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Resource Depletion and Capital Accumulation under Catastrophic Risk: Policy Actions against Stochastic Thresholds and Stock Pollution

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Resource Depletion and Capital Accumulation under Catastrophic Risk: Policy Actions against Stochastic Thresholds and Stock Pollution

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Abstract:

An intertemporal optimal strategy for accumulation of reversible capital and management of an exhaustible resource is analyzed for a global economy when resource depletion generates discharges that add to a stock pollutant that affects the likelihood for hitting a tipping point or threshold of unknown location, causing a random "disembodied technical regress". We characterize an optimal strategy by imposing the notion "precautionary tax" on current extraction for preventing a productivity shock driven by stock pollution and a capital subsidy to promote capital accumulation so as to build up a buffer for future consumption opportunities should the threshold be hit. The precautionary tax will internalize the expected welfare loss should a threshold be hit, whereas the capital subsidy will internalize the expected post-catastrophic long-run return from current capital accumulation.

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1. **Introduction**

Environmental hazards or threats that have not yet materialized should be met by precautionary actions today so as to reduce the likelihood for catastrophic outcomes in the future. Whereas there is a close correspondence in time and space between some types of emissions and environmental damages, there are some emissions that enter into a stock of pollutant that might be harmful in the future. Emissions of GHGs like CO2 or methane into the atmosphere may increase the likelihood of a future environmental disaster. Because emissions of GHGs do not necessarily add to current damages, and hence no traditional static externality to be accounted for, the long-run environmental hazards caused by, say, current consumption of fossil fuels, should be internalized by current decision makers. Under a system where emissions as well as damages interact (almost) simultaneously, taxes, quotas or permits, have been shown to improve on the efficiency of the resource allocation. However, when there is a time lag, usually of random length, between current emissions and damages of unknown magnitude, taxes cannot be directly related to current damages or costs, but must be related to expected welfare costs in the future. In order to take account of these hazards, we introduce a term "precautionary taxation", which is nothing more than just a way of defining how to tax a random stock pollutant. (A similar approach has been taken by Tsur and Zemel (2008) who introduces the notion "Pigouvian hazard tax" on such preemptive taxation when future random damages are of main concern. The paper by van der Ploeg and de Zeeuw (2013) has a similar approach to modeling random environmental catastrophes as ours, but the production structure of the economy is quite different. That paper should be considered as a complement to ours.[2])

A large number of environmental problems are caused by stock pollution; e.g., problems related to emissions of GHGs and anthropogenic climate change. There is a vast literature on such issues; see for instance Ayong Le Kama et al. (2010), Barrett (2011), Becker et al. (2010), Brito and Intriligator (1987), Clarke and Reed (1994), Cropper (1976), Gjerde et al. (1999), Hoel and Karp (2001, 2002), Keeler et al. (1971),

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[2] The concept "precautionary taxation" has been used in the public finance literature to illustrate tax instruments that improve on government budgets or increase government savings, when times are uncertain or when government debt is too high.

Nævdal (2003, 2006), Pindyck (2007), Tahvonen and Withagen (1996), Torvanger (1997), Tsur and Zemel (1996, 1998, 2008), and, de Zeeuw and Zemel (2012). In addition, the issue of what rate of discount factor should be used for how to value these environmental costs or hazards that might be incurred far into the future, has been subject to an intense debate in the literature; see Arrow (2009), Barro (2013), Dasgupta (2008), Gollier (2002), Heal (2005), Nordhaus (2007), Nævdal and Vislie (2010), Stern (2007), Tsur and Zemel (2009) and Weitzman (2007, 2009), just to mention some of the participants in that debate.

Although costs or environmental damages do not appear until accumulated stocks have reached some specific levels or thresholds (tipping points), we are seldom in a position to know exactly the location of these levels or when these critical values are reached; an issue that is discussed thoroughly in some of the cited papers. An analogy is driving a car in darkness knowing that somewhere in front of you there is a hole, of unknown size, and with a location being unknown to the driver, as long as the car is still on the road. Being aware of this possibility should normally affect a rational driver's caution, say, by slowing down the speed. A similar danger or hazard might be in front of us as well. We believe that present current economic activity generates emissions that will increase the stock of a number of pollutants in the future. The amount of accumulated stocks may not be harmful today, but we have some qualified opinion or belief that if (and when) some stock reaches a critical level, a natural disaster might be triggered, and hence be harmful to subsequent generations. However, even though we have some opinion about the relationship between the size of a stock and the disaster, we cannot accurately forecast what level of atmospheric GHGs will trigger such an event, say through a sufficiently rise in temperature or in the sea-level.[3] The knowledge that the well-being of future generations might be severely affected, or, in a worst case, their mere existence is threatened, should – from an ethical and normative point of view – affect current generation's behavior.

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[3] In reality, the threshold is more likely to be a temperature level than a GHG stock level. We simplify from this complication, but see Nævdal and Oppenheimer (2007) for a discussion.

The present paper is just about such policy issues within the context of optimal capital accumulation and resource extraction when future environmental costs are uncertain.

To study this problem we consider an economy at a very high level of aggregation where current consumption of a resource-intensive commodity is the only "input" in the generation of a stock of emissions. Along with a resource-intensive good, there is also a "composite" commodity that is produced by capital equipment alone; this commodity can be used for gross investment and current consumption. Preferences are related only to consumption profiles, consisting of the two consumption goods. Accumulated stock of emissions could enter as an argument in the utility flow, but here we take a different approach as to the relationship between a stock pollutant and a catastrophe: We assume that the likelihood for a disaster (like a sharp rise in temperature, a flood or sea-level rise) is assumed to be a function of the accumulated stock of pollutant, generated from the consumption of the resource-intensive commodity. Hitting a critical value or a threshold, whose location is random, will trigger a disaster in the sense that the economy will be inflicted a cost through a real and persistent productivity shock; cf. Torvanger (op.cit.). This shock can be conceived of as a random technical disembodied regress or as a fall in total factor productivity; cf. van der Ploeg and de Zeeuw (op.cit.). Such an event will lower the production capacity in all sectors uniformly through destruction. Hence, because the sequence of consumption choices of the resource-intensive commodity will affect the likelihood for a future disaster, an infinitely-lived planner should take this hazard into account when balancing the preferences of current and future generations. In this simplified world, the only way of postponing a likely disaster, is through lowering current consumption of the resource-intensive commodity or deplete the exhaustible resource at a slower rate.[4] (This type of inaction might be seen an application of the

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[4] For simplicity, we rule out any option for abatement. Also the stock itself is not subject to any natural decay. Both assumptions are of course restrictive, but have been introduced for analytical tractability. Also, incorporating a positive decay rate for carbon distracts from the fact that this number is not very large.

precautionary principle, as discussed by Gollier et al. (2000), and by Weisbach (2011).) Because consumption of the resource-intensive good prior to a disaster does not cause any harm, only benefits, but will affect the likelihood of some future adverse event, there is no current or static externality in the traditional sense to take into account, but a stochastic future dynamic externality. Due to the assumption that current consumption of the resource-intensive commodity enhances future risk, current generation's consumption of that good should be taxed according to the expected future cost of a disaster or, in a worst case, a catastrophe. This tax on current consumption of the resource-intensive commodity is explicitly derived; see also Tsur and Zemel (2008), and van der Ploeg and de Zeeuw (op.cit.). In addition to curbing current extraction of fossil fuels, so as to reduce the likelihood for a future disaster, the future productivity shock will induce the planner to increase capital accumulation, as compared with no shock, as long as no threshold is hit. The rationale for such excessive capital accumulation is found in the assumption that capital is reversible[5]. When anticipating a (severe) productivity shock, the planner will have a strong motive to create a buffer so as to secure future consumption opportunities for the period after a threshold has been hit. The way of implementing stronger incentives for capital accumulation is through a capital subsidy, because "myopic" producers will in an unregulated environment have no incentive to take long-run, risky consequences into account. Hence, the planner must affect their incentives. This is done by imposing a capital subsidy.

The paper is organized as follows: In section 2 we present the model. The optimal contingency consumption-investment-extraction plan is derived in section 3 and expressed as a modified Ramsey-Hotelling Rule. This rule will appear as decision rules for capital accumulation and resource depletion as long as no threshold is hit, and will of course be affected by the expected future shadow values of the capital stock and the remaining resource. We also derive the capital subsidy for stimulating

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[5] Capital is reversible in the sense that capital equipment can eventually be transformed into ordinary consumption goods.

current capital accumulation to build a buffer against a shock and the precautionary tax on the consumption of a resource-intensive commodity so as to get any current generation to internalize the expected welfare loss caused by hitting a threshold in the future. Section 4 concludes and points to some extensions.

2. **The Model**

The global planner's objective is to derive an intertemporal long-run optimum for an economy with two production sectors and one resource-extracting sector. One of the sectors – the one producing the resource-intensive commodity – uses a flow of a non-reproducible natural resource (oil or fossil fuel), v, as input in producing the consumption good (as denoted c_2). The other sector – the capital-intensive sector – uses only capital as input to produce a composite good, according to a production function, $Af(k)$. The aggregate output is allocated to consumption (c_1) and gross investment (J). The resource-intensive sector's output is assumed to be of the linear type, $Ag(v) \equiv Av$, with v as the input-flow of a non-reproducible natural resource (equal to the output-flow), whose remaining stock at t is given by $R(t)$. The parameter $A \in [0,1]$ is a random variable and is related to a post-event permanent downgrading of the production capacity, common to both sectors. The "kernel" production function f is bounded, twice continuously differentiable, strictly increasing and strictly concave, with $f(0) = 0$, $\lim_{k \to \infty} f'(k) = 0$, and $\lim_{k \to 0} f'(k)$ being positive but finite.

At some point in time in the future the economy *might* suffer from a natural disaster or a catastrophe that is triggered by hitting a threshold whose location is not known ex ante. The hitting outcome is caused by having reached a critical amount of accumulated waste generated from resource extraction. Once the disaster occurs the value of A is randomly drawn from a known distribution with a realized (and permanent) value $a \in [0,1]$, with $A \equiv 1$ prior to the shock.

The following relations characterize the economy, where $\dot{k}(t) := \dfrac{dk(t)}{dt}$ and δ is a fixed depreciation rate per unit capital equipment:

(1) $Af(k(t)) = c_1(t) + J(t) = c_1(t) + \dot{k}(t) + \delta k(t) \quad \forall t \in [0, \infty)$ and $k(0) = k_0$

(2) $c_2(t) = A v(t) \quad and \quad \dot{R}(t) = -v(t)$ with $R(t) = R_0 - \int_0^t v(s)ds$, R_0 given

We rule out demographic factors, by assuming a fixed population, with stable preferences for "all generations", given by an additive, separable felicity function $U(c_1(t), c_2(t)) = u(c_1(t)) + w(c_2(t))$. Both functions, (u, w), are twice continuously differentiable, bounded, strictly increasing and strictly concave, with $\lim_{c_1 \to 0} u'(c_1) = \infty$. However, we assume that $\lim_{c_2 \to 0} w'(c_2) > 0$ and finite.[6]

In producing the resource-intensive good a flow of emissions will be generated, say of CO2 per unit of time that accumulates into a stock of pollutant that is the main cause of a future environmental hazard or downgrading. If the stock of waste at t is given by $z(t)$, we have a growth function,

(3) $\dot{z}(t) := \dfrac{dz(t)}{dt} = D(v(t))$, with $z(0) = 0$

The growth per unit of time at t in the stock of pollutant is directly related to the use of the non-renewable resource as input in the resource-intensive commodity at t. We assume that D is twice continuously differentiable, strictly increasing and convex, with $D(0) = 0$. Because the stock is subject neither to any natural decay nor any abatement, it is non-degradable, according to Dasgupta (1982a). Again this might be regarded as a too restrictive assumption, but "no natural decay" might be justified by assuming the average lifetime of the pollutant exceeds the relevant time scale.

In this stylized economy the global planner has an objective to maximize expected present discounted utility; $E\left[\int_0^\infty e^{-rt} \left[u(c_1(t)) + w(c_2(t))\right]dt\right]$, subject to the relevant

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[6] Note that a higher value of a realized value of A can be regarded as "higher wealth". With our utility function there is a positive correlation between consumption of each good at some point in time and wealth.

constraints, where r is a non-negative pure rate of time preference or felicity discount rate.

A threshold is introduced into the model by defining a (subjective) probability function $F(z(t)) := \Pr(Y \le z(t))$ for the location of the stochastic threshold, given by the random variable Y. Here it is assumed that F is invariant to calendar time, increasing and twice continuously differentiable, with $F(0) = 0$ and also, because we assume that a threshold will exist, we let $\lim_{z \to \infty} F(z) = 1$. (Irreversible thresholds and environmental issues are studied in Boucekkine et al. (2012) as well, but within the framework of choosing *when* to switch to another regime as a known critical threshold is hit.)

Because consumption of the resource-intensive good is positive at any point in time as long as the productivity shock has not "wiped out both production functions", z will be strictly increasing over time, due to non-degradibility. Hence we have

(4) $\Pr(Y \le z(t)) = \Pr(z^{-1}(Y) \le t) := \Pr(T \le t) = \Omega(t)$

where T is the random point in time of hitting a threshold. The probability density for the event T ("the point in time of hitting the threshold") to occur in a small interval $\left[t, t + dt\right]$, is therefore $F'(z(t)) \cdot \dot{z}(t)dt = F'(z(t)) \cdot D(v(t))dt := \Omega'(t)dt$. If the threshold is hit, the economy will face an irreversible event in the sense that the economy enters a new (permanent) regime with a realized value of the productivity parameter, characterized, ex ante, by a distribution function $G(a; z(t)) = \Pr(A \le a \,\big|\, Y \le z(t))$, and a corresponding positive density function $g(a; z(t)) := \dfrac{\partial G(a; z(t))}{\partial a}$ for all $a \in \left[0, 1\right]$. To simplify even further we assume that G and g will be independent of accumulated waste; hence in the subsequent discussion we ignore the argument z in these functions. In that case only the expected value of A, conditional on hitting the threshold, will matter for the planner, as long as no threwhold has been hit.

3. **The contingency plan**

We look at the problem from the perspective of a global planner with the aim of maximizing discounted present value of expected welfare, taking account of all relevant constraints. The real big issue is what kind of precautionary actions can or should be taken so as to prevent the occurrence of a disaster – or postpone the catastrophe as far out into the future as possible. Of course, the character of these precautionary actions will depend on the prior (subjective) beliefs about the consequences of hitting a threshold. With only one possible (or major) catastrophe, the time line will have two disjoint intervals; the last one, extending over $[\tau, \infty)$, is characterized by having all uncertainty resolved when the threshold is hit at some arbitrary point in time, τ, and with a new and permanent value, a, of the stochastic shift parameter A being realized. Once the threshold is hit, the stock pollutant is assumed *not* have any further impact – it does no longer play any role in the cost-benefit calculations because no more damage can be made and all risk will from then on be eliminated. The only effect caused by the threhold is its impact on future production opportunities in (1) and (2). (The assumption of no more than one catastrophe or disaster could be justified by having more thresholds bunched closely together on the time line; cf. Barrett (op.cit.).)

Our first task is to characterize optimality in the continuation or post-catastrophe regime. Thereafter we derive the full strategy.

3-i The continuation regime

The value function for the continuation regime, called the "continuation payoff", with a fixed initial state, as given by $\{k_\tau, R_\tau; a, \tau\}$, starting at some point in time τ with a realized productivity parameter a, and with capital stock and remaining resource stock given by $k_\tau = k(\tau)$ and $R_\tau = R(\tau)$, respectively, is found as the solution to the following standard dynamic optimization problem: For any (feasible) pair $\{\tau, a\}$ being realized, with a corresponding pair of state variables $\{k, R\}$, the solution tells us what to do from τ and onwards, as summarized by the value function, which is independent of τ itself:

$$V(a, k_\tau, R_\tau) = Max_{(c_1, v)} \int\limits_\tau^\infty e^{-r(t-\tau)} \Big[u(c_1(t)) + w(av(t)) \Big] dt$$

$$s.t. \begin{cases} k(\tau) = k_\tau, R(\tau) = R_\tau \\ \dot{k}(t) := \dfrac{dk(t)}{dt} = af(k(t)) - c_1(t) - \delta k(t), \lim_{t\to\infty} k(t) \geq 0 \\ \dot{R}(t) := \dfrac{dR(t)}{dt} = -v(t), \lim_{t\to\infty} R(t) \geq 0 \end{cases}$$

The current value Hamiltonian for this problem is given by:

$$H\Big(c_1, v, k, R, t; a\Big) = u(c_1) + w(av) + p\Big(af(k) - c_1 - \delta k\Big) - qv$$

The planner's control variables are $\{c_1, v\}$; i.e. the consumption flow of the capital-intensive good and the extraction rate; both non-negative, while $\{k, R\}$ are state variables. Because the problem is a standard control problem it is not necessary to go into details. With p as the current shadow value of capital and q the current shadow value of the resource stock, we have that the continuation regime, given by a sample path $\{\hat{c}_1, \hat{v}, \hat{k}, \hat{R}\}$, will be characterized by a Ramsey Rule and a Hotelling Rule:

$$(5-i) \quad af'(\hat{k}(t)) - \delta = r + \hat{\omega}(\hat{c}_1(t)) \cdot \frac{\dot{\hat{c}}_1(t)}{\hat{c}_1(t)}$$

$$(5-ii) \quad aw'(a\hat{v}(t)) = q(\tau) e^{r(t-\tau)}$$

(Here $\hat{\omega}(\hat{c}_1)$ is the intertemporal elasticity of substitution or the absolute value of the elasticity of $u'(\hat{c}_1)$.)

Optimality requires that $u'(\hat{c}_1(t)) = p(t)$, with $p(t)$ obeying $r - \dfrac{\dot{p}(t)}{p(t)} = af'(\hat{k}(t)) - \delta$,

with $r - \dfrac{\dot{p}(t)}{p(t)}$ as the social rate of discount when aggregate output (or consumption) is the numéraire.

(5-i) is the standard Ramsey Rule for balancing current consumption and capital accumulation, whereas *(5-ii)* is the Hotelling Rule, with $q(t)$ as the current shadow value of the remaining resource stock at t, increasing at a rate equal to r.

Note that if the realized productivity shock should be very severe (with a realized a being close to zero), real capital is almost turned into a non-renewable resource that will gradually be reduced as we consume and let the remaining stock depreciate. In that case the scarcity value of capital $p(\tau)$ will be "high", whereas the remaining resource stock will lose its future value as its marginal productivity then becomes close to zero according to our assumptions. On the other hand, if realized shock is "almost invisible", with a being close to one, the shadow value of the resource will be higher than if $a \approx 0$, whereas the shadow value of real capital will be smaller than what it will be under a "doomsday" scenario. We therefore have that the shadow values of the state variables at the outset of *any* continuation regime will be a function of a as well as the values of the initial state variables; i.e. $q(\tau) = \hat{q}(a, k_\tau, R_\tau)$, with $\hat{q}(0, k_\tau, R_\tau) = 0$ and q being increasing in a, and $p(\tau) = \hat{p}(a, k_\tau, R_\tau)$, with p being decreasing in a.

For any realized pair of state variables (k_τ, R_τ) at the outset of a new regime, the shadow prices have the following well-known interpretations:

(6)
$$\begin{cases} \dfrac{\partial V(a, k_\tau, R_\tau)}{\partial k_\tau} = \hat{p}(a, k_\tau, R_\tau) \\ \dfrac{\partial V(a, k_\tau, R_\tau)}{\partial R_\tau} = \hat{q}(a, k_\tau, R_\tau) \end{cases}$$

(A standard result from optimal control theory; see Seierstad and Sydsæter (1987; chapt. 3.5, Eq (142)) is that

$$\frac{\partial V}{\partial a} = \int_\tau^\infty \frac{\partial H(\hat{c}_1(t), \hat{v}(t), \hat{k}(t), \hat{R}(t), t; a)}{\partial a} dt = \int_\tau^\infty \left[q(t)\hat{v}(t) + p(t)f(\hat{k}(t)) \right] dt > 0.$$ This welfare

gain is realized by consuming more of each normal commodity.)

As seen from the ex ante-stage, A is stochastic. Hence, before we enter the continuation regime when the true value of A is realized, only the expected future benefit will be relevant for evaluating what to do as long as no disaster has occurred. Therefore we have to define the expected maximal future benefit or the expected value function, as defined by $W(k_\tau, R_\tau) = \int_0^1 V(a, k_\tau, R_\tau) g(a) da$.[7] Let

$$\frac{\partial W(k_\tau, R_\tau)}{\partial k_\tau} = \int_0^1 \frac{\partial V(a, k_\tau, R_\tau)}{\partial k_\tau} g(a) da := \overline{p}(k_\tau, R_\tau), \text{ for short written } \overline{p}(\tau), \text{ be the}$$

expected marginal shadow value of continuation capital should the shock occur at τ, calculated at t = 0. Also, $\dfrac{\partial W(k_\tau, R_\tau)}{\partial R_\tau} = \overline{q}(k_\tau, R_\tau)$, for short $\overline{q}(\tau)$, is the expected marginal shadow value of the remaining resource at the beginning of the continuation period starting at some τ. These shadow prices will play an important role for the determination of the full strategy. Once the economy should enter a continuation regime, with some realization of A below one, there will normally be a jump from the path derived as part of the optimal strategy prior to the continuation regime, based on expected shadow values, to the one that will hold for the realized value of a. Let us therefore turn to optimal policy prior to reaching the threshold, which in conjunction with the continuation regime is termed "The Full Strategy".

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[7] One might perhaps have some objection to this approach. Taking the precautionarty principle seriously, the planner, if sufficiently risk-averse, might take only the worst-case scenario into account, by making a plan as if the realized value of A will become zero with probability one.

3-ii The Full Strategy

As seen from the beginning of the planning period, an optimal strategy is found as the solution to the following constrained optimization problem:

$$Max_{(c_1,v)} \int_0^\infty \Omega'(\tau) \left\{ \int_0^\tau e^{-rt}[u(c_1(t)) + w(v(t))]dt + e^{-r\tau}W(k_\tau, R_\tau) \right\} d\tau$$

s.t.

$$\dot{k}(t) = f(k(t)) - c_1(t) - \delta k(t) \quad \forall t \in [0, \tau], \ k(0) = k_0, \ f'(k_0) > r + \delta$$

$$\dot{R}(t) = -v(t); \forall t \in [0, \tau], R(0) = R_0$$

$$\dot{z}(t) = D(v(t)), \forall t \in [0, \tau], z(0) = 0$$

$$\Omega'(\tau)d\tau = F'(z(\tau)) \cdot D(v(\tau))d\tau, \text{ where } z(s) = \int_0^s D(v(t))dt,$$

No conditions on $z(\infty)$; with k_τ, R_τ both given, and $k(\infty) \geq 0, R(\infty) \geq 0$

To get some idea about the trade-offs the planner has to undertake we should note that even though discounting should favour current consumption of both commodities, the planner will take into account the cost or consequences of hitting a threshold. Suppose we have a planner holding very pessimistic beliefs in the sense that the expected consequences of hitting a threshold are severe, with a significantly big expected productivity shock. In that case we conjecture that an optimal solution should involve less consumption of the resource-intensive commodity as long as the shock has not occurred. This is the only way to avoid or postpone hitting a threshold as the growth of accumulated emissions then is decreased. To compensate current generation one might therefore open up for more consumption of the capital-intensive commodity. However, then current investment in capital equipment will be reduced. The implication is that future generations are made more vulnerable in the continuation regime as the economy then will start with constrained production possibilities, because the capital stock in that pessimistic case is expected to serve as a non-renewable resource during the continuation period. This way of compensating current generations might therefore neither be very smart nor optimal if a *substantial* downgrading of future production capacity is expected. Therefore, part of the optimal

strategy against anticipating a severe shock might be to build a buffer of a sufficiently high capital stock to be exploited during the continuation period, implemented by *subsidizing current capital accumulation*, while at the same time postpone the point in time of hitting a threshold by impoing a *precautionary tax on the consumption of the emission-generating commodity* (fossil fuel). To protect the living conditions for future (perhaps unlucky) generations, current generation's consumption of both commodities should normally be lowered.

One might wonder whether there are any countervailing forces. A somewhat cynical point of view, if the anticipations held are very pessimistic, might be that there is no reason to provide future generations with natural resources or capital equipment because "the world is expected to collapse any way". Therefore it might then be better to consume as long as we can, especially if we believe or expect that the future marginal productivity of the resource will be close to zero. Implementing such a cynical strategy will, on the other hand, push us closer to the cliff or "doomsday" itself, as the likelihood for a catastrophe then will increase. This scenario is excluded from our solution, even if the planner should hold extremely pessimistic beliefs.

Integrating the objective function by parts, when using that $\Omega(t) = F(z(t))$, we have:

$$\int_0^\infty \Omega'(\tau) \left\{ \int_0^\tau e^{-rt} [u(c_1(t)) + w(v(t))] dt + e^{-r\tau} W(k_\tau, R_\tau) \right\} d\tau$$
$$= \int_0^\infty e^{-rt} \left[(1 - F(z(t))) \cdot [u(c_1(t)) + w(v(t))] + F'(z(t)) \cdot D(v(t)) \cdot W(k_t, R_t) \right] dt$$

This transformed objective function is to be maximized subject to the constraints above, with (c_1, v) as non-negative control variables, and with (k, R, z) as state variables in the pre-disaster regime. Here z is a "public" bad that affects the probability distribution for hitting a threshold with an associated productivity shock of unknown size. The integrand can be regarded as the planner's expected utility during a small interval of time $[t, t + dt]$. The first part is the expected utility of consumption at t, as long as no disaster has occurred. The second term is the expected utility if the threshold should be hit during $[t, t + dt]$, which takes place, as

seen from $t = 0$, with probability $F'(z(t)) \cdot D(v(t))dt$, with a benefit given by the expected continuation payoff. (We have marginal damage being equal to zero both before and after the adverse event, with the constant post-event damage being included in the $W-$function.)

The consumption flows are the immediate contribution to welfare. However, the higher these consumption flows are, the smaller are the future stocks of both types of capital, but with accumulated stock of waste being higher. For any given future value of the shift parameter, a, the more constrained will future consumption opportunities be the more is being consumed early. Also, because current waste accumulation will increase, the likely occurrence of a disaster is accelerated. However there is a "first-order" way out, depending on what expectations the planner should hold about the severity of hitting a threshold: postpone or delay consumption of the resource-intensive commodity while stimulating capital accumulation so as to build a buffer for future consumption as long as no threshold has been hit. In that case the likelihood of a disaster is reduced and a catastrophe is pushed farther into the future, and at the same time, if a (severe) shock should occur, there is a sufficiently high capital stock that can be exploited by future generations for consumption.

Rather than committing to a single consumption path, as would be the case with no intertemporal risk, it is well known that the optimal solution in the present context has the character of a strategy that balances the current marginal benefit at any instant of time, conditional on not yet having entered the continuation phase, with the future expected marginal cost, taking into account that the threshold can be hit in the "very near future". To be more precise; if one contemplates a unit increase in the consumption of the resource-intensive commodity at some point in time when no threshold so far has been hit, the current marginal benefit has, first of all, to balance the marginal loss caused by a lower resource stock in the future. With no risk at all, this would be the sole counterbalancing element to the immediate marginal benefit. However, within the present context, we have additional cost elements associated with a higher consumption of the resource-intensive good. The rate of growth of

accumulated emissions will increase, causing a higher value of z, with the consequence of increased likelihood for a catastrophe to occur. If the threshold should be crossed "in the very near future", the associated outcome is the present value of the expected future loss in welfare from switching regime. The increased likelihood of hitting the threshold will also affect other decisions. The decision to consume the capital-intensive commodity at some point in time prior to a disaster is related to the expected future value of the stock of capital equipment. A higher expected shadow value of capital for the continuation regime should induce more capital accumulation today, in the same way as a higher expected shadow value of the natural resource for the continuation regime should lower current extraction.

Hitting a threshold may be prevented by consuming less or slow down the extraction of the exhaustible resource. When and if an adverse event occurs, the results from the preceding section specifies what to do from the start of the continuation regime.[8] As long as no threshold has been hit we need a strategy for what to do, taking into account the current state and the conditional expected continuation payoff.

In order to derive this strategy introduce (P, Q, m) as a triple of current value costate variables associated with the state equations for (k, R, z), with an associated current value Hamiltonian, as given by:

$$H(c_1, v, k, R, z, t) = (1 - F(z)) \cdot [u(c_1) + w(v)] + F'(z) \cdot D(v) \cdot W(k, R)$$
$$+ P \cdot (f(k) - c_1 - \delta k) - Q \cdot v - m \cdot D(v)$$

Here P is the current shadow value of capital, Q is the current shadow value of the remaining resource, whereas m is the shadow *cost* of accumulated waste; all three conditional on the non-occurrence of a productivity shock.

An optimal solution, marked by a star, given that no disaster has yet occurred, with strictly positive consumption flows at any point in time, has to obey the following conditions that are derived directly from the Pontryagin's maximum principle:

[8] Similar problems have been analyzed in the R&D-literature – see e.g. Kamien & Schwartz (1977, 1978), Dasgupta (1982b), Dasgupta & Stiglitz (1981), Davison (1978), ando Reed and Heras (1992).

$(7-i)$ $\quad (1-F(z^*(t))\cdot u'(c_1^*(t))-P(t)=0 \Leftrightarrow u'(c_1^*(t))=\dfrac{P(t)}{1-F(z^*(t))}:=\pi(t)$

$(7-ii)$ $\quad r-\dfrac{\dot\pi(t)}{\pi(t)}=f'(k^*(t))-\delta+\lambda(z^*(t))\cdot D(v^*(t))[\dfrac{\overline{p}(t^+)}{\pi(t)}-1]$

$(7-iii)$ $\quad (1-F(z^*(t))\cdot w'(v^*(t))+F'(z^*(t))\cdot D'(v^*(t))\cdot W^+ - Q(t)-m(t)\cdot D'(v^*(t))=0$

$(7-iv)$ $\quad r-\dfrac{\dot Q(t)}{Q(t)}=F'(z^*(t))\cdot D(v^*(t))\cdot\dfrac{\overline{q}(t^+)}{Q(t)}$

$(7-v)$ $\quad r-\dfrac{\dot m(t)}{m(t)}=\dfrac{F'(z^*(t))}{m(t)}\cdot\Big[u(c_1^*(t))+w(c_2^*(t))\Big]-\dfrac{F''(z^*(t))\cdot D(v^*(t))\cdot W^+}{m(t)}$

(In (7-ii) we have introduced the "spot" price $\pi(t)$; see below, and also the hazard rate in the domain of z, as $\lambda(z)dz:=\dfrac{F'(z)dz}{1-F(z)}$. Direct use of the maximum principle requires that $r-\dfrac{\dot P(t)}{P(t)}=r-\dfrac{\dot\pi(t)}{\pi(t)}+\lambda(z(t))D(v(t))=f'(k(t))-\delta+F'(z(t))D(v(t))\dfrac{\overline{p}(t^+)}{P(t)}$, which is rewritten as (7-ii).[9] We have also used the previously defined continuation payoff, for short denoted W^+, and the expected shadow values of real capital and resource stock, respectively, at the beginning of the continuation regime, denoted $\overline{p}(t^+)$ and $\overline{q}(t^+)$, while also taking into account that at some t, given that no shock has yet occurred, there is some probability for a shock to occur "just after t".) We now will use these conditions to make more precise the true nature of an optimal strategy.

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[9] We have two ways of expressing the notion "the social rate of discount". One is the rate of discount as seen from t = 0, expressing the required "ex ante" rate of return from saving at date t, taking into account the probability that no shock has occurred by t. This rate of discount is given by $r-\dfrac{\dot P(t)}{P(t)}$.

The concept used in (7-ii) is the conditional rate of discount, expressing the required rate of return, valued at t = 0, from saving at t, conditional on the non-occurrence of a shock by t.

3-iii Results

The conditions above can be interpreted in different ways: One way is to interpret $e^{-rt}P(t)$ in (7-i) as the price paid at $t = 0$ for delivery of a unit of c_1 at t, if no threshold has been crossed by t. An alternative interpretation is to consider what to pay for each commodity at some point in time t as long as no threshold has been hit prior to that date. For this purpose we have introduced $\pi(t) := \dfrac{P(t)}{1 - F(z(t))}$ as the conditional "spot" price per unit of c_1 delivered at t, given that no threshold has been hit prior to t. Then the condition $u'(c_1^*(t)) = \pi(t)$ is balancing the immediate utility gain from a marginal increase in consumption of the capital-intensive commodity and the expected cost due to lower capital accumulation at t, conditional on not having entered the continuation regime by t. Along the same line we can define the resource spot price $\mu(t) := \dfrac{Q(t)}{1 - F(z(t))}$, or the conditional shadow value of the remaining resource at t, given that no threshold has been hit prior to this point in time.

Let the capital-intensive good be our numéraire. As noted in footnote 11, we can define a social rate of discount (or the required rate of return from not consuming the capital-intensive commodity) as $r - \dfrac{\dot{\pi}(t)}{\pi(t)} = r + \hat{\omega}(c_1^*(t)) \dfrac{\dot{c}_1^*(t)}{c_1^*(t)}$, which for an intertemporal optimum must be equal to the rate of return from capital investment at t, conditional on no shock prior to t, as given by the RHS of *(7-ii)*. We observe that the conditional expected rate of return from capital accumulation at *t*, which is set equal to the social rate of discount, consists of two terms: $f'(k^*(t)) - \delta$ and $\lambda(z^*(t)) \cdot D(v^*(t))[\dfrac{\overline{p}(t^+)}{\pi(t)} - 1]$. The first term is the standard net return at t from a unit investment given that no threshold has been crossed prior to t. The second term can be considered as "an expected buffer gain", should the threshold be crossed "just after" *t*. For an optimal strategy, the expected marginal return on investment should be equal the required rate of return on saving. On combining *(7-i)* and *(7-ii)*, we get a modified version of the Ramsey Rule:

$$(8) \qquad \hat{\omega}(c_1^*(t))\frac{\dot{c}_1^*(t)}{c_1^*(t)} = f'(k^*(t)) - \delta - r + \lambda(z^*(t)) \cdot D(v^*(t)) \cdot \left[\frac{\overline{p}(t^+)}{\pi(t)} - 1\right]$$

To see how consumption and investment will be affected by a randomly located threshold, let us assume that the intertemporal elasticity of substitution $\hat{\omega}(c)$ is almost constant (and above one so as to get some consumption smoothening). As seen from any point in time as long as no threshold yet has been crossed, the expected shadow value of capital at the beginning of any continuation period, $\overline{p}(t^+)$, will exceed $\pi(t)$, and approach π as a lower bound, when $A = 1$ with full certainty in the continuation regime. Our reason for claiming that $\overline{p}(t^+) \geq \pi(t)$ is the following: Absent any long-term planning, myopic price-taking producers will maximize present discounted cash-flows from investing in capital equipment at some t, with "rational expectations" about the spot price π and the immediate value appreciation, by acting according to the standard "myopic" neo-classical rule $f'(k(t)) = r + \delta - \frac{\dot{\pi}(t)}{\pi(t)}$.[10]

Our optimality condition differs from this unregulated market condition. Let us therefore impose a capital subsidy σ as given by:

$$(9) \qquad \sigma(t) := -\lambda(z^*(t))D(v^*(t))[\frac{\overline{p}(t^+)}{\pi(t)} - 1] \leq 0$$

If being confronted with this capital subsidy, the producers are motivated to internalize the long-run beneficial impact of their current capital accumulation on expected welfare. Their investment choice will in that case be made compatible with the modified Ramsey Rule in (8). Hence, implicit in our planning solution there is a capital subsidy. Due to the expected buffer effect from current capital accumulation, capital equipment will have a higher marginal scarcity value at the beginning of any continuation regime than "just before", with the optimal growth rate of consumption being adjusted upwards the higher is the future shadow value of capital, as seen from

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[10] The unregulated price function will deviate from the "optimal price" derived from the full solution. However, the point to be stressed is that the decision rule in the regulated solution differs from the one in an unregulated market solution.

(8). In an unregulated market solution, no producer will have any incentive to take this long-run buffer effect from their current decisions into account. We have identified a positive externality which should be internalized through subsidizing capital accumulation at a rate $\sigma(t)$ as specified in (9). This optimal subsidy is positively correlated with the magnitude of the hazard rate or conditional probability density for a shock, and the rate of return of current investment on future increase in continuation payoff. We therefore have:

Proposition 1

Suppose that the planner is very optimistic in the sense that only a minor productivity shock is anticipated should a threshold be hit. In that case $\overline{p}(t^+) \approx \pi(t)$, for any t, prior to a shock. The planner will then pursue a policy that is close to the standard Ramsey Rule in (8) with no capital subsidy.

On the other hand we have:

Proposition 2

Suppose that the planner is very pessimistic, anticipating a significant productivity shock if a threshold is hit. These beliefs are then translated into $\overline{p}(t^+) > \pi(t)$, as future capital is expected to provide consumption opportunities as though we are extracting a non-renewable resource. Then the expected scarcity value of capital equipment at the outset of the continuation regime will be higher than the current shadow price "just before" a disaster. Current consumption will be lowered and capital accumulation higher, so as to produce a large capital buffer for the continuation regime. In this case a "high" subsidy $\sigma(t)$ will stimulate capital accumulation in the desired direction.

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A planner with pessimistic beliefs must find a way of how to finance the capital subsidy. One might imagine some lump-sum taxation. However, within the present context there is implicit a tax instrument that might help financing the subsidy of

building up a capital buffer. This tax is a Pigouvian tax that is optimally set so as to internalize the expectecd damages from switching regime caused by accumulated emissions from resource extraction. This tax can be determined by looking at the Hotelling Rule of the problem, when remembering that we have a randomly located threshold with a likelihood of hitting that depends on accumulated emissions generated from the resource-extracting industry. This issue is therefore directly related to "the beast of our story"; the rate of extraction of the resource used as input in the production of a resource-intensive commodity. The relevant cost-benefit calculation that has to be made at some point in time t, given that no threshold has been crossed by then, is found from (*7-iii*) and can be expressed as

$$(10) \quad w'(v^*(t)) = \mu(t) + \frac{D'(v^*(t))}{1 - F(z^*(t))}\Big[m(t) - F'(z^*(t)) \cdot W^+\Big]$$

The current marginal utility of the resource-intensive good at some point in time as long as no threshold has been hit, has to be balanced against the overall conditional marginal cost (in utility terms), consisting of the conditional shadow value of the remaining resource stock ($\mu(t) := \dfrac{Q(t)}{1 - F(z^*(t))}$), and the conditional net marginal cost of hitting the threshold in the near future, as given by the second term on the RHS of (10). This term shows the increased likelihood of hitting the threshold due to a marginal increase in extraction, multiplied by the expected net cost of hitting the threshold. (Note that $m(t)$ is the current shadow cost of accumulated emissions at t. This cost is adjusted by the expected welfare should the threshold be hit.)

To see how resource extraction is affected by a stochastic threshold with a probability distribution that is influenced by a stock pollutant caused by resource extraction, we can consider the conditional shadow price of the remaining resource stock ("the producers' spot price") $\mu(t)$, along with the "risk" term in (10).

First, from (*7-iv*) we can derive

$$(11) \quad \frac{\dot{\mu}(t)}{\mu(t)} = r + \lambda(z^*(t))D(v^*(t))\left[\frac{\mu(t) - \overline{q}(t^+)}{\mu(t)}\right]$$

From before we have that $\overline{q}(t^+)$ is the expected shadow value of the remaining resource at the start of a continuation regime, as long as no threshold has been hit prior to t. At any t in the pre-disaster regime we must have $\mu(t) \geq \overline{q}(t^+)$, with strict inequality if a productivity shock is anticipated. The reason is that once the economy hits the threshold, with a realized value of the shock less than one, the true marginal productivity of the resource as input in the resource-intensive sector will jump below the current productivity; the expected shadow value must be less than the current one as long as $EA < 1$; hence $\overline{q}(t^+) < \mu(t)$. Prior to any disaster the conditional expected shadow value of the resource will increase at a rate above the utility discount rate, with a corresponding "low" initial value $\mu(0)$. The return from leaving the resource unextracted is increased above the utility discount rate. Because the global planner would like to postpone resource extraction which is the only measure, within the model, to reduce the probability of entering a continuation regime, the incentive for resource owners to delay extraction must be provided. We therefore have:

Proposition 3

If the planner holds very optimistic beliefs, anticipating an insignificant (or no) productivity shock when hitting a threshold, the preferred policy is derived from having $\mu(t) \approx \overline{q}(t^+)$, with a depletion profile obeying the standard Hotelling Rule. On the other hand, if the beliefs are highly pessimistic, anticipating a severe productivity shock, then the expected shadow value of the resource at the outset of a continuation period is below the spot price; i.e., $\overline{q}(t^+) < \mu(t)$. Given these beliefs the spot price will increase at a rate above the utility discount rate as long as $\lambda(z)D(v)$ is positive, so as to provide incentives for delaying extraction. The initial producers' spot price $\mu(0)$ is then lowered as compared to the scenario with optimistic beliefs so as to avoid the possibility of depletion too early.

<p align="center">***</p>

Our condition (10) can be then regarded as a decision rule for consuming the resource-intensive good; hence this condition will determine the consumers'

conditional spot price (measured in units of utility) of this good. (We could have divided through by $\pi(t)$ to get the spot price in units of the numeraire.) To get an explicit expression for the spot price (in utils) we now use our assumption that there are no conditions on $\lim_{t \to \infty} z(t)$, with an associated transversality condition that can be expressed as $\lim_{t \to \infty} e^{-rt} m(t) = 0$. (Remember that the stock pollutant has no welfare impact once the economy enters a continuation regime.) As the the continuation payoff, W^+, is independent of when we enter the new regime, we can solve the differential equation in (7-v) to get a closed form solution for the current shadow cost of accumulated emissions $m(t)$, as given by:

$$(12) \quad m(t) = F'(z^*(t)) \cdot W^+ + \int_t^\infty e^{-r(s-t)} F'(z^*(s)) \Big[u(c_1^*(s)) + w(v^*(s)) - rW^+ \Big] ds$$

Using (12) along with (10), we can rewrite the optimality condition for the extraction and consumption of the resource-intensive good ("the pricing condition") to become:

$$(13) \quad w'(v^*(t)) = \mu(t) + \frac{D'(v^*(t))}{1 - F(z^*(t))} \int_t^\infty e^{-r(s-t)} F'(z^*(s)) \Big[u(c_1^*(s)) + w(v^*(s)) - rW^+ \Big] ds$$

At any point in time t, as long as no threshold has been hit, current marginal utility from consuming the resource-intensive commodity – defining the consumers' spot price in utils – is made up of two terms: The first one is the standard marginal cost (here measured in utils) from using the resource today, as given by the producers' shadow price of the remaining resource stock, given no shock by then. The second term on the RHS of (13) can be interpreted as the expected net marginal environmental utility cost due to higher extraction at t, conditional on no threshold yet being hit. A unit increase in the consumption of the resource-intensive commodity before any shock will require an equivalent increase in resource extraction. A higher extraction rate will add to accumulated emissions, whose growth path is shifted upwards by $D'(v)$ per unit increase in v. As seen from some time t, still outside the continuation regime, the present value of expected loss in welfare is then the integral term, with the continuation payoff transformed to a flow, when density itself follows

endogenously from accumulated emissions. (At some point in time t, the conditional expected present value of future welfare losses from a regime shift, is given by

$\int\limits_{t}^{\infty} e^{-r(s-t)} \dfrac{F'(z^*(s))}{1-F(z^*(t))} \Big[u(c_1^*(s)) + w(v^*(s)) - rW^+\Big] ds$, where the term within square

brackets is the flow of current welfare loss should the new regime occur during a short interval of length ds. As time goes by, the probability beliefs are revised because the planner learns that no threshold yet has been crossed, as z increases. Once the economy moves into the continuation regime, the continuation payoff is reaped and the shadow cost of accumulated emissions (m) drops (permanently) to zero. The expected marginal benefit that is reaped when entering the continuation regime will therefore modify the overall marginal utility cost of higher emissions. In a regulated market regime, this net marginal environmental cost should appear as a Pigouvian tax or as we have called it, a precautionary tax, which ideally should capture the expected welfare cost of a productivity shock, caused by a higher stock of accumulated waste from a marginal increase in resource extraction at some point in time, as long as no disaster has yet occurred. A tax on current consumption of the resource-intensive commodity should therefore reflect future expected net damage or welfare loss caused by hitting the randomly located threshold. This tax, which is state- and time-dependent, will slow down the pace of extraction in the early phase of the planning period, prior to a disaster, and hence encourage resource saving. The stock of accumulated waste will then increase not too fast, and a catastrophe is (hopefully) postponed.

A reasonable strategy for delaying a catastrophe is therefore accomplished by changing the price structure facing final users through taxing the consumption of the resource-intensive commodity while at the same time stimulate capital accumulation through a capital subsidy. Imposing a precautionary tax will reduce the resource price facing producers and at the same time increase the price facing final users, relative to an unregulated situation. We can summarize this in the following proposition:

Proposition 4

Suppose that the real capital market is properly regulated with the optimal capital subsidy as specified in (9). If the global government then imposes a state– and time dependent tax on current consumption of the resource–intensive commodity, with a marginal tax rate at some pre-disaster date t as given by

$$\frac{D'(v^*(t))}{1-F(z^*(t))}\int\limits_{t}^{\infty} e^{-r(s-t)}F'(z^*(s))\Big[u(c_1^*(s)) + w(v^*(s)) - rW\Big]ds\,,\text{ the optimal solution is}$$

implemented. The resources are depleted more slowly and accumulation of capital is higher than in the unregulated situation prior to a disaster.

We also have the following corollary relating the tax structure to the magnitude of the continuation payoff itself, in the extreme case asociated with "Doomsday":

Corollary

Suppose the planner has "Doomsday-beliefs", anticipating a complete destruction of production capacity should a threshold be hit, with a continuation payoff as given by $W^+ = 0$. The associated optimal policy should then induce high capital accumulation in an early phase (Proposition 2, with $\pi(t) << \overline{p}(t^+)$), low resource extraction (Proposition 3 with $\mu(t) >> \overline{q}(t^+)$), and a high marginal precautionary tax rate (Proposition 4 with $W^+ = 0$) so as to reduce the growth in accumulated emissions. Anticipating doomsday is then turned away from being self-fulfilling.

In the opposite scenario, anticipating only a minor shock, with an expected value of the shock being close to one, the optimal solution will obey the standard Ramsey-and Hotelling Rules in an early phase. The drawback or cost to such beliefs is that the economy might hit the threshold "too early" and the economy will be negatively surprised should the realized shock be far below one.

4. **Some conclusions and final remarks**

The purpose of this paper has been twofold: First we wanted to derive an optimal strategy for global capital accumulation and global resource extraction for a global economy that is facing a future random disaster – modeled as a persistent productivity shock – caused by a stock pollutant which affects the future probability distribution for hitting a threshold. Secondly, we derived the associated tax structure so that an underlying market outcome can implement the optimal solution. The problem has been put into a standard context of optimal saving in a neo-classical multi-commodity growth model, with a standard Ramsey-model, supplemented by extraction of an exhaustible resource and the accumulation of emissions affecting the likelihood of switching regime. The unregulated market outcome will normally undertake too little capital accumulation and extract too much of fossil fuel prior to a productivity shock, because the market participants will normally not be motivated to take into account the long-term consequences of their current actions. To prevent the occurrence of a productivity shock, triggered by accumulated emissions hitting a threshold, we have imposed a corrective tax, coined a precautionary tax on current consumption of a resource-intensive good, not to correct for a current externality, but to internalize future expected welfare losses caused by the stochastic productivity shock. This tax rate is higher the more the truncated probability distribution for a disaster is affected by a marginal increase in the rate of extraction and the higher is the expected welfare loss should the economy move into a continuation regime. The optimal strategy prior to any continuation regime should also stimulate capital accumulation, so as to build up a buffer against future income loss in a continuation regime. To implement this "building-up-a-buffer"-target, a capital subsidy is imposed. This subsidy is higher the more severe is the anticipated shock, and the more likely it is to hit the threshold in the near future. For some given anticipation, the tax instruments will delay a shock and, in addition, if the shock should occur, make the economy better prepared or less vulnerable in a new regime.

One important aspect of the preceding discussion has been the assumption that capital is reversible. What would be the outcome if capital is irreversible in the sense

that gross investment cannot be negative and capital cannot be turned into consumption goods?[11] First, in the irreversibility case there is no longer a strong incentive for building a buffer stock that can be turned into consumption in the continuation regime, should the shock be severe. In that case we conjecture that capital accumulation will be slower in a phase prior to a disaster, as compared to the reversibility case. On the other hand, with pessimistic beliefs about the future shock, more capital will be required for the continuation regime so as to support future production of the capital-intensive commodity. Hence, with capital being irreversible there seem to be countervailing forces when it comes to pre-disaster capital accumulation. But one has to take into account when considering capital accumulation in the contingency phase, that if hitting a threshold with a shock more severe than anticipated – say with a close to zero – then one might find oneself with too much capital at the beginning of the continuation regime, as the benefits from previous savings then cannot be reaped.

Another critical assumption of the preceding model is the existence of only one threshold. If all these thresholds can be bunched together in time, as suggested by Barrett (op.cit.), then our approach seems to have some merit. (The same approach is taken by van der Ploeg and de Zeeuw (op.cit.). Their paper bears some resemblance to our paper, but should be considered as complementary to ours.) On the other hand if there is a sequence of possible thresholds, located at various points along the time line, while also depending on each other, then our modeling framework is of course too simple. We hope to come back to both the irreversibility case and a sequence of thresholds in future works. But so far, the present paper has hopefully provided some additional insight into how to design optimal policy measures when accumulated emissions are affecting future welfare through a random event of hitting a threshold that will cause some permanent productivity shock.

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[11] See Arrow and Kurz (1970) for a formal discussion in a standard neoclassical growth model..

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