem is solved, with initial investment E_* and retrofit trigger level θ^* chosen optimally, however this time under the assumption that at the hitting time τ^* for the level θ^*, the emission level is reduced not necessarily to zero, but to some optimized level $E^* \in [0, E_*)$, where it is kept forever; this yields a discounted environmental damage from τ^* on, of $\theta^* E^* / (r - \alpha)(r + \delta - \alpha)$. At τ^*, the optimal E^* must therefore minimize the sum of this damage and the retrofit cost which we now write as two variables, $\tilde{K}(E_*, \eta)$. Sufficient for choosing $\eta = 0$ is then convexity, or alternatively that $\tilde{K}'_2 + \theta^* / (r - \alpha)(r + \delta - \alpha)$ is nonnegative for all $\eta \in (0, E_*)$. Inserting for θ^*, this is ensured if

$$-\mathcal{E}\ell_2\, \tilde{K}(E_*, \cdot) \le \frac{\gamma}{\gamma - 1} \tag{46}$$

Arguably, a natural generalization of assumption (D) (all emissions eliminated by the retrofit), where we have assumed a cost $K(E)$ of reducing emissions to zero, would be to instead impose a functional form $\tilde{K}(E, \eta) = K(E - \eta)$ for reducing emissions from E to η. Then (46) is a direct generalization of the condition $\mathcal{E}\ell K(E_*) \le \gamma / (\gamma - 1)$ under which the presence of the retrofit option leads to higher level of initial emissions under conditions (A)–(D). The interpretation of this is that there will not be the same incentive to adapt for lower retrofit cost, if those lower costs may be attained nevertheless, at the expense of *(linear!)* cost of climate damage.

Note also that the allowable maximum value $\gamma / (\gamma - 1)$ for the elasticity is due to the constraints on the retrofit action. Likely, a model admitting more general strategies would not share this property.

A2. Abandoning the Infrastructure (Relaxing Assumption (B))

Thus far we have taken as given that the infrastructure is operated forever. If we drop that assumption, the cost K is capped at V, as one can at any time close down, abandoning the services from the infrastructure. We shall see that in this case, the model will very often be losing its validity, as the optimal choice when initial time is non-negotiable, could be to invest at level $E_* = +\infty$ and then immediately abandon the infrastructure. Assume that the cap becomes effective at some $\tilde{E}$; then h gets an upward jump $h(\tilde{E}^+) - h(\tilde{E}^-) = K'(\tilde{E}^-) \cdot (\theta / \theta^*)^\gamma$. If this is positive, then W' will be discontinuous unless ϕ' has an equal jump; assuming that ϕ' continuous, we have $W'(\tilde{E}^+) - W'(\tilde{E}^-) = K'(\tilde{E}^-) \min\{1, \theta / \theta^*\}^\gamma$. Obviously, we do not have concavity. There might be a local max to the left of $\tilde{E}$, but we know nothing of whether it will be optimal. Assuming that $K(E) = V$ for all $E \geq \tilde{E}$, then with parsimonious convex ϕ, W will to the right of V/k be a difference between two convex functions, and any hope for uniqueness of any stationary point $> V/k$ would require further conditions or specification of ϕ. For given θ, notice that for large enough E (making θ^* decrease), $W'(E) = -\phi'(E) > 0$. Further assumptions have to be made to ensure $E_* < \infty$. However, $E_* = \infty$ would lead to total payoff $W^* = -\phi(\infty) - K(\infty) < 0$, and it becomes absurd to assume non-negotiable initial time. This leads to the next subsection: what if initial time is negotiable?

A3. Endogenizing Initial Time (Relaxing Assumption (A))

Introducing initial time as another choice variable will arguably adds another level of complexity to the problem, but it will resolve the objectionable properties of the previous subsection; it guarantees nonnegative value. Although the deduction is somewhat beyond the scope of this paper, the optimal rule will be to wait for for the first time τ_* for which $\theta_{\tau_*} \leq$ some sufficiently low value θ_* – chosen subject to optimized choices of E_* and τ^*. Obviously, this will ensure $\tau^* > \tau_*$ and prevent immediate retrofit/closedown action.

A4. Availability of Retrofit Technology (Relaxing Assumption (C))

Thus far, we have assumed that the initial technology E is freely available instantly. It is hot hard to accommodate into the model the restriction to a compact interval of admissible emission levels. However, what happens if the availability of technology changes over time?

Consider a situation where the new technology will not be available until some future time T, which then lower bounds the intervention time τ. We require the initial investment to be committed at time zero, but the optimal stopping problem has to be restarted at T, with the value

$$K(E) \cdot g(\frac{\Theta_T}{\theta^*}) \tag{47}$$

if $\Theta_T \geq \theta^*$, and otherwise:

$$\frac{(\gamma-1)^{\gamma-1}}{\gamma^\gamma}(\frac{\Theta_T}{(r-\alpha)(r+\delta-\alpha)})^\gamma \cdot E^\gamma K(E)^{1-\gamma} - \frac{\Theta_T \cdot E}{(r-\alpha)(r+\delta-\alpha)} \tag{48}$$

At time zero, one has to discount this by $\exp(-rT)$ and take the expectation; this applies even if T is a stochastic time. The positive probability that $\Theta_T > \theta^*$ together with the split definition makes this analytically intractable, however, we still have the form (cf. (23)and the subsequent discussion)

$$W(E) = V - \phi(E) + \mathsf{E}[e^{-rT} g(\frac{\Theta_T}{\theta^*})] \cdot K(E) \tag{49}$$

The following result can now be shown:

Proposition 7: *Suppose that* (1) *has been maximized over all* $\tau \geq T$, *where* T *is a nonnegative finite random variable with distribution not depending on* E *nor the cost function* ϕ. *Then*

$$E \mapsto \mathsf{E}[e^{-rT} g(\Theta_t / \theta^*)] \cdot K(E) \tag{50}$$

is concave if K *is convex, and affine if* K *is proportional.*

Remark: This result shows that the form of the value function, and of the optimization wrt emission level, are maintained, at least to a certain degree. However, there is not much

hope that the conclusion of Proposition 2 (namely, dependence merely upon k, γ and θ/θ^*) will carry over, as g is nonlinear and with a split definition.

A different approach could be to model the retrofit unit cost as a stochastic process (reasonably, a supermartingale after discounting). This is work in progress.

Appendix B: Proofs

Only propositions 4–6 require several lines of calculation. Proposition 1 follows Pindyck (2000). Proposition 2 is straightforward: For the first part, the problem only depends on the parameters in question. For the second part, consider the first-order condition or the derivative at zero. Proposition 3 is straightforward calculation, and the proof is omitted.

B.1. Proof of Proposition 5

We calculate $dE_* / d\gamma$ by differentiating the first-order condition, writing $\rho = \theta/\theta^*$ (<1):

$$
\begin{aligned}
-W''(E_*)\frac{dE_*}{d\gamma} &= \frac{\partial}{\partial\gamma}\Big[\Big(\frac{(\gamma-1)\theta E_*}{\gamma(r-\alpha)(r+\delta-\alpha)K(E_*)}\Big)^\gamma \cdot \frac{K(E_*)}{E_*}[\frac{\gamma}{\gamma-1}-\mathcal{E}\ell K(E_*)]\Big] \\
&= \frac{K(E_*)}{E_*}\Big[[\frac{\gamma}{\gamma-1}-\mathcal{E}\ell K(E_*)]\rho^\gamma\frac{\partial}{\partial\gamma}[\gamma\ln\frac{(\gamma-1)\theta E_*}{\gamma(r-\alpha)(r+\delta-\alpha)K(E_*)}]-\frac{1}{(\gamma-1)^2}\rho^\gamma\Big] (51) \\
&= \frac{K(E_*)}{E_*}\rho^\gamma\Big[[\frac{\gamma}{\gamma-1}-\mathcal{E}\ell K(E_*)](\frac{1}{\gamma-1}-\ln\frac{1}{\rho})-\frac{1}{(\gamma-1)^2}\Big]
\end{aligned}
$$

and the result follows by gathering terms and multiplying by $-\Delta$. Then for θ^*:

$$
\frac{d\theta^*}{d(\sigma^2)} = \theta^*\cdot\Big[-\Delta\frac{\partial}{\partial\gamma}\ln\frac{\gamma}{\gamma-1}+\frac{dE_*}{d(\sigma^2)}\cdot\frac{\partial}{\partial E_*}\ln\frac{K(E_*)}{E_*}\Big] \tag{52}
$$

and the rest is straightforward. Now by the envelope theorem, $dW^*/d\gamma$ can be calculated by partially differentiating W from (23):

$$
\begin{aligned}
\frac{dW^*}{d\gamma} &= K(E_*)\frac{\rho^\gamma}{\gamma-1}\cdot\frac{\partial}{\partial\gamma}\Big[\gamma\ln\frac{(\gamma-1)\theta E_*}{\gamma(r-\alpha)(r+\delta-\alpha)K(E_*)}+\ln(\gamma-1)\Big] \\
&= K(E_*)\frac{\rho^\gamma}{\gamma-1}\Big[\ln\rho+\gamma[\frac{1}{\gamma-1}-\frac{1}{\gamma}]-\frac{1}{\gamma-1}\Big]
\end{aligned} \tag{53}
$$

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and everything but the ln term vanishes from the bracket.

Finally, consider the environmental damage, which by the form of $\mathsf{E}[e^{-r\tau^*}]$ and assuming proportional retrofit costs ($K(E) = kE$), turns out to be

$$D = \frac{\theta E_*}{(r-\alpha)(r+\delta-\alpha)} - \frac{(\gamma-1)^{\gamma-1}}{\gamma^\gamma}(\frac{\theta}{(r-\alpha)(r+\delta-\alpha)k})^\gamma \cdot kE_*\gamma \tag{54}$$

The effect of volatility enters by way of E_* and γ:

$$
\begin{aligned}
\frac{\mathrm{d}\,D}{\mathrm{d}(\sigma^2)} &= \frac{D}{E_*}\frac{\mathrm{d}\,E_*}{\mathrm{d}(\sigma^2)} + \Delta k E_* \frac{\partial}{\partial\gamma}\Big[\frac{(\gamma-1)^{\gamma-1}}{\gamma^{\gamma-1}}(\frac{\theta}{(r-\alpha)(r+\delta-\alpha)k})^\gamma\Big] \\
&= \frac{D}{E_*}\frac{\Delta k \rho^\gamma}{-W''(E_*)}(\frac{\gamma}{\gamma-1}-1)\ln\frac{1}{\rho} + \frac{\Delta\theta E_*}{(r-\alpha)(r+\delta-\alpha)}\frac{\partial}{\partial\gamma}\Big[(\frac{\theta(\gamma-1)}{\gamma(r-\alpha)(r+\delta-\alpha)k})^{\gamma-1}\Big] \\
&= \frac{D}{E_*}\frac{\Delta k \rho^\gamma}{\phi''(E_*)(\gamma-1)}\cdot\ln\frac{1}{\rho} + \frac{\Delta\theta E_*\rho^{\gamma-1}}{(r-\alpha)(r+\delta-\alpha)}\Big[\frac{1}{\gamma}+\ln\rho\Big] \\
&= \frac{\Delta k \rho^\gamma}{\gamma-1}\Big\{\frac{D}{E_*\phi''(E_*)}\cdot\ln\frac{1}{\rho} + \Big[1-\gamma\ln\frac{1}{\rho}\Big]E_*\Big\}
\end{aligned}
\tag{55}
$$

This expression is positive for ρ near 1. Choose therefore ρ so small that the bracket is negative. Fix an E_* and construct a sequence of strictly convex investment cost functions ϕ which all lead to E_* being the optimal choice, i.e. with the same $\phi'(E_*)$; but with the sequence of $\phi''(E_*)$ values tending to $+\infty$.

B.2. Proof of Proposition 6

Differentiation is straightforward, and whenever $\mathcal{E}\ell K(E_*) \in [1, \gamma/(\gamma-1)]$, $\mathsf{E}[\tau^*]\cdot E_*$ is a product of two nonnegative increasing functions. It remains to show that the derivative of expected peak pollution stock can take either sign, assuming proportional retrofit cost κE. We first manipulate differentials:

$$\mathrm{d}\mathsf{E}[\hat{M}] = \frac{1}{\delta}(1-\rho^\epsilon)\mathrm{d}E - \Big[\frac{E}{\delta}-m\Big]\mathrm{d}(\rho^\epsilon) \tag{56}$$

where, by slight abuse of notation, we interpret " dE " to be zero if the initial investment is already made, and dE^* if it can still be optimized. By manipulating logarithmic derivatives, we get

$$dE[\hat{M}] = \frac{1-\rho^\epsilon}{\delta} dE + \left[\frac{E}{\delta} - m\right]\rho^\epsilon \cdot \epsilon \cdot \left\{dL^* + L^* \cdot d\ln\epsilon\right\}. \tag{57}$$

Observe that when taking derivatives wrt σ^2, all terms are nonnegative except *possibly* $d\ln\epsilon$; this does not depend on ρ, so in the limit $\rho \nearrow 1$, we would get a positive derivative wrt volatility. This is due to θ^* increasing.

Consider $d\ln\epsilon / d(\sigma^2)$. Write $\pi(\sigma^2) = \alpha/\sigma^2 - 1/2$, so that $\pi' = -\alpha\sigma^{-4}$, and write also $\ln\epsilon$ in terms of volatility:

$$\ln\epsilon = \ln 2 + \ln\delta - \frac{1}{2}\ln(\sigma^2) - \ln\left[\sqrt{\pi(\sigma^2)^2 + 2\delta} + \pi(\sigma^2)\right] \tag{58}$$

so that

$$\frac{d\ln\epsilon}{d(\sigma^2)} = -\frac{1}{2\sigma^2} + \frac{\alpha\sigma^{-4}}{\sqrt{\pi^2 + 2\delta}} = \frac{1}{2\sigma^2}\left[-1 + \frac{2\alpha}{\sqrt{(\alpha - \frac{1}{2}\sigma^2)^2 + 2\delta\sigma^4}}\right] \tag{59}$$

which is nonnegative if (and only if)

$$\delta \leq \frac{1}{2}\left(\frac{\alpha}{\sigma^2} + \frac{1}{2}\right)\left(3\frac{\alpha}{\sigma^2} - \frac{1}{2}\right) \tag{60}$$

To establish that $dE[\hat{M}]/d(\sigma^2)$ also takes negative values, let δ grow, but adjust θ to keep ρ constant (and sufficiently close to 0). For given level of volatility, γ and k, this will fix E_*, and therefore $dE_*/d(\sigma^2)$; choose a cost function ϕ so that $\phi''(E_*) = +\infty$ and $dE_*/d(\sigma^2)$ vanishes. Then the sign of $dE[\hat{M}]/d(\sigma^2)$ depends only on the sign of the braced expression of (57) that is, the sign of

$$\frac{2(\sigma^2)}{L^*}\frac{\mathrm{d}L^*}{\mathrm{d}(\sigma^2)}-1+\frac{2\alpha}{\sqrt{(\alpha-\frac{1}{2}(\sigma^2))^2+2\delta(\sigma^2)^2}}. \tag{61}$$

Only the last term depends on δ, and vanishes as $\delta\to\infty$ when everything else is fixed. Choosing ρ sufficiently small, will keep the first term <1.

B.3. Proof of Proposition 7

It suffices to prove that $K(E)\cdot g(\Theta_T/\theta^*)$ is concave for every $\Theta_T>0$. For $\Theta_T\geq\theta^*$ we have concavity if K is convex. For $\rho_T=\Theta_T/\theta^*<1$, we differentiate wrt E. Using that $\mathrm{d}\rho_T/\mathrm{d}E=-\rho_T(\mathcal{E}\ell K-1)/E$, we obtain

$$K'(E)g(\rho_T)-\frac{K\rho_T}{E}g'(\rho_T)\cdot(\mathcal{E}\ell K-1) \tag{62}$$

Now insert for the function g and write $K\rho_T/E$ as $(\gamma-1)\Theta_T/\gamma(r-\alpha)(r+\delta-\alpha)$ to get, after some simplifications:

$$-K'(E)\rho_T^{\gamma}-\frac{\Theta_T}{(r-\alpha)(r+\delta-\alpha)}(\rho_T{}^{\gamma-1}-1) \tag{63}$$

The pasting of this with $-K'(E)$ across $\rho_T=1$, is continuous. For concavity, it therefore suffices to prove that this expression decreases in E; for affinity, we merely need to observe that this is constant in E, since ρ_T is constant. Differentiating once more, we get:

$$-K''(E)\rho_T^{\gamma}-\frac{\rho_T}{E}(\mathcal{E}\ell K-1)\cdot\left[K'(E)\gamma\rho_T^{\gamma-1}-\frac{(\gamma-1)\Theta_T}{(r-\alpha)(r+\delta-\alpha)}\rho_T{}^{\gamma-2}\right]$$
$$=-K''(E)\rho_T^{\gamma}-\frac{\rho_T{}^{\gamma}}{E}(\mathcal{E}\ell K-1)\cdot\left[\gamma K'(E)-\gamma\rho_T\frac{K(E)}{E}\right] \tag{64}$$
$$=-K''(E)\rho_T^{\gamma}-\gamma\frac{\rho_T{}^{\gamma}K(E)}{E^2}(\mathcal{E}\ell K-1)^2$$

which is negative if $K''\geq0$.