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Keep on Fighting: Dynamic Win Effects in an All-Pay Auction

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Last 10 Memoranda

No 22/14	John K. Dagsvik and Zhiyang Jia *Labor Supply as a Choice among Latent Jobs: Unobserved Heterogeneity and Identification*
No 21/14	Simen Gaure *Practical Correlation Bias Correction in Two-way Fixed Effects Linear Regression*
No 20/14	Rolf Aaberge, Tarjei Havnes and Magne Mogstad *A Theory for Ranking Distribution Functions*
No 19/14	Alice Ciccone *Is It All About CO2 Emissions? The Environmental Effects of Tax Reform for New Vehicles in Norway*
No 18/14	Mikolaj Czajkowski, Nick Hanley and Karine Nyborg *Social Norms, Morals and Self-interest as Determinants of Pro-environment Behaviours*
No 17/14	Karine Nyborg *Reciprocal Climate Negotiators: Balancing Anger against Even More Anger*
No 16/14	Karen Evelyn Hauge and Ole Røgeberg *Contribution to Public Goods as Individuals versus Group Representatives: Evidence of Gender Differences*
No 15/14	Moti Michael and Daniel Spiro *Skewed Norms under Peer Pressure: Formation and Collapse*
No 14/14	Daniel Spiro *Resource Prices and Planning Horizons*
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Keep on Fighting: Dynamic Win Effects in an All-Pay Auction*

Derek J. Clark, Tore Nilssen, and Jan Yngve Sand†

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Abstract

We investigate a multi-period contest model in which a contestant's present success gives an advantage over a rival in the future. How this win advantage affects contestants' efforts, and whether the laggard gives up or keep on fighting are key issues. We find that the expected effort of the laggard will always be higher than the rival at some stage in the series of contests, and this is most likely to happen when at a large disadvantage or at a late stage in the series.

Keywords: win advantage, motivation, contest, discouragement
JEL codes: D74, D72

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1 Introduction

Rivalry is prevalent - in the market, on the sports field, and in the workplace, to mention a few. Rivalry also stretches over time. When rivalry across time is interlinked, it is important to understand how the rivals' incentives to stay in the fight – to keep on fighting – develop as the rivalry progresses. We investigate a multi-period contest model in which a contestant's present success gives an advantage over a rival in the future. This win advantage creates an intertemporal effect, and we study how it affects contestants' efforts and whether the laggard gives up or keeps on fighting.

In practice, one may find several sources of win advantage in contests. For a sales force, it is not uncommon for the more successful agents to be given less administrative duties, better access to back-office resources, or more training than the less successful (Farrell and Hakstian, 2001; Krishnamoorthy, *et al.*, 2005). Skiera and Albers (1998) find that successful sellers may be given the best territories, giving a basis for future success. Another source of win advantage could be successful agents having access to different prizes than less successful (Megidish and Sela, 2014). Che and Gale (2003) note that successful researchers have more grant opportunities. A further source of win advantage may be psychological (Krumer, 2013). Experimental studies by Reeve, *et al.*, (1985) and Vansteenkiste and Deci (2003) show that winners feel more competent than losers, and that winning facilitates competitive performance and contributes positively to an individual's motivation. A winner may also adopt the role of incumbent, with a loser becoming a disadvantaged challenger (Konrad, 2002).[1]

How does such a win advantage affect contestants' incentives to exert effort? In each period of our contest model, there is a prize to win. We find that winning in any particular period gives the winner an advantage, over and above the prize itself, that has two aspects. One is the immediate effect, through the win advantage, of having a greater chance to win future contests and therefore to get hold of future prizes. The other is the value of being leader rather than laggard, *i.e.*, the leader's expected value of future prizes relative to what the laggard expects. While the former effect grows as a contestant nets more and more wins, the latter effect diminishes as the game moves toward the end. Thus, over time, the two effects interact in interesting ways that are spelled out below.

The immediate effect of the win advantage is to discourage future efforts by the contestants, but more so for the leader, since he now can win a contest with less expected effort than before because of his win advantage. The other effect works in the opposite direction: a large future value of being the leader discourages the laggard, since even with a win, he will

[1] See also Ofek and Sarvary (2003) and Mehlum and Moene (2006, 2008) for incumbency contests in various settings.

continue being the laggard, or at best be able to even the score. As the game moves towards the end, however, the future value of being the leader disappears and only the immediate effect prevails. Thus, in the last period, it is the laggard who puts in the more expected effort. On balance, we find that the laggard is discouraged when there are many rounds left to play and he is lagging only slightly behind the leader.

Our findings are in accordance with empirical evidence that laggards can exert more effort than leaders. Thus, Berger and Pope (2011) find evidence in professional and college basketball games that a team that is slightly behind at half-time has a discontinuous increase in its overall win probability. In a two-period experiment, they corroborate this finding. Tong and Leung (2002) conduct several experiments related to different specifications of a dynamic tournament model and suggest that "slacking off among those who trail is probably not common" (p. 417).

On the other hand, there are theoretical studies of races and best-of-t contests that emphasize the discouragement of the disadvantaged, leading to a "discouragement effect" of trailing behind; see Konrad (2009, 2012). In the situations modelled in this line of work, there is typically no period prize, as we have here. Rather, the link between contests over time is caused by a prize for the player who first score the required number of wins. This final prize means that the value of leading never wears off as the game moves to an end, as it does in our setting. Klumpp and Polborn (2006), for example, show, in a best-of-three election model, that the winner of the first election increases his probability of winning in the second to 75%. Deck and Sheremeta (2012) correspondingly show, in a game of siege, that a defender who wins early battles is most likely to be able to ward off future attempts by the attacker. Interestingly, Konrad and Kovenock (2009) show how the discouragement effect is dampened by the introduction of period prizes in a best-of-t contest, a result serving as a bridge over to our approach.

The paper is organized as follows. Section 2 presents the model, whereas Section 3 looks at a single-stage contest with an advantaged player. The equilibrium is characterized in Section 4, and we examine effort encouragement and discouragement in Section 5. Section 6 concludes. Some of our proofs are relegated to an Appendix.

2 Sequential contests

There are two identical players, $i = 1, 2$, who compete in a series of $T \geq 2$ all-pay auctions for a prize of v in each contest by making irreversible outlays $x_{i,t} \geq 0$, $t = 1, 2, ..., T$. The probability of winning for player 1 in contest t depends on current effort as well as on the history so far, summarized by the number of wins that player 1 has in the previous $t - 1$

contests. Every previous win makes it possible for him to win the current contest with less effort. In particular, the score for player 1 in contest t is given by the sum of his current effort $x_{1,t}$ and his cumulated win advantage that winning previous contests confers on him. Denote the win advantage from winning a previous contest by

$$s \in \left(0, \frac{v}{T-1}\right). \tag{1}$$

The upper bound will be assumed to hold throughout and is there to make sure that no subgame can occur in which no effort is exerted.

After having won m_t of the previous $t-1$ contests, player 1 has a current contest score of $x_{1,t} + m_t s$, whilst the other player has a score of $x_{2,t} + (t-1-m_t)s$. The contestant with the larger score wins the current contest; in particular, player 1 wins if $x_{1,t} + m_t s > x_{2,t} + (t-1-m_t)s$. The win probability for player 1 in contest t can thus be written as:

$$p_{1,t} = \begin{cases} 1 \text{ if } m_t s + x_{1,t} > (t-1-m_t)\, s + x_{2,t} \\ \frac{1}{2} \text{ if } m_t s + x_{1,t} = (t-1-m_t)\, s + x_{2,t} \\ 0 \text{ if } m_t s + x_{1,t} < (t-1-m_t)\, s + x_{2,t} \end{cases}$$

where $m_1 = 0$. The probability of player 2 winning is defined similarly.

For the analysis that follows, it is convenient to think of the net number of wins that a player has achieved. For player 1, define this as $M_t = m_t - (t-1-m_t)$. Without loss of generality, we shall assume that $M_t \geq 0$. Then the probability that player 1 wins contest t can be written

$$p_{1,t} = \begin{cases} 1 \text{ if } M_t s + x_{1,t} > x_{2,t} \\ \frac{1}{2} \text{if } M_t s + x_{1,t} = x_{2,t} \\ 0 \text{ if } M_t s + x_{1,t} < x_{2,t} \end{cases}$$

At contest t, the maximum number of net wins for player 1 is $t-1$, meaning that this player has won all of the previous $t-1$ contests. If player 1 has won all but one of the previous $t-1$ contests, then his net win advantage is $t-3$, whereas the net win advantage is $t-5$ if player 1 has won all but two of the previous contests, and so on.

3 A single contest with advantage

To get to grips with the series of contests, it is instructive to first look at one. Consider a single all-pay auction contest in which one player is advantaged in the double sense of achieving a probability of winning with a lower effort than the rival and having a larger value of the prize if he wins. Two players compete over a prize of value $v_1 = v + a$ for player 1 and $v_2 = v$ for player 2, where $v > 0$ and $a \geq 0$, by making irreversible outlays

$x_i, i = 1, 2$; the marginal cost of an outlay is fixed at 1. The probability that player 1 wins is given by

$$p_1 = \begin{cases} 1 \text{ if } z + x_1 > x_2 \\ \frac{1}{2} \text{ if } z + x_1 = x_2 \\ 0 \text{ if } z + x_1 < x_2 \end{cases},$$

where $z \geq 0$ is a bias parameter. The expected payoff for player 1 is then given as

$$E\pi_1 = \left[\Pr\left(z + x_1 > x_2\right) + \frac{1}{2} \Pr\left(z + x_1 = x_2\right) \right] v_1 - x_1,$$

with that of player 2 defined similarly.

Let $F_i(x_i)$ be the cumulative distribution function of player i's mixed strategy, $i = 1, 2$. The following proposition characterizes the unique Nash equilibrium (Clark and Riis, 1995; Konrad, 2002):

Proposition 1 *i) If $z \geq v$, then $x_1 = x_2 = 0$.*

ii) If $z < v$, then the unique mixed-strategy Nash equilibrium of the game is

$$F_1(0) = \frac{z}{v}; \quad F_1(x_1) = \frac{z + x_1}{v}, \quad x_1 \in [0, v - z]; \tag{2}$$

$$F_2(0) = \frac{z + a}{v + a}; \quad F_2(x_2) = \frac{x_2 + a}{v + a}, \quad x_2 \in [z, v]. \tag{3}$$

In this equilibrium, the expected amounts of effort of the players are

$$Ex_1^* = \frac{(v - z)^2}{2v}, \text{ and } Ex_2^* = \frac{v^2 - z^2}{2(v + a)}; \tag{4}$$

expected net surpluses are

$$E\pi_1^* = z + a, \text{ and } E\pi_2^* = 0; \tag{5}$$

and probabilities of winning are

$$p_1^* = 1 - \frac{v^2 - z^2}{2v\,(v + a)}, \text{ and } p_2^* = \frac{v^2 - z^2}{2v\,(v + a)}.$$

Quite unsurprisingly, we see from (5) that the advantaged player has more to gain from the contest. More interestingly, we see from (2) and (3) that the disadvantaged player 2 on one hand has a higher probability of being inactive but that he, conditional on being active, has a higher expected effort. This translates, by way of (4), into the following:

Corollary 1 *The disadvantaged player has the larger expected effort of the two if and only if*

$$a < \frac{2vz}{v-z}. \tag{6}$$

This says that the laggard has more effort than his rival when his disadvantage in terms of the value of winning is sufficiently weak relative to the prize and the disadvantage in terms of the win probability. This is evident from (2) and (3): whereas v and z affect the two players more or less in the same manner, a affects the disadvantaged player's effort only – the more disadvantaged he is in terms of the value of winning, the higher is the probability that he is inactive.

These results are used in the next sections to solve and analyze our model. In terms of the series of contests, z relates to the win advantage in a particular contest, whilst a will be the extra amount that the leader can win in the continuation of the game.

4 Equilibrium

The model is solved by backwards induction to find a Nash equilibrium at each stage of the game, using the results from the previous section. We present the structure of the solution for contest T, and then for a contest $t \geq 2$, before solving for the first contest.

4.1 Contest T

Let expected payoff be given by the function $u_{i,T}(M_T)$. Since this is the final contest, expected payoffs are

$$\begin{aligned} u_{1,T}(M_T) &= p_{1,T}v - x_{1,T}; \\ u_{2,T}(M_T) &= (1-p_{1,T})v - x_{2,T}. \end{aligned}$$

In the language of Proposition 1, this is a case where $a = 0$ and $z = M_T s$. Thus, expected efforts and payoffs in equilibrium are

$$\begin{aligned} Ex^*_{1,T}(M_T) &= \frac{(v - M_T s)^2}{2v}, \quad Ex^*_{2,T}(M_T) = \frac{v^2 - (M_T s)^2}{2v}; \\ Eu^*_{1,T}(M_T) &= M_T s, \quad Eu^*_{2,T}(M_T) = 0. \end{aligned} \tag{7}$$

Note that, from (7) – and in line with Corollary 1 – we can state the following:

Corollary 2 *The laggard has the higher effort in the last contest for any* $M_T \geq 1$.

Furthermore, total expected effort in contest T is

$$Ex^*_{1,T}(M_T) + Ex^*_{2,T}(M_T) = v - M_T s.$$

If $M_T = 0$, so that each player has won equally many of the previous contests, then the all-pay auction is symmetric and we have

$$\begin{aligned} Ex^*_{1,T}(M_T &= 0) = Ex^*_{2,T}(M_T = 0) = \frac{v}{2}; \\ Eu^*_{1,T}(M_T &= 0) = Eu^*_{2,T}(M_T = 0) = 0. \end{aligned}$$

4.2 Contest $t \in \{2, ..., T-1\}$

Consider now any contest $t = 2, ..., T-1$ in which $M_t \geq 1$, *i.e.*, player 1 has at least one more win than player 2 so far. The expected payoff for player 1 is now given by:

$$u_{1,t}(M_t) = p_{1,t}\left[v + Eu^*_{1,t+1}(M_t + 1)\right] + (1 - p_{1,t})\left[Eu^*_{1,t+1}(M_t - 1)\right] - x_{1,t};$$

That is, either he wins, receives the prize v for this contest, and improves his score; or he loses, receives no prize, and worsens his score. Quite straightforwardly, we can rewrite this as

$$u_{1,t}(M_t) = Eu^*_{1,t+1}(M_t - 1) + p_{1,t}(v + a_t) - x_{1,t},$$

where

$$a_t \equiv Eu^*_{1,t+1}(M_t + 1) - Eu^*_{1,t+1}(M_t - 1).$$

Note that, if $M_t = 1$, then $Eu^*_{1,t+1}(M_t - 1) = 0$, since the contest in $t+1$ becomes symmetric if the advantaged player 1 loses contest t in this case.

Player 2 is at a disadvantage, being at least one net win down. If he wins the current contest, then he gains the stage prize v and improves his score, or rather worsens the score of his rival. But even with a win, he will continue as the disadvantaged player earning zero, or at best – if winning at $M_t = 1$ – getting even, but still earning zero. Thus, the payoff to player 2 is given by

$$u_{2,t}(M_t) = (1 - p_{1,t})\, v - x_{2,t}.$$

At contest t, $z = M_t s$ measures the bias in the probability of winning, and $a = a_t$ is the extra prize that player 1 has, relative to player 2, from winning the current stage. Note that the advantaged player has an expected gross payoff of $Eu^*_{1,t+1}(M_t - 1)$, no matter the outcome of the stage contest.

If $M_t = 0$, then the game is symmetric. Neither player has a bias in the win probability, implying that the expected equilibrium payoff from the current stage is zero. In this case, the expression for player i's payoff needs to be modified to

$$u_{i,t}(M_t = 0) = p_{i,t}\left[v + Eu_{i,t+1}(1)\right] - x_{i,t}, \qquad (8)$$

since the continuation payoff of losing from this state is 0. In this case, the contest is symmetric over a prize of $[v + Eu_{i,t+1}(1)]$ for each player, and each player has an expected effort of $\frac{1}{2}[v + Eu_{i,t+1}(1)]$, with an expected payoff of 0. Since, by definition, $M_1 = 0$, (8) holds for the first contest at $t = 1$.

4.3 The full game

Proposition 2 summarizes the equilibrium expected efforts and expected payoffs of the T sequential contests. The proof, which is based on Proposition 1, is in the Appendix.

Proposition 2 *In the subgame perfect equilibrium, in contest $t = 2, ..., T$, with $M_t \geq 1$, the expected efforts of the players are given by*

$$Ex^*_{1,t}(M_t) = \frac{(v - M_t s)^2}{2v}, \tag{9}$$

$$Ex^*_{2,t}(M_t) = \frac{v^2 - (M_t s)^2}{2(v + a_t)} = \frac{v^2 - (M_t s)^2}{2[v + 2(T - t)s]}; \tag{10}$$

with equilibrium expected payoffs

$$Eu^*_{1,t}(M_t) = s\left[(T - t + 1) M_t + \sum_{j=0}^{T-t} j\right], \tag{11}$$

$$Eu^*_{2,t}(M_t) = 0.$$

For contest $t = 1$, and for any contest in which $M_t = 0$, expected efforts and payoffs are given by

$$Ex^*_{1,t}(M_t = 0) = Ex^*_{2,t}(M_t = 0) = \frac{1}{2}\left[v + s\left(T - t + \sum_{j=0}^{T-t-1} j\right)\right]; \tag{12}$$

$$Eu^*_{1,t}(M_t = 0) = Eu^*_{2,t}(M_t = 0) = 0. \tag{13}$$

At the outset, $t = 1$, the contest is symmetric; the contestants have an expected effort that far exceeds the value of the stage prize v since they each want to be the advantaged player in contest $t = 2$, with the possibility of compounding this early win advantage. The expected payoff in equilibrium for the game as a whole is zero, so that the players compete away the whole surplus in the course of the game. This leads to the following corollary:

Corollary 3 *The total expected effort over the T contests is vT.*

From any symmetric state $M_t = 0$, (12) indicates that there is intense competition to get the game onto a favorable track. When the contest is asymmetric, two factors play a role: the bias in the probability function, $z_t = M_t s$, and the difference in the prize between the two players,[2]

$$a_t = 2(T - t)s. \tag{14}$$

Note that the extra payoff to winning contest t, a_t, depends on the number of contests left after this contest and the size of the advantage per win and is independent of the number of net wins at that contest. Whereas an increase in the bias z_t decreases the expected efforts of both players, increasing the prize difference a_t only affects the expected effort of the laggard, and negatively so. Hence the lead in the contest as measured by M_t reduces the expected effort of the leader and the laggard; the fact that the leader has more to gain due to a positive continuation payoff only reduces the effort of the laggard. Note from (12) that both players fight hard to win the first contest, to get the contest on to a favorable track, with expected effort in sum that exceeds the size of the stage prize. Intense competition characterizes any symmetric contest (i.e., where $M_t = 0$), again by (12); however, expected efforts fall when this happens further into the contest. The less future there is in the contest, the less value there is to being the leader. To illustrate this, consider an example.

Example 1 $v = 1, T = 8, s = 0.05$

Write $EX_t^*(M_t = 0) = Ex_{1,t}^*(M_t = 0) + Ex_{2,t}^*(M_t = 0)$. This gives the following table of total expected effort for tied states:

Contest	$EX_t^*(M_t = 0)$
1	2.4
3	1.75
5	1.3
7	1.05

The expected payoff of the advantaged player from contest t has a simple form, as indicated by (11). In this expression, $T - t + 1$ is the number of contests remaining when we reach contest t. Hence, the expected payoff in equilibrium to the player with a net win advantage is conveniently expressed as a function of the number of remaining contests, the number of net wins at that stage, and the size of the advantage per win.

[2] This expression for a_t is shown in the Appendix.

5 Effort encouragement and discouragement

We can use Proposition 2 together with Corollary 1 to show the following two results:

Corollary 4 *In any contest $t \geq 2$ where $M_t \geq 1$, the laggard has higher expected effort than the leader if and only if*

$$T - t < \frac{vM_t}{v - M_t s}. \tag{15}$$

Corollary 5 *When $T = 3$, the expected effort of the laggard is larger than the leader at $t = 2$.*

Together, Corollaries 2 and 5 deal with cases of a short series of contests. When the series consists of two contests, the laggard will always exert more effort in expectation than the leader in the final contest. When the series consists of three contests, the laggard will always have more expected effort than the leader in the second contest, and also in the final one, should he still be disadvantaged at this stage. From (9) and (10), it can be verified that the win advantage, as measured by M_t, reduces the expected effort of the leader by more than the laggard. Modifying this effect is the fact that the winner of the first contest has more to fight for as measured by a_2, which is zero when $T = 2$, and $2s$ when $T = 3$. Hence there is no effect on the expected effort of the laggard through this channel in the former case, and a negative effect in the latter. In sum, however, the expected effort of the leader falls most in such short series of contests.

Corollary 4 deals with the more general case. From this we can conclude that the laggard has more effort than the leader in cases where

- he is at a large disadvantage (large M_t),
- there are a low number of contests left (low $T - t$),
- the stage prize v is low.

These results reflect the findings in Section 3 above: When there are relatively few contests left, the difference in valuation between winning and losing, a_t, becomes small. The value of a_t affects the laggard's effort negatively but does not affect the leader's effort, whereas the bias M_t affects both expected efforts negatively. It can easily be verified that the negative effect that increasing M_t has on the leader's effort is larger in magnitude than the reduction in that of the laggard. Hence the leader slacks off by more than the laggard is discouraged following an increase in the net win. The role of the size of the win advantage s is more subtle, since it leads to more bias in the probability function, causing less effort by both competitors, at the same time as it increases a_t which reduces only the laggard's effort. The larger is s, the more a_t falls in each successive contest, which raises the effort of the laggard. Hence, although increases in M_t and s lead to a higher likelihood that the laggard will have more effort, they work through different channels.

We also note that our results are partly driven by the fact that competitors can win a prize at each stage. This will generally raise the expected effort level for both players. The comparative-static properties of (9) and (10) show that an increase in v will tend to raise the expected effort of the leader relative to the follower when there are many contests left, and that the laggard's effort will be raised the most in later stages of the contest. Early in the series of contests, a leader has a great deal to fight for, since $a_t = 2(T - t)s$ is large, and increasing v strengthens this effect. Later on, a_t falls, giving the laggard more to fight for.

5.1 The unluckiest loser

Many trajectories of the game are possible, depending upon who wins each stage. One extreme case is that of the "unluckiest loser", *i.e.*, a player who has lost each contest to date. Suppose that, at the start of contest t, player 2 has lost each previous contest so that $M_t = t - 1$. One's intuition might dictate that this loser will eventually give up. Corollaries 2 and 5 indicate that this is not the case in a short series of contests. When $T = 2$, the unluckiest loser loses the first contest and has more effort in expectation than the rival. When $T = 3$, the loser of the first contest has more expected effort in contest 2, as well as in contest 3 should he lose the second contest. The next proposition extends this to longer series of contests, showing that there will always come a time, before the last contest, at which the effort of the unluckiest loser outstrips that of his winning opponent. Furthermore, the laggard who keeps losing will have more expected effort for the duration of the contest. The proof is in the Appendix.

Proposition 3 *Suppose that $T \geq 4$.*

(i) There exists a $\widehat{t} \in \{3, ..., T-1\}$ such that, if $M_t = t-1$ for some $t \in \{2, ..., T\}$, then $Ex^*_{1,t}(M_t) > Ex^*_{2,t}(M_t)$ if $t < \widehat{t}$, and $Ex^*_{2,t}(M_t) > Ex^*_{1,t}(M_t)$ if $t > \widehat{t}$.

(ii) The time $\widehat{t}$ decreases in s and increases in T.

In part (i) of Proposition 3, we find a contest denoted by $\widehat{t}$ such that the expected effort of the unluckiest loser will outstrip that of the leader. Furthermore, continuing to lose gives a higher effort in expectation from the laggard. Consider the forms of (9) and (10), and suppose player 2 is lagging behind. Each of these expressions is decreasing in the bias term M_t.

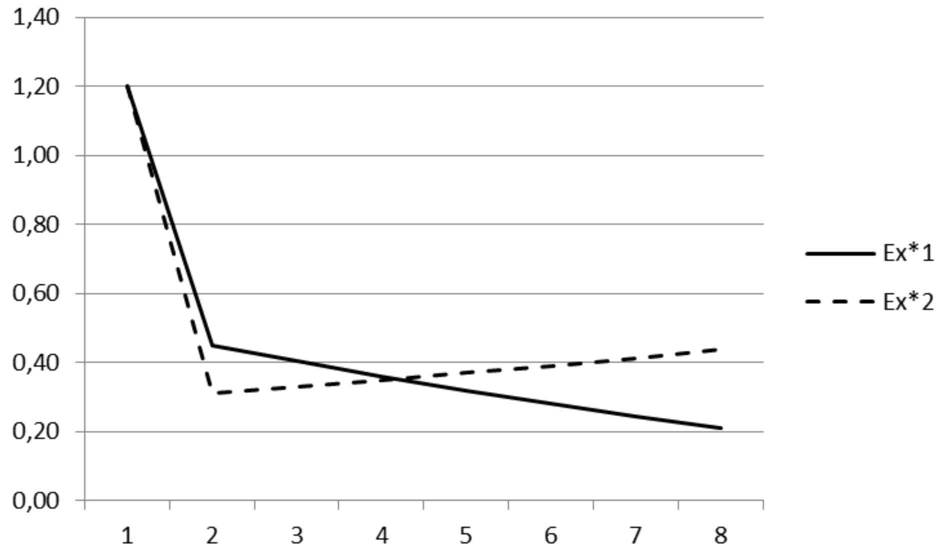

Figure 1: Efforts in the case of the unluckiest loser

As the sequence of contest progresses and the bias gets bigger, this exerts downward pressure on each of the players' efforts. Note, however, that the expected effort of player 2 also increases relative to previous contests, since $a_t = 2(T - t)s$ falls as we progress through the contest sequence; $T - t$ measures the number of contests remaining. The continuation value to winning for the advantaged player falling as the game progresses spurs the unluckiest loser on to more effort, causing the expected effort paths to cross. This is illustrated in Figure 1, where $T = 8, v = 1$, and $s = 0.05$.

Initially both players have a high expected effort in order to become the advantaged player from contest $t = 2$. After this, the expected effort of each player falls, with the loser of the first contest having the largest fall. As the bias increases, the luckiest winner decreases expected effort successively; this effect also exerts downward pressure on the expected effort of the laggard, but the positive effect – that winning matters less and less to the advantaged player – outweighs this. Hence, the effort of the laggard increases across contests. In the example, the unluckiest loser has the larger expected effort in each period from $t = 5$ on.

The first effect in part (ii) of Proposition 3 says that the crossing of expected effort will be earlier, the higher is s. This is due to the fact that a large s gives both a large win bias in the probability, and a large continuation value of winning to the leader. The former effect makes both players exert less effort, with the larger effect on the leader. The latter effect makes the leader's continuation value fall quickly so that the leader has less to gain from successive wins. This encourages even the unluckiest loser.

5.2 Relative effort of winners and losers

In this section we look at how being a winner or a loser affects relative efforts of the two competitors. Above, it was noted that a player who is disadvantaged in the final contest always has the larger expected effort. We now analyze the relative efforts of the advantaged and the disadvantaged player. The case in which $T \in \{2,3\}$ has been dealt with above, so that longer sequences are considered here. The results are summed up in the following proposition, with proof in the Appendix.

Proposition 4 *Suppose* $T \geq 4$.

(i) There is always one contest t *in the series such that* $t \leq T-1$, $M_t \geq 1$, *and* $Ex^*_{2,t}(M_t) > Ex^*_{1,t}(M_t)$.

(ii) If $t \leq T-1$, $M_t \geq 1$, *and* $Ex^*_{2,t}(M_t) > Ex^*_{1,t}(M_t)$, *then* $Ex^*_{2,t+1}(M_t+1) > Ex^*_{1,t+1}(M_t+1)$.

(iii) If $t \leq T-2$, $M_t \geq 2$, *and* $Ex^*_{2,t}(M_t) > Ex^*_{1,t}(M_t)$, *then* $Ex^*_{1,t+1}(M_t-1) > Ex^*_{2,t+1}(M_t-1)$.

(iv) If $t \leq T-1$, $M_t \geq 2$, *and* $Ex^*_{2,t}(M_t) > Ex^*_{1,t}(M_t)$, *then it is possible to have* $Ex^*_{1,t+1}(M_t-1) > Ex^*_{2,t+1}(M_t-1)$.

Part (i) states that the expected effort of a laggard will always be larger than that of the advantaged player at some stage in the series of contests before the final stage. Hence, the results from the example of the unluckiest loser hold in the game generally, and are not just due to the specific trajectory chosen there. Again the intuition is based upon the two effects: the bias which reduces both efforts, and that of the leader more, and the reduction in the continuation payoff for the leader in the series, which encourages the laggard.

Part (ii) states that, if the laggard has more expected effort in contest t and loses, then he will also have more expected effort in the following contest. The transition from contest t to $t+1$ here implies an increased win bias causing more slacking off by the leader, while the progression of the contest lowers the continuation value of the leader.

Part (iii) looks at the case in which the leader has the more expected effort in contest t; should he lose this contest, then, given that he is still advantaged, he will continue to have the more effort in the next contest, as long as the game by then has not reached the final contest; recall that the laggard always has more effort in contest T. In this case, the transition of the contest from t to $t+1$ implies a smaller win bias; both expected efforts increase, affecting the leader more.

Part (iv) looks at the case in which the laggard has more expected effort in a contest; if he *wins* the contest and is still disadvantaged, then it is possible for this player to have less expected effort than the rival in the next contest.

Parts (ii) and (iii) of Proposition 4 can be combined to show that the sign of the difference in efforts of the players is invariant to loss in the following sense:

Corollary 6 *Suppose $T \geq 4$. Irrespective of who has the more expected effort in contest t, with $M_t \geq 1$, if this player loses that contest, then he will have more expected effort also in contest $t+1$, unless $t = T-1$.*

6 Conclusion

By using the all-pay auction as a basic stage contest, we have investigated the effects of winning and losing on effort in a finite series of contests. This extends previous work on contest series since much of this relies on a two-period structure; often the two-period structure is analyzed since a stage contest is often defined in the Tullock form which is cumbersome to work with when periods become interlinked.[3] Clark et al (2014) investigate the win advantage in a two-period model with the Tullock structure, with the focus on the optimal division of a prize mass. Megidish and Sela (2014) look similarly at the effect of making different prizes available for winners and losers. Ridlon and Shin (2013) consider whether the winner or loser in a first contest should be helped or handicapped in the second when ability is not perfectly observable. In our all-pay auction model, we have assumed *ex-ante* symmetric players who then become asymmetric in the game setting. The expected total amount of effort is equal to the total prize mass, making this mechanism quite efficient at giving effort incentives. Our concern here has been with the distribution of effort over time between the leader and the laggard as a contribution to the debate about whether a favorite slacks off, and whether a laggard gets discouraged. Our results show that the disadvantage can be a spur to extra relative effort, and that – at some stage – we should expect a weak performer to outstrip the rival in terms of effort.

The win advantage that we introduce here works through two channels. First, being a net winner gives a player an increased probability of success in the next contest. Several sources of such bias can be suggested related to psychological effects, access to higher prizes for proven contestants, or more focussed contestants. This type of bias tends to make both the advantaged and the disadvantaged player slack off, but is shown to affect the advantaged player more. This is one channel through which the laggard may be able to win in spite of the disadvantage. Additionally, the win advantage gives the leader a larger prize to fight for; this player can not

[3]The Tullock win probability typically defines the probability of winning as a player's effort relative to the sum of efforts. See Skaperdas (1996).

only win a stage prize, but has a positive continuation payoff, whereas the laggard only fights for the stage prize. This is shown to only affect the expected effort of the laggard, so that the larger this continuation payoff, the less effort that this player exerts in expectation. However, the size of the continuation payoff shrinks as the series of contests progresses so that the laggard becomes less discouraged in each contest. These two effects combine to give the result that at some stage in the contest, the laggard will have more expected effort than the leader. This even happens in the case of the "unluckiest loser" in which a player has lost each previous contest. That even this contestant does not simply give up is a stark result from the analysis.

Our results have relevance for worker motivation, and in particular for managers who want to design motivational schemes that keep all workers encouraged throughout. But the analysis is also pertinent in settings where firms compete for customers with the introduction of a new product or a new technology. There may be an advantage to winning an early contest, due to winning a customer base, for example. An established base of customers benefits the firm in market with network effects but may also play a role with new products as winning customers and being able to show that the new product catches on may result in an advantage in subsequent rounds. Examples here can be the competition between the different operating systems Symbian, Android, and iOS for customers buying mobile handsets or tablets. As more customers adopt handsets or tablets with Android technology, more apps are developed that increase the value for customers in the next contest and subsequently raise the potential value from producing and selling an Android handset or tablet. This generates an advantage for the firms using Android OS in their products. Another variant of this is the use of loyalty programmes, leading to firms competing to lock-in customers in their programmes, and then compete for volume of business (either using price, quantity, quality of other product attributes). Customers becoming locked-in causes competition for volume to be less intense than without such locked-in customers. This may be seen as an advantage for the firms, although Chen and Xie (2007) show that an asymmetry in customer loyalty may have both a positive and a negative effect.

A Appendix

A.1 Proof of Proposition 2

Consider contest $T-1$. If $M_{T-1} \geq 1$, then the expected payoffs in contest $T-1$ are

$$\begin{aligned}
u_{1,T-1}(M_{T-1}) &= p_{1,T-1}\left[v + Eu_{1,T}\left(M_{T-1}+1\right)\right] \\
&\quad + \left(1-p_{1,T-1}\right) Eu_{1,T}\left(M_{T-1}-1\right) - x_{1,T-1} \\
&= Eu_{1,T}\left(M_{T-1}-1\right) \\
&\quad + p_{1,T-1}\left[v + Eu_{1,T}\left(M_{T-1}+1\right) - Eu_{1,T}\left(M_{T-1}-1\right)\right] - x_{1,T-1} \\
u_{2,T-1}(M_{T-1}) &= \left(1-p_{1,T-1}\right) v - x_{2,T-1}
\end{aligned}$$

Through the win advantage, player 1 has a guaranteed payoff of $Eu_{1,T}(M_{T-1}-1)$ if he loses contest $T-1$. If player 1 wins contest $T-1$, then he gets the instantaneous prize v and the continuation value in contest T, with $M_T = M_{T-1}+1$. Should player 1 lose contest $T-1$, then he gets no instantaneous prize but receives the continuation value from the net number of wins $M_T = M_{T-1}-1$ in the next contest.

Since $M_{T-1} \geq 1$, then, if player 2 wins, he receives the instantaneous prize v, and the net win for player 1 is $M_{T-1}-1 \geq 0$ in contest T; the continuation value for player 2 is zero in the final contest whatever.

The extra value to player 1 from winning contest $T-1$ is thus given by $Eu_{1,T}\left(M_{T-1}+1\right) - Eu_{1,T}\left(M_{T-1}-1\right)$; commensurate with the notation in Section 3, denote this extra value to winning by a_{T-1}. Using the results from contest T in Section 4.1, we have that $a_{T-1} = 2s$; note that this is independent of the number of net wins in this contest. From Proposition 1, we now find expected efforts and payoffs in contest $T-1$ as

$$\begin{aligned}
Ex_{1,T-1}(M_{T-1}) &= \frac{\left(v - M_{T-1}s\right)^2}{2v} \\
Ex_{2,T-1}(M_{T-1}) &= \frac{v^2 - \left(M_{T-1}s\right)^2}{2\left(v+2s\right)} \\
Eu_{1,T-1}(M_{T-1}) &= Eu_{1,T}\left(M_{T-1}-1\right) + \left(M_{T-1}+2\right) s \\
&= \left(M_{T-1}-1\right) s + \left(M_{T-1}+2\right) s \\
&= \left(2M_{T-1}+1\right) s \\
Eu_{2,T-1}(M_{T-1}) &= 0
\end{aligned}$$

Using (5), we can stipulate the form of the equilibrium expected payoff for player 1 in contest t to be:

$$\begin{aligned}
Eu^{*}_{1,t}(M_t) &= Eu_{1,t+1}\left(M_t-1\right) + a_t + M_t s \\
&= Eu_{1,t+1}\left(M_t+1\right) + M_t s
\end{aligned}$$

Calculating the expected payoffs recursively backwards reveals a pattern for the equilibrium expected payoff in each contest

$$\begin{aligned}
Eu_{1,T}(M_T) &= M_T s \\
Eu_{1,T-1}(M_{T-1}) &= (2M_{T-1}+3)\,s \\
Eu_{1,T-2}(M_{T-2}) &= (3M_{T-2}+6)\,s \\
Eu_{1,T-3}(M_{T-3}) &= (4M_{T-3}+10)\,s \\
&\;. \\
&\;. \\
Eu_{1,T-k}(M_{T-k}) &= \left((k+1)\,M_{T-k}+\sum_{j=0}^{k} j\right) s
\end{aligned}$$

This is rewritten in the more convenient form (11) in the Proposition.

In order to examine the equilibrium expected efforts for the advantaged and disadvantaged player, we simply need to identify the parameters in (4) for each contest. The bias term z is $M_t s$, and we need to calculate the difference to the leader from winning and losing the current contest, a_t.

It is convenient to consider how a_t is determined using (11):

$$a_t = Eu_{1,t+1}(M_t+1) - Eu_{1,t+1}(M_t-1) \tag{A1}$$

From (11), we have

$$Eu_{1,t+1}(M_{t+1}) = \left([T-(t+1)+1]\,M_{t+1} + \sum_{j=0}^{T-(t+1)} j\right) s \tag{A2}$$

Applying (A2) in (A1), replacing M_{t+1} by first M_t+1 and then M_t-1, gives

$$\begin{aligned}
a_t &= [T-(t+1)+1]\,[M_t+1-(M_t-1)]\,s \\
&= 2(T-t)s
\end{aligned}$$

Putting $z = M_t s$ and $a = a_t$ into (4) gives the expected efforts in the Proposition assuming that player 1 is leader.

Symmetric cases, in which $M_t = 0$, are dealt with in the main text.

A.2 Proof of Proposition 3

Part (i). Consider contest t, and suppose player 2 has lost all the previous $t-1$ contest, so that $m_t = M_t = t-1$. The difference in effort between leader and laggard is, from Proposition 2,

$$
\begin{aligned}
& Ex^*_{1,t}(t-1) - Ex^*_{2,t}(t-1) \\
= & \frac{[v-(t-1)s]^2}{2v} - \frac{v^2-(t-1)^2 s^2}{2[v+2(T-t)s]} \\
= & \frac{s[v-s(t-1)]}{v[v+2s(T-t)]}\left\{st^2 - [s(T+1)+2v]t + [v+T(s+v)]\right\}
\end{aligned}
$$

By the assumption in (1), $v - s(t-1) > 0$. It follows that the above expression has the same sign as the one inside curly brackets. Disregarding for now that t is integer, that expression, in turn, is a convex function of t, with negative slope and positive value at zero. It thus has two real roots in t, both positive, which we call $\bar{t} > \underline{t} > 0$. Moreover, $Ex^*_{1,t}(t-1) - Ex^*_{2,t}(t-1) < 0$ if and only if $\bar{t} > t > \underline{t}$.

In order to prove the Proposition, we need to show that $\bar{t} > T$, and that $2 < \underline{t} < T$. It is readily verified that

$$
\begin{aligned}
\bar{t} &= \frac{1}{2s}\left[2v + s(T+1) + \sqrt{s^2(T-1)^2 + 4v^2}\right], \text{ and} \\
\underline{t} &= \frac{1}{2s}\left[2v + s(T+1) - \sqrt{s^2(T-1)^2 + 4v^2}\right]. \qquad \text{(A3)}
\end{aligned}
$$

We first show that $\bar{t} > T$. Consider

$$
\begin{aligned}
\bar{t} &> T \\
&\iff \frac{1}{2s}\left[2v + s(T+1) + \sqrt{s^2(T-1)^2 + 4v^2}\right] - T > 0 \\
&\iff \frac{1}{2s}\left[2v - s(T-1) + \sqrt{s^2(T-1)^2 + 4v^2}\right] > 0 \\
&\iff \sqrt{s^2(T-1)^2 + 4v^2} + 2v > s(T-1)
\end{aligned}
$$

By (1), the right-hand-side of the inequality is at most v, whilst the left-hand-side is at least $4v$. Hence $\bar{t} > T$.

We next show that $\underline{t} < T$. Consider

$$
\begin{aligned}
T &> \underline{t} \\
&\iff T - \frac{1}{2s}\left[2v + s(T+1) - \sqrt{s^2(T-1)^2 + 4v^2}\right] > 0 \\
&\iff \frac{1}{2s}\left[-2v + s(T-1) + \sqrt{s^2(T-1)^2 + 4v^2}\right] > 0 \\
&\iff s(T-1) + \sqrt{s^2(T-1)^2 + 4v^2} > 2v
\end{aligned}
$$

where $\sqrt{s^2(T-1)^2 + 4v^2} \geq 2v$, so the inequality holds.

We finally show that $\underline{t} > 2$. Consider

$$\begin{aligned} \underline{t} &> 2 \\ \iff & \frac{1}{2s}\left[2v + (T+1)s - \sqrt{s^2(T-1)^2 + 4v^2}\right] - 2 > 0 \\ \iff & \frac{1}{2s}\left[2v + (T-3)s - \sqrt{s^2(T-1)^2 + 4v^2}\right] > 0 \end{aligned}$$

In the proposition we have $T \geq 4$. Let

$$\Phi(s,T,v) \equiv 2v + (T-3)s - \sqrt{s^2(T-1)^2 + 4v^2}.$$

Note that Φ is continuous in s, that $\Phi(0,T,v) = \Phi(v\frac{T-3}{T-2},T,v) = 0$, and that $\Phi(s,T,v) > 0$ for $v\frac{T-3}{T-2} > s > 0$. By (1), we have $\frac{v}{T-1} > s$. Since $v\frac{T-3}{T-2} > v\frac{1}{T-1}$ for any $T \geq 4$, we have $\Phi(s,T,v) > 0$ for permissible parameter values, proving $\underline{t} > 2$.

It follows that $T > \underline{t} > 2$. This must also hold if we make the restriction to integer values. Thus, $\widehat{t} \in \{3,...,T-1\}$.

Part (ii). Differentiations in (A3) give $\frac{\partial \underline{t}}{\partial s} < 0$ and $\frac{\partial \underline{t}}{\partial T} > 0$. With the restriction to integer values, the signs of the differences still hold, although weakly so.

A.3 Proof of Proposition 4

Part (i). The laggard has more expected effort if the condition in (15) is fulfilled. This is least likely to be satisfied for $M_t = 1$, in which case the condition can be written as

$$t > T - \frac{v}{v-s}.$$

Clearly, $T - \frac{v}{v-s} < T - 1$, since $\frac{v}{v-s} > 1$.

Part (ii). The laggard having more expected effort means, from (15), that

$$M_t\left[v + s(T-t)\right] - v(T-t) > 0. \tag{A4}$$

If the laggard loses, then $M_{t+1} = M_t + 1$, and the left hand side of the inequality for contest $t+1$ can be written as

$$\begin{aligned} (M_t+1)\left[v + s(T-t-1)\right] - v(T-t-1) &= \\ \left[M_t(v + s(T-t)) - v(T-t)\right] + \left[2v - M_t s\right] + s(T-t-1) &> 0 \end{aligned}$$

where the inequality follows since the first square-bracketed term is positive by (A4), and the second one is positive by (1).

Part (iii). In contest t, we have $M_t\left[v + s(T-t)\right] - v(T-t) < 0$, since the leader has more effort in this period. By the leader losing we get

$M_{t+1} = M_t - 1$, and the left hand side of the inequality for period $t+1$ becomes

$$\begin{aligned} (M_t - 1)(v + s(T - t - 1)) - v(T - t - 1) &= \\ [M_t (v + s(T - t)) - v(T - t)] - M_t s - s(T - t - 1) &< 0. \end{aligned}$$

Part (iv). If the laggard has more effort in contest t, then

$$T - t < \frac{vM_t}{v - M_t s},$$

by (15). If the laggard wins this contest, then $M_{t+1} = M_t - 1$, and the leader has more effort in contest $t+1$ if

$$T - t - 1 > \frac{v(M_t - 1)}{v - (M_t - 1)s}.$$

For these two inequalities to be consistent requires

$$\begin{aligned} \frac{v(M_t - 1)}{v - (M_t - 1)s} + 1 &< \frac{vM_t}{v - M_t s} \Longleftrightarrow \\ \frac{v(M_t - 1)}{v - (M_t - 1)s} - \frac{v(M_t - 1) + M_t s}{v - M_t s} &< 0 \Longleftrightarrow \\ -s\frac{v(M_t - 1) + [v - (M_t - 1)s] M_t}{[v - (M_t - 1)s](v - M_t s)} &< 0, \end{aligned}$$

which is clearly true, by (1).

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