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Intergenerational Egalitarianism

Paolo Giovanni Piacquadio[1]

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Abstract

I study the egalitarian way of distributing resources across generations. Distributional equity deeply conflicts with the Pareto principle: efficient allocations cannot guarantee that *i)* each generation be assigned a consumption bundle that is at least as large as an arbitrarily small fraction of the bundle assigned to any other generation and that *ii)* each generation finds its assigned bundle at least as desirable as an arbitrarily small fraction of the bundle assigned to any other generation with same preferences. Overcoming such tension unveils a new ethical dilemma for intergenerational equity: the *short-term/long-term inequality trade-off.* The egalitarian ethical observer can choose between: *i)* "weak equity" among all generations (at the cost of possibly large inequalities among proximate ones) and *ii)* "strong equity" among few successive generations (at the cost of possibly large inequalities among distant ones). The last alternative is specific to the dynamic framework and provides a rationale to treat generations differently based on their time order.

Keywords: Intergenerational justice, allocation rules, fairness.

JEL classification codes: D63, D71, Q56.

[1]Department of Economics, University of Oslo, Moltke Moes vei 31, 0851 Oslo, Norway. Email: p.g.piacquadio@econ.uio.no; Tel: +47 22 85 72 37. The author wishes to thank particularly G. Asheim, B. Harstad, F. Maniquet, W. Thomson, R. Boucekkine, and an anonymous referee. Helpful discussions with C. d'Aspremont, D. de la Croix, M. Fleurbaey, A. Gosseries, P. Hammond, L. Lauwers, V. Manjunath, H. Moulin, J. Roemer, Y. Sprumont, B. Tungodden, G. Valletta, R. Vohra, S. Zuber are gratefully acknowledged. This paper is part of the research activities at the Centre for the Study of Equality, Social Organization, and Performance (ESOP) at the Department of Economics at the University of Oslo. ESOP is supported by the Research Council of Norway through its Centres of Excellence funding scheme, project number 179552. This research is part of the ARC project 09/14-018 on "sustainability" (French speaking community of Belgium).

1 Introduction

This paper investigates the egalitarian way of distributing resources over time. The egalitarian alternative doesn't need to be selected for allocating resources, but it is a necessary reference point for evaluating inequality of allocations and inequality aversion of different theories of intergenerational justice.

I consider a dynamic model of production, consumption, and investment. In each period, production transforms available capital goods in output. Output can be partly allocated for the consumption of the currently living generation and, for the remaining part, invested as capital goods for use in the following period. The egalitarian distribution of resources is identified by an (allocation) **rule**, i.e. a correspondence that selects a subset of feasible allocations for each intergenerational distribution problem.[1]

A new impossibility result for intergenerational justice arises. Let fairness be interpreted by the following two requirements: *no-domination* requires that no generation is given less consumption than any other generation; *equal treatment of equals* requires that no generation finds its consumption less desirable than that assigned to any other generation with same preferences. These equity conditions are together not compatible with Pareto efficiency, even in a finite time horizon.[2] More strikingly, even if we were to accept considerably weaker version of such axioms, in fact infinitely weaker, the impossibility result remains.[3]

The main result is to show that overcoming such tension is possible and determines a new ethical trade-off. The egalitarian planner has to make a choice: on the one hand, some allocations satisfy strong equity conditions among proximate generations, but allow for large inequalities among distant generations (*long-term inequality*); on the other hand, some allocations satisfy sufficiently weak equity conditions among all generations, but allow for some inequalities among proximate generations (*short-term inequality*). I name this ethical dilemma the *long-term/short-term inequality trade-off*.

[1] Differently from the majority of contributions on intergenerational equity, this approach belongs to the literature on fair allocation theory. In this setting, the social choice is described by a rule and the appeal of a rule is judged by the social relevance of the axioms it satisfies. For a survey on fair allocation theory, see Thomson (2011). I discuss how the present contribution relates to the literature in the next subsection.

[2] Well-known difficulties characterize the infinite horizon framework (see the seminal contribution of Diamond, 1965, and the recent review by Asheim, 2010). To distinguish the sources of tension between equity and efficiency, I first analyze the finite time horizon, and then extend the framework to the infinite time horizon.

[3] This clash is similar to Pazner and Schmeidler (1974); similarities and differences are discussed in the next subsection.

Along the lines of such ethical choice, two families of rules arise: the "time independent rules" and the "sequential rules". Time independent rules are rules that treat each generation independently of the time they live in. Belonging to this family, an adapted version of the "budget constrained Pareto optimal" method, introduced by Moulin (1991), and the "egalitarian equivalent" solution, by Pazner and Schmeidler (1978). The first rule guarantees that no generation is given less than any other generation (*no-domination*), but cannot guarantee that each generation finds its consumption bundle at least as desirable as an (arbitrarily small) fraction of what is assigned to any other generation with the same preferences. The second rule guarantees that generations with same preferences are treated alike (*equal treatment of equals*), but cannot guarantee that each generation be given more than an (arbitrarily small) fraction of what is given to any other generation.

Sequential rules select allocations that satisfy both fairness requirement, i.e. *no-domination* and *equal treatment of equals*, among *pairs* of successive generations. Equitable distribution of resources among proximate generations comes, however, at the cost of long-term inequalities: equity cannot be guaranteed among more distant generations. Although long-term inequalities might be inevitable, the interest of present and far-future generations is not to be sacrificed and all generations can be treated with equal concern.

1.1 The basic difficulty

Let the distributional equity requirement be **no-envy**: an allocation satisfies *no-envy* if no member of the society would be better-off with the bundle assigned to someone else (see Foley, 1967; Kolm, 1972; Varian, 1974). This axiom is much stronger than those discussed in the paper, but proves particularly interesting for two reasons. First, it is ethically appealing and compatible with Pareto efficiency in static resource distribution problems. Second, the tension between *no-envy* and efficiency highlights the mechanisms responsible for the main impossibility result. I illustrate the issue with an example of economy in which no Pareto efficient allocation satisfies *no-envy*.

There are two agents, A and B, living in two different locations, L_A and L_B.[4] The preferences of agent A over goods 1 and 2 are represented by the function $u_A(x_A) = x_A^1 + \frac{3}{2}x_A^2$; the preferences of agent B are represented by

[4] Locations can be indifferently interpreted over space or time.

$u_B(x_B) = \frac{3}{2}x_B^1 + x_B^2$.[5] All goods to distribute, say $\Omega \equiv (2,1) \in \mathbb{R}^2_{++}$, are at location L_A: these goods can be assigned to agent A, but need to be transported to location L_B before being assigned to agent B. Transportation is assumed to involve a (linear) transformation of goods.[6] Let $\rho \equiv \left(\frac{1}{2}, 2\right) \in \mathbb{R}^2_{++}$ be the coefficients of such transformation: then, an allocation is feasible if $x_A^1 + 2x_B^1 \leq 2$ and $x_A^2 + \frac{1}{2}x_B^2 \leq 1$.[7] The set of Pareto efficient allocations is such that all resources are distributed and that if $x_A^1 < 2$ then $x_A^2 = 0$ (and correspondingly if $x_B^2 < 2$ then $x_B^1 = 0$). Consider the efficient allocation $\bar{x}_A \equiv (2,0)$, $\bar{x}_B \equiv (0,2)$. Since $u_A(\bar{x}_A) = 2 < 3 = u_A(\bar{x}_B)$ and $u_B(\bar{x}_B) = 2 < 3 = u_B(\bar{x}_A)$, *no-envy* fails to hold: agent A would be better-off with x_B, while B would be better-off with x_A. Moreover, all other efficient allocations are such that either *i)* $x_A^1 < 2$ (and thus $x_B^1 > 0$) and consequently $u_A(x_A) < u_A(\bar{x}_A) < u_A(\bar{x}_B) < u_A(x_B)$, or *ii)* $x_A^2 > 0$ (and thus $x_B^2 < 2$) and thus $u_B(x_B) < u_B(\bar{x}_B) < u_B(\bar{x}_A) < u_B(x_A)$. In both cases *no-envy* is not satisfied.

The intuition of the result goes as follows. At efficient allocations, the relative scarcity of goods differs across location and defines the social cost of each agent's consumption bundle. Thus, when an agent prefers most the good that is for him relatively scarce, it might be too expensive (and thus not feasible) to assign to each his favorite bundle.[8]

This tension is also similar to the one highlighted by Pazner and Schmeidler

[5]Throughout the paper, preferences are allowed to differ among generations. Such generality provides a more flexible and, arguably, more realistic framework to study intergenerational justice: in the philosophical literature, Barry (1999) suggests that we should take into account that different generations have different views of what constitutes a good life. First, since each generation represents many different agents (with different preferences), a change in generation's composition over time will modify it's aggregate preferences. Second, indifference curves change among generations as a result of an evolutionary process (see Alger and Weibull, 2013) or might represent varying tastes, when commodity's social attributes change how fixed underlying preferences manifest (see Karni and Schmeidler, 1990). Third, preferences can vary according to the different motivational states – determined by motivationally salient properties like varying technology and environment – through which alternatives are ranked (see Dietrich and List, 2013). Forth, commodities might be interpreted as Lancaster's characteristics (intrinsic properties of goods) for which different preferences represent varying consumption technology (Lancaster, 1966).

[6]Such transformation can be interpreted as a transportation (iceberg) cost, when the transfer is over space, or returns on savings/investments, when the transfer is over time.

[7]It is important that resources can be continuously transferred across agents: goods can be continuously taken away from the "richer" agent and a corresponding amount (transformed by ρ) can be assigned to the poorer one, independently of how the rich/poor relation is defined. A problematic case, ruled out by the model, holds when in location B there are resources $\Omega_B > \Omega$ that cannot be transferred to A; as agent A can never be assigned as much as agent B at any efficient allocation, egalitarianism is a too strong requirement.

[8]Importantly, this difficulty holds for any ρ such that $\frac{\rho^1}{\rho^2} \neq 1$: thus, the transformation of the left-over might describe a transfer cost ($\rho \leq \mathbf{1}$), a productive return ($\rho \geq \mathbf{1}$), or a combination of them.

(1974).[9] A crucial differenceis that generations do not differ in terms of working skills; however, since generations live in different periods and resource scarcity naturally varies over time (even with a constant technology), each generation faces a different opportunity cost (as with heterogeneous skills). Thus, time does not only identify each generation, but it is also a crucial non-transferable characteristic: as a consequence, it might be considered ethically relevant for defining intergenerational egalitarianism, as the dilemma highlights.

1.2 Related literature

The axiomatic literature on intergenerational equity has its roots in the seminal contributions of Koopmans (1960) and Diamond (1965). Diamond, in particular, establishes a key negative result: there is no continuous ordering that is Pareto efficient and treats all generations equally. The egalitarian concern is interpreted as "finite anonymity;" it requires the ranking to be invariant to permutations of utilities of a finite number of generations.[10] The underlying assumption of the utility streams literature is that generations are treated equally when they achieve the same index of well-being (which requires comparability of welfare across generations).

The present work is complementary to the utility streams literature as it addresses the meaning of equality in terms of distribution of resources. Proving how difficult it is to assign consumption bundles such that generations are treated equally, the paper emphasizes that equality of well-beings (independently of how these are measured) cannot correspond to a very egalitarian distribution of resources. Second, if equal well-being should be attributed to generations when their consumptions are equitably assigned, the long-term/short-term inequality trade-off imposes a crucial constraint on how to construct such well-being indexes.

In a one-commodity framework, it is straightforward to identify inequalities by how much each generation is assigned.[11] In such a setting, Epstein (1986) axiomatizes utilitarianism and egalitarianism and establishes a dilemma between development and equality; Asheim (1991) defines a quasi-ordering of unjust allocations and shows the importance of restricting the analysis to specific classes

[9] As in Pazner and Schmeidler (1974), the difficulty is not due to externalities or non-convexities, which are ruled out in the specification of the model.

[10] Further advances of the utility streams literature are surveyed in Asheim (2010).

[11] The same natural comparability applies when preferences are constant over time: inequalities are identified by differences in achieved well-beings. This is the case, among many others, of the seminal contribution of Koopmans, 1960).

of economies. More recently, Asheim et al. (2010) characterize choice functions (in an approach similar to the one presented here) that select efficient paths with non-decreasing consumptions based on procedural and consequentialist equity principles.

In overlapping generations settings, Fleurbaey (2007) singles out a welfare criterion that endogenously determines a comparable well-being measure. This is an ordering extension of a "time independent rule": the allocation corresponding to equal well-beings satisfies *efficiency* and *equal treatment of equals*, but not *no-domination*. However, when more goods and overlapping-generation agents are introduced, even *efficiency* and *equal treatment of equals* are not compatible (see Isaac and Piacquadio (2012)). The present work combines intertemporal redistribution of resources via production and investment with a multidimensional commodity framework with no comparable information about preferences. As in Asheim et al. (2010), the incompatibility between equity and efficiency in the infinite horizon framework is avoided by imposing specific domain restriction.

The remainder of the paper is organized as follows. In Section 2, I present the finite-time model of the economy. In Section 3, I discuss the tension between equity and efficiency. In Section 4, I present the ethical dilemma of the egalitarian ethical observer and show which egalitarian rules can be defined. In Section 5, I extend the model and the results to the infinite horizon framework. In Section 6, I conclude. Longer proofs are gathered in the appendix.

2 The model

The economy spans a finite number of periods, i.e. $T \equiv \{0, 1, ..., \bar{t}\}$ with finite $\bar{t} > 0$. Let $L \geq 2$ be a finite set of infinitely divisible and privately appropriable goods.[12]

For each $t \in T$, there is a vector $k_t \in \mathbb{R}_+^L$ of capital goods available in period t. These resources are used as input in production. For each $t \in T$, let $F_t : \mathbb{R}_+^L \to 2^{\mathbb{R}_+^L}$ be the production correspondence and let $y_t \in \mathbb{R}_+^L$ denote the output. Let $\mathcal{F}$ be the set of correspondences F_t satisfying the following restrictions:[13]

[12] The notation for vector inequalities is as follows: $x \geq y$ means that $x_i \geq y_i$ for all i; $x > y$ means that $x_i \geq y_i$ for all i and $x \neq y$; $x \gg y$ means that $x_i > y_i$ for all i. With a slight abuse of notation, the sets and their cardinality are indicated with the same capital letter.

[13] I do not assume that the technology to be a transformation of netputs, for which for some net amount of goods to be created (output larger than input), some other goods need to be destroyed (input larger than output). This would provide a realistic description of reality, but

For each $k_t \in \mathbb{R}^L_+$:

i) $F_t(k_t)$ is non-empty;

ii) $F_t(k_t)$ is compact; and

iii) $F_t(k_t)$ is convex: for each pair $y_t, y'_t \in F_t(k_t)$ and each $\alpha \in [0,1]$, $\alpha y_t + (1-\alpha_t) y'_t \in F_t(k_t)$.

Moreover:

iv) F_t is upper-hemicontinous: it has a closed graph and the images of compact sets are bounded;[14]

v) F_t is strictly monotonic: for each pair $k_t, k'_t \in \mathbb{R}^L_+$, if $k_t > k'_t$, then for each $y' \in F_t(k'_t)$ there exists $y \in F_t(k_t)$ such that $y > y'$;

vi) F_t is convex: for each pair $k_t, k'_t \in \mathbb{R}^L_+$, each $y_t \in F_t(k_t)$, each $y'_t \in F_t(k'_t)$, and each $\alpha \in [0,1]$, $\alpha y_t + (1-\alpha) y'_t \in F_t(\alpha k_t + (1-\alpha) k'_t)$;[15] and

vii) F_t satisfies no-free lunch: $\{0\} = F_t(0)$.

A profile of technologies is a list $(F_0, ..., F_{\bar{t}}) \in \mathcal{F}^T$.

For each $t \in T$, the output y_t can be used in two ways: part of it is used for consumption at t – let $x_t \in \mathbb{R}^L_+$ be the vector of goods used in this way – and the remaining part determines the capital goods to be used as input in production at $t+1$ – let $k_{t+1} \le y_t - x_t$ be this remainder.[16]

would also require L to include all existing commodities. The absence of such a restriction is more in line with the macroeconomic literature, where the net return on capital can be larger than one. The results of the finite horizon model do not depend on this restriction. In the infinite horizon version, a lower bound on productivity is necessary to combine equity and efficiency even in a one-dimensional framework (see Asheim et al., 2010). As an example, assume a unit of good has to be shared among infinite many agents – corresponding to the case of one commodity with unitary net return on capital: equality, requiring all agents to be assigned the same quantity, is not compatible with efficiency, requiring the good to be distributed to the agents.

[14]Formally, the correspondence has a closed graph if for each series $k^n_t \in \mathbb{R}^L_+$ converging to $k^*_t \in \mathbb{R}^L_+$, and each $y^n_t \in F_t(k^n_t)$, it holds that $y^*_t \in F_t(k^*_t)$. The image of compact sets are bounded if for each compact set $\mathcal{B} \in 2^{\mathbb{R}^L_+}$, the set $\{y_t \in \mathbb{R}^L_+ | y_t \in F_t(k_t) \text{ for some } k_t \in \mathcal{B}\}$ is bounded.

[15]Requirement *iii)* that $F_t(k_t)$ be convex is a restriction on the set of outputs that can be produced for each given amount of inputs. Requirement *vi)* is instead about the convexity of the production correspondence: a linear combination of outputs can be produced with the corresponding linear combination of inputs.

[16]For the sake of simplicity, resources, output, and consumption are defined on the same commodity space. The results extend to the case in which resources and consumption are defined in (possibly different) subsets of the output space.

Each generation lives for one period. For each $t \in T$, generation t has a preference relation R_t defined over $\mathbb{R}^L_+$. Let I_t and P_t be the symmetric and asymmetric relations induced by R_t. Let $\mathcal{R}$ be the set of preference relations R_t that are complete, transitive, continuous, strictly monotonic, and convex.[17] A profile of preferences is a list $(R_0, ..., R_{\bar{t}}) \in \mathcal{R}^T$.

Let $\underline{k} \in \mathbb{R}^L_+$ be the initial capital endowment. Let $\bar{k} \in \mathbb{R}^L_+$ be the capital that (imperatively) needs to be left over at the end of the time horizon. Requiring $\bar{k} = 0$ does not affect the results, but $\bar{k} \geq 0$ is needed for the later extension to infinite time horizon, when such capital is required to produce goods for later generations.[18]

An **economy** is a list $E \equiv \left(\underline{k}, \bar{k}, \{F_t\}_{t\in T}, \{R_t\}_{t\in T}\right) \in \mathbb{R}^{2L}_+ \times \mathcal{F}^T \times \mathcal{R}^T$. A (feasible) **allocation for *E*** is a list $a \equiv \left(\{k_t, y_t, x_t\}_{t\in T}\right) \in \mathbb{R}^{3LT}_+$ such that

$$\begin{cases} x_t + k_{t+1} \leq y_t & \forall t \in T, \\ y_t \in F_t(k_t) & \forall t \in T \\ k_0 = \underline{k} \\ k_{t+1} = \bar{k} \end{cases} \tag{2.1}$$

Let $\mathcal{E}$ be the domain of economies with at least one feasible allocation. For each $E \in \mathcal{E}$, let $A(E)$ be the set of **feasible allocations of *E***.

The assumptions on $\mathcal{E}$ guarantee that resources can be smoothly transferred across generations: *i)* each can be assigned any bundle of goods between zero and all resources; *ii)* reducing the bundle assigned to a generation allows giving (continuously) more to any other generation. This smoothness is fundamental for the analysis: it makes an equitable and efficient allocation an appealing benchmark for defining intergenerational egalitarianism.

Let $E \in \mathcal{E}$. An allocation $a \in A(E)$ is **(Pareto) efficient for *E*** if there is no $a' \in A(E)$ such that for each $t \in T$, $x_t \; R_t \; x'_t$, and for some $t \in T$, $x_t \; P_t \; x'_t$. Let $P(E)$ be the set of **efficient allocations of *E***.

An allocation rule, or simply a **rule**, is a mapping that associates to each

[17] The formal definition of these properties is as follows. Completeness requires that for each pair $x, x' \in \mathbb{R}^L_+$, either xR_tx' or $x'R_tx$ (or both). Transitivity requires that for each triple $x, x', x'' \in \mathbb{R}^L_+$, if xR_tx' and $x'R_tx''$, then xR_tx''. Continuity requires that for each $x \in \mathbb{R}^L_+$, the sets $\left\{x' \in \mathbb{R}^L_+ | x'R_tx\right\}$ and $\left\{x' \in \mathbb{R}^L_+ | xR_tx'\right\}$ are closed. Strict monotonicity requires that for each pair $x, x' \in \mathbb{R}^L_+$, if $x > x'$, then xP_tx'. Convexity requires that for each pair $x, x', x'' \in \mathbb{R}^L_+$, if xI_tx' and $x'' = \alpha x + (1-\alpha)x'$ for some $\alpha \in [0,1]$, then $x''R_tx$.

[18] As will become clear in Section 5, the analysis can be thought as a two-step approach: in the first step, I study how to distribute resources among a finite number of generations for each final resources $\bar{k}$; in the second step, I use the results obtained over the finite horizon to optimally determine $\bar{k}$ in an infinite horizon extension.

economy a non-empty subset of its set of feasible allocations. The generic notation for a rule is the letter ψ, i.e. $\psi : E \to 2^{A(E)}$.

3 The tension between equity and efficiency

In this section, I show how deep the tension between equity and *efficiency* is. I suggest equity be interpreted by two very weak axioms that compare the bundle of goods generations are assigned. The axioms are parametrized by a scalar $\varepsilon \in [0,1]$: the larger ε, the stronger the axiom.[19] Importantly, even in their strongest version (when $\varepsilon = 1$), these axioms remain much weaker than *no-envy*.

Let $\varepsilon \in [0,1]$. The first axiom requires that no generation be assigned less than a fraction ε of the bundle assigned to each other generation. When $\varepsilon = 1$, the axiom is equivalent to **no-domination** and requires no generation be assigned less of each good than another (this was introduced by Thomson, 1983, in a static framework). As ε decreases the requirement becomes weaker and weaker. When $\varepsilon = 0$, the axiom is vacuous.

> *Let $\varepsilon \in [0,1]$. For each $E \in \mathcal{E}$, $a = \left(\{k_t, y_t, x_t\}_{t\in T}\right) \in A(E)$ satisfies* ***ε-no-domination*** *if for each pair $t, t' \in T$, $x_t \not\ll \varepsilon x_{t'}$.*

Let $\varepsilon \in [0,1]$. The second axiom requires that no generation be assigned a bundle that it finds less desirable than a fraction ε of the bundle assigned to each other generation with the same preferences. When $\varepsilon = 1$, the axiom is equivalent to **equal treatment of equals** and requires generations with same preferences to be indifferent between their consumptions.[20] As ε decreases the requirement becomes weaker and weaker. When $\varepsilon = 0$, the axiom is vacuous.

> *Let $\varepsilon \in [0,1]$. For each $E \in \mathcal{E}$, $a = \left(\{k_t, y_t, x_t\}_{t\in T}\right) \in A(E)$ satisfies* ***ε-equal treatment of equals*** *if for each pair $t, t' \in T$ such that $R_t = R_{t'}$, $x_t \; R_t \; \varepsilon x_{t'}$.*

The next result shows that *efficiency* is not compatible with both $\varepsilon-$*no-domination* and $\varepsilon'-$*equal treatment of equals*, when $\varepsilon, \varepsilon' > 0$. To strengthen this impossibility result, I prove it on the restricted domain of economies with linear and

[19] The idea to parametrize equity axioms was introduced by Moulin and Thomson (1988) and used in the intergenerational framework by Isaac and Piacquadio (2012).

[20] *Equal treatment of equals*, in the dynamic setting, demands the time generations live in to be irrelevant, as soon as these are indistinguishable in terms of personal traits (i.e. preferences).

time-invariant technologies. Let B be a positive semi-definite diagonal matrix $L \times L$. Let $\bar{\mathcal{F}}$ be the set of linear technologies; that is, technologies $F \in \mathcal{F}$ such that for each $k \in \mathbb{R}_+^L$, $F(k) = \left\{ y \in \mathbb{R}_+^L \,|y \leq Fk \right\}$. Let $\bar{\boldsymbol{\mathcal{E}}} \subset \mathcal{E}$ be the domain of economies with linear and time-invariant technologies; that is, technologies such that for each $E \in \mathcal{E}$ and each pair $t, t' \in T$, $F_t = F_{t'} \in \bar{\mathcal{F}}$.

Theorem 1. *On the domain $\bar{\mathcal{E}}$ and for each pair $\varepsilon, \varepsilon' \in (0, 1]$, no rule satisfies Pareto efficiency, ε-no-domination, and ε'-equal treatment of equals.*

Proof. Let $\varepsilon, \varepsilon' > 0$. For each $E \in \bar{\mathcal{E}}$ and each $t \in T$, let generation t's locus of efficient allocations be defined as:

$$C_t \equiv \left\{ x_t \in \mathbb{R}_+^L \left| \exists a \equiv \left(\{k_t, y_t, x_t\}_{t \in T} \right) \in P(E) \right. \right\}.$$

Step 1. Let $E \in \bar{\mathcal{E}}$ be such that $t \in T = \{0, 1, 2, 3\}$, $L = \{c, w\}$,[21] and $\begin{pmatrix} y^c \\ y^w \end{pmatrix} = \begin{pmatrix} \rho^c & 0 \\ 0 & \rho^w \end{pmatrix} \begin{pmatrix} k^c \\ k^w \end{pmatrix}$. Let $\rho \equiv \frac{\rho^c}{\rho^w}$. Preferences are differentiable and strictly convex. Let $R_0 = R_2$ be represented by utility $u(x) = c^\alpha w^{1-\alpha}$ and let $R_1 = R_3$ be represented by utility $v(x) = c^\beta w^{1-\beta}$ with $\alpha, \beta \in (0, 1)$ and $\frac{\alpha}{1-\alpha} = \rho \frac{\beta}{1-\beta}$. Assume $A(E)$ contains allocations $a \equiv \left(\{k_t, y_t, x_t\}_{t \in T} \right) \in A(E)$ such that $x_t > 0$ for each $t \in T$.

Consider allocations that assign strictly positive consumption bundles to each generation. Then, C_t, restricted to positive consumptions, are (a portion of) linear functions described by $w = r_t c$ and are such that $r_0 = r_1 = \rho^2 r_2 = \rho^2 r_3$. These are represented in Figure 3.1 as dash-dotted lines.

Step 2. Let $a \equiv \left(\{k_t, y_t, x_t\}_{t \in T} \right) \in A(E)$ be such that:

i) $a \in P(E)$, i.e. $x_t \in C_t$ for each $t \in T$;

ii) $x_1 = \varepsilon x_0$, i.e. a (minimally) satisfies *ε-no-domination* between generations 0 and 1;

iii) $u(x_0) = u(\varepsilon' x_2)$ and $v(\varepsilon' x_1) = v(x_3)$, i.e. a (minimally) satisfies *ε'-equal treatment of equals* between generations 0 and 2 and between generations 1 and 3.

Step 3. By strict convexity of preferences, a is the only feasible allocation for E satisfying *i)-iii)*. By contradiction, assume $\bar{a} \equiv \left(\left\{ \bar{k}_t, \bar{y}_t, \bar{x}_t \right\}_{t \in T} \right) \in A(E)$ with $\bar{a} \neq a$ satisfies the same properties.

Case 1. If $\bar{x}_0 > x_0$, $\bar{a}$ is such that for each $t \in T$, $\bar{x}_t \geq x_t$: since $\bar{a} \neq a$, either $\bar{a} \notin A(E)$ or $a \notin P(E)$.

[21] I will denote by c and w also the quantities of these goods assigned to generations.

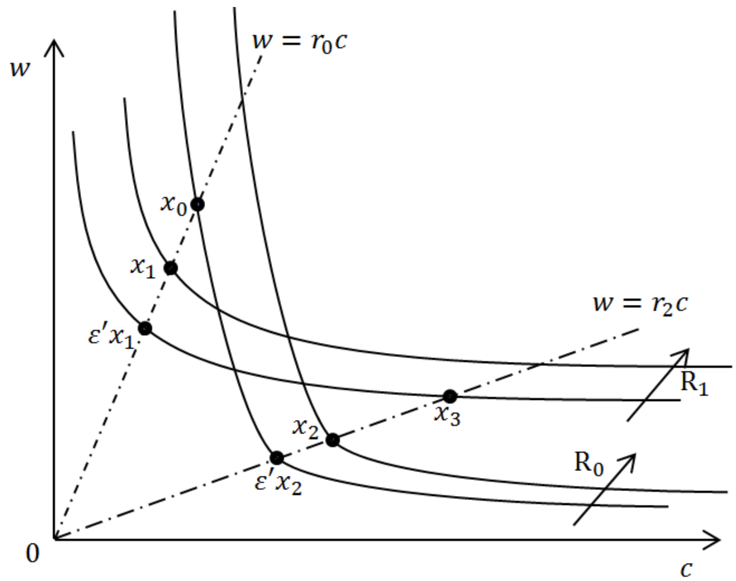

Figure 3.1: Clash between *Pareto efficiency*, ε-*no-domination*, and ε'-*equal treatment of equals*.

Case 2. If $\bar{x}_0 < x_0$, $\bar{a}$ is such that for each $t \in T$, $\bar{x}_t < x_t$: since $\bar{a} \neq a$, $\bar{a} \notin P(E)$.

Case 3. If $\bar{x}_0^c > x_0^c$ and $\bar{x}_0^w < x_0^w$, $\bar{a}$ is such that for each $t \in T$, $\bar{x}_t^c > x_t^c$ and $\bar{x}_t^w < x_t^w$; thus, $\bar{a} \notin P(E)$.

Case 4. If $\bar{x}_0^c < x_0^c$ and $\bar{x}_0^w > x_0^w$, $\bar{a}$ is such that for each $t \in T$, $\bar{x}_t^c < x_t^c$ and $\bar{x}_t^w > x_t^w$; thus, $\bar{a} \notin P(E)$.

Step 4. Let $x_0 \equiv (c_0, w_0) \in C_0$ be assigned to 0. *Efficiency* and $u(x_0) = u(\varepsilon' x_2)$ imply that $x_2 \equiv (c_2, w_2)$ is such that $c_2 = (\varepsilon')^{-1} \rho^{2(1-\alpha)} c_0$ and $w_2 = (\varepsilon')^{-1} \rho^{-2\alpha} w_0$. Since $x_1 = (\varepsilon c_0, \varepsilon w_0)$, *efficiency* and $v(\varepsilon' x_1) = v(x_3)$ imply that $x_3 \equiv (c_3, w_3)$ is such that $c_3 = \varepsilon\varepsilon' \rho^{2(1-\beta)} c_0$ and $w_3 = \varepsilon\varepsilon' \rho^{-2\beta} w_0$.

A conflict arises with ε-*no-domination* since for each pair $\varepsilon, \varepsilon' > 0$, there is ρ such that $x_2 < \varepsilon x_3$. To show this, let $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{1+\rho}$. When $(\varepsilon\varepsilon')^2 \rho^{\frac{\rho-1}{1+\rho}} > 1$, it follows that:

$$c_2 = (\varepsilon')^{-1} \rho c_0 < \varepsilon c_3 = \varepsilon^2 \varepsilon' \rho^{2\frac{\rho}{1+\rho}} c_0$$

$$w_2 = (\varepsilon')^{-1} \rho^{-1} w_0 < \varepsilon w_3 = \varepsilon^2 \varepsilon' \rho^{-\frac{2}{1+\rho}} w_0.$$

□

This result implies the same negative conclusion between *efficiency*, *no-domination, and equal treatment of equals*, which corresponds to the case $\varepsilon = \varepsilon' = 1$. It also implies the clash between *efficiency* and *no-envy* presented in the introduction (as *no-envy* is stronger than *no-domination* and *equal treatment of equals*).

Corollary 1. *On the domain $\bar{\mathcal{E}}$, no rule satisfies Pareto efficiency, no-domination,* and *equal treatment of equals.*

Corollary 2. *On the domain $\bar{\mathcal{E}}$, no rule satisfies Pareto efficiency and no-envy.*

This impossibility result is particularly strong. The ethical observer cannot commit to both distributing efficiently resources and treat generations equitably according to the above fairness criteria.

A key role is played by the Pareto principle. It forces the decision-maker to take into account that generations face different economic conditions, i.e. the relative scarcity of goods.[22] While *efficiency* tells that time should matter, the equity requirements constrain inequalities in terms of assigned goods and impede the decision-maker to place much importance to the time generations live in. Furthermore, the two axioms operate differently: *ε-no-domination* introduces an evaluation of goods based on physical amounts; *ε-equal treatment of equals* introduces evaluations based on generations preferences. The richness of the dynamic framework with at least 4 generations (as in the proof above) impedes defining a common-ground evaluation of goods consistent with both efficiency and both equity axioms, independently of how weak (but not vacuous) these are.[23]

[22]The importance of investigating egalitarianism without weakening *efficiency* can be motivated as follows. Assume preferences are represented by some comparable utility measure. Since the utility possibility frontier for any pair of generations is a strictly decreasing function, it is always possible to select an efficient and equal-utility allocation. Combining equity and efficiency is thus a way to investigate what allocation should correspond to this egalitarian reference when the planner does not know how to compare utilities.

[23]The clash is avoided when the relative scarcity of goods at *efficient* allocations is constant, requiring that the relative productivity of goods be constant. Let $L = \{c, w\}$ and $F \equiv (F_0, ..., F_{\bar{t}}) \in \mathcal{F}^T$; for each $t \in T$, let $F_t \equiv \begin{pmatrix} \rho_t^c & 0 \\ 0 & \rho_t^w \end{pmatrix}$ and define the relative productivity $\rho_t \equiv \frac{\rho_t^c}{\rho_t^w}$. Then, a rule that combines *efficiency* and *no-envy* exists on a subdomain of $\bar{\mathcal{E}}$ such that $\rho_t = \rho_{t'} = 1$ for each $t, t' \in T$. An example of such a rule is similar to the equal-split Walrasian (see (Varian, 1974; Thomson, 2011)), where each is assigned its most preferred alternative in the same budget set; this set is here identified by competitive prices and by endowing each generation with a share of initial resources $\underline{k}$ that is inversely proportional to the cumulative productivity of resources, corresponding to a factor $s_t \equiv \left(\frac{\prod_{\tau=0}^{t} \rho_\tau^c}{\sum_{t \in T} (\prod_{\tau=0}^{t} \rho_\tau^c)} \right)^{-1}$ for each $t \in T$.

4 The long-term/short-term inequality trade-off

In this section, I study how much it is necessary to weaken equity in order to restore compatibility with *efficiency*.

Let an axiom be **comparative** if it requires an allocation to satisfy a property assessed in terms of binary comparison between consumption bundles assigned to generations.[24] In this framework, the strength of a comparative axiom can be measure along two directions, named "trait" and "scope".

The **trait** of a comparative axiom is defined as the condition that the axiom requires on the bundles assigned to a pair of generations. Reducing the parameter ε of, for example, $\varepsilon-$*no-domination* weakens the *trait* of the axiom.

The **scope** of a comparative axiom is defined as the set of all pairs of generations among which the axiom's condition is imposed. For each comparative axiom, say "equity", let n-period "equity" be the axiom obtained by imposing the condition of "equity" to all the pairs of generations belonging to each subset of n successive generations. As n decreases the axiom becomes weaker and weaker. When $n = 1$ no binary comparison is allowed and the axiom is vacuous. Let $n \geq 2$ and $\varepsilon \in [0, 1]$. Then, **n-period ε-no domination** requires that no two generations living apart for fewer than n periods be assigned less than a fraction ε of the others' consumption bundle. Similarly, **n-period ε-equal treatment of equals** requires that no two generations with the same preferences living apart for fewer than n periods be assigned bundles that one finds less desirable than a fraction ε of the others' bundle.

Inequalities can be roughly measured by: *i)* the number of violations of the equity condition of an axiom (in terms of the *scope* parameter n); and *ii)* by the intensity of such violations (in terms of the *trait* parameter ε). While Theorem 1 tells that some inequalities are unavoidable, the next result identifies which inequalities can be avoided and which cannot. More precisely, it introduces a new ethical dilemma for egalitarianism: the **long-term/short-term inequality trade-off**.

A natural choice for the ethical observer is to give up either $\varepsilon-$*no-domination* or $\varepsilon-$*equal treatment of equals*. Although this allows satisfying the remaining

[24]Comparative axioms belong to the category that Thomson (2001) identifies as "punctual inter-personal" axioms. Punctual axioms are those that apply to each economy separately; these are opposed to "relational" axioms that involve comparisons between economies. Interpersonal axioms satisfy comparisons between bundles assigned to more than one agent; these are opposed to "intra-personal" axioms that hold on an agent-by-agent basis. Comparative axioms restrict the punctual inter-personal requirements to binary comparisons of bundles, i.e. involving only one pair of generations at a time.

axiom with full *trait*, i.e. $\varepsilon = 1$, it is not possible to avoid large inequalities among proximate-living generations (**short-term inequalities**). Allocations that satisfy *efficiency* and *no-domination* do not guarantee that each generation be assigned a bundle it considers as desirable as the $\varepsilon > 0$ fraction of what was assigned to the previous (or successive) generation with the same preferences, no matter how small ε is. Similarly, allocations that satisfy *efficiency* and *equal treatment of equals* do not guarantee that each generation be assigned a bundle that is at least as large as the $\varepsilon > 0$ fraction of what was assigned to the previous generation, no matter how small ε is.

A further alternative is however possible: both axioms can be satisfied with full *trait* among pairs of successive generations (with $n = 2$). The flip side of the coin is that possibly large inequalities arise among more distant generations (**long-term inequalities**). Allocations that satisfy *efficiency* and both $2-$*period no-domination* and $2-$*period equal treatment of equal* do not guarantee that both $\varepsilon-$*no-domination* and $\varepsilon'-$*equal treatment of equals* be satisfied among the other generations, no matter how small (but positive) ε and ε' are.[25]

Theorem 2. *Let $\varepsilon, \varepsilon' \in [0, 1]$ and $n, n' \geq 2$. On the domain $\mathcal{E}$, the following results hold.*

1) a) for each n, ε such that $(n-2)\,\varepsilon \neq 0$ there exists a rule satisfying efficiency and n-period $\varepsilon-$no-domination. Furthermore, no such rule satisfies n'-period $\varepsilon'-$equal treatment of equals, for $\varepsilon' > 0$;

b) for each n', ε' such that $(n'-2)\,\varepsilon' \neq 0$ there exists a rule satisfying efficiency and n'-period $\varepsilon'-$equal treatment of equals. Furthermore, no such rule satisfies n-period $\varepsilon-$no-domination, for $\varepsilon > 0$;

2) for each $\varepsilon, \varepsilon'$ such that $\varepsilon\varepsilon' \neq 0$ there exists a rule satisfying efficiency, 2-period $\varepsilon-$no-domination, and 2-period $\varepsilon'-$equal treatment of equals. Furthermore, no such rule satisfies n-period $\varepsilon-$no-domination and n'-period $\varepsilon'-$equal treatment of equals, for $n, n' > 2$.

Proof. The existence part is proven constructively in the rest of this section

[25] An attractive feature of this alternative is that "stronger" equality among successive generations is more likely to achieve policy implementability. Generations can immediately compare their-selves with proximate generations; thus, it is easier for the planner to obtain their agreement when the underlying egalitarian distribution of resources treats these proximate generations similarly enough. It appears more arduous to convince the next generation to accept a smaller bundle (compared to the present generation's one) justified by the idea that in the far future some generation with same preferences will achieve the same indifference curve. In the same way, it seems unlikely to convince the next generation (with same preferences) to be worse-off than the present one to guarantee that each future generation is assigned non-dominated bundles.

(Lemmata 1-2-3). The impossibility part is proven in Appendix A. □

The dilemma highlighted by Theorem 2 is new to the literature on intergenerational justice. When information about generations well-being is comparable, it is natural to interpret equal consideration for all generations as an independence of achieved utilities with respect to the time generations live in (usually expressed in the form of invariance to permutations and referred to as "anonymity").

By addressing distributional equity with non-comparable information about preferences, I question which bundles generations should be assigned at the egalitarian distribution of resources (which can then possibly be used to attribute an equal measure of well-beings). The dilemma is thus orthogonal to independence to permutation of well-beings: anonymity operates at the level of evaluating streams of comparable well-being levels; the long-term/short-term inequality trade-off arises at the level of the construction of such comparable indexes.

It is thus not inconsistent to attribute the same utility level at allocations selected by rules in *2)* of Theorem 2 and, together, to rank equally two streams of utilities when one is the permutation of the other. Such an ethical observer agrees that the value of goods might change over time and accepts such flexible evaluation to better compare proximate-living generations. At the same time, this ethical observer treats inequalities of achieved utilities independently of the time generations live in.

4.1 Time independent rules

Let **time-independent rules** be those that satisfy equity axioms with full *scope*.[26] These rules satisfy the feeling of justice according to which selected

[26] A formal definition of time-independence as an axiom is given hereafter. It is formalized as an anonymity property that relies on permuting agents. The additional difficulty is the need of correspondingly vary the economy such that the set of feasible consumption allocations for the new economy is the permutation of the initial one.

Let $E \in \mathcal{E}$. Denote by $X(E)$ be the set of such consumption plans; a (feasible) consumption plan is a vector $x \equiv \left(\{x_t\}_{t\in T}\right) \in X(E)$ such that there exists an allocation $\bar{a} \equiv \left(\{\bar{k}_t, \bar{y}_t, \bar{x}_t\}_{t\in T}\right) \in A(E)$ with $x_t = \bar{x}_t$ for each $t \in T$. For each $x \in X(E)$ and each π, let $x_\pi \equiv \left(\{x_{\pi(t)}\}_{t\in T}\right)$ be the permuted consumption plan of x with respect to π. For each $E \in \mathcal{E}$ and each π, the **permuted economy** (of E with respect to π) is $E_\pi \equiv \left(\underline{k}', \bar{k}', \{F'_t\}_{t\in T}, \{R'_t\}_{t\in T}\right)$ with $R_t = R'_{\pi(t)}$ and such that $x_\pi \in X(E_\pi)$ if and only if $x \in X(E)$, where x_π is the permuted consumption plan of x with respect to π. *A rule ψ satisfies* ***time independence*** *if for each $E \in \mathcal{E}$ and for each $\pi : T \to T$, $a \equiv \left(\{k_t, y_t, x_t\}_{t\in T}\right) \in \psi(E)$ implies that, whenever $E_\pi \in \mathcal{E}$, there exists $a' \equiv \left(\{k'_t, y'_t, x'_t\}_{t\in T}\right) \in \psi(E_\pi)$ such that* allocations should not discriminate by generation names: a generation t and a generation t' should be assigned goods independently of what the times t and t' are.

Among the rules that satisfy *efficiency* and *no-domination,* I adapt to the dynamic framework one proposed by Moulin (1991): the "budget constrained Pareto optimal method". Define the rule ψ^{bcpo} as the one that, for each economy $E \in \mathcal{E}$, selects an *efficient* allocation that assigns to each generation a bundle of goods with equal value, according to some given price vector $p \in \mathbb{R}^L_{++}$. Formally, for each $E \in \mathcal{E}$:

$$\psi^{bcpo}(E) \equiv \{a \in A(E) \,|a \in P(E) \text{ and } p \cdot x_t = p \cdot x_{t'} \; \forall t, t' \in T\}.$$

The next result shows that the rule ψ^{bcpo} exists.

Lemma 1. *On the domain $\mathcal{E}$, the rule ψ^{bcpo} is well-defined.*

Proof. Let $E \in \mathcal{E}$. Let the set of feasible budgets at prices $p > 0$ be

$$B \equiv \{b \in \mathbb{R}_+ \,|a \in A(E) \text{ s.t. } p \cdot x_t = p \cdot x_{t'} = b \forall t, t' \in T\}.$$

The set B is non-empty and compact. Let $\bar{b} \equiv \max B$ and let $a \equiv \left(\{k_t, y_t, x_t\}_{t \in T}\right) \in A(E)$ be such that $p \cdot x_t = \bar{b}$ for each $t \in T$. By contradiction, assume that a is not *efficient* for E and let $a' \equiv \left(\{k'_t, y'_t, x'_t\}_{t \in T}\right) \in A(E)$ be such that for each $t \in T$, $x'_t \; R_t \; x_t$ and for some $t \in T$, $x'_t \; P_t \; x_t$. By continuity and monotonicity of the preferences and upper hemicontinuity and strict monotonicity of the production correspondences, there is $a'' \equiv \left(\{k''_t, y''_t, x''_t\}_{t \in T}\right) \in A(E)$ with $b'' > \bar{b}$ such that for each $t \in T$, $x''_t \; R_t \; x_t$ and for some $t \in T$, $x''_t \; P_t \; x_t$. □

An example of a rule that satisfies *efficiency* and *equal treatment of equals* is the "egalitarian equivalent rule," introduced by Pazner and Schmeidler (1978). It selects efficient allocations such that each agent finds his consumption as desirable as a reference bundle that is named egalitarian equivalent. For each $E \in \mathcal{E}$, the *egalitarian equivalent* solution is defined by:

$$\psi^{ee}(E) = \left\{a \in A(E) \bigcap P(E) \,\middle|\, \exists z \in \mathbb{R}^L_+ Z \text{ s.t. } x_t \; I_t \; z \; \forall t \in T\right\}$$

The next result shows that the rule ψ^{ee} exists.

Lemma 2. *On the domain $\mathcal{E}$, the rule ψ^{ee} is well-defined.*

$x' = x_\pi$. All time-independent rules satisfy this axiom.

Proof. For each $E \in \mathcal{E}$, assumptions *i)*, *ii)*, *iv)*, *v)*, and *vii)* of the technology guarantee that $A(E)$ is compact. Together with strict monotonicity and continuity of preferences, it implies that the result of Pazner and Schmeidler (1978, Prop. 1) applies. □

4.2 Sequential rules

The family of rules I present next satisfy both *no-domination* and *equal treatment of equals* among pairs of successive generations.[27]

I first introduce an equity axiom that, close in spirit to *no-envy*, implies both 2-*period no-domination* and 2-*period equal treatment of equals*. Consider a pair of successive generations t and $t+1$. The axiom is satisfied if switching their respective consumption bundles either makes them both better-off or makes them both worse-off.

For each $E \in \mathcal{E}$, $a = \left(\{k_t, y_t, x_t\}_{t\in T}\right) \in A(E)$ satisfies ***sequential permutation solidarity*** *if for each $t \in \{0, ..., \bar{t}-1\}$:*

i) $x_t \; P_{t+1} \; x_{t+1} \Longleftrightarrow x_{t+1} \; P_t \; x_t$;

ii) $x_t \; P_t \; x_{t+1} \Longleftrightarrow x_{t+1} \; P_{t+1} \; x_t$.

Let the **sequential rules** be the family of rules satisfying *efficiency* and *sequential permutation solidarity*. The existence of such rules, together with Lemmas 1 and 2, completes the proof of Theorem 2.

Lemma 3. *One the domain $\mathcal{E}$, the sequential rules are well-defined.*

Proof. See Appendix B. □

[27]The impossibility of satisfying equity restrictions among more distant generations (as proven in Theorem 2) might lead to allocations that are biased in favor of present or future generations. Such issue is related to the axioms of "no-dictatorship of the present" and "no-dictatorship of the future" introduced by Chichilnisky (1996) for the infinite utility streams literature: these require the social evaluation to be sensitive to the well-being of the far future and of the present generations. In the present framework, it is possible to guarantee equal concern to present and future generations by requiring a rule to be independent with respect to the only permutation that preserves the proximity of generations. This permutation is such that the last generation becomes the first one, the second-last generation becomes the second one, etc... As for *time independence*, the permutation is equally applied to the technology, such that the economic circumstances of each generation remain invariant. For each $E \in \mathcal{E}$, let $\pi^{ti} : T \to T$ be the permutation defined by $\pi^{ti}(t) = \bar{t} - t$. *A rule ψ satisfies* ***time inversion invariance*** *if for each economy $E \in \mathcal{E}$, $a \equiv \left(\{k_t, y_t, x_t\}_{t\in T}\right) \in \psi(E)$ implies that there exists $a' \equiv \left(\{k'_t, y'_t, x'_t\}_{t\in T}\right) \in \psi\left(E_{\pi^{ti}}\right)$ such that $x' = x_\pi$.* Time inversion invariance is satisfied by the non-empty subset of sequential rules that treat pairs of successive generations symmetrically.

5 Extension to the infinite horizon

The infinite-horizon economy spans periods $T \equiv \{\underline{t}, \underline{t}+1, ...\}$ with $\underline{t} \geq 0$. Assumptions *i)-vii)* on technology and assumptions *i)-iii)* on preferences hold: a profile of technologies is a list $\left(F_{\underline{t}}, F_{\underline{t}+1}, ...\right) \in \mathcal{F}^T$ and a profile of preferences is a list $\left(R_{\underline{t}}, R_{\underline{t}+1}, ...\right) \in \mathcal{R}^T$.

An infinite-horizon economy is a list $E \equiv \left(\underline{k}, \{F_t\}_{t\in T}, \{R_t\}_{t\in T}\right) \in \mathbb{R}^L_+ \times \mathcal{F}^T \times \mathcal{R}^T$. The domain of such economies is denoted by $\mathcal{E}^\infty$. Let $E \in \mathcal{E}^\infty$. The set of feasible allocations for E is denoted by $A(E)$: it satisfies conditions (2.1). The set of *efficient* allocations for E is denoted by $P(E)$. Finally, for each $a \in A(E)$, denote by $E_{\geq t}\left(\underline{k}_t\right)$ the **reduced economy of E with endowment $\underline{\boldsymbol{k}}_{\boldsymbol{t}}$**, obtained by taking as initial time $t \geq \underline{t}$ and as initial resources $\underline{k}_t = k_t$ (defined in a).

To deal with the infinite horizon, two alternative domain restrictions are introduced on preferences and technology. The idea is to define a finite time $\bar{t}$ after which the issues of comparability of preferences and the tension between equity and efficiency are ruled out. First is the assumption of existence of an efficient distribution of consumption that sustains in the long-run an equal budget – measured at time invariant prices; second is the assumption that preferences are homogeneous in the long-run and that there is an efficient distribution of consumption that treats them alike (these are indifferent between their assigned consumptions).

A1. *For each $E \equiv \left(\underline{k}, \{F_t\}_{t\in T}, \{R_t\}_{t\in T}\right) \in \mathcal{E}^\infty$, there is a finite time $\bar{t} \in T$ and prices $\pi \in \mathbb{R}^L_{++}$ such that for each $k_{\bar{t}} \in \mathbb{R}^L_+$, there exists $a_{\geq \bar{t}} \equiv \left(\{k_t, y_t, x_t\}_{t\in[\bar{t},\infty)}\right) \in P\left(E_{\geq \bar{t}}(k_t)\right)$ such that $\pi \cdot x_t = \pi \cdot x_{t'}$ for each $t, t' \geq \bar{t}$.*

A2. *For each $E \equiv \left(\underline{k}, \{F_t\}_{t\in T}, \{R_t\}_{t\in T}\right) \in \mathcal{E}^\infty$, there is a finite time $\bar{t} \in T$ such that:*

i) *$R_t = R_{t'} \equiv \bar{R}$ for each $t, t' \geq \bar{t}$;*

ii) *for each $k_{\bar{t}} \in \mathbb{R}^L_+$, there is $a_{\geq \bar{t}} \equiv \left(\{k_t, y_t, x_t\}_{t\in[\bar{t},\infty)}\right) \in P\left(E_{\geq \bar{t}}(k_t)\right)$ such that $x_t \bar{I} x_{t'}$ for each $t, t' \geq \bar{t}$.*

Let $\mathcal{E}^\infty_{A1}$ and $\mathcal{E}^\infty_{A2}$ be the sub-domain of economies that satisfy assumptions A1 and A2 respectively. In the following, I first extend the impossibility result of Theorem 1 and then state the ethical dilemma of Theorem 2 for the infinite-horizon domain of economies. The proofs of all results in this section are in Appendix C.

Theorem 3. *On the domain $\mathcal{E}^\infty_{A1} \bigcap \mathcal{E}^\infty_{A2}$ and for each pair $\varepsilon, \varepsilon' \in (0, 1]$, no rule satisfies Pareto efficiency, ε-no-domination, and ε'-equal treatment of equals.*

Theorem 4. *Let $\varepsilon, \varepsilon' \in [0,1]$ and $n, n' \geq 2$. On the domain $\mathcal{E}^{\infty}_{A1} \bigcap \mathcal{E}^{\infty}_{A2}$, the following results hold.*

1) a) for each n, ε such that $(n-2)\,\varepsilon \neq 0$ there exists a rule satisfying efficiency and n-period $\varepsilon-$no-domination. Furthermore, no such rule satisfies n'-period $\varepsilon'-$equal treatment of equals, for $\varepsilon' > 0$;

b) for each n', ε' such that $(n'-2)\,\varepsilon' \neq 0$ there exists a rule satisfying efficiency and n'-period $\varepsilon'-$equal treatment of equals. Furthermore, no such rule satisfies n-period $\varepsilon-$no-domination, for $\varepsilon > 0$;

2) for each $\varepsilon, \varepsilon'$ such that $\varepsilon\varepsilon' \neq 0$ there exists a rule satisfying efficiency, 2-period $\varepsilon-$no-domination, and 2-period $\varepsilon'-$equal treatment of equals. Furthermore, no such rule satisfies n-period $\varepsilon-$no-domination and n'-period $\varepsilon'-$equal treatment of equals, for $n, n' > 2$.

The proof of the impossibility result is immediate: it is sufficient to note that for each finite horizon economy introduced for proving the corresponding finite-horizon result, one can extend the economy such that efficiency and equity impose a specific allocation for all (infinite) following generations, parametrized by the final capital stock $\bar{k}$.

Assumption A1 is introduced to guarantee that the existence result of Lemma 1 extends. Assumption A2 is similarly introduced to extend the existence results of Lemmas 2 and 3.

Lemma 4. *On the domain $\mathcal{E}^{\infty}_{A1}$, the rule ψ^{bcpo} is well-defined.*

Lemma 5. *On the domain $\mathcal{E}^{\infty}_{A2}$, the rule ψ^{ee} is well-defined.*

Lemma 6. *On the domain $\mathcal{E}^{\infty}_{A2}$, sequential rules are well-defined.*

6 Conclusions

In the economic literature, intergenerational justice is generally addressed by ranking streams of a comparable index of well-being. This is the case of the discounting literature (Nordhaus, 2007; Stern, 2007) and of the utility streams literature (Diamond, 1965; Svensson, 1980; Basu and Mitra, 2003; Zuber and Asheim, 2012).[28] In such frameworks, it is natural to call egalitarian each allocation for which well-being is equalized among generations. The present contribution, instead, attempts to define intergenerational egalitarianism in terms

[28]For a general discussion of ethical approaches to intergenerational justice see Page (2007) and Fleurbaey and Blanchet (2013).

of distributional fairness, i.e. by direct comparison of the consumption bundles assigned to generations.

In a discrete time model of investment, production, and consumption, I show that a strong clash arises between efficiency and distributional equity. This difficulty unveils a deep "equity gap": equalizing well-beings – however these are measured – cannot guarantee that the corresponding allocation distributes resources in a satisfactory way. In particular, it is not possible to distribute resources efficiently and at the same time ensure that:

1. each generation's assignment is at least as large as an arbitrarily small fraction of the bundle assigned to any other generation (ε-*no-domination*);

2. each generation finds its assignment at least as desirable as an arbitrarily small fraction of the bundle assigned to any other generation with the same preferences (ε-*equal treatments of equals*).

Responsible for such difficulty is neither technological change nor the time horizon, but the evolution of resource scarcity over time. When goods have a different productivity (even if time-invariant), resource scarcity changes over time. Then, Pareto efficiency forces the ethical observer to account for this evolution in terms of marginal rates of substitutions; equity conditions, instead, impede the ethical observer to make too much of a difference of such change to avoid discrimination across generations. Egalitarian allocations can be defined only with weaker requirements.

The main contribution of the paper is to draw the exact boundaries between impossibility and possibility results. While a more conventional strategy suggests adopting only one of the above equity requirements, in this dynamic framework there is a further option: reducing the number of successive generations among which equity is imposed. This shows that egalitarianism can be defined along the lines of a newly identified ethical dilemma: the *short-term/long-term inequality trade-off.* The ethical observer can choose between satisfying stronger equity conditions among pairs of successive generations or weaker equity conditions among all (and more distant) generations.

A Proof of Theorem 2

Proof. (impossibility result) 1a) I show that there is no rule satisfying *efficiency*, 3-*period* ε-*no-domination, and* 2-*period* ε'-*equal treatment of equals* for

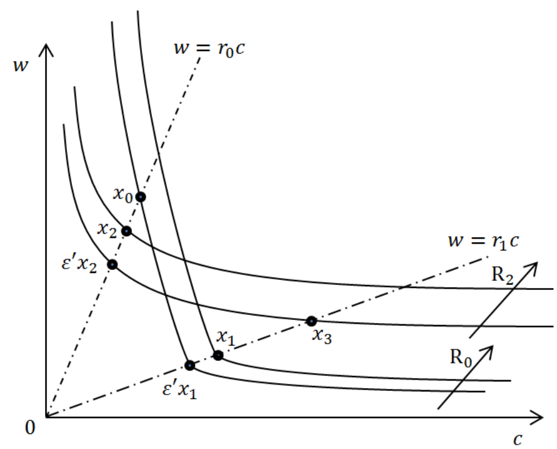

Figure A.1: Clash between *efficiency*, *3-period* $\varepsilon-$*no-domination*, and *2-period* $\varepsilon'-$*equal treatment of equals.*

any $\varepsilon, \varepsilon > 0$. This is sufficient for the impossibility as the equity axioms with larger scope ($n > 3$ and $n' > 2$) are stronger than the ones considered here. The proof is similar to the one of Theorem 1.

Step 1. Let $E \in \bar{\mathcal{E}}$ be such that $t \in T = \{0,1,2,3\}$, $L = \{c, w\}$, and $\begin{pmatrix} y^c \\ y^w \end{pmatrix} = \begin{pmatrix} \rho^c & 0 \\ 0 & \rho^w \end{pmatrix} \begin{pmatrix} k^c \\ k^w \end{pmatrix}$. Let $\rho \equiv \frac{\rho^c}{\rho^w}$. Preferences are differentiable and strictly convex. Let $R_0 = R_1$ be represented by utility $u(x) = c^\alpha w^{1-\alpha}$ and let $R_2 = R_3$ be represented by utility $v(x) = c^\beta w^{1-\beta}$ with $\alpha, \beta \in (0,1)$ and $\frac{\alpha}{1-\alpha} = \rho^2 \frac{\beta}{1-\beta}$. Assume $A(E)$ contains allocations $a \equiv \left(\{k_t, y_t, x_t\}_{t \in T}\right) \in A(E)$ such that $x_t > 0$ for each $t \in T$.

Consider allocations that assign strictly positive consumption bundles to each generation. Then, the contract curve C_t (defined in the proof of Theorem 1), restricted to positive consumptions, is a (portion of) linear function described by $w = r_t c$ and is such that $r_0 = r_1 = \rho r_2 = \rho r_3$. These are represented in Figure A.1 as dash-dotted lines.

Step 2. Let $a \equiv \left(\{k_t, y_t, x_t\}_{t \in T}\right) \in A(E)$ be such that:

i) $a \in P(E)$, i.e. $x_t \in C_t$ for each $t \in T$;

ii) $x_2 = \varepsilon x_0$, i.e. a (minimally) satisfies 3-*period* ε-*no-domination* between generations 0 and 2;

iii) $u(x_0) = u(\varepsilon' x_1)$ and $v(\varepsilon' x_2) = v(x_3)$, i.e. a (minimally) satisfies 2-*period* ε'-*equal treatment of equals* between generations 0 and 1 and between generations 2 and 3.

Step 3. By strict convexity of preferences, a is the only feasible allocation for E satisfying *i)-iii)*. By contradiction, assume $\bar{a} \equiv \left(\left\{\bar{k}_t, \bar{y}_t, \bar{x}_t\right\}_{t\in T}\right) \in A(E)$ with $\bar{a} \neq a$ satisfies the same properties.

Case 1. If $\bar{x}_0 > x_0$, $\bar{a}$ is such that for each $t \in T$, $\bar{x}_t \geq x_t$: since $\bar{a} \neq a$, $\bar{a} \notin A(E)$ or $a \notin P(E)$.

Case 2. If $\bar{x}_0 < x_0$, $\bar{a}$ is such that for each $t \in T$, $\bar{x}_t < x_t$: since $\bar{a} \neq a$, $\bar{a} \notin P(E)$.

Case 3. If $\bar{x}_0^c > x_0^c$ and $\bar{x}_0^w < x_0^w$, $\bar{a}$ is such that for each $t \in T$, $\bar{x}_t^c > x_t^c$ and $\bar{x}_t^w < x_t^w$; thus, $\bar{a} \notin P(E)$.

Case 4. If $\bar{x}_0^c < x_0^c$ and $\bar{x}_0^w > x_0^w$, $\bar{a}$ is such that for each $t \in T$, $\bar{x}_t^c < x_t^c$ and $\bar{x}_t^w > x_t^w$; thus, $\bar{a} \notin P(E)$.

Step 4. Let $x_0 \equiv (c_0, w_0)$ be assigned to 0. *Efficiency* and $u(x_0) = u(\varepsilon' x_1)$ imply that $x_1 \equiv (c_1, w_1)$ is such that $c_1 = (\varepsilon')^{-1} \rho^{1-\alpha} c_0$ and $w_1 = (\varepsilon')^{-1} \rho^{-\alpha} w_0$. Since $x_2 = \varepsilon x_0$, *efficiency* and $v(\varepsilon' x_2) = v(x_3)$ imply that $x_3 \equiv (c_3, w_3)$ is such that $c_3 = \varepsilon\varepsilon' \rho^{1-\beta} c_0$ and $w_3 = \varepsilon\varepsilon' \rho^{-\beta} w_0$.

The conflict arises with 3-*period* ε-*no-domination* since for each $\varepsilon, \varepsilon' > 0$, there is ρ such that $x_1 < \varepsilon x_3$. To show this, let $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{1+\rho^2}$. When $(\varepsilon\varepsilon')^2 \rho^{\frac{\rho^2-1}{2(1+\rho^2)}} > 1$, it follows that:

$$c_1 = (\varepsilon')^{-1} \rho^{\frac{1}{2}} c_0 < \varepsilon c_3 = \varepsilon^2 \varepsilon' \rho^{\frac{\rho^2}{1+\rho^2}} c_0$$

$$w_1 = (\varepsilon')^{-1} \rho^{-\frac{1}{2}} w_0 < \varepsilon w_3 = \varepsilon^2 \varepsilon' \rho^{-\frac{1}{1+\rho^2}} w_0.$$

1b) The proof is identical to that of Theorem 1, where the axiom of 3-*period* ε'-*equal treatment of equals* is substituted for ε'-*equal treatment of equals* and 2−*period* ε-*no-domination* is substituted for ε-*no-domination*..

2) The above result in *1a)*, or equivalently *1b)*, also shows that no rule satisfies *efficiency*, 3-*period* ε'-*no-domination*, and 3-*period* ε-*equal treatment of equals,* for $\varepsilon, \varepsilon' > 0$.

(existence result) The existence part is shown by constructing rules satisfying the specific version of the axioms. The corresponding results are presented in Lemmas 1-3. □

B Proof of Lemma 3

Proof. Let $E \in \mathcal{E}$. I first introduce a cardinalization of a generation's gain/loss should it be assigned the bundle of another generation instead of its own. Denote the Euclidean distance between any pair $x, \bar{x} \in \mathbb{R}_+^L$ by $d(x, \bar{x}) \equiv \sqrt{\sum_{\ell \in L} (x_\ell - \bar{x}_\ell)^2}$. Let the **permutation gain (loss) of generation t towards t' at allocation a** be the (inverse of the) minimum distance between the bundle assigned to the other generation $x_{t'}$ and the indifference curve of generation t at x_t:[29]

$$g_t(x_t, x_{t'}) \equiv \begin{cases} \min_{\bar{x}_t I_t x_t} d(\bar{x}_t, x_{t'}) & \text{if } x_{t'} R_t x_t \\ -\min_{\bar{x}_t I_t x_t} d(x_t, x_{t'}) & \text{otherwise.} \end{cases}$$

Let the **sequential permutation equivalent allocations**, denoted by $G(E) \subseteq A(E)$, be the set of allocations such that the permutation gain/loss are equalized among successive generations, i.e.

$$G(E) \equiv \{a \in A(E) \,|\, d_t(x_t, x_{t+1}) = d_{t+1}(x_{t+1}, x_t) \;\; \forall t \in \{0, ..., \bar{t} - 1\}\}.$$

Clearly, if $a \in G(E)$ then a satisfies *sequential permutation solidarity.* It remains to prove that, for each economy in the domain, $G(E) \bigcap P(E) \neq \emptyset$.

Let the function $u_0 : \mathbb{R}_+^L \to \mathbb{R}$ represent the preferences of generation 0, i.e. for each pair $x_0, \bar{x}_0 \in \mathbb{R}_+^L$, $x_0 R_0 \bar{x}_0$ if and only if $u_0(x_0) \geq u_0(\bar{x}_0)$. Define the utility possibility set of 0, restricted to sequential permutation equivalent allocations be $U_0 = \{u_0(x_0) \,|\, a \in G(E)\}$. The set U_0 is non-empty: by assumptions on E, $A(E) \neq \emptyset$; by Assumptions *i)*, *iv)* and *vii)* on technology, there exists $a \in A(E)$ such that $x_t = 0$ for each $t \in T$; clearly, $a \in G(E)$ and thus $U_0 \neq \emptyset$. The set U_0 is bounded: by strict monotonicity of preferences, there is no $a \in A(E)$ for which $u_0(x_0) > max_{y_0 \in F_0(k_0)} u_0(y_0)$. The set U_0 is compact: by Assumptions *i)-vii)* on technology, the set of feasible allocations is compact; by continuity of the preferences and of the permutation gains, $G(E)$ is compact and therefore also U_0 is. Let $\bar{u}_0$ be the maximal element of U_0 and let $\bar{a} \in G(E)$ be such that $u_0(\bar{x}_0) = \bar{u}_0$. By contradiction assume that $\bar{a} \notin P(E)$. Then, by assumption on technology, there exist $\delta > 0$ and $a' \in A(E)$ such that $x'_t > \bar{x}_t + \delta \mathbf{1}$ for each $t \in T$. Let the function $\gamma_1 : \mathbb{R}_+ \to \mathbb{R}_+$ be such that for each

[29] Note that the permutation gain is continuous. Moreover it is such that: *i)* t achieves no permutation gain when t is indifferent between x_t and $x_{t'}$, i.e. $d_t(x_t, x_{t'}) = 0$ for each $x_{t'} I_t x_t$; and *ii)* the permutation gain is independent of which bundle the generation was assigned among those that it finds equally good, i.e. $d_t(x_t, x_{t'}) = d_t(\bar{x}_t, x_{t'})$ for each $x_t I_t \bar{x}_t$.

$\delta_0 > 0$, $\delta_1 \equiv \gamma_1(\delta_0)$ satisfies $g_1(\bar{x}_1 + \delta_1\mathbf{1}, \bar{x}_0 + \delta_0\mathbf{1}) = g_0(\bar{x}_0 + \delta_0\mathbf{1}, \bar{x}_1 + \delta_1\mathbf{1})$; since g_1 (g_0) is continuous, strictly increasing in δ_0 (δ_1), and strictly decreasing in δ_1 (δ_0), γ_1 is well-defined, continuous, and strictly increasing. Define a similar function γ_t for each $t \in [2, \bar{t}]$ and let $\Gamma : \mathbb{R}_+ \to \mathbb{R}_+^T$ be the function that associates to each $\delta_0 > 0$ a vector $(\bar{\delta}_0, \bar{\delta}_1, ..., \bar{\delta}_{\bar{t}}) \in \mathbb{R}_+^T$ such that $\bar{\delta}_0 = \delta_0$ and $\bar{\delta}_t = \gamma_t(\bar{\delta}_{t-1})$ for each $t \in [1, \bar{t}]$. By continuity of Γ (following from continuity of each γ_t), there is a sufficiently small $\delta_0^* > 0$ such that there exists $a'' \in A(E)$ satisfying: $x_t'' = \bar{x}_t + \Gamma_t(\delta_0^*) \leq x_t'$. Since $a'' \in G(E)$ (by construction of Γ) and $u_0(x_0'') > \bar{u}_0$, a contradiction arises. □

C Proofs of the results in Section 5

Theorem 3.

Proof. The proof is similar to the one of Theorem 1. Let $\varepsilon, \varepsilon' > 0$.

Step 1. Let $E \in \mathcal{E}_{A1}^\infty \bigcap \mathcal{E}_{A2}^\infty$ be such that $t \in T = \{0, 1, ...\}$ and $L = \{c, w\}$. The technology is such that $\begin{pmatrix} y_t^c \\ y_t^w \end{pmatrix} = \begin{pmatrix} \rho^c & 0 \\ 0 & \rho^w \end{pmatrix} \begin{pmatrix} k_t^c \\ k_t^w \end{pmatrix}$ for each $t \in \{0, 1, 2, 3\}$ and $\begin{pmatrix} y_t^c \\ y_t^w \end{pmatrix} = \lambda \begin{pmatrix} k_t^c \\ k_t^w \end{pmatrix}$ for each $t \geq 4$ with $\lambda > \max\left[\frac{1}{\varepsilon}, \frac{1}{\varepsilon'}\right]$. Let $\rho \equiv \frac{\rho^c}{\rho^w}$. Preferences are differentiable and strictly convex. Let $R_0 = R_2$ be represented by utility $u(x) = c^\alpha w^{1-\alpha}$ and let $R_1 = R_3 = R_t$ for each $t \geq 4$ be represented by utility $v(x) = c^\beta w^{1-\beta}$ with $\alpha, \beta \in (0, 1)$ and $\frac{\alpha}{1-\alpha} = \rho \frac{\beta}{1-\beta}$. Assume $A(E)$ contains allocations $a \equiv (\{k_t, y_t, x_t\}_{t \in T}) \in A(E)$ such that $x_t > 0$ for each $t \in T$.

Consider allocations that assign strictly positive consumption bundles to each generation. Then, the contract curve C_t, restricted to positive consumptions, is a (portions of) linear function described by $w = r_t c$ and is such that $r_0 = r_1 = \rho^2 r_t$ for each $t \geq 2$.

Step 2. Let $a \equiv (\{k_t, y_t, x_t\}_{t \in T}) \in A(E)$ be such that:

i) $a \in P(E)$, i.e. $x_t \in C_t$ for each $t \in T$;

ii) $x_1 = \varepsilon x_0$, i.e. a (minimally) satisfies $\varepsilon-$*no-domination* between generations 0 and 1;

iii) $u(x_0) = u(\varepsilon' x_2)$ and $v(\varepsilon' x_1) = v(x_3)$, i.e. a (minimally) satisfies $\varepsilon'-$*equal treatment of equals* between generations 0 and 2 and between generations 1 and 3;

iv) $\frac{x_{t+1}}{\max[\varepsilon,\varepsilon']} \leq x_t \leq \max[\varepsilon,\varepsilon'] \, x_{t+1}$ and $\frac{x_{t+2}}{\max[\varepsilon,\varepsilon']} \leq x_t \leq \max[\varepsilon,\varepsilon'] \, x_{t+2}$ for each $t \geq 3$, i.e. satisfies $\varepsilon-$*no-domination* and $\varepsilon'-$*equal treatment of equals* for each $t, t' \geq 3$.

Step 3. Such allocations a exists, but is not necessarily unique. For each $a \in A(E)$, let $a_{<\bar{t}} \equiv \left(\{\omega_t, y_t, x_t\}_{t \in [\underline{t}, \bar{t}-1)} \right)$ be the allocation up to time $\bar{t}-1$ such that $a = \left(a_{<\bar{t}} | a_{\geq \bar{t}}\right)$. Step 3 of the proof of Theorem 1 shows that there is a unique distribution of resources $a_{<4}$ for each k_4 (final capital goods defining the finite horizon economy limited to generations 0 to 3). Note that the proof does not rely on the total amount of resources as long as feasible allocations with strictly positive consumption exist and thus extends to this framework. Assume $k_4 \in \mathbb{R}^L_+$ is so large that no positive consumption can be assigned to generations 0 to 3, then the equity restriction *iv*) can not be satisfied between generations 3 and 4 and allocation a does not satisfy conditions *i)-iv)* set in Step *2).* Let a be otherwise.

Step 4. Let $x_0 \equiv (c_0, w_0)$ be assigned to 0; *efficiency* and $u(x_0) = u(\varepsilon' x_2)$ imply that $x_2 \equiv (c_2, w_2)$ is such that $c_2 = (\varepsilon')^{-1} \rho^{2(1-\alpha)} c_0$ and $w_2 = (\varepsilon')^{-1} \rho^{-2\alpha} w_0$. Since $x_1 = (\varepsilon c_0, \varepsilon w_0)$, *efficiency* and $v(\varepsilon' x_1) = v(x_3)$ imply that $x_3 \equiv (c_3, w_3)$ is such that $c_3 = \varepsilon\varepsilon' \rho^{2(1-\beta)} c_0$ and $w_3 = \varepsilon\varepsilon' \rho^{-2\beta} w_0$.

A conflict arises with ε-*no-domination* since for each $\varepsilon, \varepsilon' > 0$, there is ρ such that $x_2 < \varepsilon x_3$. To show this, let $\alpha = \frac{1}{2}$ and $\beta = \frac{1}{1+\rho}$. When $\varepsilon (\varepsilon')^2 \rho^{\frac{\rho-1}{1+\rho}} > 1$, it follows that:

$$c_2 = (\varepsilon')^{-1} \rho c_0 < \varepsilon c_3 = \varepsilon\varepsilon' \rho^{2\frac{\rho}{1+\rho}} c_0$$

$$w_2 = (\varepsilon')^{-1} \rho^{-1} w_0 < \varepsilon w_3 = \varepsilon\varepsilon' \rho^{-\frac{2}{1+\rho}} w_0.$$

Since this contradiction arises for each k_4 such that $a_{<4}$ assigns strictly positive bundles to each generation $t < 4$, the proof is completed.

□

Theorem 4.

Proof. (impossibility part)

<u>1a)</u> I show that there is no rule satisfying *efficiency*, 3-*period* ε-*no-domination, and* 2-*period* ε'-*equal treatment of equals* for any $\varepsilon, \varepsilon > 0$. This is sufficient for the impossibility as the equity axioms with larger scope ($n > 3$ and $n' > 2$) are stronger than the ones considered here.

Step 1. Let $E \in \mathcal{E}^\infty$ be such that $t \in T = \{0, 1, ...\}$ and $L = \{c, w\}$. The technology is such that $\begin{pmatrix} y_t^c \\ y_t^w \end{pmatrix} = \begin{pmatrix} \rho^c & 0 \\ 0 & \rho^w \end{pmatrix} \begin{pmatrix} k_t^c \\ k_t^w \end{pmatrix}$ for each $t \in \{0, 1, 2, 3\}$

and $\begin{pmatrix} y_t^c \\ y_t^w \end{pmatrix} = \lambda \begin{pmatrix} k_t^c \\ k_t^w \end{pmatrix}$ for each $t \geq 4$ with $\lambda > \max\left[\frac{1}{\varepsilon}, \frac{1}{\varepsilon'}\right]$. Let $\rho \equiv \frac{\rho^c}{\rho^w}$. Preferences are differentiable and strictly convex. Let $R_0 = R_1$ be represented by utility $u(x) = c^\alpha w^{1-\alpha}$ and let $R_2 = R_3 = R_t$ for each $t \geq 4$ be represented by utility $v(x) = c^\beta w^{1-\beta}$ with $\alpha, \beta \in (0,1)$ and $\frac{\alpha}{1-\alpha} = \rho^2 \frac{\beta}{1-\beta}$. Assume $A(E)$ contains allocations $a \equiv \left(\{k_t, y_t, x_t\}_{t\in T}\right) \in A(E)$ such that $x_t > 0$ for each $t \in T$.

Consider allocations that assign strictly positive consumption bundles to each generation. Then, the contract curve C_t, restricted to positive consumptions, is a (portion of) linear function described by $w = r_t c$ and is such that $r_0 = r_1 = \rho r_t$ for each $t \geq 2$.

Step 2. Let $a \equiv \left(\{k_t, y_t, x_t\}_{t\in T}\right) \in A(E)$ be such that:

i) $a \in P(E)$, i.e. $x_t \in C_t$ for each $t \in T$ or, equivalently, $m_0(x_0) = \rho^t m_t(x_t)$ for each $t \in T$;

ii) $x_2 = \varepsilon x_0$, i.e. a (minimally) satisfies 3-*period ε-no-domination* between generations 0 and 2;

iii) $u(x_0) = u(\varepsilon' x_1)$ and $v(\varepsilon' x_2) = v(x_3)$, i.e. a (minimally) satisfies 2-*period ε-equal treatment of equals* between generations 0 and 1 and between generations 2 and 3;

iv) $\frac{x_{t+1}}{\max[\varepsilon,\varepsilon']} \leq x_t \leq \max[\varepsilon,\varepsilon'] x_{t+1}$ and $\frac{x_{t+2}}{\max[\varepsilon,\varepsilon']} \leq x_t \leq \max[\varepsilon,\varepsilon'] x_{t+2}$ for each $t \geq 3$, i.e. satisfies 3-*period ε'-no-domination* and *2-period ε'-equal treatment of equals* for each $t, t' \geq 3$.

Step 3. Such allocation a exists, but is not necessarily unique. For each $a \in A(E)$, let $a_{<\bar{t}} \equiv \left(\{k_t, y_t, x_t\}_{t\in[\underline{t},\bar{t}-1)}\right)$ be the allocation up to time $\bar{t}-1$ such that $a = \left(a_{<\bar{t}} | a_{\geq\bar{t}}\right)$. Step 3 of the proof of Theorem 1 shows that there is a unique distribution of resources $a_{<4}$ for each k_4 (final capital goods defining the finite horizon economy limited to generations 0 to 3). Note that the proof does not rely on the total amount of resources as long as feasible allocations with strictly positive consumption exist and thus extends to this framework. Assume $k_4 \in \mathbb{R}_+^L$ is so large that no positive consumption can be assigned to generations 0 to 3, then the equity restriction *iv*) can not be satisfied between generations 3 and 4 and allocation a does not satisfy conditions *i)-iv)* set in Step *2)*. Let a be otherwise.

Step 4. Let $x_0 \equiv (c_0, w_0)$ be assigned to 0; *efficiency* and $u(x_0) = u(\varepsilon' x_2)$ imply that $x_1 \equiv (c_1, w_1)$ is such that $c_1 = (\varepsilon')^{-1} \rho^{1-\alpha} c_0$ and $w_1 = (\varepsilon')^{-1} \rho^{-\alpha} w_0$. Since $x_2 = \varepsilon x_0$, *efficiency* and $v(\varepsilon' x_2) = v(x_3)$ imply that $x_3 \equiv (c_3, w_3)$ is such that $c_3 = \varepsilon\varepsilon' \rho^{1-\beta} c_0$ and $w_3 = \varepsilon\varepsilon' \rho^{-\beta} w_0$. The conflict arises with 3-*period ε-*

no-domination since for each $\varepsilon, \varepsilon' > 0$, there is ρ such that $x_1 < \varepsilon x_3$. To show this, let $\alpha = \frac{1}{2}$ and, thus,$\beta = \frac{1}{1+\rho^2}$. When $\varepsilon (\varepsilon')^2 \rho^{\frac{\rho^2-1}{2(1+\rho^2)}} > 1$, it follows that:

$$c_1 = (\varepsilon')^{-1} \rho^{\frac{1}{2}} c_0 < c_3 = \varepsilon\varepsilon' \rho^{\frac{\rho^2}{1+\rho^2}} c_0$$

$$w_1 = (\varepsilon')^{-1} \rho^{-\frac{1}{2}} w_0 < w_3 = \varepsilon\varepsilon' \rho^{-\frac{1}{1+\rho^2}} w_0.$$

Since this contradiction arises for each k_4 such that $a_{<4}$ assigns strictly positive bundles to each generation $t < 4$, the proof is completed.

1b) The proof is identical to the one of Theorem 3, where the axiom of 3-*period* ε-*no-domination* is substituted for ε-*no-domination* and 3-*period* ε'-*equal treatment of equals* is substituted for ε'-*equal treatment of equals.*

2) The above result in *1a),* or equivalently *1b)*, also shows that no rule satisfies *efficiency*, 3-*period* ε-*no-domination*, and 3-*period* ε'-*equal treatment of equals* with $\varepsilon, \varepsilon' > 0$.

(existence part) The existence part is shown by constructing rules satisfying the specific version of the axioms. The corresponding results are presented in Lemmas 4-6. □

Lemma 4.

Proof. Let $E \in \mathcal{E}^{\infty}_{A1}$ be such that, w.l.g., $\underline{t} = 0$. For each $a \in A(E)$, let $a_{<\bar{t}} \equiv \left(\{k_t, y_t, x_t\}_{t\in[0,\bar{t}-1)}\right)$ be the allocation up to time $\bar{t}-1$ such that $a = \left(a_{<\bar{t}} | a_{\geq\bar{t}}\right)$. By Assumption A1, there exists $\bar{t} \in T$ and prices $\pi \in \mathbb{R}^L_{++}$ such that for each $k_{\bar{t}} \in \mathbb{R}^L_+$, $a_{\geq\bar{t}} \equiv \left(\{k_t, y_t, x_t\}_{t\in[\bar{t},\infty)}\right) \in P\left(E_{\geq\bar{t}}(k_t)\right)$ with $\pi \cdot x_t = \pi \cdot x_{t'}$ for each $t, t' \geq \bar{t}$. For each allocation $a \equiv \left(\{k_t, y_t, x_t\}_{t\in[0,\infty)}\right) \in A(E)$ (specifying also $k_{\bar{t}}$), define $\bar{E}(\bar{k}_{\bar{t}}) \equiv \left(\bar{k}_0, \bar{k}_{\bar{t}}, \{\bar{F}_t\}_{t\in[0,\bar{t}-1]}, \{\bar{R}_t\}_{t\in[0,\bar{t}-1]}\right)$ such that $\bar{k}_0 = k_0$, $\bar{k}_{\bar{t}} = k_{\bar{t}}$, $\bar{F}_t = F_t$ and $\bar{R}_t = R_t$ for each $t \in [0, \bar{t}-1]$. Clearly, $\bar{E}(k_{\bar{t}}) \in \mathcal{E}$.

Following the proof of Lemma 1, there exists a rule ψ^{bcpo} for each prices $p \in \mathbb{R}^L_{++}$ and for each economy in the domain $\mathcal{E}$, thus including $\bar{E}(k_{\bar{t}})$ for each $k_{\bar{t}} \in \mathbb{R}^L_+$. Let $a \in P(E)$ be such that *i)* $\pi \cdot x_t = \pi \cdot x_{t'} \equiv \gamma_{<\bar{t}}$ for each $t, t' \in [0, \bar{t}-1]$ and *ii)* $\pi \cdot x_t = \pi \cdot x_{t'} \equiv \gamma_{\geq\bar{t}}$ for each $t, t' \geq \bar{t}$. The existence of such allocations is granted by $a_{<\bar{t}} \in \psi^{bcpo}\left(\bar{E}(k_{\bar{t}})\right)$ for condition *i)* and Assumption A1 for condition *ii).*

Finally, upper-hemicontinuity of production correspondences and continuity of preferences guarantee that there exists an allocation such that $\gamma_{<\bar{t}} = \gamma_{\geq\bar{t}}$, proving the result. □

Lemma 5.

Proof. The reasoning of the proof is similar to the above. Assumption A2 allows satisfying *equal treatment of equals* for each generation from $\bar{t}$ onwards for each $k_{\bar{t}} \in \mathbb{R}^L_+$. The existence result of Lemma 2 guarantees that for each $k_{\bar{t}} \in \mathbb{R}^L_+$ this is also the case for generations before $\bar{t}$. Assumptions on production and preferences guarantee that it is possible to change $k_{\bar{t}}$ continuously such that the allocation is efficient and all generations are indifferent to the same egalitarian equivalent bundle z, as required by the rule. □

Lemma 6.

Proof. Let $E \in \mathcal{E}^{\infty}_{A2}$ be such that, w.l.g., $\underline{t} = 0$. For each $k_{\bar{t}} \in \mathbb{R}^L_+$, there is $a \in P(E)$ such that: *i)* $a_{<\bar{t}}$ (defined above) satisfies *sequential permutation solidarity* among generation before $\bar{t}$ (due to Lemma 3); *ii)* $a_{\geq \bar{t}}$ treats all generations equally from $\bar{t}$ onwards (by A2), which is a stronger requirement than *sequential permutation solidarity*. Furthermore, assumptions on production and preferences guarantee that there is $k_{\bar{t}}$ such that, on top of *i)* and *ii)*, *sequential permutation solidarity* is also satisfied for $\bar{t}-1$ and $\bar{t}$. This proves the existence of an allocation satisfying *efficiency* and *sequential permutation solidarity* for each economy in the domain $\mathcal{E}^{\infty}_{A2}$. □

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