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Last 10 Memoranda

No 29/14	Lars Kirkebøen, Edwin Leuven and Magne Mogstad *Field of Study, Earnings, and Self-Selection*
No 28/14	Erik Biørn *Serially Correlated Measurement Errors in Time Series Regression: The Potential of Instrumental Variable Estimators*
No 27/14	Erik Biørn *The Price-Quantity Decomposition of Capital Values Revisited: Framework and Examples*
No 26/14	Olav Bjerkholt *Econometric Society 1930: How it Got Founded*
No 25/14	Nils Chr. Framstad *The Effect of Small Intervention Costs on the Optimal Extraction of Dividends and Renewable Resources in a Jump-Diffusion Model*
No 24/14	Leif Andreassen, Maria Laura Di Tommaso and Steinar Strøm *Wages Anatomy: Labor Supply of Nurses and a Comparison with Physicians*
No 23/14	Derek J. Clark, Tore Nilssen and Jan Yngve Sand *Keep on Fighting: Dynamic Win Effects in an All-Pay Auction*
No 22/14	John K. Dagsvik and Zhiyang Jia *Labor Supply as a Choice among Latent Jobs: Unobserved Heterogeneity and Identification*
No 21/14	Simen Gaure *Practical Correlation Bias Correction in Two-way Fixed Effects Linear Regression*
No 20/14	Rolf Aaberge, Tarjei Havnes and Magne Mogstad *A Theory for Ranking Distribution Functions*

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Public Education and Pensions in Democracy: A Political Economy Theory*

Francesco Lancia Alessia Russo

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Abstract

This paper presents a dynamic politico-economic theory of fiscal policy to explain the simultaneous existence of public education and pensions in modern democracies. The driving force of the model is the intergenerational conflict over the allocation of the public budget. Successive generations of voters choose fiscal policies through repeated elections. The political power of elderly voters creates the motive for adults to support public investment in the human capital of future generations, since it expands future pension possibilities. We characterize the Markov perfect equilibrium of the voting game in a small open economy. The equilibrium can reproduce qualitative and quantitative features of intergenerational fiscal policies in modern economies.

Keywords: Intergenerational conflict, Markov perfect equilibrium, pension, public education, repeated voting, small open economy.

JEL Classification: D72, E62, H23, H30, H53.

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1 Introduction

In all democracies today, a central concern for governments is the public financing of education and pensions. These two programs are targeted towards different cohorts. While public education is an investment in future generations, public pensions are a transfer to the past generation. When both programs are financed by the current generation via taxation, an intergenerational conflict over the financing and the allocation of the public budget arises. As in modern democracies agents vote on fiscal policy at the election, but neither do future generations vote, nor is the past generation the majority of the electorate, the following question naturally arises. Why do democratic institutions implement public education and pensions?

Ever since the seminal paper of Pogue and Sgotz (1977), a vast literature has intensely debated the link between public education and pensions. The normative approach in this literature justifies the existence of the link between the two spending programs as a means to support complete market allocations (see, e.g., Becker and Murphy, 1988; Boldrin and Montes, 2005). Public education and pensions are treated as exogenous policies. In modern democracies, however, fiscal policies are endogenously determined by policymakers without committing to future policies. The positive approach in this literature provides relevant explanations for the endogenous emergence of public education and pensions. Some are based on altruism (see, e.g., Tabellini, 1991), while others focus on reputational concerns (see, e.g., Bellettini and Berti Ceroni, 1999; Rangel, 2003). They build on the idea that collective decisions are based on generosity, and transfers are linked to past contributions. In real world, however, voters are anonymous and governments have short-term mandates. An alternative and more macro-oriented positive approach examines the endogenous emergence of policies in the absence of altruism and reputational concerns. It highlights price channeling as the incentive device through which intergenerational exchanges emerge (see, e.g., Gonzalez-Eiras and Niepelt, 2012). In modern economies, however, financial markets are integrated and worldwide prices are at most weakly responsive to domestic policies.

We aim to construct a model with endogenous fiscal policies in the absence of altruism, commitment, reputation, and price channeling. Our view of modern democracies is as follows. Short-term mandate governments need to be attentive to the well-being of adults and the elderly because individuals in both groups can vote. Adult voters are motivated to support investments in the human capital of future generations insofar as they anticipate that this investment will expand the financing possibilities of social security programs. Elderly voters are motivated to support social security programs. Short-term mandate governments, therefore, use their fiscal authority to finance both programs. The allocation of the public budget serves policymakers' goals of winning elections by addressing the economic needs of constituents.

The key feature of this intergenerational mechanism is the government's strategic use of human capital. Policymakers can directly manipulate the human capital stock inherited by their successors through their choice to invest in education and in turn impact future pensions. This strategic use of fiscal authority allows policymakers to buy the vote of adults without jeopardizing votes from the elderly. A government's strategic use of human capital, however, depends fundamentally on the distribution of political power among all currently-living voters. Indeed, human capital investments are not provided if only adults or only elderly voters exert political power. In the former case, the incentive for adults to invest in future generations disappears if a political rent for the elderly in the form of pension benefits cannot be extracted from younger generations. In the latter case, the elderly fiercely oppose spending on public education, as they receive no direct benefit from it. When the distribution of power across cohorts does not favor one generation or the other, the existence of a political rent for the elderly can stimulate public education insofar as it garners political support for growth-oriented policies. The relation between human capital and pensions can be observed in the correlations from our regressions for OECD countries (see Appendix A.1). The regressions show that an increase in the level of human capital increases the generosity of pension transfers.[1]

We embed this intergenerational mechanism in a dynamic model of human capital accumulation à la Boldrin and Montes (2005). The basic structure is that of an overlapping generations world where individuals live for three periods: young, adult, and elderly. They acquire skills during the first period, offer elastic labor and partially save their proceeds in the working-age period, and receive a pension benefit on top of return from private savings in the retirement age period. Except for their age and economic role in society, agents are identical.

We depart from Boldrin and Montes (2005) set-up in two main respects. First, we endogenize public education and pension policies by introducing electoral competition. At any date, adults and the elderly appoint a short-term mandate government according to a majority rule. We model electoral competition using a probabilistic voting model (Lindbeck and Weibull, 1987), where the government maximizes a weighted sum of currently-living voters' utility, with no concern for the well-being of unborn generations. The role of the policymaker is to establish fiscal budgets to finance education and pensions. The government has full commitment during its tenure, but cannot commit on behalf of its successors. Second, we focus on a small open economy, where the in-

[1] This is the case after other factors that would be expected to influence the size of social security system are controlled for.

terest rate is determined at the worldwide level and capital accumulation is independent of domestic saving thereby excluding price channeling as a possible determinant for the emergence of fiscal policies.

We restrict attention to stationary Markov perfect equilibria of this dynamic game. With this equilibrium concept, we rule out equilibria in which current political outcomes are directly dependent on past outcomes. We believe that this is an appropriate concept of our model, where a period is long (around 30 years) and political competition among agents takes place in each period. The equilibrium refinement makes it easy to perform interesting comparative statics exercises and explore the quantitative aspects of equilibrium policies, such as the consequence of demographic transition.

We demonstrate the existence of a unique Markov perfect equilibrium characterized by public education and pensions under alternative economic scenarios. We start with the simplest case, where human capital is the sole payoff-relevant state variable, to highlight the main insights of the model. We then add a private saving technology and show that the strategic role of human capital is still relevant even when the presence of private financial wealth reduces the demand for public pensions. In this context, we show that the crowding in effect of public education on pensions prompts short-term mandate governments to internalize the technological spillover of human capital, although benefits from public investments occur beyond the life-time of current living voters. This result suggests that even if credit market constraints are absent the government may improve upon the market allocation by providing education. Finally, we examine the case of distortionary taxation to test the robustness of the analytical results. A calibrated version of the model delivers empirically plausible values of intergenerational fiscal policies in OECD countries. Although the baseline model is quite stylized, the main predictions remain substantively unchanged even with the addition of more realistic features. In particular, the qualitative predictions remain unaltered when capital taxation is introduced and a realistic number of life periods is added.[2]

The model produces predictions consistent with existing empirical evidence. First, the model predicts that the political power of the elderly has a negative impact on spending for anything other than social security. Second, the model predicts that public education receives the strongest political support when power is evenly distributed among voters. Finally, the model predicts that a demographic transition, consisting of a baby boom

[2] In a small open economy, assets would move after the announcement of capital income tax. Therefore, the tax rate in the political equilibrium would be necessarily zero. In a framework with T periods, the working-age cohorts support human capital investment to increase their future wealth, just as they were willing to accumulate physical capital in the presence of storage technology. These incentives are lost at retirement. Our mechanism, therefore, remains in place from a qualitative perspective. Indeed, the working-aged have incentives to support public education before retiring to increase rent opportunities after retiring.

followed by a baby bust, is accompanied by an initial drop in spending on education per student and pension per retiree, and a subsequent rapid recovery of total expenditure. After such a transition, social security eventually contracts as the young cohort shrinks. This pattern resembles the post-war dynamics for public education investments and social security programs in OECD countries.

In terms of policy implications, our findings suggest that a transition from a tax-based, pay-as-you-go system to an investment-based retirement system—as advocated for in the ongoing policy debate—can be harmful for all future generations. This is because after such a shift, political incentives to provide human capital investments disappear and the economy shrinks, converging on intergenerational autarky. This bad equilibrium emerges because currently desirable policies can have unintended political consequences. For example, the destruction of political rents for the elderly makes growth unsustainable. To this end, our theory demonstrates why policy recommendations should not only address present failures but also take into account subsequent political consequences.

Our paper is not only related to the already cited contributions. We also contribute to the growing literature that analyzes endogenous fiscal policies in an environment similar to ours; namely, a politico-economic framework without altruism, commitment, reputation, and price channeling. However, this literature looks at a different set of questions with respect to the one we posit in this paper. Some authors study the intergenerational conflict over taxes and transfers in the presence of private assets including private financial wealth or private human capital (see, e.g., Azariadis and Galasso, 2002; Bassetto, 2008; Chen and Song, 2014; Grossman and Helpman, 1998; Hassler, Storesletten, and Zilibotti, 2007).[3] Others examine the sustainability of redistributive fiscal policies in the presence of public assets including public debt (see, e.g., Song, Storesletten, and Zilibotti, 2012). In our model, governments manipulate both private and public assets in the form of private financial wealth and public human capital through fiscal policies. We show here that private assets do not offset the strategic use of public human capital by policymakers.

Finally, our paper is also related to the literature that investigates the relation between redistribution and growth. Politico-economic models of growth (see, e.g., Alesina and Rodrick, 1994; Azzimonti, 2011; Battaglini and Coate, 2007; Krusell, Quadrini, and Ríos-Rull, 1997; Persson and Tabellini, 1994) suggest that political conflict over the allocation of the public budget leads to extensive redistribution, which depresses growth. They show that politicians tend to be endogenously short-sighted as long as parties compete to retain

[3] Worth mentioning is the literature that investigates the determination of social security programs in a closed economy with physical capital accumulation (see, e.g., Forni, 2005; Gonzalez-Eiras and Niepelt, 2008; Mateos-Planas, 2008). The current pensions have a positive influence on future social security benefits and the returns to savings, providing the incentives for adult voters to support intergenerational transfers.

power via the democratic process. As a result, the economy experiences underinvestment in productive assets. Our model offers a different perspective. As long as redistribution is crucial to buying political support for growth-oriented policies, intergenerational conflict over the allocation of the public budget may stimulate growth and improve welfare.

Our paper proceeds as follows. Section 2 describes the model. Section 3 defines the equilibrium concept. Section 4 presents the main results and reviews the equilibrium policy in the absence of economic friction. Section 5 illustrates the quantitative experiments. Section 6 concludes and the Appendixes provide all the proofs and additional results not included in the text.

2 The Model

Time is discrete, indexed by t, and runs from zero to infinity. The model consists of a small open economy populated by overlapping generations of three-period-lived agents: young, adult, and elderly. Agents acquire skills in the first period, work in the second period, and retire in the last period. Every individual has n children, born at the beginning of the working-age period, who survive to old age with the probability of one. At the beginning of every period, two political candidates run for office. Each candidate proposes a fiscal platform to maximize the probability of winning the election without committing to future policies.

Production At each time t, the economy produces private goods via market and household production technologies. Market production combines physical capital, k_t, and effective labor as inputs in the constant returns to scale production function $y_t^M = Qk_t^\alpha (l_t h_t)^{1-\alpha}$, where l_t and h_t denote the firms' labor demand and human capital respectively and Q is the total factor productivity. Physical capital is perfectly mobile and depreciates fully after one period. We denote R the world interest rate and w the workers' pre-tax wage per unit of efficiency labor service. In a perfectly competitive factor market, $k_t = (\alpha Q/R)^{1/(1-\alpha)} l_t h_t$ and $w = (1-\alpha) Q (\alpha Q/R)^{\alpha/(1-\alpha)}$. Household production uses labor and human capital as inputs in the production function $y_t^H = F(l_t) h_t$. The function $F(l_t)$ is assumed to be continuously differentiable and with derivatives $F_l < 0$, $F_{ll} \leq 0$, and $F_{lll} < 0$.

Human Capital The human capital technology combines physical resources invested in education, f_t, and parental human capital as inputs in the constant returns to scale production function $h_{t+1} = H(f_t, h_t) \equiv Ah_t^\theta f_t^{1-\theta}$. The function $H(f_t/h_t, 1)$ describes the growth rate of human capital: the larger the human capital investments, the higher the future labor productivity. Moreover, more investments today reduce the need to invest in the next period, implying technological spillover. We assume that markets, in which young agents can borrow physical resources to support their education, do not exist. Thus, in the absence of government intervention, the lack of borrowing opportunities for the young generation implies that $f_t = 0$ for all dates t.[4]

Household Agents derive utility from $\left(c_t^a, c_{t+1}^o\right)$, denoting consumption as adults and elderly respectively. Neither consumption when young nor the welfare of descendants affects the lifetime utility.[5] We assume additively separable logarithmic preference over private consumption. Thus, the utility of an adult at time t can be written as $\log\left(c_t^a\right) + \beta \log\left(c_{t+1}^o\right)$, where $\beta \in (0,1)$ is the discount factor.[6] Individuals use the total after-tax labor income—equal to the sum of labor income taxed at the rates τ_t and z_t and home income—for consumption and saving, s_t, so that when retired, they consume pension benefits, b_t, and capitalized savings at the rate R. The per-period budget constraints for adults and the elderly, therefore, read $c_t^a + s_t \leq (1 - \tau_t - z_t)\, w l_t h_t + F(l_t)\, h_t$ and $c_{t+1}^o \leq R s_t + b_{t+1}$ respectively. We note that the government cannot tax household production. Hence, taxation distorts the time agents work in the market.[7] At the initial time $t = 0$, the economy is endowed with an exogenous amount of human capital h_0, held by the first generation of adults, and private financial wealth s_{-1}, held by the first generation of the elderly.

Competitive Economic Equilibrium Given prices and fiscal policies, agents allocate their time to maximize the total after-tax labor income and allocate their savings to equalize the interest factor and the marginal rate of substitution in consumption. Hence, the equilibrium labor and saving functions are $l_t = L(\tau_t, z_t) \equiv -\left(F_l\right)^{-1}\left((1 - \tau_t - z_t)\, w\right)$ and $s_t = (\beta / (1+\beta))\left((1 - \tau_t - z_t)\, w L(\tau_t, z_t) + F\left(L(\tau_t, z_t)\right)\right) h_t - (1/R(1+\beta))\, b_{t+1}$ respectively. Ignoring irrelevant constants, homogeneous utility function implies that the indirect utility of adults and the elderly can be expressed, respectively, as

$$\mathcal{U}^a\left(\tau_t, z_t, h_t; b_{t+1}\right) = (1+\beta) \log\left(\Upsilon\left(\tau_t, z_t, h_t; b_{t+1}\right)\right) \tag{1}$$

and

$$\mathcal{U}^o\left(b_t, s_{t-1}\right) = \log\left(R s_{t-1} + b_t\right) \tag{2}$$

[4]In reality, credit markets to finance educational investments are rare. The reasons for such a lack of privately-provided credits are various and widely studied. See, Pogue and Sgontz (1977) and Becker and Murphy (1988) for a classical discussion, and Kehoe and Levine (2001) and Boldrin and Montes (2005) for a more recent one.

[5]Adding such considerations would modify the quantitative but not the qualitative prescriptions of the model. Therefore, we disregard them to avoid the notational burden.

[6]Log utility is used for tractability. In a companion paper (Lancia and Russo, 2012), we generalize the analysis to CES-utility function.

[7]Specifying this type of preference, which is a special case of Greenwood, Hercowitz, and Huffman (1988), is standard in the literature. Adopting this formulation results in the absence of wealth effect on labor supply decisions.

where $\Upsilon(\tau_t, z_t, h_t; b_{t+1}) \equiv ((1 - \tau_t - z_t) wL(\tau_t, z_t) + F(L(\tau_t, z_t))) h_t + b_{t+1}/R$ is the present value of after-tax lifetime income.

Government At the beginning of every period, short-term mandate governments, democratically elected by their constituents, use their fiscal authority to transfer income across generations. The allocation of the public budget simultaneously serves the political scope of the elected representatives and the economic needs of their constituents. The government runs two public programs: education and social security. Fiscal revenue to fund human capital investment is raised using the labor income tax rate $\tau_t \in [0, 1]$. Similarly, the social security system to finance retirement benefits is funded by the payroll tax rate $z_t \in [0, 1]$. Assuming that politicians are prevented from borrowing, the public budget constraints read

$$\tau_t wL(\tau_t, z_t) h_t = nf_t \tag{3}$$

and

$$z_t wL(\tau_t, z_t) h_t = (1/n) b_t \tag{4}$$

Fiscal feasibility requires that revenues must be sufficient to cover expenditures with nonnegative human capital investment and pension transfers.

Definition 1 *Given initial conditions $\{h_0, s_{-1}\}$, an* ***Equilibrium Feasible Allocation*** *is a sequence of allocations $\{c_t^a, c_t^o, l_t, s_t, k_{t+1}, h_{t+1}\}_{t=0}^{\infty}$ and policies $\{\tau_t, f_t, z_t, b_t\}_{t=0}^{\infty}$ such that for all dates $t \geq 0$: (i) agents maximize utility subject to their budget constraints; (ii) firms maximize profits; (iii) the public budget constraints are satisfied and the fiscal feasibility conditions hold; (iv) the markets for capital, labor, and final goods clear.*[8]

Election Ruling governments are chosen by repeated elections according to majority rule. The young have no political power.[9] Before the election, two office-seeking candidates simultaneously and non-cooperatively propose fiscal platforms that satisfy the budget constraints (3) and (4). We model the electoral competition as a probabilistic voting model à la Lindbeck and Weibull (1987), adapted to an overlapping generations world with intergenerational transfers. In this case, the candidates promise to implement the same equilibrium fiscal platform. This maximizes a weighted sum of currently-living

[8] In a small open economy, the clearing market condition for physical capital implies that the stock of net foreign assets held by the private sector must be equal to the aggregate domestic saving and aggregate physical capital stock demanded by domestic firms.

[9] Our assumption matches the empirical evidence that young agents have lower voter turnout in elections than adults or the elderly. For example, in a study of U.S. elections, Galasso and Profeta (2004) show the turnout rate among those aged 60-69 is twice as high as among the young aged 19-29 years. Mulligan and Xala-i-Martin (1999) suggest that young citizens disperse their political interests among different and often contrasting issues, while their older counterparts are likely to focus their voting decisions on fewer programs such as Social Security and Medicare.

voters' utility, with no concern for the well-being of unborn generations. We provide an explicit microfoundation of the probabilistic voting game in the supplementary material in Appendix B.[10] Formally, the political objective function is given by

$$\mathcal{U}\left(\tau_t, z_t, b_t, h_t, s_{t-1}; b_{t+1}\right) = \mathcal{U}^a\left(\tau_t, z_t, h_t; b_{t+1}\right) + (\phi/n)\ \mathcal{U}^o\left(b_t, s_{t-1}\right) \tag{5}$$

where the weight $\phi \in [0; \infty)$ captures the political power of elderly voters relative to adult voters.[11] In the limit, when ϕ approaches infinity, the dominance of elderly voters shapes the institutional process. In contrast, when $\phi = 0$, adult voters hold the only relevant political position. As Eq. (5) shows, adopting a probabilistic voting framework accounts for political factors as well as demographic characteristics. The population component is summarized by the dependency ratio $1/n$.

3 Politico-Economic Equilibrium

At the beginning of every period, a political candidate is democratically elected to implement the promised fiscal platform. The elected candidate has full commitment during its tenure, but cannot commit on behalf of its successors. By implementing their fiscal platform, elected candidates create dynamic linkages across policy-making periods as asset variables evolve. Fully rational voters anticipate how a government's fiscal platform will affect the decisions of future policymakers.

Markov Perfect Equilibrium Constructing policies that are contingent upon history and enforced by reputation mechanisms allows for multiple subgame-perfect equilibria. We, however, rule out such mechanisms. We instead restrict attention to differentiable stationary Markov perfect equilibria (hereafter, MPE) where strategies are conditioned only on the current payoff-relevant state variables of the economy. For governments, the relevant state variables are the assets held by pivotal constituents, i.e., human capital held by adults and financial wealth held by the elderly. Equilibria characterized here correspond to limits of finite-horizon MPE (Fudenberg and Tirole, 1991). The equilibrium objects we are interested in, therefore, are the fiscal policy rules and the rules governing the evolution of the asset variables. Hereafter, unless otherwise specified,

[10] Probabilistic voting has been studied extensively both theoretically and empirically. For example, Stromberg (2008) uses probabilistic voting to study presidential elections in the United States, and shows that the model explains candidates'behavior. See Persson and Tabellini (2000, p.52-58) for a formal discussion.

[11] The weight ϕ is a measure of the effectiveness of intergenerational political power. It reflects the existence of formal institutions which guarantee active and passive political participation. These institutions include electoral rules as age restrictions for candidates, lobby power, and voter enfranchisement. It also measures how informal institutions alter the representativeness of an age group. These informal institutions include civil society, clientelism, corruption, and social norms.

we omit time indexes and switch to a recursive notation with primes denoting next-period variables and s_{-} denoting the assets held by the currently-living elderly.

Definition 2 *Given initial conditions* $\{h_0, s_{-1}\}$*, a differentiable stationary* ***Markov Perfect Equilibrium*** *is an equilibrium feasible allocation such that the private saving rule* $\mathcal{S} : [0,1] \times \mathbb{R}_+ \times [0,1] \times \mathbb{R}_+ \to \mathbb{R}_+$ *and the fiscal policy rules,* $\mathcal{T} : \mathbb{R}_+ \times \mathbb{R}_+ \to [0,1]$ *equal to the labor income tax rule,* $\mathcal{Z} : \mathbb{R}_+ \times \mathbb{R}_+ \to [0,1]$ *equal to the payroll tax rule,* $\mathcal{F} : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ *equal to the education investment rule, and* $\mathcal{B} : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ *equal to the pension transfer rule, satisfy the following points:*

i. *The map* $\mathcal{S}(\tau, f, z, h)$ *is the private saving choice* s *that solves*

$$s = (\beta/(1+\beta))((1-\tau-z)\, wL(\tau, z) + F(L(\tau, z)))\, h - (1/R(1+\beta))\, \mathcal{B}(h', s)$$

where $b' = \mathcal{B}(h', s)$*, with* $h' = H(f, h)$*;*

ii. *Given the political objective function* $\mathcal{U}(\tau, z, b, s_{-}, h; b')$ *where* $b' = \mathcal{B}(h', s)$*, with* $h' = H(f, h)$ *and* $s = \mathcal{S}(\tau, f, z, h)$*, the equilibrium fiscal policies solve*

$$\{\mathcal{T}(h, s_{-}), \mathcal{F}(h, s_{-}), \mathcal{Z}(h, s_{-}), \mathcal{B}(h, s_{-})\} = \underset{\{\tau, f, z, b\}}{\arg\max}\ \mathcal{U}(\tau, z, b, h, s_{-}; \mathcal{B}(s, h'))$$

subject to the public budget constraints

$$\mathcal{T}(h, s_{-})\, wL(\mathcal{T}(h, s_{-}), \mathcal{Z}(h, s_{-}))\, h = n\mathcal{F}(h, s_{-})$$

and

$$\mathcal{Z}(h, s_{-})\, wL(\mathcal{T}(h, s_{-}), \mathcal{Z}(h, s_{-}))\, h = (1/n)\, \mathcal{B}(h, s_{-})$$

Point (i) defines a functional equation that maps current fiscal policies and human capital stock to the optimal private saving, $s = \mathcal{S}(\tau, f, z, h)$. This equation describes the private sector's responsiveness to a variation of τ, f, and z, when voters rationally anticipate that future pension policy will be set according to the equilibrium rule $\mathcal{B}(h', s)$. Governments consistently choose intergenerational fiscal policies, which are subject to the public budget constraints and the private response of individuals, with the expectation that future governments will act according to the MPE rule. Even though governments have short-term mandates and do not have access to a commitment technology, they can strategically use their fiscal authority to manipulate the decisions of future policymakers. This is because a government's fiscal policies affect the amount of human capital and private financial wealth available to its successors. Such a dynamic strategic link, endogenously obtained by combining the Markov policy rules with the human capital technology and the private investment decision, will serve as the cornerstone of our analysis.

Definition 3 *An MPE is said to be a* ***Political Intergenerational State*** (*hereafter, PIS*) *if it implements both public education and pensions.*

Implicit return for PIS The outlined electoral process emphasizes political pressure exercised by different age groups. PIS, therefore, may arise even though a large part of the electorate has no economic motive to support it. Indeed, office-seeking candidates may prefer to sacrifice economic principles for the political object of been elected. Accordingly, we introduce a measure that operationalizes the economic viability of PIS. The criterion for economic viability requires that all generations at all times support PIS given the alternative of ending the fiscal program. Clearly, elderly voters support PIS, which awards them a pension at zero cost. Adults, however, gain from the implementation of PIS if and only if the corresponding implicit rate of return is larger than the return on private saving. The existence of a private saving technology implies that individuals may finance their retirement with earnings from their invested savings at the world interest rate R. The opportunity cost of contributing to the public programs is, therefore, high for an adult with a valuable outside option. For such adults to support PIS, a fiscal platform must offer a greater share of the total fiscal surplus by allowing them to pay lower taxes. In the literature on social security systems, the implicit rate of return for a pay-as-you-go system is equal to the ratio of pensions received and taxes paid to retirement financing. Our model, however, suggests that democratic institutions strategically link public education and pension transfers. Accordingly, we specialize the implicit rate of return for PIS by including the labor income tax to finance education as an additional cost. This rate, therefore, corresponds to

$$\frac{b'}{(\tau + z)\, wL(\tau, z)\, h + d(\tau, z)} \tag{6}$$

where $d(\tau, z) = ((wL(0,0) + F(L(0,0))) - (wL(\tau, z) + F(L(\tau, z))))\, h$ measures the consumption loss suffered by taxpayers as a result of the distortionary effect of taxing labor supply. This loss is increasing and convex in each tax rate and equal to zero when labor supply is inelastic. In the following analysis, we use the difference between Eq. (6) and the world interest rate to evaluate the economic viability of PIS.

4 Inelastic Labor Supply

In this section, we use a tractable version of the model to analytically characterize the conditions for the emergence of PIS. We simplify the full model described in the previous section by assuming that agents' labor supply is inelastic. To clarify the driver of the main mechanism, first, we specialize the set-up with the human capital as the sole payoff-relevant state variable (Subsection 4.1); second, we test the robustness of the proposed mechanism when agents can perfectly substitute private savings for public pensions (Subsection 4.2).

4.1 Strategic Role of Human Capital

This section analyzes an economy where individuals are prevented from saving privately. During their working lifetime, individuals may support human capital investments. In a small open economy and absent markets for loans, however, individual investment in the human capital of future generations is non-appropriable, since it provides investors with no wealth claims that can be exercised during their retirement-age periods. Therefore, non-altruistic agents have no incentives to support investments in future generations, although it is technically and economically feasible to do so. In this context, we show how the design of an electoral institution that guarantees political participation by all currently-living voters can render an otherwise impracticable positive level of public investments politically viable.

Politico-Economic Equilibrium Specializing Definition 2 to an environment with only human capital yields the following system of functional equations:

$$\frac{c^a}{c^o} = \frac{1}{\phi} \tag{7}$$

and

$$\frac{b'}{b} = \frac{\beta}{n\phi}\frac{d\mathcal{B}\left(h'\right)}{df} \tag{8}$$

where $b' = \mathcal{B}(h')$, with $h' = H(f, h)$.[12] The total derivative $d\mathcal{B}(h')/df = \mathcal{B}_{h'} \cdot H_f$ captures the dynamic strategic link between current investment in human capital and next-period pension transfers. $\mathcal{B}_{h'}$ operationalizes the strategic role of human capital. Eq. (7) yields the intra-temporal trade-off between the marginal cost of taxation borne by adults and the marginal benefits of pension transfers enjoyed by the elderly. Such a trade-off is entirely determined by the political power of currently-living constituents and reveals a conflict of interest between adult and elderly voters. The stronger the power of the elderly (i.e., higher ϕ), the greater the reduction of c^a/c^o ratio and, in turn, the more unbalanced the distribution of consumption in their favor. Eq. (8) yields the generalized Euler condition for human capital investment. Its right-hand side captures the compensatory effect claimed by adults who anticipate that increasing human capital

[12] The technical details to obtain the functional equations (7) and (8) are provided in the supplementary material in Appendix B.

investment will expand the financing possibilities of social security programs. Such an effect fundamentally hinges on the adults' prospect of politically claiming a share of the future returns of public investments via pension transfers, as illustrated in the following Propositions.

Proposition 1 *If $\phi = 0$, there exists a unique MPE, which implements intergenerational autarky.*
Proof. *(See Appendix).* ■

The MPE has a stark property. When elderly voters have no political power, democratic institutions fail to implement PIS. The economy, therefore, reverts to intergenerational autarky even though a growth-enhancing technology is at disposal. The intuition for this result is straightforward. When pension benefits are not provided—as implied by setting $\phi = 0$ in Eq. (7)—there are neither economic nor political motives for adults to invest in public education.[13] Hence, no public capital good is produced. In contrast, if the elderly voters actively participate in the voting process (i.e., $\phi > 0$), then they extract an electoral rent in the form of pension transfers by exerting their political power. Accordingly, forward-looking adults support growth-oriented policies as they are democratically entitled to claim a share of the human capital of future generations. Thus, granting political power to the elderly creates a motive for adults to publicly save for retirement by investing in the human capital of the young.

Proposition 2 *Given an initial condition h_0, if $\phi > 0$, there exists a unique PIS, which is characterized by the following set of policy functions and law of motion of human capital:*

i. $f = \mathcal{F}(h) \equiv \frac{\beta(1-\theta)}{\phi+n(1+\beta(1-\theta))} wh$;

ii. $\tau = \mathcal{T}(h) \equiv (n/w)(\mathcal{F}(h)/h)$;

iii. $b = \mathcal{B}(h) \equiv \frac{n\phi}{\phi+n(1+\beta(1-\theta))} wh$;

iv. $z = \mathcal{Z}(h) \equiv (1/wn)(\mathcal{B}(h)/h)$;

v. $h'/h = A\left((\mathcal{F}(h)/h)\right)^{1-\theta}$.

Proof. *(See Appendix).* ■

[13] This can be easily seen by setting $\mathcal{B}(h) = 0$ in condition (8), which implies that $f = 0$ for all generations.

The equilibrium result for the intergenerational fiscal policies is easily illustrated. The education investment rule and the pension transfer rule are increasing and linear functions of the labor income of adults. Accordingly, the wealthier the society, the larger the amount of fiscal revenue devoted to public education and pensions.

Corollary 1 *The strategic role of human capital is stronger where the political power of elderly voters is larger.*
Proof. *(See Appendix).* ■

As highlighted in the previous section, the dynamic strategic link across policies motivates the current government to manipulate future asset holdings and, in turn, future fiscal platforms. The model predicts that when elderly voters hold stronger political power, the strategic role of human capital is magnified. Indeed, a larger ϕ improves the future political ability of adults to reap a fraction of the return on public investment, which will be embedded in the labor productivity of future generations.

Implicit return for PIS The next step of the analysis explores the economic viability of implementing public education and pensions as two arms of a unique social policy package deal. In this framework, PIS allows adults to consume during their retirement-age period given the lack of any nonperishable goods. This fiscal program then is desirable not only for the initial generation of retirees, who gain initial benefits without having paid any taxes, but also for all future generations. Inserting the equilibrium policy rules into Eq. (6) yields the implicit rate of return for PIS:

$$\frac{b'}{wh\left(\tau+z\right)}=\underbrace{\frac{n\phi}{\phi+n\beta\left(1-\theta\right)}}_{Appropriability}\underbrace{\frac{h'}{h}}_{Growth} \tag{9}$$

Eq. (9) has an interesting property. The implicit rate of return is made up of two distinct elements: the growth component and the appropriability component. As standard in the literature, the growth component measures the endogenous growth rate of the tax base.[14] As distinctive feature of our model, the appropriability component captures the ability of retirees to exercise a political claim on the human capital investment they made as adults. Remarkably, the implicit rate of return for PIS is lower than the growth rate of the economy. Furthermore, it is a non-monotonic function of the political power held by the elderly, as highlighted in the following Corollary.

Corollary 2 *The implicit rate of return for PIS is hump-shaped in ϕ.*
Proof. *(See Appendix).* ■

[14]See Feldstein and Liebman (2002) for a formal discussion on the implicit rate of return for an unfunded pension system.

A variation in the distribution of political power across voters has a two-fold impact on Eq. (9). On the one hand, an increase in ϕ strengthens the appropriability component. On the other hand, a higher ϕ depresses growth because it reduces public education (see Proposition 2). The economic viability of PIS depends critically upon which of these two effects dominates. With sufficiently low ϕ, the former effect dominates, whereas with sufficiently high ϕ, the latter effect prevails. Moreover, Eq. (9) is null in the two extreme scenarios. When ϕ is equal to zero, adults are precluded from extracting rent when old. When ϕ approaches infinity, elderly voters fiercely oppose growth. Together, these effects imply Corollary 2.

Real-world legislators do not explicitly consider public education and pensions a social policy package deal. Hence, it becomes relevant to ask how beneficial human capital investments are for adult voters. For this purpose, we consider an endowment economy in which governments decide to only fund pension benefits. Clearly, pensions are provided as long as elderly voters hold political power. The corresponding return for pensions is the population growth rate. We then compare the population growth rate with Eq. (9) to evaluate the conditions under which public education and pension receive stronger political support from adults.

Corollary 3 *The implicit rate of return for PIS is larger than the population growth rate when the productivity of the human capital technology is sufficiently high for any* $\phi \in \left(\underline{\phi}, \overline{\phi}\right)$*, with* $\underline{\phi} > 0$ *and* $\overline{\phi} < \infty$*.*

Proof. *(See Appendix).* ■

Corollary 3 establishes that, for adult voters, PIS is desirable compared to a fiscal program with only pensions when the human capital technology is sufficiently productive and the distribution of power across cohorts does not favor one generation or the other. Indeed, an increase in the productivity of the public technology increases the marginal return of human capital investment and, in turn, the rate of growth for labor income. This increase raises the return of the fiscal program. Since the rate of return for PIS is zero at the extreme values of ϕ, the result is achieved when political power is distributed evenly across generations. One implication of the theory is that public education receives the strongest political support when power is evenly distributed among voters.

4.2 Private Financial Wealth as Substitute for Pensions

Given the preceding discussion, would the presence of a private saving technology jeopardize incentives to support public education? Unlike human capital investment, private investment for retirement is fully appropriable because it provides investors with a wealth claim on the investment returns. Adults, therefore, may have incentives to substitute public pensions with private savings when the world interest rate is sufficiently high. In this section, we argue that this avenue does not undermine the incentives to implement public education and pensions. Moreover, the model generates two additional results: (i) short-term mandate governments internalize human capital spillover; and (ii) PIS emerges only if the corresponding implicit rate of return is smaller than the world interest rate.

Politico-Economic Equilibrium The introduction of private storage technologies modifies the previous analysis in one major respect. Private financial wealth held by retirees is a potential payoff-relevant state variable. As a consequence, changes to the public budget will affect future fiscal platforms through changes in the amount of both public and private asset holdings. Applying Definition 2 yields the following system of functional equations:

$$\frac{c^a}{c^o} = \frac{1}{\phi}\left(1 - \frac{1}{whR}\frac{d\mathcal{B}\left(h', s\right)}{dz}\right) \tag{10}$$

and

$$R = \frac{1}{wh}\frac{d\mathcal{B}\left(h', s\right)}{d\tau} + \frac{1}{n}\frac{d\mathcal{B}\left(h', s\right)}{df} \tag{11}$$

where $b' = \mathcal{B}\left(h', s\right)$, with $h' = H\left(f, h\right)$ and $s = \mathcal{S}\left(\tau, f, z, h\right)$. The total derivatives $d\mathcal{B}\left(h', s\right)/dz = \mathcal{B}_s \cdot \mathcal{S}_z$, $d\mathcal{B}\left(h', s\right)/d\tau = \mathcal{B}_s \cdot \mathcal{S}_\tau$, and $d\mathcal{B}\left(h', s\right)/df = \mathcal{B}_{h'} \cdot H_f + \mathcal{B}_s \cdot \mathcal{S}_f$ operationalize the dynamic strategic links between current fiscal policies and the next-period pension transfers. Eq. (10) yields the trade-off between taxpayers and tax recipients. The key difference with respect to Eq. (7) is that, while in the absence of private asset holdings, c^a/c^o ratio is constant, this changes with taxes when adults can save privately. Agents respond to a fiscal amendment of the social security budget through variations in their saving decisions. This variation affects the stock of private financial wealth held by future constituents and in turn the future amount of pension benefits. The possibility of private saving also has an interesting implication for the generalized Euler condition for human capital investment. In the absence of private storage technology, there are no investment alternatives to human capital investments. In contrast, agents benefit from a wider investment portfolio when they have access to private capital markets. According to Eq. (11), therefore, the returns of human capital investment must offset the returns of private savings.

Proposition 3 *If $\phi = 0$, there exists a unique MPE, which implements intergenerational autarky.*

The preclusion of rent extraction through the exercise of political power when elderly removes the incentives of adult voters to support public education and, in turn, prevents growth. This result follows the discussion carried out in the absence of private financial wealth. The proof is easily demonstrated in view of the equilibrium refinement we adopt. Consider a finite-horizon economy and solve by backward induction. At the terminal period, adults have no future and pensions are not provided since it is not in the adults' interest. In the previous period, adults anticipate that the expense of financing education will not be compensated for by retirement benefits. Public education and pensions, therefore, are not politically viable in any period.

Proposition 4 *Given initial conditions $\{h_0, s_{-1}\}$, if ϕ is larger than a certain threshold $\tilde{\phi}$, there exists a unique PIS such that:*

1. *The set of policy rules and laws of motion of assets are:*

i. $f = \mathcal{F}(h, s_-) \equiv \psi h$;

ii. $\tau = \mathcal{T}(h, s_-) \equiv (n/w)\,\psi$;

iii. $b = \mathcal{B}(h, s_-) \equiv a_s s_- + a_h h$;

iv. $z = \mathcal{Z}(h, s_-) \equiv (1/wn)\,(\mathcal{B}(h, s_-)/h)$;

v. $h'/h = A\psi^{1-\theta}$;

vi. $s/h' = e_s\,(s_-/h) + e_h$.

where $\psi = w(1-\theta)/(n(1-\theta)+(\phi/\beta))$, $a_s \equiv -R(1+\beta)/(1+\beta+(\phi/n))$, $a_h \equiv (\phi/(1+\beta+(\phi/n)))\,(w+n\psi\theta/(1-\theta))$, $e_s \equiv -(a_s\beta R/(n(a_s+R(1+\beta))))\,h/h'$, *and* $e_h \equiv (\beta R/(a_s+R(1+\beta)))\,(w-n\psi-(a_h/n))h/h' - a_h/(a_s+R(1+\beta))$;

2. *The condition for balanced growth is* $R = R^*(\phi) \equiv (n+(\phi/\beta))\,A\psi^{1-\theta}$.[15]

Proof. *(See Appendix).* ∎

Proposition 4 predicts that governments implement public education and pensions when the political power held by the elderly voters is sufficiently strong.[16] Interestingly, human capital maintains its strategic role, which increases with ϕ. The presence of private assets, therefore, does not fully offset the strategic link between human capital and pensions, which is necessary to implement PIS.[17] The equilibrium policies share

[15] To prove existence of MPE, we assume that the world interest rate is sufficiently large. See Appendix A.2 for details.

[16] Note that there cannot be $\phi < \tilde{\phi}$. Otherwise, the nonnegativity constraint of pension transfers would bind in equilibrium. This would occur because the elderly would be required to transfer a share of their private wealth to subsidize adults' consumption.

[17] This conclusion is fundamentally different from the result that, in an economy with public debt, the presence of savings offsets the strategic effect of public assets on future policies. Since public and private bonds yield the same return, the agents' private response to a fiscal adjustment neutralizes the strategic valence of public debt. See, Song, Storesletten, and Zilibotti (2012).

properties similar with those reported in Proposition 2. Here, we highlight the main differences. First, the pension transfer rule is a linear and decreasing function of private financial wealth. This is because private savings are substitutes for public pensions. Second, human capital investment is muted to changes in private asset holdings. This result occurs because private financial wealth does not affect the present value of after-tax lifetime income per se. Therefore, it is not payoff-relevant state variable for adult voters, who finance education via taxation and benefit from this investment via pensions. Finally, the condition $R = R^*(\phi)$ guarantees that the economy moves along a balanced growth path. Under this scenario, the economy itself lacks transitional dynamics since it is on the balanced growth path from the initial period onwards.[18]

We next emphasize that democratic institutions which guarantee the political participation of all active constituents not only implement public education and pensions, but also allow policymakers to internalize technological spillover. This occurs although benefits from public education emerge beyond the life-time of currently-living voters.

Proposition 5 *Short-term mandate governments internalize human capital spillover.*
Proof. *(See Appendix).* ■

This result fundamentally hinges on the absence of commitment technology when private capital is perfectly mobile. The intuition is easily illustrated. If a government invests too little in public education, subsequent policymakers necessarily provide less pension transfers to the generation that was alive under the former government. Since human capital and pensions are positively related, the lack of public investments by one government restricts the public budget possibilities of future governments. The less the former government cares for future generations, the harsher the budget restrictions. Anticipating smaller pension transfers when elderly, adults save more. This depresses the expected retirement benefits even further and reduces the demand for public education in the first instance. Private savings continue to adjust up to the point at which no pensions are provided. Such an allocation is not desirable for constituents when ϕ is sufficiently large. Underinvestment in public education therefore is not a profitable deviation for governments. The unique equilibrium strategy prescribes allocating the maximum amount of fiscal revenue to human capital investments that is compatible with the provision of pensions for the elderly.[19] Then, it is interesting to ask how well governments do.

[18]In a world comprising a set of small open and homogenous economies, there exists a unique equilibrium interest rate such that the world asset market clears. MPE, therefore, must feature $R = R^*(\phi)$. Note that there can no be $R < R^*(\phi)$. Otherwise, the economy would accumulate an ever-increasing deficit. Analogously, it is impossible that $R > R^*(\phi)$. Otherwise, the economy would accumulate an ever-increasing surplus.

[19]This proposition is different from the well-known result that in a closed economy the social security

Corollary 4 *For any $\phi > \tilde{\phi}$, there exists a Pareto weight $\delta \in (0,1)$, such that if $\delta = \beta n / (\phi + \beta n)$, short-term mandate governments implement the optimal level of investment in public education.*

Proof. *(See Appendix).* ■

According to Corollary 4, short-term mandate governments can implement the level of human capital investments chosen by a social planner who discounts the utility of future generations with a discount factor δ.[20] These findings suggest that even if credit market constraints are absent a government can improve upon the market allocation by publicly financing education.[21]

Implicit return for PIS We next examine the condition for the economic viability of implementing the fiscal program described in Proposition 4. Inserting the equilibrium fiscal policies evaluated along the balanced growth path into Eq. (6) yields the implicit rate of return for PIS

$$\frac{b'}{wh(\tau + z)} = \underbrace{\frac{a_s(s/h') + a_h}{n\psi + (1/n)(a_s(s/h') + a_h)}}_{Appropriability} \underbrace{\frac{h'}{h}}_{Growth} \tag{12}$$

Eq. (12) is made up of the components of appropriability and growth, as previously discussed. The comparison of the implicit rate of return for PIS with the world interest rate yields the following result.

Proposition 6 *In the presence of private financial wealth, the implicit rate of return for PIS is necessarily lower than R.*

Proof. *(See Appendix).* ■

In contrast to the previous section, adult voters lose out on the joint implementation of public education and pensions when private investment possibilities are available. Adults bear the cost associated with the fiscal program because they do not have enough power

program relaxes the incentives to implement public education. This occurs because higher human capital investment increases the domestic interest rate. The resulting reduction of the present value of pension benefits lowers the demand for public education. As a consequence, governments do not internalize human capital spillover (Gonzalez-Eiras and Niepelt, 2012).

[20] Gonzalez-Eiras and Niepelt (2008) show that in the absence of binding non-negativity constraints on tax rates, tax distortions, and intragenerational inequality, the Ramsey policy with full commitment supports the social planner allocation. The social planner's objective function is given by $(\beta/\delta)\log(c_0^o) + \sum_{t=0}^{\infty} \delta^t \left(\log(c_t^a) + \beta \log(c_{t+1}^o)\right)$.

[21] With complete markets, market institutions can at most comply with short-term projects failing to internalizing technological spillover. Political institutions, therefore, outperform market institutions. The presence of economic frictions partially jeopardizes the policymakers' possibility to internalize the technological spillover. A detailed analysis of the case with distortionary taxation is provided in Section 5.

to oppose the arrangement. If adults suddenly become pivotal voters and the elderly did not exert political pressure, governments would dismantle the intergenerational fiscal program to allow agents to pursue higher returns from private investments. While perhaps surprising at first, the result is straightforward. Suppose that the world interest rate was smaller than Eq. (12). Adults would then have no incentive to privately save and the economy would revert to a welfare regime with human capital only, as discussed in Subsection 4.1. Private financial wealth, therefore, is payoff-relevant for the determination of PIS insofar as it is economic profitable.

These findings have important policy implications. Modern economies around the world are debating fundamental structural reforms to their social security systems. Most of the reforms entail a gradual shift from a tax-based, pay-as-you-go system to an investment-based retirement system. Advocates for such reforms argue that the implicit rate of return for existing social security schemes falls short of the market interest rate. This discrepancy will only grow as populations age. Our model, however, shows that this reform is not necessarily desirable. Indeed, the proposed transition would certainly improve the allocation of resources for currently-living adults (short-term gain), as highlighted in Proposition 6. The cost of the reform, however, would be borne by both the currently-living elderly and all future generations since the economy would fall into intergenerational autarky after one period (long-term loss). This bad equilibrium occurs because currently desirable policies have unintended negative effects on future policies, such as, for example, when the destruction of economic rents for the elderly wipes out the incentive to provide public education and, in turn, makes growth unsustainable. Thus, our theory demonstrates why policy recommendations should not only address present failure, but also take into account subsequent political ramifications of initial period policies.

5 Elastic Labor Supply

The framework above rests on the assumption that the labor supply during the pre-retirement period is arbitrarily fixed and does not respond to the imposition of taxation. A more realistic analysis recognizes that individuals do modify their behavior in response to changes in the labor income tax rates and social security contributions. This induces a deadweight loss borne by each generation of taxpayers, which affects the opportunities for financing both public education and pensions. We introduce the concept of elastic labor supply by parametrizing the household production technology as $F(l) = (\xi/(1+\xi))\, X \left(1 - l^{1+1/\xi}\right) h$, where $\xi > 0$ is the Frisch elasticity and X measures the productivity of household technology. Thus, the equilibrium labor function is

$l = L(\tau, z) \equiv ((1 - \tau - z) w/X)^{\xi}$. An MPE is, therefore, characterized by a system of two functional equations:

$$\frac{c^a}{c^o} = \frac{\mu^z}{\phi} \left(1 - \frac{1}{RwhL(\cdot)} \frac{d\mathcal{B}(h', s)}{dz} + \frac{1}{nR} \frac{\mu^\tau - 1}{\mu^\tau} \frac{d\mathcal{B}(h', s)}{df} \right) \tag{13}$$

and

$$R \left(1 + \phi \frac{\mu^z - 1}{\mu^z} \frac{c^a}{c^o} \right) = \frac{1}{whL(\cdot)} \frac{d\mathcal{B}(h', s)}{d\tau} + \frac{1}{n\mu^\tau} \frac{d\mathcal{B}(h', s)}{df} \tag{14}$$

where $\mu^z = (1 - \tau - z) / (1 - \tau - (1 + \xi) z)$ and $\mu^\tau = (1 - \tau - z) / (1 - z - (1 + \xi) \tau)$ denote the marginal costs of public funds.[22] Eqs. (13) and (14) encompass the case of lump-sum taxes, namely Eqs. (10) and (11), as a particular case, where $L(\tau, z) = 1$ and $\mu^\tau = \mu^z = 1$. There are two main differences from the case with inelastic labor supply. First, the tax distortions increase the cost of financing b and f, as captured by μ^z and μ^τ. Second, the fiscal amendment of one budget affects the other budget because it distorts the labor supply. Specifically, an increase of the labor income tax rate negatively influences the social security budget. This adverse fiscal interaction effect is operationalized by $(1 - 1/\mu^\tau)$ in Eq. (13). By analogy, an increase in social security contributions tightens the public investment possibilities. This adverse fiscal interaction effect is measured by $(1 - 1/\mu^z)$ in Eq. (14). These differences have a relevant impact on the determination of the equilibrium policy rules. The presence of frictions in the labor market fuel even more the intergenerational conflict over the allocation of public resources. Then, the deadweight loss associated with the distortionary taxation partially jeopardizes the gains associated with the fiscal programs.

A full analytical characterization of MPE under elastic labor supply is not available; therefore, we must resort to numerical analysis. The computational strategy uses a standard projection method with Chebyshev collocation to approximate the private saving rule $\mathcal{S}(\tau, f, z, h)$ and the government policy rules $\mathcal{T}(h, s_-)$, $\mathcal{F}(h, s_-)$, $\mathcal{Z}(h, s_-)$, and $\mathcal{B}(h, s_-)$, exploiting the equilibrium conditions (13) and (14) in tandem with the Euler condition for savings and the government budget constraints (3) and (4).

5.1 Quantitative Analysis

This section illustrates the properties of the model with elastic labor supply and shows that a reasonably calibrated version of the model is consistent with key features of modern economies. We use the calibrated model to run numerical experiments. The algorithm

[22] The marginal cost of public fund measures the marginal cost of raising an additional unit of revenue from taxes to finance social security and education respectively. Such a cost is increasing and convex in the corresponding tax rate, and is equal to unity plus the deadweight loss induced by taxation.

adopted to solve for the equilibrium is described in the supplementary material in Appendix B. We take one period in the model to correspond to 30 years in the data.

Target Observation		Parameters	
GDP growth rate	2.5	A	3.347
Education-GDP ratio	5.4	θ	0.846
Pension-GDP ratio	7.16	β	0.997^{30}
Ratio of labor income to total income for adults	33/51	X	1.360
Capital-output ratio (annualized)	3	R	1.04^{30}
Tax rate corresponding to the top of the Laffer curve	0.60	ξ	2/3
Population growth rate		n	1.006^{30}
Political power of elderly relatively to adults		ϕ	0.8

Table 1: Calibration.

We assume an annual gross population growth rate of 1.006, corresponding to the average OECD rate during the period 1995–2009. We fix a capital share of output of $\alpha = 1/3$ and an annualized capital-output ratio of 3. These parameters imply an annual interest rate of 4%, which is standard in quantitative macroeconomics. We normalize Q so that w is equal to unity. In line with Trabandt and Uhlig (2011), we set $\xi = 2/3$ so that the top of the Laffer curve is at 60%. The literature provides no guidance on the distribution of political power across generations. We set ϕ to 0.8, indicating that adult and elderly voters have approximately the same per-capita influence (Gonzalez-Eiras and Niepelt, 2008). We calibrate the parameters A, θ, and β to fit the following key moments in OECD economies. We pool the data from the period 1995–2009, and set the GDP growth rate and the GDP-share of education and pension transfers to the value of 2.5%, 5.4%, and 7.16% respectively. X targets the ratio of market labor earning to total income, including home income, for adults. This ratio of 33/51 is the ratio of market hours worked to total hours for US working-age households (Aguiar and Hurst, 2007). Matching jointly these moments yields $A = 3.347$, $\theta = 0.846$, $\beta = 0.9138$, and $X = 1.360$. Despite the simplicity of the model, our calibrated political economy generates plausible values. For example, the annual discount factor corresponds to 0.997. Moreover, the calibrated value of X is consistent with Song, Storesletten, and Zilibotti (2012). Table 1 summarizes the parameters.

Figure 1 plots the stationary equilibrium policy rules of the calibrated economy. We exploit the homogeneity property of the utility function to make the state space unidimensional. The ratio of private financial wealth held by the elderly to human capital

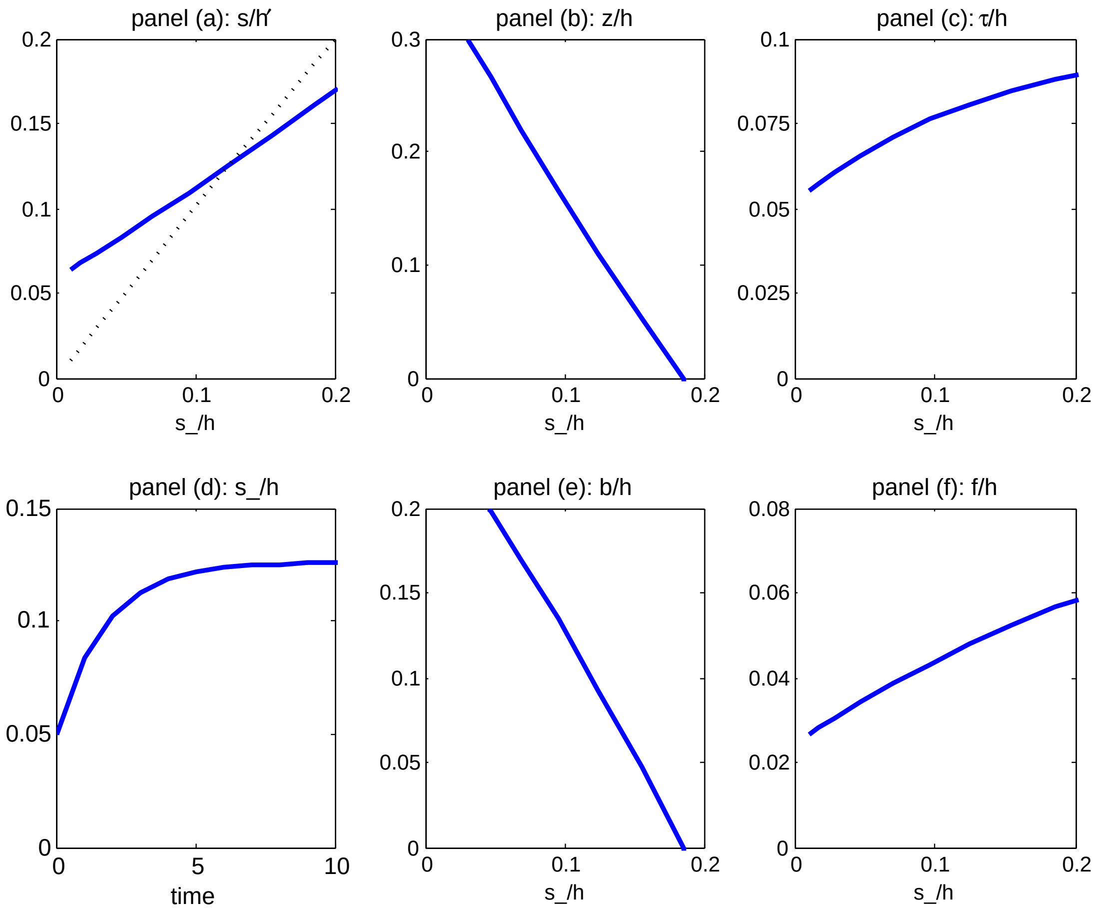

Figure 1: Equilibrium policy rules

held by adults, s_{-}/h, is the payoff-relevant state variable. Accordingly, the private and public equilibrium policy rules are expressed in per efficiency units. Consider first the private saving rule in per efficiency units. When labor is inelastic, it coincides with the 45-degree line and then any initial condition s_{-1}/h_0 is a steady-state for the economy. As panel (a) illustrates, in the calibrated equilibrium with elastic labor supply, the private saving rule in per efficiency units is a concave function, which converges monotonically to the interior steady-state level. Panel (d) graphs the corresponding equilibrium time path. Panel (b) and (e) plot the equilibrium payroll tax rule and the pension transfer rule respectively in per efficiency units. Similar to the case of inelastic labor supply, both policy rules are decreasing in s_{-}/h. We note that the steady-state rate of social security contribution is 10.7%, which is in line with the average rate for the OECD countries during the period 1996-2009. The two sides of the public investment budget, however, are qualitatively different from the case of inelastic labor supply. Both the labor income tax rate (panel (c)) and the public investment rule (panel (f)) in per efficiency units are increasing nonlinear functions of s_{-}/h. This positive equilibrium relation relies on the observation that an increase of private financial wealth reduces the payroll tax for pension financing. Given the adverse fiscal interaction effect between the two public programs, the fiscal adjustment of the social security budget expands public investment possibilities. The equilibrium predictions conform with evidence that financially weak constituents in developed countries are more supportive of universal social insurance programs and less attentive to growth-oriented policies.[23]

Comparative statics We next study how the long-run equilibrium responds to changes in fundamental parameters of the economy. We focus on two key parameters. First, we examine changes in the Frisch elasticity, as quantified by ξ, to evaluate the impact of distortionary taxes. Second, we consider a change in the distribution of the political power across generations, as measured by ϕ, to evaluate changes in the potential of elderly voters to extract rent.

The upper part of Figure 2 depicts the long-run equilibrium policy rules for $\xi \in [0.3, 0.9]$ holding all other parameters constant. As ξ increases, the higher fiscal distortion provokes a government shirking effect characterized by declining pensions per retiree (panel (a)) and education spending per student (panel (b)), in tandem with a reduction in the implicit rate of return for PIS (panel (c)). The result is intuitive and supported by empirical evidence. The lower part of Figure 2 shows the outcome of a political experiment where the baseline ϕ is changed in a range from 0.7 to 2.1. A number of interesting features of the equilibrium's long-run allocations emerge. Consistent with the case of inelastic labor, an increase in ϕ leads to more generous pension transfers per retiree (panel (d)). As the elderly become more powerful, short-term mandate governments try to maximize the political consensus by addressing the needs of the crucial voting group with larger transfers.[24] The induced adverse fiscal interaction effect depresses human capital investments (panel (e)). Finally, the implicit rate of return for PIS turns out to be hump-shaped in ϕ (panel (f)), in line with Corollary 2. We note that PIS is politically sustained although the corresponding implicit return falls short the world interest rate. This numerical solution reinforces the analytic results presented in the previous section.

[23] Perotti and von Thadden (2006) theoretically and empirically argue that the distribution of private financial wealth influences political support for rents. They provide evidence for OECD economies, which shows that political support may shift away from free markets and toward a more corporatist governance system in response to the loss of financial wealth.

[24] This prediction is consistent with evidence discussed by Mulligan and Xala-i-Martin (1999). The authors argue that the political consciousness of the elderly has increased as the bulk of baby boomers approach retirement age and, in turn, public pension benefits have risen.

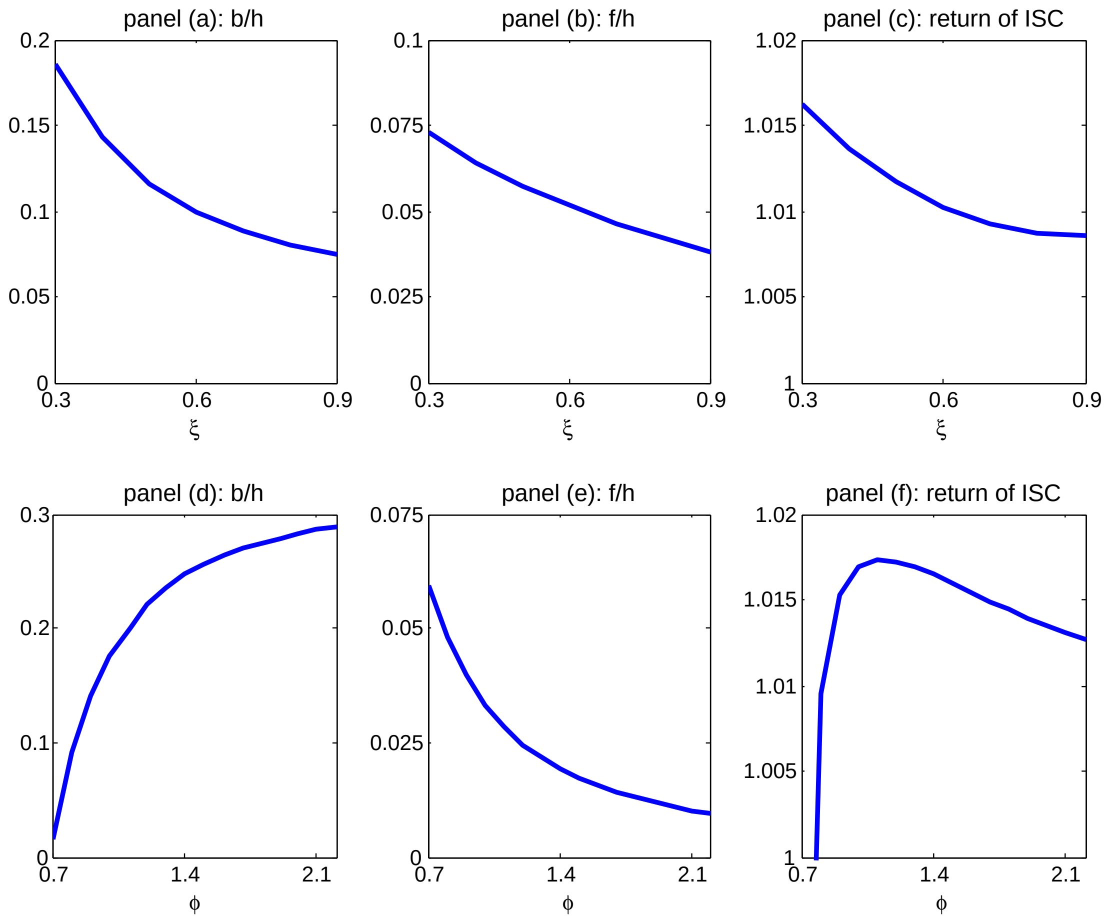

Figure 2: Comparative statics at the steady state

5.2 Demographic Shocks

The model also delivers interesting predictions for how human capital investments and pensions respond to demographic changes.[25] The following experiment aims to qualify the political sustainability of intergenerational fiscal programs when they are affected by changes to economic fundamentals.

Suppose the economy is hit by a temporary unexpected demographic boom. The fertility rate increases from its baseline value 1.006^{30} to 1.02^{30}. A demographic bust follows, which reestablishes the initial fertility condition. All other parameters correspond to the baseline values indicated in Table 1. The shock occurs at the beginning of the period, before the government sets the fiscal policy and agents take private decision. Figure 3 plots the time path of the policies adjustments.

In the first period, the government reacts by reducing both education spending and pensions per recipient. An unexpected baby boom is accompanied by a fall in education

[25]We describe the impact of a financial shock on MPE in the supplementary material in Appendix B.

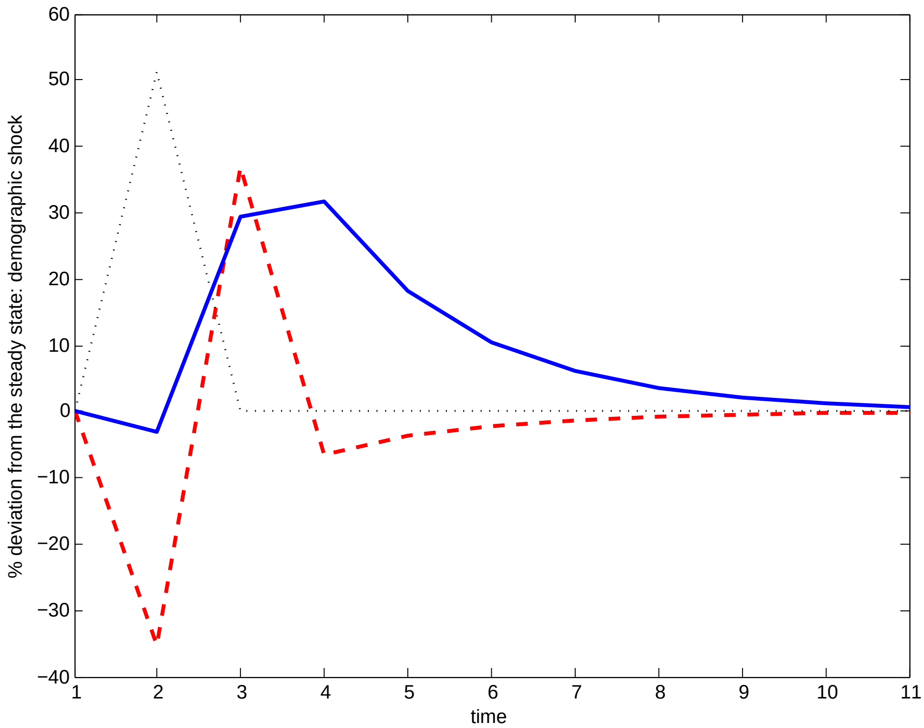

Figure 3: Demographic transition. The figure shows impulse-response functions of a demographic baby boom-bust shock. The solid(dashed) line denotes the dynamics of pensions(public education). The dotted line represents the demographic dynamics.

per student and a boost in the aggregate education spending, implying a quality-quantity trade-off is at work. The consequent increase of the income tax to finance education adversely affects the social security budget. Therefore, pension benefits per retiree also fall. In the second period, the fertility rate reverts to the baseline value, but the policies do not yet converge toward the initial steady state. The baby boomers become workers, whereas the number of students and retirees are at the baseline level. This demographic transition expands the financing of both programs. Moreover, adult voters rationally anticipate that they will form the largest and most politically influential cohort when they will be elderly. The possibility of extrapolating a larger electoral rent then provides additional motives for sustaining the public education program. In the third period, the baby boomers retire. As anticipated, the government raises social security benefits per retiree and, in turn, pension contributions. The resulting distortion of labor supply pushes public human capital investments downward. From period $t = 4$ onward, the population reverts to the initial stationary level. Pensions fall and public investments grow. Eventually, both programs converge to the steady state. This pattern resembles the post-war dynamics for public education and social security programs in modern economies. Lindert (1996) shows that for OECD countries during the period 1960-1981: (i) growing number of children meant diluted education expenditure per-child (baby boom squeeze on schooling per child); (ii) countries with more adults tended to spend more on tax-bases social program, even per recipient; (iii) the ratio of social security contributions to GDP and the proportion of retired individuals in the population are positively correlated. Similar evidence are also borne in Persson and Tabellini (2003) and are perfectly in line with our quantitative experiment. These results are, however, in contrast to Razin, Sadka, and Swagel (2002) who argue that the dependency ratio is negatively related to per-retiree pension transfers.

6 Conclusions

In this paper, we show why democratic institutions in modern economies implement public education and pensions as two arms of a unique social policy package deal. Altruism, commitment, reputation, and price channeling are absent. In a small open economy, the political power of elderly voters creates the motive for adult voters to support growth-oriented policy. The power of the theory lies in the ability of short-term mandate governments to sustain long-term growth and internalize technological spillover. Two fundamental features of the model drive our results: (i) the nature of short-term relations among politicians and voters; and (ii) the prospect of follow-up intergenerational fiscal programs, which serves as the incentive device to implement current policies.

In this paper, we assume that governments run a balanced budget and do not have access to financial credit markets. Adding public debt would break the link between taxation and expenditure, allowing governments to shift the fiscal burden to future generations. Then, governments could manipulate the state of the world inherited by their successors through human capital investment and public debt. These two strategic channels have counteracting forces. On the one hand, increasing human capital investment crowds in social security, because it raises labor productivity and reduces the fiscal burden borne by future constituents, as this paper shows. On the other hand, increasing public debt crowds out future pension benefits, because it increases the fiscal burden borne by future generations (Song, Storesletten, and Zilibotti, 2012). The strategic interaction between these two fiscal channels is an extension that is worth pursuing.

7 Appendix A

In this Appendix, we provide the anecdotal evidence that was referred to in the Introduction (see Appendix A.1) and the proofs (see Appendix A.2).

A.1 ANECDOTAL EVIDENCE

In this Appendix, we examine the empirical relationship between social security and human capital. We show that the data are broadly consistent with the main implication of the theory: an increase in the level of human capital increases the generosity of pension transfers. We do not provide a structural test of our theory. The empirical results should then be seen as only suggestive of the pension-human capital nexus.

	$\Delta\log(b_{it})$					
	[1]	[2]	[3]	[4]	[5]	[6]
$\Delta\log(h_{it})$	0.047**	0.042**	0.074**	0.048**	0.045**	0.074**
	(0.018)	(0.019)	(0.030)	(0.018)	(0.018)	(0.037)
$\Delta\log(wealth_{it})$		0.021				0.031
		(0.037)				(0.043)
$\Delta\log(tax_{it})$			0.060			0.051
			(0.066)			(0.067)
$\Delta\log(unemp_{it})$				0.043		0.026
				(0.035)		(0.048)
$\Delta\log(open_{it})$					0.037	0.107
					(0.118)	(0.137)
Time Fixed	YES	YES	YES	YES	YES	YES
Cluster	Country	Country	Country	Country	Country	Country
Observations	324	261	271	324	324	223
Countries	26	24	25	26	26	22
Adj. R-squared	0.327	0.417	0.395	0.34	0.33	0.447

Table 2: Regression. Data on pensions, tax revenue, and private financial wealth are from OECD database (2013). Data on human capital come from Barro and Lee (2010). Data on population, GDP, and unemployment rate are from World Bank database. The index of financial openness comes from Penn World Table. Variables are expressed in constant US dollar 2005 and concern 26 OECD countries, which were OECD members over the period 1995-2009. *significant at 10% **significant at 5% ***significant at 1%.

We estimate regressions in which social security, the dependent variable, is a function of human capital, as suggested by our theory, and additional control variables. We employ the following baseline empirical specification:

$$\Delta\log b_{it} = \alpha_0 + \alpha_1\Delta\log h_{it} + \mu_t + \delta_i + \upsilon_{it}$$

where Δ stands for the first-difference operator of the variable of interest, t denotes the year, i denotes the country, δ_i is a country fixed effect, and μ_t is a year fixed effect. The dependent variable b_{it} is measured by the pension generosity defined as the ratio pension per retiree over GDP per capita. Using such a proxy, we control for demographic and technological factors that can also drive variations in pension transfers. The proxy for human capital h_{it} is defined as labor force participation with tertiary education as percentage of total labor force. Table 2 provides the results of regressions. Column (1) shows the result for the baseline regression. The following additional controls are gradually introduced in the model: private financial wealth, $wealth_{it}$, defined as total financial net worth household per capita, to capture the presence of a private saving technology as a substitute for public pension; total tax revenue per capita as a share of GDP, tax_{it}, to check for the size of government; unemployment rate, $unemp_{it}$, to control for business cycle effects; and financial openness, $open_{it}$, to control for the adverse effects of external shocks on pensions. All variable are expressed in logarithmic form. The results are reported in columns (2)-(5).

A.2 PROOF

Proposition 1. Eq. (7) holds with $c^o = 0$ when $\phi = 0$. Hence, $b = 0$ in each period. The first-order condition with respect to f then yields $-n/c^a < 0$, which implies that $f = 0$. ■

Proposition 2. Using the public budget constraints, $\tau_t w h_t = n f_t$ and $z_t w h_t = (1/n)\, b_t$, yields $c_t^a = w h_t - n f_t - (1/n)\, b_t$ and $c_t^o = b_t$. Hence, the outcome of the political maximization program is $\{\mathcal{F}(h_t), \mathcal{B}(h_t)\} = \underset{\{f_t, b_t\}}{\arg\max} \log(c_t^a) + \beta \log\left(c_{t+1}^o\right) + (\phi/n)\log(c_t^o)$. Starting from a sufficiently large and finite T, we first compute the first-order conditions with respect to the intergenerational transfers. Second, we solve by backward induction for each time $T - j$, with $j = 0, 1, ..., T$, subject to the next-period policy rules. Finally, we take the limit of the finite-horizon equilibrium for T that tends to infinity.

At the terminal date T, adults have no future. Hence, the political objective function is $\log(c_T^a) + (\phi/n)\log(c_T^o)$. The first-order conditions with respect to f_T and b_T are $-n/c_{T-1}^a < 0$ and $c_T^a/c_T^o = 1/\phi$ respectively. This implies that $f_T = \mathcal{F}(h_T) \equiv 0$ and $b_T = \mathcal{B}(h_T) \equiv (n\phi/(\phi+n))\, w h_T$. At time $T-1$, adults have a two-period temporal horizon. Hence, the political objective function is $\log\left(c_{T-1}^a\right) + \beta\log(c_T^o) + (\phi/n)\log\left(c_{T-1}^o\right)$. The first-order conditions with respect to f_{T-1} and b_{T-1} are, respectively, Eqs. (7) and (8) where $d\mathcal{B}(h_T)/df_{T-1} = w\,(n\phi/(\phi+n))\, A\,(1-\theta)\,(h_{T-1}/f_{T-1})^{\theta}$. Standard algebra implies that $f_{T-1} = \mathcal{F}(h_{T-1}) \equiv (\beta(1-\theta)/(\phi + n(1+\beta(1-\theta))))\, w h_{T-1}$ and $b_{T-1} = \mathcal{B}(h_{T-1}) \equiv (n\phi/(\phi + n(1+\beta(1-\theta))))\, w h_{T-1}$. Iterating the maximization program for period $T-2$, we find that the equilibrium policy rules are structurally equal to the previous-period policies. Then, we conclude that for every time t

$$f_t = \mathcal{F}(h_t) \equiv \frac{\beta(1-\theta)}{\phi + n(1+\beta(1-\theta))} w h_t \tag{A.1}$$

and

$$b_t = \mathcal{B}(h_t) \equiv \frac{n\phi}{\phi + n(1+\beta(1-\theta))} w h_t \tag{A.2}$$

Replacing Eqs. (A.1) and (A.2) in the public budget constraints, we obtain $\tau_t = \mathcal{T}(h_t) \equiv n\beta(1-\theta)/(\phi + n(1+\beta(1-\theta)))$ and $z_t = \mathcal{Z}(h_t) \equiv \phi/(\phi + n(1+\beta(1-\theta)))$. Finally, $h_{t+1}/h_t = A(w\beta(1-\theta)/(\phi + n(1+\beta(1-\theta))))^{1-\theta}$ is obtained by inserting Eq. (A.1) into the human capital technology. ■

Corollary 1. Inspecting Eq. (A.2) yields $\mathcal{B}_{h_{t+1}} = wn\phi/(\phi + n(1+\beta(1-\theta)))$. Differentiating it with respect to ϕ, we obtain $\mathcal{B}_{h_{t+1},\phi} \propto wn^2(1+\beta(1-\theta))$, which is positive for any parameter value. ■

Corollary 2. From Eq. (9), we denote $\rho(\phi) \equiv b'/(wh(\tau+z))$. It is straightforward to show that $\lim_{\phi\to 0}\rho(\phi) = \lim_{\phi\to\infty}\rho(\phi) = 0$. Differentiating $\rho(\phi)$ with respect to ϕ and equating it to zero, we obtain $\tilde{\phi} \equiv (n/2)\left(\beta\theta + \sqrt{\beta(\beta\theta^2 + 4(1+\beta(1-\theta)))}\right)$ as the unique positive solution. Finally, taking the second-order derivative of $\rho(\phi)$ with respect to ϕ and evaluating it in $\tilde{\phi}$ yields $\rho_{\phi,\phi}\left(\tilde{\phi}\right) \propto -n\sqrt{\beta\left(4+\beta(\theta-2)^2\right)}$, which is negative for any parameter value. Then, we conclude that $\rho(\phi)$ is hump-shaped in ϕ. ■

Corollary 3. Consider an economy where adults are endowed with $w>0$. In a probabilistic voting, the political maximization program is $\max_b \log(w(1-z)) + (\phi/n)\log(b)$ subject to the social security budget constraint $wz = (1/n)b$. Hence, the equilibrium pension rule is $b = n\phi w/(n+\phi)$. The corresponding implicit rate of return is the population growth rate, i.e., $b'/wz = n$. The implicit rate of return for PIS, as described in Eq. (9), is strictly larger than n only if $A > \overline{A}$ with

$$\overline{A} \equiv \frac{(\phi + n\beta(1-\theta))}{\phi}\left(\frac{(\phi + n(1+\beta(1-\theta)))}{w\beta(1-\theta)}\right)^{1-\theta}$$

From Corollary 2, the implicit rate of return for PIS tends to zero when ϕ approaches zero and infinity. This implies that there exist two levels $\underline{\phi} > 0$ and $\overline{\phi} < \infty$ such that for any value of $\phi \in \left(\underline{\phi}, \overline{\phi}\right)$ Eq. (9) is larger than n. ■

Proposition 4. The resolution strategy involves three steps. First, we implement the backward induction maximization procedure considering a T-period economy. Second, we determine the condition for the existence of an MPE that corresponds to the limit of the finite horizon equilibrium for T approaching infinity. Third, we identify the condition for the existence of a balanced growth path. Using the public budget constraints, the present value of after-tax lifetime income reads $\Upsilon(f_t, b_t, h_t; b_{t+1}) = wh_t - nf_t - (1/n) b_t + b_{t+1}/R$. Hence, the outcome of the political maximization program is $\{\mathcal{F}(h_t, s_{t-1}), \mathcal{B}(h_t, s_{t-1})\} = \arg\max_{\{f_t, b_t\}} (1+\beta) \log(\Upsilon(\cdot)) + (\phi/n) \log(c_t^o)$, subject to the Euler condition for savings.

First step (Backward Induction) At the terminal date T, adults have no future. Hence, $s_T = 0$. The political objective function is $\log(c_T^a) + (\phi/n) \log(c_T^o)$ where $c_T^a = wh_T - nf_T - (1/n) b_T$ and $c_T^o = Rs_{T-1} + b_T$. The first-order conditions with respect to f_T and b_T are $-n/c_T^a < 0$ and $c_T^a/c_T^o = 1/\phi$ respectively. This implies that $f_T = \mathcal{F}(h_T, s_{T-1}) \equiv 0$ and

$$b_T = \mathcal{B}(h_T, s_{T-1}) \equiv -\frac{1}{1+(\phi/n)} Rs_{T-1} + \frac{\phi}{1+(\phi/n)} wh_T \tag{A.3}$$

At time $T-1$, adults have two-period temporal horizon. Using Eq. (A.3) to eliminate b_T, we can write the equilibrium private saving

$$s_{T-1} = \frac{\beta(1+(\phi/n))}{(\phi/n)+\beta(1+(\phi/n))} (wh_{T-1} - nf_{T-1} - (1/n) b_{T-1}) - \frac{\phi}{R((\phi/n)+\beta(1+(\phi/n)))} wh_T$$

The first-order conditions with respect to f_{T-1} and b_{T-1} are $nR = d\mathcal{B}(h_T, s_{T-1})/df_{T-1}$ and $\Upsilon(\cdot)/c_{T-1}^o = ((1+\beta)/\phi)(1-(n/R) d\mathcal{B}(h_T, s_{T-1})/db_{T-1})$ where $d\mathcal{B}(\cdot)/db_{T-1} = (R\beta/n)/((\phi/n)+\beta(1+(\phi/n)))$ and $d\mathcal{B}(\cdot)/df_{T-1} = n\beta R/((\phi/n)+\beta(1+(\phi/n))) + (w\phi(1+\beta)/((\phi/n)+\beta(1+(\phi/n)))) H_{f_{T-1}}$. Let denote $\psi_{(1)} \equiv (wA(1-\theta)/R)^{1/\theta}$ where the subscript in the parenthesis denotes the number of iterations. Standard algebra shows that $f_{T-1} = \mathcal{F}(h_{T-1}, s_{T-2}) \equiv \psi_{(1)} h_{T-1}$ and

$$b_{T-1} = \mathcal{B}(h_{T-1}, s_{T-2}) \equiv -\frac{1+\beta}{1+\beta+(\phi/n)} Rs_{T-2} + \frac{n\phi}{1+\beta+(\phi/n)} \left(\frac{w}{n} + \frac{\theta}{1-\theta}\psi_{(1)}\right) h_{T-1} \tag{A.4}$$

Next, consider time $T-2$. Using Eq. (A.4), the equilibrium private saving reads

$$s_{T-2} = \frac{\beta(1+\beta+(\phi/n))}{(1+\beta)(\beta+(\phi/n))} (wh_{T-2} - nf_{T-2} - (1/n) b_{T-2}) - \frac{n\phi}{R(1+\beta)(\beta+(\phi/n))} \left(\frac{w}{n} + \frac{\theta}{1-\theta}\psi_{(1)}\right) h_{T-1}$$

The first-order conditions with respect to f_{T-2} and b_{T-2} are structurally equal to those of $T-1$ where $d\mathcal{B}(h_{T-1}, s_{T-2})/db_{T-2} = \beta R/(n(\beta + (\phi/n)))$ and $d\mathcal{B}(h_{T-1}, s_{T-2})/df_{T-2} = n\beta R/(\beta + (\phi/n)) + (n\phi/(\beta + (\phi/n)))\left((w/n) + (\theta/(1-\theta))\psi_{(1)}\right) H_{f_{T-2}}$. Let denote $\psi_{(2)} \equiv \left(A\left(w(1-\theta) + n\theta\psi_{(1)}\right)/R\right)^{1/\theta}$. Standard algebra implies that $f_{T-2} = \mathcal{F}(h_{T-2}, s_{T-3}) \equiv \psi_{(2)} h_{T-2}$ and

$$b_{T-2} = \mathcal{B}(h_{T-2}, s_{T-3}) \equiv -\frac{1+\beta}{1+\beta+(\phi/n)} R s_{T-3} + \frac{n\phi}{1+\beta+(\phi/n)} \left(\frac{w}{n} + \frac{\theta}{1-\theta}\psi_{(2)}\right) h_{T-2}$$

Iterating the maximization problem for any $j > 2$, we find that the equilibrium policy rules are structurally equal to those of $j = 2$.

Second step (Fixed Point) The MPE, characterized as the limit of the finite-horizon equilibrium, exists if and only if $\lim_{j\to\infty} \psi_{(j)}$ exists and is finite, where $\psi_{(j)} = m(\psi_{(j-1)}) \equiv \left(A\left(w(1-\theta) + n\theta\psi_{(j-1)}\right)/R\right)^{1/\theta}$ with $\psi_{(1)}$ as initial condition. Note that the mapping $m\left(\psi_{(j-1)}\right)$ is twice continuously differentiable with $m(0) > 0$, $m_\psi > 0$, and $m_{\psi\psi} > 0$. Moreover, denoting $\bar{\psi} \equiv (1/\theta)\left((R/nA)^{1/(1-\theta)} - (w(1-\theta)/n)\right)$ the value of $\psi_{(j)}$ such that $m_\psi\left(\bar{\psi}\right) = 1$, we obtain $m(\bar{\psi}) \equiv (R/nA)^{1/(1-\theta)}$. Then, $m(\bar{\psi}) < \bar{\psi}$ for $R > \left(An^\theta\left((1-\alpha)Q(\alpha Q)^{\alpha/(1-\alpha)}\right)^{1-\theta}\right)^{\frac{1-\alpha}{1-\alpha\theta}}$. As a consequence, the first-order difference equation $\psi_{(j)} = m(\psi_{(j-1)})$ supports two solutions (ψ^1, ψ^2) where $\psi^1 \leq \psi^2$ with ψ^1 as the unique, locally stable fixed-point. Henceforth, $\psi^1 \equiv \psi$ with

$$\psi = \left(\frac{A}{R}(w(1-\theta) + n\theta\psi)\right)^{\frac{1}{\theta}} \tag{A.5}$$

At every date t, the equilibrium policy rules are equal to

$$f_t = \mathcal{F}(h_t, s_{t-1}) \equiv \psi h_t \tag{A.6}$$

and

$$b_t = \mathcal{B}(h_t, s_{t-1}) \equiv a_s s_{t-1} + a_h h_t \tag{A.7}$$

where $a_h \equiv (\phi n/(1+\beta+(\phi/n)))((w/n) + (\theta/(1-\theta))\psi)$ and $a_s \equiv -R(1+\beta)/(1+\beta+(\phi/n))$. Moreover, replacing Eqs. (A.6) and (A.7) in the public budget constraints, we obtain $\tau_t = \mathcal{T}(h_t, s_{t-1}) \equiv (n/w)\psi$ and $z_t = \mathcal{Z}(h_t, s_{t-1}) \equiv (1/wn)(a_s(s_{t-1}/h_t) + a_h)$.

Third step (Balanced Growth) Inserting Eqs. (A.6) and (A.7) into the human capital production technology and equilibrium private saving yields, respectively,

$h_{t+1}/h_t = A\psi^{1-\theta}$ and

$$\frac{s_t}{h_{t+1}} = -\frac{a_s \beta R (h_t/h_{t+1})}{n(a_s + R(1+\beta))} \frac{s_{t-1}}{h_t} + \frac{R\beta (w - n\psi - (a_h/n))(h_t/h_{t+1}) - a_h}{a_s + R(1+\beta)}$$

The economy then is on its balanced growth if

$$R = (n + (\phi/\beta)) A\psi^{1-\theta} \tag{A.8}$$

Solving simultaneously Eqs. (A.5) and (A.8) yields $\psi = (w(1-\theta)) / (n(1-\theta) + (\phi/\beta))$ and $R = (n + (\phi/\beta)) A ((w(1-\theta)) / (n(1-\theta) + (\phi/\beta)))^{1-\theta}$. Inserting these two values into Eq. (A.7) yields the restriction on ϕ such that $b_t > 0$, i.e.,

$$\phi > (w(1-\theta))^{1-\theta} \left(n(1-\theta) + \frac{\phi}{\beta} \right)^{\theta} \frac{(1+\beta) A}{w} \frac{s_{-1}}{h_0} \tag{A.9}$$

The right-hand side of Eq. (A.9) is increasing and concave in ϕ, and larger than zero for $\phi = 0$. It implies that there exists a unique level $\tilde{\phi}$, such that if $\phi > \tilde{\phi}$, the inequality (A.9) is satisfied and fiscal feasibility is guaranteed. ■

Proposition 5. Consider the basic set-up presented in Subsection 4.2, but assume that competitive markets exist in which young agents can borrow physical resources, e_t, to invest in education. The individual maximization problem for an agent born in period $t-1$ is $\max_{\{e_{t-1}, s_t\}} \log(c_t^a) + \beta \log(c_{t+1}^o)$ subject to the feasibility constraint $0 \leq e_{t-1} \leq wh_t/R$, the individual resource constraints $c_t^a + s_t + Re_{t-1} \leq wh_t$ and $c_{t+1}^o \leq Rs_t$, and the human capital technology $h_t = H(h_{t-1}, e_{t-1}) \equiv Ah_{t-1}^{\theta} e_{t-1}^{1-\theta}$. The first-order conditions with respect to e_{t-1} and s_t are $H_{e_{t-1}} = R/w$ and $c_{t+1}^o / c_t^a = \beta R$. Solving for the unknown variables yields $e_t/h_t = (w(1-\theta)A/R)^{\frac{1}{\theta}}$ and $s_t/h_t = w\theta\beta/(1+\beta)$. Note that the economy is on the balanced growth path from the initial period onwards. We aim to compare the equilibrium amount of public education provided by governments and the equilibrium level of private education delivered in the presence of complete credit markets. For this purpose, evaluate the equilibrium private education when condition (A.8) is satisfied, others things equal. Thus, inserting Eq. (A.8) into the expression for private education yields $e_t/h_t = \left(\left((w(1-\theta))^{\theta} (n(1-\theta) + (\phi/\beta))^{1-\theta} \right) / (n + (\phi/\beta)) \right)^{1/\theta}$. Comparing it with the public investment rule, i.e., $f_t/h_t = (w(1-\theta)) / (n(1-\theta) + (\phi/\beta))$, we obtain that f_t/h_t is always strictly larger than e_t/h_t. The difference $f_t/h_t - e_t/h_t = n\theta$ quantifies the amount of technological spillover internalized by policymakers. ■

Corollary 4. Consider the choice of a social planner of a small open economy who chooses a sequence of allocations $\{c_t^a, c_t^o, s_t, f_t, h_{t+1}\}$ to maximize the discounted utility of all generations. Following Farhi and Werning (2007), the planner attaches geometrically decaying Pareto weight, $\delta \in (0,1)$, on the discounted utility of each generation. Given initial conditions $\{h_0, s_{-1}\}$, the sequential formulation of the planner's problem reads

$$\max_{\{c_t^a, c_t^o, s_t, f_t, h_{t+1}\}_{t=0}^{\infty}} (\beta/\delta) \log(c_0^o) + \sum_{t=0}^{\infty} \delta^t \left(\log(c_t^a) + \beta \log\left(c_{t+1}^o\right)\right)$$

subject to the aggregate resources constraint and the human capital technology

$$\begin{aligned} c_t^a + (c_t^o/n) + n f_t + s_t &\leq w h_t + R(s_{t-1}/n) && \forall t \ \ (\varkappa_t \delta^t) \\ h_{t+1} - H(f_t, h_t) &\leq 0 && \forall t \ \ (\varrho_{t+1}\delta^{t+1}) \end{aligned}$$

where $(\varkappa_t \delta^t)$ and $(\varrho_{t+1}\delta^{t+1})$ are the associated Lagrangian multipliers. The first-order conditions with respect to c_t^a, c_t^o, s_t, f_t, and h_{t+1} yield

$$\begin{aligned} c_t^a &: 1/c_t^a = \varkappa_t \\ c_t^o &: (\beta/\delta)(1/c_t^o) = (1/n)\varkappa_t \\ s_t &: \varkappa_t = (R/n)\varkappa_{t+1}\delta \\ f_t &: n\varkappa_t = \varrho_{t+1}\delta H_{f_t} \\ h_{t+1} &: \varrho_{t+1} = w\varkappa_{t+1} + \varrho_{t+2}\delta H_{h_{t+1}} \end{aligned}$$

together with the transversality conditions, $\lim_{t\to\infty} \varkappa_t \delta^t s_t = 0$ and $\lim_{t\to\infty} \varrho_{t+1}\delta^{t+1} h_{t+1} = 0$. Eliminating the multipliers from the first-order conditions, the following conditions for optimal policies hold

$$c_{t+1}^o / c_t^a = R\beta \tag{A.10}$$

and

$$c_t^a / c_t^o = \delta / n\beta \tag{A.11}$$

and

$$R = \left(w + n\frac{H_{h_{t+1}}}{H_{f_{t+1}}}\right) H_{f_t} \tag{A.12}$$

where $H_{h_t} = \theta A (f_t/h_t)^{1-\theta}$ and $H_{f_t} = (1-\theta) A (h_t/f_t)^{\theta}$. We then guess and verify that the optimal education investment rule is $f_t = \widehat{\psi} h_t$. Inserting the guess into Eq. (A.12) yields

$$\widehat{\psi} = \left(A\left(w(1-\theta) + n\theta\widehat{\psi}\right)/R\right)^{1/\theta} \tag{A.13}$$

Using Eqs. (A.10) and (A.11), the economy is on its balanced growth where all variables grow at the rate $h_{t+1}/h_t = A\widehat{\psi}^{1-\theta}$ if

$$R = A\widehat{\psi}^{1-\theta}(n/\delta) \tag{A.14}$$

Solving simultaneously Eqs. (A.13) and (A.14) yields $\widehat{\psi} = \delta w(1-\theta)/n(1-\delta\theta)$ and $R = A(\delta w(1-\theta)/n(1-\delta\theta))^{1-\theta}(n/\delta)$. We now compare the levels of investment in public education provided by a government (see, Proposition 4) and a social planner. It is straightforward to show that $\widehat{\psi} = \psi$ if $\delta = \beta n/(\phi + \beta n)$ with $\phi > \tilde{\phi}$. ■

Proposition 6. For contradiction, suppose that $b'/wh(\tau + z) > R$ along the balanced growth path. Since the implicit rate of return for PIS, Eq. (12), is a monotonic decreasing function of the state variable s/h', then $b'/wh(\tau + z)$ must necessarily be larger than R when $s/h' = 0$. Using Eq. (12), the inequality holds only if $a_h/(n\psi + (1/n)a_h) > n + (\phi/\beta)$ with $a_h \equiv n\phi/(1 + \beta + (\phi/n))(w/n + \psi\theta/(1-\theta))$. Simple algebra shows that this condition is never satisfied, which proves the contradiction. Then, we conclude that implicit rate of return for PIS must necessarily be smaller than R. ■

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8 Appendix B (Not For Publication)

In this Appendix, we provide some supplementary material. Section B.1 presents the political microfoundation of the model. Section B.2 derives the first-order conditions of the politico-economic optimization program. Section B.3 describes the details of the numerical strategy. Finally, Section B.4 shows the details of the numerical analysis in the case of a financial shock.

B.1 PROBABILISTIC VOTING MODEL

The political equilibrium discussed in the paper has an explicit microfoundation in terms of the voting model based on Lindbeck and Weibull (1987) and applied to a dynamic OLG environment with intergenerational transfers. The electoral competition takes place between two office-seeking candidates belonging to parties $\iota \in \{\mathcal{L}, \mathcal{R}\}$. Candidates and voters move sequentially. First, candidates announce their political platform, $q_\iota \equiv \{\tau_\iota, f_\iota, z_\iota, b_\iota\}$, constrained to the per-period public budgets and fiscal feasibility. As the election takes place each period, the candidates cannot credibly commit to future policies. Second, voter j belonging to cohort $i \in \{a, o\}$ choose the preferred candidate based on the fiscal announcements and her ideology. Agents vote for party $\mathcal{R}_t$ as long as the idiosyncratic ideological bias, σ^i_j, is larger than the difference in the indirect utility achieved from voting for the alternative platforms, net of the aggregate shock η. It implies that $\sigma^i_j \geq \sigma^i(h_t, s_{t-1}) \equiv \mathcal{U}^i_{\mathcal{L}_t} - \mathcal{U}^i_{\mathcal{R}_t} - \eta$, where $\sigma^i(h_t, s_{t-1})$ identifies the swing voters of each cohort, i.e., the voter who is indifferent between the two candidates. We assume that σ^i_j is drawn from a symmetric and cohort-specific uniform distribution on $\left[-1/\left(2\boldsymbol{\sigma}^{\mathbf{i}}\right), 1/\left(2\boldsymbol{\sigma}^{\mathbf{i}}\right)\right]$. Similarly, the i.i.d. random variable η is uniformly distributed in the support $[-1/(2\boldsymbol{\eta}), 1/(2\boldsymbol{\eta})]$. The assumption of uniform distribution is for simplicity (see Banks and Duggan, 2005).[26] Conditional on η, the share of voters belonging to cohort i and supporting party $\mathcal{R}_t$ is $\lambda^i_t \equiv (1/2) - \boldsymbol{\sigma}^i\left(\mathcal{U}^i_{\mathcal{L}_t} - \mathcal{U}^i_{\mathcal{R}_t} - \eta\right)$. Under majority rule, party $\mathcal{R}_t$ wins the election if and only if it obtains the largest share of votes, i.e., $\lambda^a_t + (1/n)\lambda^o_t > \frac{1}{2}(1 + (1/n))$. This implies that η must be larger than the threshold level $\eta(h_t, s_{t-1}) \equiv (1/n)(\boldsymbol{\sigma}^o/(\boldsymbol{\sigma}^a + \boldsymbol{\sigma}^o))\left(\mathcal{U}^o_{\mathcal{L}_t} - \mathcal{U}^o_{\mathcal{R}_t}\right) + (\boldsymbol{\sigma}^a/(\boldsymbol{\sigma}^a + \boldsymbol{\sigma}^o))\left(\mathcal{U}^a_{\mathcal{L}_t} - \mathcal{U}^a_{\mathcal{R}_t}\right)$. Hence, the objective function of party $\mathcal{R}_t$ is $\max_{q_{\mathcal{R}_t}} \Pr(\eta_t \geq \eta(h_t, s_{t-1}))$ which simplifies to:

$$\max_{q_{\mathcal{R}_t}} \frac{1}{2} - \boldsymbol{\eta}\eta(h_t, s_{t-1}) \tag{B.1}$$

[26]The random variable σ^i_j reflects the voters' opinions about the candidate's positions (e.g., civil rights, pro-market rules, religious issues) and personal characteristics (e.g., honesty, leadership, trustworthiness). As it is drawn from cohort-specific distributions, individuals belonging to the same cohort may vote differently. The additional random variable η measures the average candidates' popularity. Thus, individuals belonging to different cohorts may support the same party.

Likewise, for party $\mathcal{L}_t$, the objective function is $\max_{q_{\mathcal{L}_t}} \Pr\left(\eta_t \le \eta\left(h_t, s_{t-1}\right)\right)$ which collapses to $\max_{q_{\mathcal{L}_t}} (1/2) + \boldsymbol{\eta}\eta\left(h_t, s_{t-1}\right)$.

To prove that the two candidates' platforms converge to the same equilibrium fiscal platform, we adopt a backward procedure. Consider a two-period economy. In the last period 2, the political maximization program for party $\mathcal{R}_2$, described in Eq. (B.1), simplifies to:

$$\max_{q_{\mathcal{R}_2}} \frac{1}{2} - \boldsymbol{\eta}\left(\frac{1}{n}\frac{\boldsymbol{\sigma}^o}{\boldsymbol{\sigma}^a + \boldsymbol{\sigma}^o}\left(\mathcal{U}^o_{\mathcal{L}_2} - \mathcal{U}^o_{\mathcal{R}_2}\right) + \frac{\boldsymbol{\sigma}^a}{\boldsymbol{\sigma}^a + \boldsymbol{\sigma}^o}\left(\mathcal{U}^a_{\mathcal{L}_2} - \mathcal{U}^a_{\mathcal{R}_2}\right)\right) \tag{B.2}$$

In the last period, adults have no future. Then, $\mathcal{U}^a_{\iota_2}$ is equal to $u\left(c^a_{\iota_2}\right)$. Hence, Eq. (B.2) reduces to $\max_{q_{\mathcal{R}_2}} u\left(c^a_{\mathcal{R}_2}\right) + (1/n)\left(\boldsymbol{\sigma}^o/\boldsymbol{\sigma}^a\right) u\left(c^o_{\mathcal{R}_2}\right)$. Following the same logic, the political objective function for party $\mathcal{L}$ at time 2 turns out to be $\max_{q_{\mathcal{L}_2}} u\left(c^a_{\mathcal{L}_2}\right) + (1/n)\left(\boldsymbol{\sigma}^o/\boldsymbol{\sigma}^a\right) u\left(c^o_{\mathcal{L}_2}\right)$. This implies that office-seeking candidates propose the same equilibrium platform, prescribing $\tau_{\mathcal{L}_2} = \tau_{\mathcal{R}_2}$, $f_{\mathcal{L}_2} = f_{\mathcal{R}_2}$, $z_{\mathcal{L}_2} = z_{\mathcal{R}_2}$, and $b_{\mathcal{L}_2} = b_{\mathcal{R}_2}$. It follows that $\mathcal{U}^i_{\mathcal{L}_2} = \mathcal{U}^i_{\mathcal{R}_2}$. Replicating the same argument at date 1, the maximization program for party $\mathcal{R}_1$ reads

$$\max_{q_{\mathcal{R}_1}} \frac{1}{2} - \boldsymbol{\eta}\left(\frac{1}{n}\frac{\boldsymbol{\sigma}^o}{\boldsymbol{\sigma}^a + \boldsymbol{\sigma}^o}\left(\mathcal{U}^o_{\mathcal{L}_1} - \mathcal{U}^o_{\mathcal{R}_1}\right) + \frac{\boldsymbol{\sigma}^a}{\boldsymbol{\sigma}^a + \boldsymbol{\sigma}^o}\left(\mathcal{U}^a_{\mathcal{L}_1} - \mathcal{U}^a_{\mathcal{R}_1}\right)\right) \tag{B.3}$$

where $\mathcal{U}^a_{\iota_1} = u\left(c^a_{\iota_1}\right) + p_{\iota_1}\left(\mathcal{U}^o_{\mathcal{R}_2} + \sigma^o_{j,2} + \eta\right) + (1 - p_{\iota_1})\,\mathcal{U}^o_{\mathcal{L}_2}$, with $p_{\iota_1} \equiv (1/2) - \boldsymbol{\eta}\eta\left(h_2, s_1; \iota_1\right)$ defined as the probability of $\mathcal{R}$ to be elected at time 2, conditional on the incumbent party ι_1. Given the equilibrium policy at time 2, we obtain $p_{\mathcal{L}_1} = p_{\mathcal{R}_1} = 1/2$. Hence, the maximization program (B.3) boils down to $\max_{q_{\mathcal{R}_1}} u\left(c^a_{\mathcal{R}_1}\right) + \beta u\left(c^o_{\mathcal{R}_2}\right) + (1/n)\left(\boldsymbol{\sigma}^o/\boldsymbol{\sigma}^a\right) u\left(c^o_{\mathcal{R}_1}\right)$. Replicating the same argument for party $\mathcal{L}_1$ the objective turns out to be $\max_{q_{\mathcal{L}_1}} u\left(c^a_{\mathcal{L}_1}\right) + \beta u\left(c^o_{\mathcal{L}_2}\right) + (1/n)\left(\boldsymbol{\sigma}^o/\boldsymbol{\sigma}^a\right) u\left(c^o_{\mathcal{L}_1}\right)$. Such an argument can be applied in every period t. Therefore, the equilibrium policy platform solves the following maximization program:

$$\max_{q_t} u\left(c^a_t\right) + \beta u\left(c^o_{t+1}\right) + (\phi/n)\, u\left(c^o_t\right)$$

where $\phi \equiv \boldsymbol{\sigma}^o/\boldsymbol{\sigma}^a$. We conclude by noting that under the assumption of Markov perfect equilibria, the probabilistic voting outlined in this Appendix applies equally to both static and dynamic models.

B.2 DERIVATION OF FIRST-ORDER CONDITIONS

In this section, we derive the first-order conditions of the politico-economic problem as described in Eqs. (13) and (14). As reported in the text $\mathcal{U}\left(\tau, z, b, h, s_-; b'\right) = (1+\beta)\log\left(\Upsilon\left(\tau, z, h; b'\right)\right) + (\phi/n)\log\left(Rs_- + b\right)$, where $\Upsilon(\cdot) = (1 - \tau - z)\, wL(\tau, z)\, h +$

$F\left(L\left(\tau,z\right)\right)h+b'/R$ with $L\left(\tau,z\right)=\left(\left(1-\tau-z\right)w/X\right)^{\xi}$. Moreover, $b'=\mathcal{B}\left(h',s\right)$ where $h'=H\left(f,h\right)$ and $s=\mathcal{S}\left(\tau,z,f,h\right)$ is the mapping that solves the recursion

$$s=\left(\beta/\left(1+\beta\right)\right)\left(\left(1-\tau-z\right)wL\left(\tau,z\right)+F\left(L\left(\tau,z\right)\right)\right)h-\left(1/R\left(1+\beta\right)\right)\mathcal{B}\left(h',s\right)$$

Hence, the outcome of the political maximization program is

$$\left\{\mathcal{T}\left(h,s_{-}\right),\mathcal{F}\left(h,s_{-}\right),\mathcal{Z}\left(h,s_{-}\right),\mathcal{B}\left(h,s_{-}\right)\right\}=\underset{\{\tau,f,z,b\}}{\arg\max}\ \mathcal{U}\left(\tau,z,b,h,s_{-};\mathcal{B}\left(s,h'\right)\right)$$

subject to the public budgets constraints $\tau whL\left(\tau,z\right)\geq nf$ and $zwhL\left(\tau,z\right)\geq\left(1/n\right)b$. Write the problem as a standard Lagrangian problem with multipliers λ and η associated with the public investment budget and the social security budget respectively. Removing the functional arguments for expositional clarity, the first-order conditions with respect to τ, f, z, and b yield

$$\begin{aligned}
\tau:0&=\frac{1+\beta}{\Upsilon\left(\cdot\right)}\frac{d\Upsilon\left(\cdot\right)}{d\tau}+whL\left(\cdot\right)\left(\lambda\frac{1-z-\left(1+\xi\right)\tau}{1-\tau-z}-\eta\frac{z\xi}{1-\tau-z}\right)\\
f:0&=\frac{1+\beta}{\Upsilon\left(\cdot\right)}\frac{d\Upsilon\left(\cdot\right)}{df}-\lambda n\\
z:0&=\frac{1+\beta}{\Upsilon\left(\cdot\right)}\frac{d\Upsilon\left(\cdot\right)}{dz}+whL\left(\cdot\right)\left(\eta\frac{1-\tau-\left(1+\xi\right)z}{1-\tau-z}-\lambda\frac{\tau\xi}{1-\tau-z}\right)\\
b:0&=\frac{\phi}{c^{o}}-\eta
\end{aligned}$$

where $d\Upsilon\left(\cdot\right)/d\tau=wh\left(L_{\tau}-L\left(\cdot\right)\right)+F_{l}L_{\tau}h+\left(1/R\right)\left(d\mathcal{B}\left(\cdot\right)/d\tau\right)$, $d\Upsilon\left(\cdot\right)/dz=wh\left(L_{z}-L\left(\cdot\right)\right)+F_{l}L_{z}h+\left(1/R\right)\left(d\mathcal{B}\left(\cdot\right)/dz\right)$ and $d\Upsilon\left(\cdot\right)/df=\left(1/R\right)\left(d\mathcal{B}\left(\cdot\right)/df\right)$. The marginal cost of public funds are $\mu^{z}=\left(1-\tau-z\right)/\left(1-\tau-\left(1+\xi\right)z\right)$ and $\mu^{\tau}=\left(1-\tau-z\right)/\left(1-z-\left(1+\xi\right)\tau\right)$. Eliminating the multipliers from the first-order conditions, i.e., $\eta=\phi/c^{o}$ and $\lambda=\left(\left(1+\beta\right)/n\right)\left(1/\Upsilon\left(\cdot\right)\right)\left(d\Upsilon\left(\cdot\right)/df\right)$, the following generalized Euler conditions for equilibrium policies hold

$$\frac{c^{a}}{c^{o}}=\frac{\mu^{z}}{\phi}\left(1-\frac{1}{RwhL\left(\cdot\right)}\frac{d\mathcal{B}\left(h',s\right)}{dz}+\frac{1}{nR}\frac{\mu^{\tau}-1}{\mu^{\tau}}\frac{d\mathcal{B}\left(h',s\right)}{df}\right)\tag{B.4}$$

and

$$R\left(1+\phi\frac{\mu^{z}-1}{\mu^{z}}\frac{c^{a}}{c^{o}}\right)=\frac{1}{whL\left(\tau,z\right)}\frac{d\mathcal{B}\left(h',s\right)}{d\tau}+\frac{1}{n\mu^{\tau}}\frac{d\mathcal{B}\left(h',s\right)}{df}\tag{B.5}$$

as Eqs. (13) and (14) in the text, where $c^{a}=\Upsilon\left(\cdot\right)/\left(1+\beta\right)$ Note that Eqs. (B.4) and (B.5) encompass Eqs. (10) and (11), as a particular case, where $L\left(\cdot\right)=\mu^{z}=\mu^{\tau}=1$.

In the absence of private saving technology and with inelastic labor supply, the political objective function reduces to $\mathcal{U}\left(\tau,z,b,h;b'\right)=\log\left(\left(1-\tau-z\right)wh\right)+\beta\log\left(b'\right)+$

$(\phi/n)\log(b)$ where $b' = \mathcal{B}(h')$ with $h' = H(f,h)$. The public budget constraints are $\tau wh \geq nf$ and $zwh \geq (1/n)\,b$. Removing the functional arguments for expositional clarity, the first-order conditions for τ, f, z, and b yield

$$
\begin{aligned}
\tau &: 0 = -\frac{1}{1-\tau-z} + wh\lambda \\
f &: 0 = \frac{\beta}{b'}\frac{d\mathcal{B}(h')}{df} - \lambda n \\
z &: 0 = -\frac{1}{1-\tau-z} + wh\eta \\
b &: 0 = \frac{\phi}{b} - \eta
\end{aligned}
$$

Eliminating the multipliers from the first-order conditions yield Eqs. (7) and (8) as described in the paper.

B.3 NUMERICAL ALGORITHM

In this section, we describe the numerical strategy to compute the private saving rule $\mathcal{S}(\tau, f, z, h)$ and the public policy rules $\mathcal{T}(h, s_{-})$, $\mathcal{F}(h, s_{-})$, $\mathcal{Z}(h, s_{-})$, and $\mathcal{B}(h, s_{-})$, that solve conditions (13) and (14) in tandem with the government budget constraints (3) and (4) and the Euler condition for savings. It is based on a standard projection method with n-order Chebyshev polynomials. Within the class of orthogonal polynomials, the Chebyshev method stands out for its efficiency in approximating smooth functions.[27] Exploiting homogeneity of the utility function with respect to the state h, the two-dimensional state Markov perfect equilibrium can be conveniently reduced to an equilibrium defined over a one-dimensional state space with $\tilde{s} \equiv s_{-}/h$ and $\tilde{s} \in [\tilde{s}_{\min}, \tilde{s}_{\max}] \subseteq \mathbb{R}_{+}$. Therefore, the computation involves solving a system of 5-n non-linear equations. We approximate the functions for public policies and saving per efficiency units as $\tilde{\mathcal{T}}(\tilde{s}; \mathbf{a}^{\tau}) = \sum_{i=1}^{n} a_i^{\tau}\theta_i(\tilde{s})$, $\tilde{\mathcal{F}}\left(\tilde{s}; \mathbf{a}^{f}\right) = \sum_{i=1}^{n} a_i^{f}\theta_i(\tilde{s})$, $\tilde{\mathcal{Z}}(\tilde{s}; \mathbf{a}^{z}) = \sum_{i=1}^{n} a_i^{z}\theta_i(\tilde{s})$, $\tilde{\mathcal{B}}\left(\tilde{s}; \mathbf{a}^{b}\right) = \sum_{i=1}^{n} a_i^{b}\theta_i(\tilde{s})$, and $\tilde{\mathcal{S}}(\tilde{s}; \mathbf{a}^{s}) = \sum_{i=1}^{n} a_i^{s}\theta_i(\tilde{s})$ where $\mathbf{a}^{\tau} = (a_1^{\tau}, .., a_n^{\tau})$, $\mathbf{a}^{f} = \left(a_1^{f}, .., a_n^{f}\right)$, $\mathbf{a}^{z} = (a_1^{z}, .., a_n^{z})$, $\mathbf{a}^{b} = \left(a_1^{b}, .., a_n^{b}\right)$, and $\mathbf{a}^{s} = (a_1^{s}, .., a_n^{s})$ are vectors of unknown coefficients, and $\theta_i(\tilde{s})$ are the Chebyshev polynomials that form the basis for the approximation. The accuracy of the approximation is assessed by the Euler equation errors. By opting for a polynomial of order 15, the errors over 1901 points uniformly distributed over the state space are below 10^{-10} in all of our numerical experiments.

[27] See Judd (1998) for a complete characterization of their properties and a rigorous exposition of projection techniques.

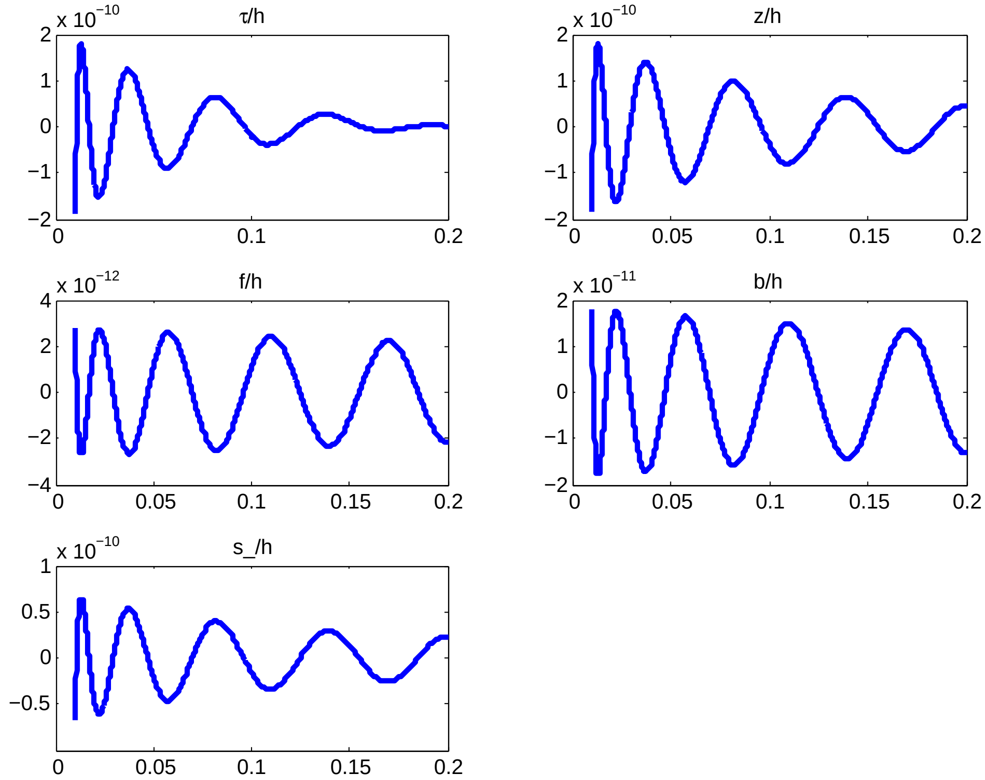

Figure 4: Errors for government's Euler conditions and the household's Euler equation.

B.4 FINANCIAL SHOCKS

We now study the effect of a temporary unexpected financial shock on the MPE. Consider the following experiment. The world interest rate is at its baseline value 1.04^{30} in the initial period. It drops temporarily in the second period to 1.03^{30}. It reverts to the baseline value from period $t = 3$ onwards. All the remaining parameter values are as in Table 1. Figure 5 shows the fiscal policies dynamics.

When the unexpected shock hits, the net present value of future pensions is positively affected. Indeed, current savings will be capitalized at a lower interest rate in the next period, which makes the opportunity cost for financing retirement benefits smaller. Anticipating this avenue, the government increases both education and pension spending. In so doing, it also influences the choice of subsequent governments by strategically manipulating the relevant asset variables. An increase in both education and pension spending depresses private savings and boosts the human capital of future generations. The evolution of the asset variables ultimately leads to higher pension benefits. In the second period, the world interest rate returns to the initial level and all expectations are realized. Retirement benefits per receipts jump at the rationally anticipated level. Increased pension contributions have the usual adverse effect on the budget, and education spending falls. Moreover, the lower net present value of the expected return on social security further discourages public investments. From period $t = 3$ onward, pensions fall and public capital goods grow, to gradually converge to the initial steady state. These predictions are consistent with empirical evidence that show the relation between the dynamics of the market interest rate and the viability of public fiscal programs (see, e.g., Rodrik, 2011). A more convincing analysis, however, should also take into account the possibility for countries to issue public bonds. This is an extension of the paper that is worth pursuing in future research.

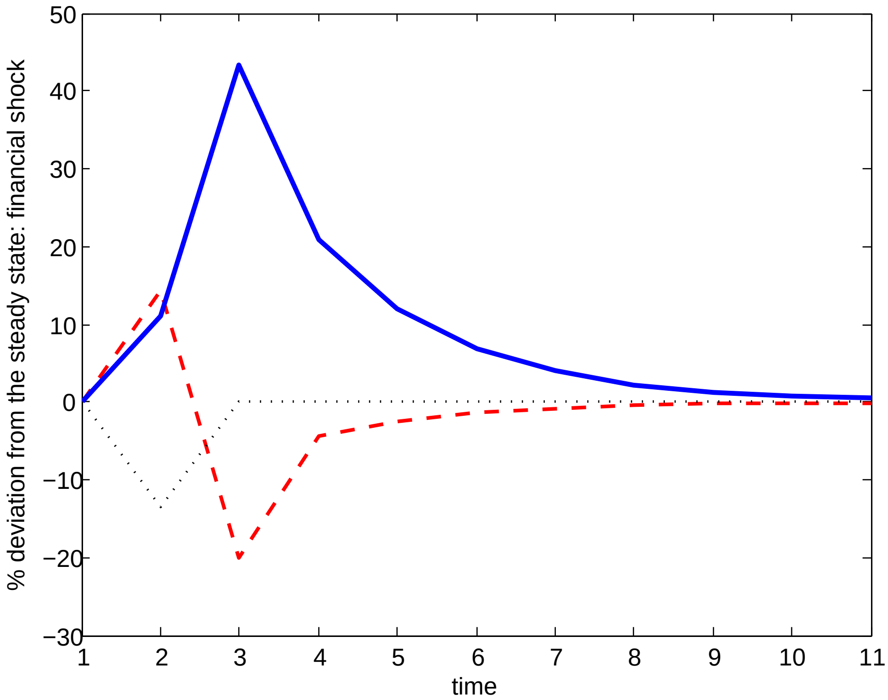

Figure 5: The figure shows impulse-response functions of a financial fall-recover of the world interest rate. The solid(dashed) line denotes the dynamics of pension(public education). The dotted line represents the dynamics of the world interest rate.

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