MEMORANDUM

No 12/2015

Threatening Thresholds?
The Effect of Disastrous Regime Shifts on the Cooperative and Non-cooperative use of Environmental Goods and Services

Florian K. Diekert

ISSN: 0809-8786

Department of Economics
University of Oslo

This series is published by the
University of Oslo
Department of Economics

P. O.Box 1095 Blindern
N-0317 OSLO Norway
Telephone: + 47 22855127
Fax: + 47 22855035
Internet: http://www.sv.uio.no/econ
e-mail: econdep@econ.uio.no

In co-operation with
The Frisch Centre for Economic Research

Gaustadalleén 21
N-0371 OSLO Norway
Telephone: +47 22 95 88 20
Fax: +47 22 95 88 25
Internet: http://www.frisch.uio.no
e-mail: frisch@frisch.uio.no

Last 10 Memoranda

No 11/15	André K. Anundsen and Ragnar Nymoen *Did US Consumers 'Save for a Rainy Day' before the Great Recession?*
No 10/15	Finn Førsund *Economic Perspectives on DEA*
No 09/15	Andreas Kotsadam, Eivind Hammersmark Olsen, Carl Henrik Knutsen and Tore Wig *Mining and Local Corruption in Africa*
No 08/15	Eric Nævdal *Catastrophes and Expected Marginal Utility – How the Value of The Last Fish in a Lak is Infinity and Why We Shouldn't Care (Much)*
No 07/15	Niklas Jakobsson and Andreas Kotsadam *The Economics of Trafficking for Sexual Exploitation*
No 06/15	Geir B. Asheim and Stéphane Zuber *Evaluating Intergenerational Risks: Probability Adjusted Rank-Discounted Utilitarianism*
No 05/15	Fridrik Mar Baldursson and Nils-Henrik von der Fehr *Natural Resources and Sovereign Expropriation*
No 04/15	Erik Biørn and Xuehui Han *Persistence, Signal-Noise Pattern and Heterogeneity in Panel Data: With an Application to the Impact of Foreign Direct Investment in GDP*
No 03/15	Alice Ciccone *Environmental Effects of a Vehicle Tax Reform: Empirical Evidence from Norway*
No 02/15	Katinka Holtsmark and Kristoffer Midttømme *The Dynamics of Linking Permit Markets*

Previous issues of the memo-series are available in a PDF® format at:
http://www.sv.uio.no/econ/english/research/unpublished-works/working-papers/

Threatening Thresholds?

The effect of disastrous regime shifts on the cooperative and non-cooperative use of environmental goods and services.

Florian K Diekert*

Abstract

This paper presents an analytically tractable dynamic game in which players jointly use a resource. The resource replenishes fully but collapses should total use exceed a threshold in any one period. The initial level of use is known to be safe. If it is at all optimal to increase resource use, the consumption frontier is pushed once. Moreover, it is shown that the degree of experimentation is decreasing in the safe value of resource use. Non-cooperative agents can take advantage of this feature and coordinate on a "cautious" equilibrium. If the status quo is sufficiently valuable, the threat of the regime shift induces the first-best. If the status-quo is not sufficiently valuable, experimentation will be inefficiently risky. But given that the threshold has not been crossed, the updated consumption frontier will, *ex post*, be socially optimal. However, there is also an pareto-inferior, "aggressive" equilibrium in which the resource is depleted immediately. Under some conditions, immediate depletion is a self-fulfilling prophecy, although the social optimum is to sustain the resource indefinitely. Closed-form solutions are provided for a specific example and it is shown that the pareto-superior, "cautious" equilibrium is risk-dominant up to a high probability that the opponents play an aggressive strategy. The result that the threat of a disastrous regime shift allows the agents to coordinate on a pareto-superior equilibrium, because it only pays to search for the location of the threshold once, is robust to extensions that account for more general resource dynamics. [244 words]

Keywords: Dynamic Games; Thresholds and Natural Disasters; Learning.
JEL-Codes: C73, Q20, Q54

*Department of Economics and Centre for Ecological and Evolutionary Synthesis, University of Oslo, PO-Box 1095, 3017 Oslo, Norway. Email: f.k.diekert@ibv.uio.no. This research is funded by NorMER, a Nordic Centre of Excellence for Research on Marine Ecosystems and Resources under Climate Change. I would like to thank Geir Asheim, Scott Barrett, Volker Diekert, Bård Harstad, Larry Karp, Matti Liski, Leo Simon, and seminar participants at the Beijer Institute, Statistics Norway, and in Oslo, Helsinki, Santa Barbara, and Berkeley for constructive discussion and comments.

1 Introduction

Everyday-experience painfully teaches that many things tolerate some stress, but if this is driven too far or for too long, they break. Not surprisingly, regime shifts, thresholds, or tipping points are a popular topic in the scientific literature. In this paper, I investigate whether and how the existence of a catastrophic threshold may be *beneficial* in the sense that it enables non-cooperative agents to coordinate their actions, thereby improving welfare over a situation where this tipping point is not present or ignored. To this end, I develop a generic model of using a productive asset that loses (some or all) its productivity upon crossing some (potentially unknown) threshold. I show that learning in this model is only affirmative. Thus, any experimentation is – if at all – undertaken in the first period. Moreover, the degree of experimentation is decreasing in the value of current use, which is known to be safe.

The model is applicable to many different settings of non-cooperative learning about the location of a threshold, but I would like to think of it broadly in terms of ecosystem services. Prime examples are the eutrophication of lakes, the bleaching of coral reefs, or saltwater intrusion (Scheffer et al., 2001; Tsur and Zemel, 1995). On a larger scale, the collapses of the North Atlantic cod stocks off the coast of Canada, or the capelin stock in the Barents Sea can – at least in part – be attributed to overfishing (Frank et al., 2005; Hjermann et al., 2004). The most important application may however be the climate system, where potential drivers of a disastrous regime shift could be a disintegration of the West-Antartic ice sheet, a shutdown of the thermohaline circulation, or a melting of Permafrost (Lenton et al., 2008).

A key feature of these examples is that the exact location of the tipping point is unknown. Most previous studies translate the uncertainty about the location of the threshold in state space into uncertainty about the occurrence of the event in time. This allows for a convenient hazard-rate formulation (where the hazard rate could be exogenous or endogenous), but it has the problematic feature that, eventually, the event occurs with probability 1. In other words, even if we were to totally stop extracting/polluting, the disastrous regime shift could not be avoided. Arguably, it is more realistic to model the regime shift in such a way that when it has not occurred up to some level, the players can avoid the event by staying at or below that level. I therefore follow the modeling approach of e.g. Tsur and Zemel (1994, 1995); Nævdal (2001); Lemoine and Traeger (2014) where – figuratively speaking – the threshold is interpreted as the edge of a cliff and the policy question is whether, when, and where to stop walking. This modeling approach leads, under certain conditions, to a socially optimal "safe minimum standard of conservation" as pointed out by Mitra and Roy (2006).

In order to isolate the effect of a catastrophic threshold on the ability to cooperate, I abstract – as a first step – from the dynamic common pool aspect of non-cooperative resource use. This allows me to obtain tractable analytic solutions and feedback Nash-equilibria for general utility functions and general continuous probability distributions. In section 3.1, I discuss the case when the location of the threshold is known to expose the underlying strategic structure of the game. I show that there is a Nash equilibrium where the resource is conserved indefinitely and a Nash equilibrium where the resource is depleted immediately. When the location of the threshold is unknown, any experimentation is undertaken in the first period only (both in the social optimum, section 3.3, and in the non-cooperative game, section 3.4). Agents can coordinate on the first-best if the socially optimal action is to use the environmental good at its current level, and if this status quo is sufficiently valuable. If preserving the status quo is not sufficiently valuable, the players may still coordinate on not depleting the resource for sure, but they will increase their consumption by an inefficiently high amount. However, provided that the increase in consumption has not caused the disastrous regime shift, the players can coordinate on keeping to the updated level of consumption, which is, *ex post*, socially optimal.

The dynamic structure of this problem is stunted because learning is only affirmative. That is, given that the agents know that the current state is safe, and they expand the current state, they will only learn whether the future state is safe or not. The agents will not obtain any new information on how much closer they have come to the threshold. Empiricists will agree that there is no learning without experiencing.

In simple words, it is pitch-dark when the agents walk towards the cliff. Consequently, there is no point to split any given distance in several steps. This basic feature of resource use under the threat of a disastrous threshold is robust to several extensions that are explored in section 4. While the threat of the threshold may may no longer induce coordination on the first-best when the common-pool externality relates to *both* the (endogenous) risk of passing the threshold and resource itself, the threshold still encourages coordination on an extraction path that is pareto-superior compared to the Nash equilibrium without a threshold.

Section 5 concludes the paper and points to important future applications of the modeling framework. All proofs are collected in the Appendix.

Relation to the literature

Advancing our understanding of how to optimally learn about the location of an irreversible threshold and how this is constrained by the strategic interactions of non-cooperative players is one of the main contributions of this paper. There are two papers that discuss optimal search in a similar modeling structure: Rob (1991) studies optimal and competitive capacity expansion when market demand is unknown. In contrast to the results that I present here, Rob finds that learning will take place over several periods and socially optimal learning is faster than competitive search. Costello and Karp (2004) investigate optimal pollution quotas when abatement costs are unknown. In line with the current paper, they find that learning takes place in the first period only.

The difference between Rob's model on the one hand and Costello and Karp (2004) and my results on the other hand is the following: In Rob (1991), the information gained by an additional unit of installed capital is small, but so is the cost. Over-shooting market demand by a lot is very costly compared to under-shooting several times. Consequently, learning takes place gradually. In the non-cooperative equilibrium, learning is even slower due to the private nature of search costs but the public nature of information. In Costello and Karp (2004), the information gain from an additional unit of quota is small as well, but search costs are very high in the beginning and then decline. In fact, the costs are zero once the quota is non-binding. Thus, although the costs of shooting are high, there is no additional harm in over-shooting and it is therefore optimal to search only once. In my model, the marginal gain from search is bounded above and decreasing, but the marginal costs from search increase. The disastrous regime shift occurs when the threshold is crossed, irrespective of how far the agents have stepped over it. As in Costello and Karp (2004), there is thus no additional harm in over-shooting (but the costs of shooting are increasing in my model), and it is therefore optimal to search only once. I show that non-cooperative learning is more aggressive than socially optimal because costs of search are public while the immediate gains are private.[1]

Moreover, I highlight how the effect of a threatening threshold depends on the initial safe value. In particular, I show that experimentation is deceasing in this state value: The more the players know that they can safely consume, the less will they be willing to risk triggering the regime shift by enlarging the set of consumption opportunities. This aspect has, to the best of my knowledge, not yet been appreciated.

The pioneering contribution analyzing the economics of regime shifts in an environmental context was by Cropper (1976). There are by now a good dozen papers on the optimal management of renewable resources under the threat of a irreversible regime shift. Recently, Polasky et al. (2011) have summarized and characterized the literature at hand of a simple fishery model. They contrast whether the regime shift implies a collapse of the resource or merely a reduction of its renewability, and whether probability of crossing the threshold is exogenous or depends on the state of the system (i.e. it is endogenous). They show that resource extraction should be more cautious when crossing the threshold implies a loss of renewability and the probability of crossing the threshold depends on the state of the system. In contrast, exploitation should be more aggressive when a regime shift implies a collapse of the resource and the probability of crossing the threshold cannot be influenced. There is no change in optimal extraction for the loss-of-renewability/exogenous-probability case and the results are ambiguous for the collapse/endogenous-probability case.

Until now the literature has been predominantly occupied with optimal management, leaving aside the central question of how agent's strategic considerations influence and are influenced by the potential to trigger a disastrous regime shift. Still, there are a few notable exceptions: Crépin and Lindahl (2009) analyze the classical "tragedy of the commons" in a grazing game with complex feedbacks, focussing on open-loop strategies. They find that, depending on the productivity of the rangeland, under- or overexploitation might occur. Kossioris et al. (2008) focus on feedback equilibria and analyze, with help of numerical methods, non-cooperative pollution of a "shallow lake". They show that, as in most differential games with renewable resources, the outcome of the feedback Nash equilibrium is in general worse than the open-loop equilibrium or the social optimum. However, they highlight that for some combinations of parameter values, a regime shift can be avoided in a feedback Nash equilibrium (although the value of the game is still substantially lower than in the first-best).

[1]Incidentally, Costello and Karp (2004) conjecture that sequential learning might be optimal in their model if they do away with their assumption that a myopic quota is binding for sure. In the current application, it follows naturally that the probability of crossing the threshold is zero when no step is being taken. An avenue for future research is to investigate whether both the fixed initial costs of searching and no additional harm from overshooting are necessary elements of the model to arrive at this immediate learning structure of search.

Fesselmeyer and Santugini (2013) introduce an exogenous event risk into a non-cooperative renewable resource game à la Levhari and Mirman (1980). As in the optimal management problem with an exogenous probability of a regime shift, the impact of shifted resource dynamics is ambiguous: On the one hand, the threat of a less productive resource induces a conservation motive for all players, but on the other hand, it exacerbates the tragedy of the commons as the players do not take the risk externality into account. Finally, Sakamoto (2014) has, by combining analytical and numerical methods, analyzed a non-cooperative game with an endogenous regime shift hazard. He shows that this setting may lead to more precautionary management, also in a strategic setting. Taken together, these studies and the current paper show that the effect of a regime shift pulls in the same direction in a non-cooperative setting as under optimal management.

In addition to this literature, the current paper is closely related to three articles that discuss the role of uncertainty about the threshold's location on whether a catastrophe can be avoided. Barrett (2013) shows that players in a linear-quadratic game are in most cases able to form self-enforcing agreements that avoid catastrophic climate change when the location of the threshold is known but not when it is unknown. Similarly, Aflaki (2013) analyzes a model of a common-pool resource problem that is, in its essence, the same as the stage-game developed in section 3. He shows that an increase in uncertainty leads to increased consumption, but that increased ambiguity may have the opposite effect. Bochet et al. (2013) confirm the detrimental role of increased uncertainty in the stochastic variant of the Nash Demand Game: Even though "cautious" and "dangerous" equilibria co-exist (as they do in my model), they provide experimental evidence that participants in the lab are not able to coordinate on the Pareto-dominant cautious equilibrium.[2] However, the models in Aflaki (2013), Barrett (2013), and Bochet et al. (2013) are all static and can therefore not address the prospects of learning. Here, I show that the sharp distinction between known and unknown location of a threshold vanishes in a dynamic context. More uncertainty still leads an increased consumption, but this is now partly driven by the increased gain from experimentation.

Analyzing how strategic interactions shape the exploitation pattern of a renewable resource under the threat of a disastrous regime shift is important beyond mere curiosity driven interest. It is probably fair to say that international relations are basically characterized by an absence of supranational enforcement mechanisms which would allow to make binding agreements. But also locally, within the jurisdiction of a given nation, control is seldom complete and the exploitation of many common pool resources is shaped by strategic considerations. Extending our knowledge on the effect of looming regime shifts by taking non-cooperative behavior into account is therefore a timely contribution to both the scientific literature and the current policy debate.

[2] Bochet et al. (2013, p.1) conclude that a "risk-taking society may emerge from the decentralized actions of risk-averse individuals". Unfortunately, it is not clear from the description in their manuscript whether the participants were able to communicate. The latter has shown to be a crucial factor for coordination in threshold public goods experiments (Tavoni et al., 2011; Barrett and Dannenberg, 2012). Hence, it may be that what they refer to as "societal risk taking" is simply the result of strategic uncertainty.

2 Basic model

To repeat, I consider a situation where several agents share a productive asset. Each agent obtains utility from using it and as long as total use is at or below a threshold, the asset remains intact. Contrarily, if total use in any period exceeds the threshold, a disastrous regime shift will occur. The players are unable to make binding agreements. In order to expose the effect of a threatening regime shift on the cooperative and non-cooperative use of environmental goods and services as clearly as possible, I consider a model where the common pool externality applies only to the risk of triggering the regime shift. While this is clearly a simplification, it allows for a tractable solution of a general dynamic model with minimal requirements on the (concavity and boundedness) utility function or the distribution of the threshold's location (continuity).

Essentially, the players face the problem of sharing a magic pie: If they do not eat too much today, they will tomorrow find the full pie replenished, to be shared again. Each player however faces the temptation to eat a little more than what was eaten yesterday since he or she will have that piece for himself, whereas the future consequences of his voracity will have to be shared by all.

A literal example for this model could be whale-watching: Imagine a group of whales coming to a certain spot to play and this sight is enjoyed by tourists. Boat operators know that the current intensity of whale-watching is tolerated by the animals. Pondering whether to offer more tours, they have to weigh the additional income against the fear that there may be some intensity beyond which the whales get disturbed and avoid the area. A second example could be saltwater intrusion: players can take fresh water from a reservoir and the water will replenish. However, once the water pressure in the reservoir has dropped too low, saltwater will enter and the reservoir will be spoiled. A third example could be giant sequoias: While our current level of pollution allows us to still marvel at these millennia-old trees, there is a risk that further pollution caused by increased consumption will irreversibly destroy the last remaining groves.

Resource dynamics

- Time is discrete and indexed by $t = 0, 1, 2, ...$
- Each period, the players can in total consume up to the available amount of the resource. There are two regimes: In the *productive regime*, the upper bound on the available resource is given by R, and in the *unproductive regime*, the upper bound is given by r (with $r \ll R$).
- The game starts in the productive regime and will stay in the productive regime as long as total consumption does not exceed a threshold T. The threshold T is the same in all periods, but it may be known or unknown.
- To highlight the effect of uncertainty about the threshold, I define a state variable s_t, denoting the upper bound of the "safe consumption possibility set" at time t. That is,

a total extraction up to s_t has not triggered a regime shift before and is hence known to not trigger a regime shift in the future (i.e. $\text{Prob}(T \leq s_t) = 0$).

Players, choices, and payoff

- Players derive utility from consuming the resource according to some general function $u(c_t^i)$. I assume that this function is continuous and bounded below by $u(0) = b$ and above by $u(R) = B$ with $u' > 0$ and $u'' \leq 0$.
- Let c_t^i be the consumption of player i at time t. For clarity, I split per-period consumption in the productive regime in two parts: $c_t^i = \alpha^i s_t + \delta_t^i$. This means:
  1. The players consume a share α^i of s_t (the amount of the resource that can be used safely).
  2. The players may choose to consume an amount δ_t^i more than $\alpha^i s_t$, effectively pushing the boundary of the production possibility set at the risk of triggering the regime shift.
- In other words, δ_t^i is the effective choice variable with $\delta_t^i \in [0, R - s_t - \delta_t^{-i}]$, where δ_t^{-i} is the extension of the production set by all other players. I denote δ without superscript i as the total extension of the safe set, i.e. $\delta_t = \sum_{i=1}^N \delta_t^i$.
- The objective of the players is to choose that sequence of extension decisions $\Delta^i = \delta_0^i, \delta_1^i, ...$ which, for given strategies of the other players Δ^{-i}, and for a given initial value s_0, maximizes the sum of expected per-period utilities, discounted by a common factor β with $\beta \in (0, 1)$. I concentrate on Markovian strategies.

The probability of triggering the regime shift

- Let the probability density of T on $[0, A]$ be given by a continuous function f such that the cumulative probability of triggering the regime shift is *a priori* given by $F(x) = \int_0^x f(\tau)\mathrm{d}\tau$.
- The variable A with $R \leq A \leq \infty$ denotes the upper bound of the support of T. When $R < A$, there is some probability $1 - F(R)$ that extracting the entire amount of the resource is actually safe and the presence of a critical threshold is immaterial. When $R = A$ extracting the entire amount of the resource will trigger the regime shift for sure. Both R and A are known with certainty.
- Knowing that a given a given exploitation level s is save, the updated density of T on $[s, A]$ is given by $f_s(\delta) = \frac{f(s+\delta)}{1-F(s)}$. The cumulative probability of triggering the regime

shift when, so to say, taking a step of distance δ from the safe value s is:

$$\begin{aligned} F_s(\delta) = \quad \int_0^\delta f_s(\tau)\mathrm{d}\tau \quad &= \quad \frac{1}{1-F(s)} \int_0^\delta f(\tau)\mathrm{d}\tau \\ = \quad \frac{1}{1-F(s)} \int_s^\delta f(s+\xi)\mathrm{d}\xi \quad &= \quad \frac{F(s+\delta)-F(s)}{1-F(s)} \end{aligned} \tag{1}$$

So that $F_s(\delta)$ is the discrete version of the hazard function.

- The key expression that I use in the remainder of the paper is $L_s(\delta)$, which I call the survival function. It denotes the probability that the threshold is not crossed when taking a step δ, given that the event has not occurred up to s. Let $L(x) = 1 - F(x)$ and

$$L_s(\delta) = 1 - F_s(\delta) = \frac{1-F(s)-(F(s+\delta)-F(s))}{1-F(s)} = \frac{L(s+\delta)}{L(s)} \tag{2}$$

- The survival function has the following properties: $L_s(\delta) \in \left[\frac{1-F(R)}{1-F(s)}; 1\right]$ (it is bounded below by the conditional probability that T is not in the interval $[s, R]$); $\frac{\partial L_s(\delta)}{\partial \delta} = \frac{-f(s+\delta)}{1-F(s)} < 0$ (intuitively, the survival probability decreases as the step size increases); and $\frac{\partial L_s(\delta)}{\partial s} = \frac{-f(s+\delta)(1-F(s))+(1-F(s+\delta))f(s)}{[1-F(s)]^2} < 0$ if $\frac{1-F(s+\delta)}{1-F(s)} < \frac{f(s+\delta)}{f(s)}$ (that is, the chances of surviving a step δ decline with s when the density of T is non-decreasing on $[0, R]$).

Preliminary clarifications and tractability assumptions

- It is well known that the static non-cooperative game of sharing a given resource has infinitely many equilibria. Here, I focus on symmetric pure-strategy equilibria and set $\alpha^i = \frac{1}{N}$. Moreover, the game requires a statement about the consequences when the sum of players' consumption plans exceed the total available resource. In this case, I assume that the resource is rationed so that each players gets an equal share.

- I consider the disastrous regime shift to be irreversible.

- Without loss of generality, I set $r = 0$ and $u(0) = 0$. That is, the regime shift implies a complete collapse of the resource, but it is not a "doomsday event".

- For now, the model abstracts from the dynamic common pool problem in the sense that the consumption decision of a player today has no effect on the consumption possibilities tomorrow, *except* that a.) the set of safe consumption possibilities may have been enlarged and b.) the disastrous regime shift may have been triggered.

- The simplicity of the resource dynamics allows to include the idea that the system is the more likely to experience a disastrous regime shift the more of the stock has been exploited via the cdf of the threshold location. However, the core model feature that a safe state s is safe (no matter how far or close to the threshold), implies that the players cannot learn about the potential proximity of the threshold by observing the current state. That is, my model does not allow for "early warning signals".

3 Social optimum and non-cooperative equilibrium

In this main part of the paper, I will first expose the underlying strategic structure of the game by analyzing the situation when the threshold is known (section 3.1). In section 3.2, I provide a short non-technical overview of the implications of uncertainty about the threshold's location. I describe the socially optimal course of action in section 3.3. In particular, I will show that any experimentation – if at all – is undertaken in the first period and that it is decreasing with the value of the consumption level that is known to be safe. In section 3.4, I then show that (in addition to the Nash equilibrium of immediate resource depletion) this feature of learning may allow for a cautious equilibrium where either the resource is conserved with probability 1 or the players experiment once. The amount of experimentation will be sub-optimally large, but if they have not crossed the threshold, staying at the updated safe level will – *ex post* – be socially optimal. In section 3.5, I provide an instructive specific example for which closed-form solution can be obtained easily. Finally, I analyze how optimal and non-cooperative resource use changes with changes in the parameters in section 3.6.

3.1 Known threshold location

What is the first-best outcome in a situation when the threshold T is known? When T is large, the first-best is to indefinitely use exactly that amount of the resource which does not cause the regime shift. However, when T is small (so that a large part of the available resource R must be foregone to ensure its continued existence) it will be socially optimal to cross the threshold and deplete the resource immediately. How small T must be for depletion to be optimal obviously depends on the discount factor β: the less one discounts the future, the more willing one is to sacrifice today's consumption to ensure consumption in the future.

The non-cooperative game then has two equilibria in pure strategies: Either the players deplete the resource immediately, or they can coordinate on staying at the threshold. For a given safe value of total consumption s, player i's value function is:

$$V(s) = \max_{\delta^i} \left\{ u(s/N + \delta^i) + I \cdot \beta V(s) \right\}, \text{ where } \begin{cases} I = 1 & \text{when} \quad s + \delta^i + \delta^{-i} \leq T \\ I = 0 & \text{when} \quad s + \delta^i + \delta^{-i} > T \end{cases} \tag{3}$$

Due to the simplicity of the model structure, it is clear that if staying at the threshold can be rationalized in any one period, it can be done so in every period. The payoff from avoiding the regime shift is $\frac{u(T/N)}{1-\beta}$. Conversely, the payoff from deviating and immediately depleting the resource when all other players' policy is to stay at the threshold is given by $u\left(R - \frac{N-1}{N}T\right)$. Staying at the threshold can thus be sustained as a Nash equilibrium whenever $\frac{u(T/N)}{1-\beta} \geq u\left(R - \frac{N-1}{N}T\right)$. Denote by $\bar{\beta}$ the value of β for which this condition just so holds with equality (i.e. $\bar{\beta}$ is the lowest discount factor for which staying at the threshold can be sustained for given values of N, T, and R). We have:

$$\bar{\beta} = 1 - \frac{u(T/N)}{u\left(R - \frac{N-1}{N}T\right)} \tag{4}$$

In fact, there will always be a parameter combination so that the first-best can be supported as a Nash-equilibrium of the game with a known threshold (Proposition 1). Given these conditions, the game exhibits the structure of a coordination game. Here, as in the static game from Barrett (2013, p.236), "[e]ssentially, nature herself enforces an agreement to avoid catastrophe."

Proposition 1. *When the location of the threshold is known with certainty, then there exists, for every combination of N, T, and R, a value of $\bar{\beta}$ such that the first-best can be sustained as a Nash-equilibrium when $\beta \geq \bar{\beta}$. The larger is N, or the closer T is to 0, the larger has to be β.*

Proof. The proof is placed in Appendix A.1 □

3.2 Unknown threshold location

I now turn to the case when the threshold's location is unknown. In spite of the uncertainty about T, the players do know that resource consumption up to the safe value s does not trigger the disastrous regime shift. Hence, the threat of a disastrous regime shift can, in principle, be eliminated by coordinating on consuming exactly the amount s. The flip-side of this coin is that by consuming s, the players do not learn about what other consumption levels would have been safe. Instead of staying at s, the players can also expand the set of safe consumption possibilities by some amount δ. When deciding on the size of this step δ, the gain from expanding the set of safe consumption possibilities to $s' = s + \delta$ has to be weighted against the probability of causing the disastrous regime shift. The effect of strategic non-cooperative interaction will manifest itself in the classic way that each player realizes that the gain from increased consumption is private while the cost in terms of a higher regime-shift hazard are borne by all.

Both in the equilibrium of the non-cooperative game, and in the social optimum, experimentation will be undertaken in the first period only. The reason is that learning is only affirmative. Having expanded the set of consumption possibilities, it is revealed whether the state s' is safe or not, but no new knowledge about the relative probability that the threshold is located at, say, s_1 or s_2 (with $s_1, s_2 > s'$) has been acquired. Therefore, it does not pay to experiment a second time. If it were optimal to learn whether the threshold is located in the realm beyond s', it would have also been optimal to do so in the first period. It is this feature of learning that opens for an equilibrium where the resource is not depleted immediately.

Moreover, the degree of experimentation is declining in s. The intuition for this effect is clear: The more valuable my current outside option, the less I can gain from an increased consumption set, but the more I can lose should the experiment trigger the regime shift. This implies that the largest step is undertaken when $s = 0$, which is reminiscent Janis Joplin's dictum that "freedom is another word for nothing left to lose".

3.3 Social optimum when the location of T is unknown

Starting from a given safe value s, the social planner has in principle two options: She can either stay at s (choose $\delta = 0$), thereby ensuring the existence of the resource in the next period (as $L_s(0)$, the probability of not crossing the threshold, is 1). Alternatively, she can take a positive step into unknown territory (choose $\delta > 0$), potentially expanding the set of safe extraction possibilities to $s' = s + \delta$, albeit at the risk of a resource collapse (as $L_s(\delta) < 1$ for $\delta > 0$). The social planner's "Bellmann equation" is thus:

$$V(s) = \max_{\delta \in [0, R-s]} \left\{ u(s+\delta) + \beta L_s(\delta) V(s+\delta) \right\} \tag{5}$$

The crux is, of course, that the value function $V(s)$ is *a priori* not known. However, we do know that once the planner has decided to not expand the set of safe extraction possibilities, it cannot be optimal to do so at a later period: If $\delta = 0$ is chosen in a given period, nothing is learned for the future ($s' = s$), so that the problem in the next period is identical to the problem in the current period. If moving in the next period would increase the payoff, it would increase the payoff even more when one would have made the move a period earlier (as the future is discounted).

To introduce some notation, let s^* be a value of s at which it is not socially optimal to expand the set of safe extraction values (as the threat of a disastrous regime shift lures too large) and let $\overline{s}^*$ be the lowest member of this set of values. In Appendix A.2, I show that $\overline{s}^*$ must exist, so that for $s \geq \overline{s}^*$, it is optimal to choose $\delta = 0$. In this case, we know $V(s)$, it is given by $V(s) = \frac{u(s)}{1-\beta}$.

This leaves three possible paths when starting from values of s_0 that are below $\overline{s}^*$. The social planner could

a.) make one step and then stay,

b.) make several, but finitely many steps and then stay,

c.) make infinitely many steps.

Suppose that a value at which it is socially optimal to remain standing is reached in finitely many steps. This implies that there must be a last step. For this last step, we can explicitly write down the objective function as we know that the continuation value of staying at s' forever is $\frac{u(s')}{1-\beta}$. Denote the social planner's valuation of taking exactly one step δ from the initial value s and then staying at s' forevermore by $\varphi(s)$ and denote by $\delta^*(s)$ the optimal choice of the last step. Formally:

$$\varphi(\delta; s) = u(s+\delta) + \beta L_s(\delta) \frac{u(s+\delta)}{1-\beta}. \tag{6}$$

This yields the following first-order-condition for an interior solution:

$$\varphi'(\delta; s) = u'(s+\delta) + \frac{\beta}{1-\beta} \left[L_s'(\delta) u(s+\delta) + L_s(\delta) u'(s+\delta) \right] = 0. \tag{7}$$

Note that we need not have an interior solution so that $\delta^*(s) = 0$ when $\varphi'(\delta; s) < 0$ for all δ and $\delta^*(s) = R - s$ when $\varphi'(\delta; s) > 0$ for all δ. That is:

$$\delta^*(s) = \max\left\{0; \min\left\{\arg\max \varphi(\delta; s); R - s\right\}\right\}. \tag{8}$$

With this explicit functional form in hand, I can show that it is better to traverse any given distance before remaining standing in one step rather than two steps. *A forteriori*, this holds for any finite sequence of steps. Also an infinite sequence of steps cannot yield a higher payoff since the first step towards s^* will be arbitrarily close to s^* and concavity of the utility function ensures that there is no gain from never actually reaching s^*.

The first-best consumption is summarized by the following proposition:

Proposition 2. *The socially optimal total use of the resource is either s_0 for all t or $s_0+\delta^*(s_0)$ for $t = 0$ and, if the resource has not collapsed, s_1 for all $t \geq 1$. In other words, any experimentation – if at all – is undertaken in the first period.*

Proof. The proof is given in Appendix A.2. □

The intuition is the following: Given that it is optimal to eventually stop at some $s^* \geq \bar{s}^*$, the probability that the threshold is located on the interval $[s, s^*]$ is exogenous. Hence the probability of triggering the regime shift when going from s to s^* is the same whether the distance is traversed in one step or in many steps. Due to discounting, the earlier the optimal safe value s^* is reached, the better. In other words, given that one has to walk out into the dark, it is best to take a deep breath and get to it.

In short, the dynamics of the consumption pattern are stunted: For initial values of s below some threshold $\bar{s}^*$, it is optimal to make exactly one step and then stay at the updated value s' forever (provided T is not located between s and s', of course). For initial values of s above $\bar{s}^*$, it is optimal to never expand the set of safe consumption possibilities.

Proposition 3 then shows that the more the social planner knows, the less she learns.

Proposition 3. *The socially optimal step size $\delta^*(s)$ is decreasing in s.*

Proof. The proof is placed in Appendix A.3. □

3.4 Non-cooperative equilibrium when the location of T is unknown

The game has two equilibria in pure strategies: an "aggressive" equilibrium where the players immediately deplete the resource, and a "cautious" equilibrium where there is a positive probability that the resource is maintained forever. In fact, there are two types of the "cautious" equilibrium, depending on the initial value s. Similar to section 3.3, I define $\bar{s}^{nc}$ to be the lowest member of the set of safe values at which a Nash equilibrium with the players choosing $\delta = 0$ can be supported. For $s \geq \bar{s}^{nc}$, the cautious equilibrium thus conserves the resource with probability 1. I define $\underline{s}^{nc}$ to be the largest member of the set at which depletion is the only equilibrium. For values of $s \in (\underline{s}^{nc}, \overline{s}^{nc})$, the "cautious" equilibrium implies that the players experiment once (and the regime shift thus occurs with positive probability).

In general, the "Bellman equation" of player i can (for a given strategy of the other players Δ^{-i}) be expressed as:

$$V^i(s, \Delta^{-i}) = \max_{\delta^i \in [0, R-s]} \left\{ u(s + \delta^i) + \beta L_s(\delta^i + \delta^{-i}) V^i(s + \delta, \Delta^{-i}) \right\} \tag{9}$$

Also here, the crux is that V^i is *a priori* unknown. Similar to above, I denote by ϕ the value for player i to take exactly one step of size δ^i and then remain standing when the other players' strategy $\Delta^{-i} = \{\delta^{-i}, 0, 0, 0, ...\}$ is to do the same:

$$\phi(\delta^i; \delta^{-i}, s) = u\left(\frac{s}{N} + \delta^i\right) + \beta L_s(\delta^i + \delta^{-i}) \frac{u\left(\frac{s+\delta^i+\delta^{-i}}{N}\right)}{1-\beta} \tag{10}$$

Let $g(\delta^{-i}, s)$ be the interior solution to the first-order-condition of maximizing $\phi(\delta^i; \delta^{-i}, s)$:

$$\begin{aligned} \phi'(\delta^i; \delta^{-i}, s) = u'\left(\frac{s}{N} + \delta^i\right) \\ + \frac{\beta}{1-\beta}\left[L_s'(\delta^i + \delta^{-i}) u\left(\frac{s + \delta^i + \delta^{-i}}{N}\right) + \frac{1}{N} L_s(\delta^i + \delta^{-i}) u'\left(\frac{s + \delta^i + \delta^{-i}}{N}\right)\right] \end{aligned} \tag{11}$$

The best-reply function for player i, $\delta^{i*}(\delta^{-i}, s)$ is then given by

$$\delta^{i*}(\delta^{-i}, s) = \begin{cases} 0 & \text{if} \quad s \geq \overline{s}^{nc} & \text{(12a)} \\ g(\delta^{-i}, s) & \text{if} \quad s \in (\underline{s}^{nc}, \overline{s}^{nc}) & \text{(12b)} \\ R - s - \delta^{-i} & \text{if} \quad s \leq \underline{s}^{nc} & \text{(12c)} \end{cases}$$

For a symmetric step size $\delta^{-i} = (N-1)\delta^i$, we have:

$$\begin{aligned} \phi'(\delta^{nc}; s) = u'\left(\frac{s}{N} + \delta^{nc}\right) \\ + \frac{\beta}{1-\beta}\left[L_s'(N\delta^{nc}) u\left(\frac{s + \delta^{nc}}{N}\right) + \frac{1}{N} L_s(N\delta^{nc}) u'\left(\frac{s + \delta^{nc}}{N}\right)\right] \end{aligned} \tag{13}$$

Proposition 4. *The set of Markov-strategies*

$$\begin{array}{ccc} 0 & \textit{if} & s \geq \overline{s}^{nc} \\ \delta^{nc}(s) & \textit{if} & s \in (\underline{s}^{nc}, \overline{s}^{nc}) \\ \frac{R-s}{N} & \textit{if} & s \leq \underline{s}^{nc} \end{array}$$

where $\delta^{nc}(s)$ is defined by the interior solution to (13)*, constitutes a feedback Nash equilibrium. That is, for $s_0 \geq \overline{s}^{nc}$ coordination to stay at s_0 can be supported as a Nash equilibrium. For $s_0 < \overline{s}^{nc}$ taking one step and then staying at $s_1 = s_0 + \delta^{nc}$ can be supported as a Nash equilibrium.*

Proof. The proof is given in Appendix A.4 □

Obviously, the best-reply for player i when all other players plan to expand the consumption set by $R - s$ is to chose $R - s$ as well. This would ensure that the player at least gets an equal share of R. I call this equilibrium in which the resource is immediately depleted the "aggressive equilibrium" and the equilibrium described in Proposition 4 the "cautious equilibrium". Note that, for a given s, both the "cautious" and the "aggressive equilibrium" are unique.[3]

In short, the game has the structure of a coordination problem where the immediate depletion of the resource may become a self-fulfilling prophecy. Indeed, for $s \leq \underline{s}^{nc}$, the immediate depletion of the resource cannot be avoided in a non-cooperative setting, despite the fact that there is a range of initial values ($s \in [\underline{s}^*, \underline{s}^{nc}]$) for which it is optimal to conserve the resource indefinitely with positive probability. For $s \in [\underline{s}^{nc}, \overline{s}^{nc}]$, the strategic interactions imply that experimentation is inefficiently large. However, should it turn out that $s' = s + N\delta^{nc}$ is safe, this consumption pattern is *ex-post* socially optimal. Figure 1 illustrates the aggregate expansion of the set of safe consumption possibilities in the cautious equilibrium and contrasts it with the social optimum.

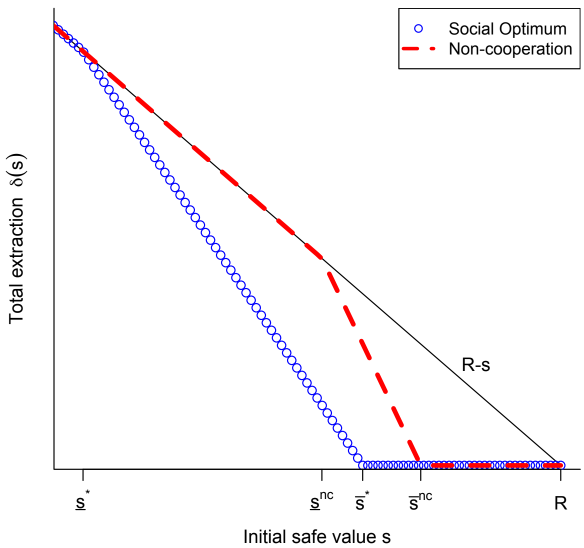

Figure 1: Illustration of policy function $\delta(s)$. The blue circles represent the socially optimal extension δ of the safe consumption set s (on the y-axis) as a function of the safe consumption set on the x-axis (where obviously $s \leq R$ and $\delta \in [0, R - s]$). For values of s below $\underline{s}^*$, it is optimal to consume the entire resource (choose $\delta(s) = R - s$). For values of s above $\overline{s}^*$, it is optimal to remain standing (choose $\delta(s) = 0$). The red dashed line plots the cautious non-cooperative equilibrium, showing how $\underline{s}^* \leq \underline{s}^{nc}$ and $\overline{s}^* \leq \overline{s}^{nc}$ (in some cases we may even have $\underline{s}^{nc} < \overline{s}^*$). It illustrates how even the "cautious" experimentation under non-cooperation implies excessive risk-taking. The figure also shows that the first-best and the non-cooperative outcome may coincide for very low and very high values of s.

[3]Uniqueness of the latter type of equilibrium simply follows from the assumption that in case of incompatible demands, the resource is shared equally among the players. Uniqueness of the symmetric "cautious equilibrium" (should it entail $\delta^{nc}(s) < \frac{R-s}{N}$) can be established by contradiction. Suppose all other players $j \neq i$ choose to expand the consumption set to a level at which – should the threshold have not been crossed – no player would have an incentive to go further. Player i's best-reply cannot be to choose $\delta^i = 0$ in this situation as the gain from making a small positive step (which are private) exceed the (public) cost of advancing a little further. Hence, the only equilibrium at which the players expand the consumption set once is the symmetric one.

Faced with this coordination problem, the question arises which of the two equilibria can we expect to be selected. Clearly, the "cautious equilibrium" pareto-dominates the "aggressive equilibrium".[4] With rational players and without strategic uncertainty, the cautious equilibrium would thus be the outcome of the game. But what happens when the players are uncertain about the other player's behavior? As the disastrous regime shift is irreversible, there is no room for dynamic processes that lead players to select the pareto-dominant equilibrium (Kim, 1996). Therefore, I turn to the static concept of risk-dominance (Harsanyi and Selten, 1988).

Since the game is symmetric, applying the criterium of risk-dominance for equilibrium selection has the intuitive interpretation that the cautious equilibrium is selected if player i prefers to play cautiously (i.e. by choosing $\delta^i(s) = \delta^{nc}(s)$) rather than playing aggressively (i.e. choosing $\delta^i(s) = R - s$) when the expected payoff from doing so exceeds the expected payoff from playing aggressively when player i assigns probability p to the other players playing aggressively. Obviously, whether the cautious or the aggressive equilibrium is risk-dominant will depend both on this probability p as well as on the safe value s. We can, for a given safe value s solve for the probability p^* at which the player is just indifferent between playing cautiously or aggressively:

$$p^* \cdot \pi_{[all\ aggressive]} + (1 - p^*) \cdot \pi_{[only\ i\ aggressive]} = p^* \cdot \pi_{[only\ i\ cautious]} + (1 - p^*) \cdot \pi_{[all\ cautious]}$$
$$\Leftrightarrow$$
$$p^* = \frac{\pi_{[all\ cautious]} - \pi_{[only\ i\ aggressive]}}{(\pi_{[all\ cautious]} - \pi_{[only\ i\ aggressive]}) - (\pi_{[only\ i\ cautious]} - \pi_{[all\ aggressive]})}$$

In the above calculation, $\pi_{[all\ aggressive]}$ refers to the payoff of playing aggressive, when all other players play aggressively, $\pi_{[only\ i\ aggressive]}$ refers to the payoff of playing aggressive, when all other players play cautiously, etc. In order to explicitly solve for the value of p^*, we need to put more structure on the problem. For the specific example developed in section 3.5 below, we can calculate and plot p^* as a function of s (see Figure 2). The grey area below the line drawn by p^* shows the set of values for which player i prefers to play the strategy that pertains to the cautious equilibrium. Figure 2 illustrates how robust this equilibrium is: Even when the players think that there is more than a 50% chance that all other players play the aggressive strategy, it still pays to play the cautious strategy for a wide range of initial values s. (Clearly, p^* is not defined for $s < \underline{s}^{nc}$ when the cautious and the aggressive equilibrium coincide.)

[4] This follows immediately from the fact that, by definition, $\delta^{nc}(s)$ is the interior solution to the symmetric maximization problem (9) (with $\delta^{-i} = (N-1)\delta^{nc}$) where the policy $\delta(s) = R - s$ was an admissible candidate.

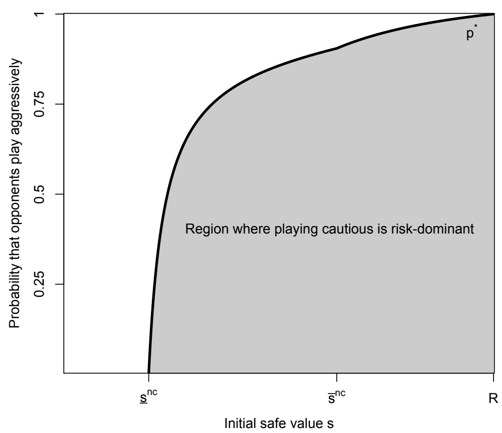

Figure 2: p^* as a function of s for $u(c) = \sqrt{c}$, $f = \frac{1}{A}$ and $\beta = 0.8$, $A = R = 1$ and $N = 10$

3.5 Specific example

For a given utility function and a given probability distribution of the threshold's location it is then possible to determine $\delta^*(s)$, $\delta^{nc}(s)$ and calculate the value function $V(s)$. Below, I do this for $u(c) = \sqrt{c}$ and a uniform probability distribution so that it is especially easy to explicitly solve (7) and (13). When the social planner thinks that every value in $[0, A]$ is equally likely to be the threshold, i.e. $f = \frac{1}{A}$, and accordingly $L_\delta(s) = \frac{A-s-\delta}{A-s}$, the optimal extension of the set of safe extraction possibilities is given by:

$$\delta^* = \frac{A - (1+2\beta)s}{3\beta} \tag{14}$$

There will only be an interior solution to (7) when $s \in [\underline{s}^*, \overline{s}^*]$. We have:[5]

$$\underline{s}^* = \max\left\{0, \frac{A - 3\beta R}{(1-\beta)}\right\}$$

$$\overline{s}^* = \min\left\{\frac{A}{1+2\beta}, R\right\}$$

Exploiting the fact that due to symmetry we have $\delta^{-i} = (N-1)\delta^i$, I solve (13) for an interior equilibrium value δ^{nc}. Total non-cooperative expansion is then given by:

$$N\delta^{nc} = \frac{\big((1-\beta)N + \beta\big)A - \big((1-\beta)N + 3\beta\big)s}{3\beta} \tag{15}$$

[5]At $\underline{s}^*$ it is optimal to consume the entire resource, so that $\underline{s}^*$ is found by solving $R - s = \frac{A-(1+2\beta)s}{3\beta}$. At $\overline{s}^*$ it is optimal to remain standing, so that $\overline{s}^*$ is found by solving $0 = \frac{A-(1+2\beta)s}{3\beta}$.

Again, there will only be an interior equilibrium for $s \in [\underline{s}^{nc}, \overline{s}^{nc}]$, where:

$$\underline{s}^{nc} = \max\left\{0, \frac{((1-\beta)N + \beta)A - 3\beta R}{(1-\beta)N}\right\}$$

$$\overline{s}^{nc} = \min\left\{\frac{((1-\beta)N + \beta)A}{(1-\beta)N + 3\beta}, R\right\}$$

From inspection of (15) it becomes clear that the total non-cooperative consumption is increasing in N: $\frac{\partial[N\delta^{nc}]}{\partial N} = \frac{(1-\beta)(A-s)}{3\beta} > 0$. This points to the "tragedy of the commons": The more players there are, the more aggressive the first-period expansion of the set of consumption possibilities. Furthermore, one can find the combination of parameters that would ensure a self-fulfilling prophecy of extirpation when staring from an initial value of $s = 0$ (no prior knowledge of a safe level of extraction). It is namely given by $N \geq \frac{\beta}{1-\beta}(3R - A)$, hence increasing in R and decreasing in β and A, as it is intuitive.

Figure 3 plots the value function for a uniform prior (with $A = R = 1$) and a discount factor of $\beta = 0.8$, illustrating how it changes as the number of player increases. The more players there are, the greater the distance of the non-cooperative value function (assuming that the players coordinate on the Pareto-dominated equilibrium, plotted by the blue solid diamonds) to the socially optimal value function (plotted by the black open circles). In particular when $N = 10$, one sees the region (roughly from $s = 0$ to $s = 0.2$) where there is no "cautious" equilibrium, and the large value of $\overline{s}^{nc}$ (roughly 0.62) when it first becomes individually rational to remain standing. All in all however, this example shows that the threat of a irreversible regime shift is very effective when the common pool externality applies only to the risk of crossing the threshold. (At least for this specific utility function and these parameter values. Note that $\beta = 0.8$ implies a unreasonably high discount rate, but it was chosen to magnify the effect of non-cooperation for a small number of players.)

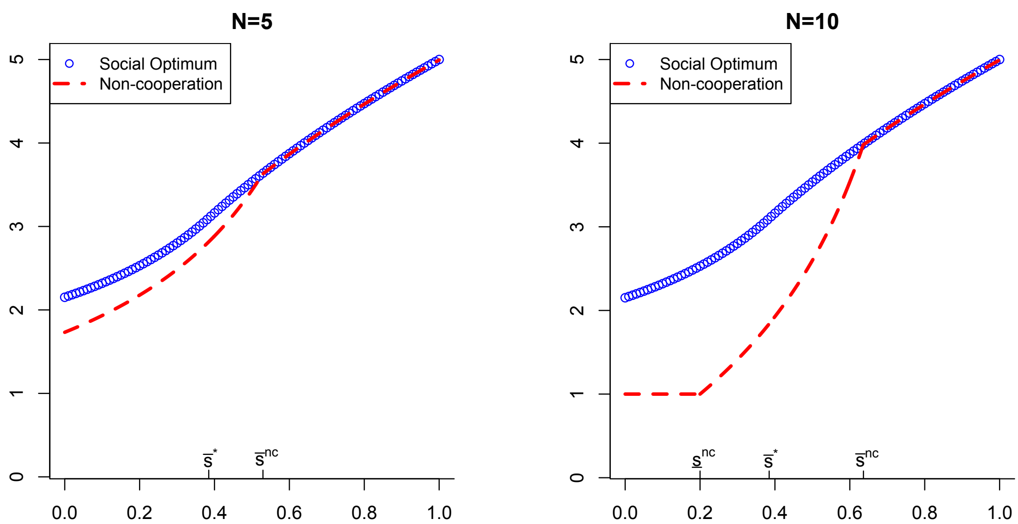

Figure 3: Illustration of cooperative and non-cooperative use of renewable resource: Value function $V(s)$ with $u(c) = \sqrt{c}$, $\beta = 0.8$, and $A = R = 1$ for $N = 5$ and $N = 10$.

3.6 Comparative statics

In order to analyze how the extraction pattern changes with changes in the parameters, I first note that δ^{nc}, the equilibrium expansion of the set of safe values, is monotonically decreasing in s. (The argument is the same as in the proof of Proposition 3 when simply replacing $\delta^*(s)$ with $N\delta^{nc}(s)$ and is therefore omitted.) This implies that the aggregate extraction pattern as a function of the prior knowledge about the set of safe consumption possibilities indeed looks qualitatively as in Figure 1 (where it was plotted for the specific functional forms analyzed in section 3.5). The effect of an increase in the fundamentals β, N, $L_s(\delta)$, and R can therefore be analyzed by investigating changes to $\phi'(\delta^{nc}, s)$.

The first comparative static result conforms with basic intuition: The more impatient the players are, the less they value the current safe consumption value, and the more aggressive they are. The second result shows that an increase in the number players may exacerbate the "tragedy of the commons", but not necessarily in all case. There are two opposing effects: On the one hand, the addition of one more players increases aggregate extraction if all players were to choose the same consumption level as before. On the other hand, the addition of another player leads all other players to decrease their individual consumption as they partly take the increase in N into account. Proposition 5 gives a sufficient condition for when the former dominates. The third comparative static result, that an increase risk of crossing the threshold leads to a larger range of the cautious equilibrium is not related to the risk aversion of the agents as such, but stems from the fact that the expected cost of experimentation increase, while the gains stay the same. Finally, an increase in the upper bound of the consumption possibility set, R, increases the range where a cautious equilibrium exists, in spite of the fact that R neither affects the interior consumption choices directly, nor the value $\bar{s}^{nc}$ at which the cautious equilibrium implies no further experimentation. The reason for the result is the following: At the old value of $\underline{s}^{nc}$, the equilibrium experimentation just coincides with choosing $R - s$. Now when the R shifts outwards, say to $\tilde{R}$, equilibrium experimentation at the old $\underline{s}^{nc}$ is strictly less than $\tilde{R} - s$.

Proposition 5. *We then have the following comparative statics results:*

(a) *The boundaries $\underline{s}^{nc}$ and $\bar{s}^{nc}$, and aggregate extraction for $s \in [\underline{s}^{nc}, \bar{s}^{nc}]$, decrease with β.*

(b) *An increase in N leads to more aggressive extraction when $\frac{N}{N+1} > u'(\frac{R}{N})/u'(\frac{R}{N+1})$.*

(c) *The more likely the regime shift (in terms of a first-order stochastic dominance), the larger the range where a cautious Nash-equilibrium exists.*

(d) *The higher the maximum potential reward R, the larger the range where a cautious Nash-equilibrium exists.*

Proof. The proofs are given in Appendix A.5. □

4 Extensions

The paper's main results do not rely on specific functional forms for utility or the probability distribution of the threshold's location. Tractability was achieved by considering extremely simple resource dynamics, namely the resource remained intact and replenishes fully in the next period as long as resource use in the current period has not exceeded T. In other words, there was no common pool externality relating to the resource dynamics itself. Alternatively, one could therefore interpret R as the upper bound of the consumption possibility set in the productive regime and r being the upper bound of the consumption possibility set in the unproductive regime. In this section, I explore to what extent the results of section 3 are robust to more general resource dynamics.

4.1 Delay in the occurrence of the regime shift

Consider a situation where the players, in a given period, observe only with some probability whether they have crossed the threshold. In fact, it is not unreasonable to model the true process of the resource as hidden and that it will manifest itself only after some delay (see Gerlagh and Liski (2014) for a recent paper that focusses on this effect in the context of optimal climate policies). Hence, as time passes the players will update their beliefs whether the threshold has been located on the interval $[s_t, s_t + \delta_t]$. How does this learning affect the optimal and non-cooperative strategies? This becomes an extremely difficult question as – due to the delay – the problem is no longer Markovian. Yet, it is possible to show the following:

Proposition 6. *Also when crossing the threshold at time t triggers the regime shift at some (potentially uncertain) time $\tau > t$, it is still socially and individually rational to experiment – if at all – in the first period only.*

Proof. The proof is given in Appendix A.6. □

In other words, the fact that the learning dynamics are stunted is robust to a delay in the occurrence of the regime-shift. This does of course not imply that the optimal decision under the two different models will be the same. They almost surely will differ, as delaying the consequences of crossing the threshold decreases the costs of experimentation. Yet, as the players only learn that they have crossed the threshold when the disastrous regime shift actually occurs, they cannot capitalize on this delay by trying to expand the set of safe consumption possibilities several times.

4.2 Growing R

Previously, the upper bound of the resource, R has been treated as known and constant. In this subsection, I shall depart from this assumption and consider the case when R increases (but f and T remain unchanged). Formally, the resource dynamics can be expressed as:

$$R_{t+1} = \begin{cases} G(R_t) & \text{if } \sum_i c_t^i \leq T \\ 0 & \text{if } \sum_i c_t^i > T \end{cases} \tag{16}$$

where $G'(R) > 0$.

In this situation, there is scope for a continued expansion of the set of safe consumption possibilities, but only as long as the upper bound of the available resource at time t, R_t, is binding. As R_t can, by construction, not exceed A and we know from the proofs of Proposition 2 and 4 that there will be some point at which it is neither socially optimal, nor a cautious Nash equilibrium to further expand the set of safe consumption possibilities.

In other words, although a growing R_t may induce several periods where $\delta_t(s_t) = R_t - s_t$, once $\delta_t(s_t) < R_t - s_t$ for some $t = \tau$, we have $\delta_t = 0$ for all $t > \tau$. The validity of this statement can be easily checked by observing that the first-order conditions for the socially optimal and individually rational choice of δ_t (equation (7) and (13), respectively) do not depend on R. Note that this argument also shows that uncertainty about R is immaterial for the optimal learning dynamics and the outcome of the game.

4.3 Non-renewable resource dynamics

The above results have been obtained by assuming that the resource replenishes fully every period unless the thresholds has been crossed. In other words, the common-pool externality related only to the risk of crossing the threshold. Here, I study the opposite case of a non-renewable resource to analyze the effect of a disastrous regime shift when the common-pool externality relates not only to the risk of crossing the threshold but also to the resource itself. Specifically, I consider the following model of extraction from a known stock of a non-renewable resource:

$$\max_{c_t^i} \sum_{t=0}^{\infty} \beta^t u(c_t^i) \quad \text{subject to: } R_{t+1} = \begin{cases} R_t - \sum_i c_t^i & \text{if} \;\; \sum_i c_t^i \leq T \\ 0 & \text{if} \;\; \sum_i c_t^i > T \end{cases} \tag{17}$$

I assume that the utility function is of such a form, that in a world without the threshold, there is a pareto-dominant non-cooperative equilibrium in which positive extraction occurs in several periods (though the players could empty the resource in one period, if they so wish). Due to discounting, it is clear that the extraction level will decline as time passes, both in the social optimum and in the non-cooperative equilibrium. Due to the stock externality, it is clear that the extraction rate in the non-cooperative equilibrium is inefficiently large.

A simple interpretation of this model could be a mine from which several agents extract a valuable resource. If aggregate extraction is too high in a given period, the structure of the shafts may collapse, making the remainder of the resource inaccessible. In spite of this natural interpretation, two things are rather peculiar about this model setup: First, any player can extract any amount up to R_t. (The option to introduce a capacity constraint on current extraction – though realistic – would come at the cost of significant clutter without yielding any apparent benefit.) Second, the assumption that R_0 is known and that T is constant means that this is *not* a problem of eating a cake of unknown size. This problem has since long been dealt with in the literature (see e.g. Kemp, 1976; Hoel, 1978) and is not considered here.

As in section 3, it is instructive to first discuss the case when the location of the threshold is known in order to expose the strategic structure that results from the potential for a disastrous regime shift. Let $\tilde{c}^{nc}(R_t)$ be the total non-cooperative extraction level (as a function of the resource stock R_t) in absence of the regime shift risk. Clearly, if $T > \tilde{c}^{nc}(R_0)$ the whole problem is not interesting, and I thus only consider the case when the known value of T is below $\tilde{c}^{nc}(R_0)$. The relevant question is therefore whether the agents can coordinate on staying at the level T in the first period or not. If they can, they will stay at the level T for an interval of time $t = 0, 1, ..., \tau$, where τ is the time at which the resource stock has been depleted to a level where the non-cooperative extraction path stays below T until the resource is exhausted. That is R_τ is defined by $\tilde{c}^{nc}(R_\tau) = T$.

The temptation to empty the resource in the first period, when all other players stay at the threshold is given by $u(R_0 - \frac{N-1}{N}T) > u(\frac{T}{N}) + \beta V(R_0 - T)$. Whether this inequality holds depends on the particular form of u and V and cannot be answered in general. Still, it is possible to prove the following:

Proposition 7. *In the game described by* (17)*, a known threshold is crossed in the first period, or never.*

Proof. The proof is given in Appendix A.7. □

Proposition 8 characterizes the case when the location of the threshold is unknown.

Proposition 8. *In the game described by* (17) *when T is unknown, there exists, in addition to the aggressive equilibrium in which the resource is exhausted in the initial period, a pareto dominant equilibrium in which experimentation – if at all – is undertaken in the first period only and $s_1 = s_0 + \delta^{nc}(s_0)$ is an upper bound on aggregate extraction for the remainder of the game.*

Proof. The proof is given in Appendix A.8. □

The intuition is the same as before. Because learning is only affirmative, it does not pay to experiment in the second (or any later) period. If the threshold has not been crossed, the extraction path will be constrained by $s_1 = s_0 + \delta^{nc}(s_0)$ up to the period where the resource stock has been depleted to a level where the non-cooperative extraction path stays below s_1 until the resource is exhausted. As the players would never expand the set of safe extraction possibilities beyond $\tilde{c}^{nc}(R_0)$ in the cautious equilibrium, the threat of a disastrous regime shift is welfare improving when the players coordinate. However, the cautious equilibrium will coincide with the first-best only in the very special case that the initially safe value s_0 is binding in each period of the game and the social optimum.

4.4 Renewable resource dynamics

Finally, consider a generic renewable resource problem, where the objective of player i is:

$$\max_{c_t^i} \sum_{T=0}^{\infty} \beta^t u(c_t^i, R_t) \quad \text{subject to: } R_{t+1} = \begin{cases} G(R_t - \sum_i c_t^i) & \text{if } \sum_i c_t^i \leq T \\ 0 & \text{if } \sum_i c_t^i > T \end{cases} \tag{18}$$

Note that the instantaneous utility function now directly depends on the resource stock (with $\frac{\partial u}{\partial R} > 0$). This could, for example, be due to stock dependent harvesting costs as it is usual for fishery models. Suppose that without the threshold, there is a unique Nash equilibrium with steady state resource stock R_∞^{nc}. Due to the negative stock externality, the socially optimal steady state resource stock, R_∞^{so}, is larger than R_∞^{nc}.

Parallel to section 4.3, the threshold applies to the total exploitation level in any given period, and not to the stock as such. While this structure may seem peculiar at first sight, it is without loss of generality here, because there is a one-to-one mapping between total harvest and escapement for a given initial stock.

Assume that the initial resource stock R_0, and the initial safe exploitation level, s_0, are above R_∞^{so}. Assume furthermore that there are no capacity constraints to harvesting, so that is any agent i can consume $R_0 - R_\infty^{nc}$ in one period should he or she wish to do so. Although we would need to put a lot more structure on this problem to solve it explicitly, we can make the following statement:

Proposition 9. *Unless $\delta_0^{nc}(s_0) = R_0 - R_\infty^{nc}$, the threat of a disastrous regime shift will strictly improve welfare as players can coordinate on a steady-state resource stock above R_∞^{nc}.*

Proof. The argument is the same as the one for Proposition 8 and is omitted. □

As in the case of non-renewable resource dynamics, the maximum extraction level that is known to be safe puts an upper bound on extraction and thereby mitigates the negative stock externality. If the initial value of s_0 is relatively low the players will experiment once and stay at the updated level, unless, of course, the players have crossed the threshold.

For example, the North-Atlantic herring fishery suggests that threat of a stock collapse may indeed be very policy relevant. For centuries, this fishery has been the economic centerpiece for many communities in Northern Europe. However, this immense stock collapsed in the late 1960s, and in spite of a complete harvest moratorium it took almost 30 years to recover. By the end of the 1990s the spawning stock biomass has reached levels above 6 million tons again, but a changed distribution pattern in the early 2000s lead to strong disagreement among the harvesting nations and severe overfishing. However, as Miller et al. (2013, p.325) conjecture, the competing nations could restore cooperation when “staring into the abyss that yawned before them.”

5 Discussion and Conclusion

The effect of potential regime shifts on the cooperative and non-cooperative use of environmental goods and services is polarizing. When the location of the threshold is known to be low, or if it is sufficiently likely that even a low level of consumption causes catastrophe, the game may exhibit prisoner-dilemma features: Although it would be optimal to sustain the resource at its current level of use, the only non-cooperative equilibrium will be the immediate extirpation of the resource.[6] In contrast, when the threshold is known to be high, or if it is sufficiently likely that the productive regime can be sustained even at a high level of consumption, the game changes into a coordination-problem: The threat of loosing the productive resource can effectively enforce the first-best consumption level. For intermediate values, the equilibrium will neither be extirpation nor status-quo consumption, but rather a one-time increase in consumption, expanding the set of safe consumption possibilities. This expansions will be inefficiently large, but if it has not caused the regime shift, the players will be able to coordinate on staying at the updated level. Staying at the updated level is *ex post* socially optimal when the externality applies only to the risk of a regime shift (i.e. any given level of safe consumption is efficiently shared among the agents). When the externality also applies to the resource use itself, the threat of the threshold loses importance. Due to the dynamic common-pool externality on the resource, non-cooperative extraction will be inefficiently high even in absence of any risk of a regime shift. This means that in particular when the threshold is believed to be above the first-best consumption pattern, its threat cannot act as a "commitment device" to ensure efficient extraction. Nevertheless, the threshold may still dampen non-cooperative extraction.

These conclusions have been derived by using a general dynamic model that has placed only minimal requirements on the utility function (concavity and boundedness) and the probability distribution of the threshold (continuity). Nevertheless, there are a number of modeling assumptions that warrant discussion.

First, a prominent aspect of this model is that the threshold itself is not stochastic. The central motivation is that this allows concentrating on the effect of uncertainty about the threshold's location. This is arguably the core of the problem: we don't know which level of use triggers the regime shift. This modeling approach implies a clear demarcation between a safe region and a risky region of the state space. In particular, it implies that the edge of the cliff, figuratively speaking, is a safe – and in many cases optimal – place to be. The alternative approach, modeling the risk of a regime shift by a hazard rate acknowledges that, figuratively speaking, the edge of a cliff is often quite windy and not a particular safe place. This however implies, that the regime shift will occur with certainty as time goes to infinity, no matter how little of the resource is used; eventually there will be a gust of wind that is strong enough to blow us over the edge, regardless of where we stand. This is of course not very realistic either. But also on a deeper level one could argue that the non-stochasticity is in effect not a flaw but a feature: Let me cite Lemoine and Traeger (2014, p.28) who argue that

[6] But note that extirpation is also the socially optimal course of action if the threshold is deemed to be very low and the status quo consumption would be of little value compared to the resource potential.

"we would not actually expect tipping to be stochastic. Instead, any such stochasticity would serve to approximate a more complete model with uncertainty (and potentially learning) over the precise trigger mechanism underlying the tipping point." This being said, it would still be interesting to investigate how the choice between a hazard-rate formulation (as in Polasky et al., 2011 or Sakamoto, 2014) or a threshold formulation influences the outcome and policy conclusions in an otherwise identical model.

Second, I have modeled the players to be identical. In the real world, players are rarely identical. One dimension along which players could differ could be their valuation of the future. However, *prima facie* it should not be difficult to show that any such differences could be smoothed out by a contract that gives a larger share of the gains from cooperation to more impatient players. One could also investigate the effect of heterogenous beliefs about the existence and location of the threshold. Agbo (2014); Koulovatianos (2015) analyze this in the framework of Levhari and Mirman (1980). In the current set-up such a heterogeneity could lead to interesting dynamics and possible multiple equilibria, where some players rationally do not want to learn about the probability distribution of T whereas other players do invest in learning and experimentation. Another dimension along which players could differ is their size or the degree to which they depend on the environmental goods or services in question. As larger players are likely to be able to internalize a larger part of the externality than smaller players, different sets of equilibria may emerge. Especially in light of the discussions surrounding a possible climate treaty (Harstad, 2012; Nordhaus, 2015), it is topical to analyze a situation where groups of players can form a coalition to ameliorate the negative effects of non-cooperation in future applications.

Third, I have assumed the regime shift to be irreversible. This is obviously a considerable simplification. Groeneveld et al. (2013) have analyzed the problem of how a social planner would learn about the location of a flow-pollution threshold in a setting where repeated crossings are allowed. In the current set-up, the introduction of several thresholds would most likely not yield significant new insights but make the analysis a lot more cumbersome. For example, if one presumes that crossing the threshold implies that one learns where it is,[7] the game turns into a repeated game. This may imply that cooperation is sustainable for sufficiently patient players (van Damme, 1989). However there could also be cases where irreversibility emerges "endogenously" when it is possible – but not an equilibrium – to move out of a non-productive regime. Hence, a fruitful avenue is to utilize the tractability of the current modeling approach to explore this issue.

A final, related, point is the fact that I have concentrated on Markovian strategies. When the players are allowed to use history-dependent strategies, the threat of a threshold may allow them to coordinate on the social optimum in all phases of the game. They could simply agree on expand the set of safe consumption possibilities by the socially optimal amount and threaten that if any player steps too far, this triggers the depletion of the resource in the

[7]Groeneveld et al. (2013) presume that crossing the threshold tells the social planner that the regime shift has occurred, but not where. This is a little bit like sitting in a car, fixing the course to a destination and then blindfolding oneself. Arguably conscious experimentation is more realistically described by saying that the course is set, but the eyes remain open.

next period. This obviously begs the question of renegotiation proofness, but it is plausible that already a contract that is binding for two periods is sufficient to achieve the first-best. The threat of a disastrous regime shift is a very strong coordinating device. This is true irrespective of whether the threshold's location is known or unknown, because the current model of uncertainty implies that it is, loosely speaking, pitch dark when the players take a step. It is only afterwards that they realize whether the disastrous regime shift has occurred or not. Would the coordinating force of a catastrophic threshold diminish when the players can learn about its location without risking to cross it? Importantly, an extension of the model along these lines would speak to the recent debate on "early warning signals" (Scheffer et al., 2009; Boettiger and Hastings, 2013) and is the task of future work.

References

Aflaki, S. (2013). The effect of environmental uncertainty on the tragedy of the commons. *Games and Economic Behavior*, 82(0):240–253.

Agbo, M. (2014). Strategic exploitation with learning and heterogeneous beliefs. *Journal of Environmental Economics and Management*, 67(2):126–140.

Barrett, S. (2013). Climate treaties and approaching catastrophes. *Journal of Environmental Economics and Management*, 66(2):235–250.

Barrett, S. and Dannenberg, A. (2012). Climate negotiations under scientific uncertainty. *Proceedings of the National Academy of Sciences*, 109(43):17372–17376.

Bochet, O., Laurent-Lucchetti, J., Leroux, J., and Sinclair-Desgagné, B. (2013). Collective Dangerous Behavior: Theory and Evidence on Risk-Taking. Research Papers by the Institute of Economics and Econometrics, Geneva School of Economics and Management, University of Geneva 13101, Institut d'Economie et Economtrie, Universit de Genve.

Boettiger, C. and Hastings, A. (2013). Tipping points: From patterns to predictions. *Nature*, 493(7431):157–158.

Costello, C. and Karp, L. (2004). Dynamic taxes and quotas with learning. *Journal of Economic Dynamics and Control*, 28(8):1661–1680.

Crépin, A.-S. and Lindahl, T. (2009). Grazing games: Sharing common property resources with complex dynamics. *Environmental and Resource Economics*, 44(1):29–46.

Cropper, M. (1976). Regulating activities with catastrophic environmental effects. *Journal of Environmental Economics and Management*, 3(1):1–15.

Fesselmeyer, E. and Santugini, M. (2013). Strategic exploitation of a common resource under environmental risk. *Journal of Economic Dynamics and Control*, 37(1):125–136.

Frank, K. T., Petrie, B., Choi, J. S., and Leggett, W. C. (2005). Trophic cascades in a formerly cod-dominated ecosystem. *Science*, 308(5728):1621–1623.

Gerlagh, R. and Liski, M. (2014). Carbon Prices for the Next Hundred Years. CESifo Working Paper Series 4671, CESifo Group Munich.

Groeneveld, R. A., Springborn, M., and Costello, C. (2013). Repeated experimentation to learn about a flow-pollutant threshold. *Environmental and Resource Economics*, forthcoming:1–21.

Harsanyi, J. C. and Selten, R. (1988). *A general theory of equilibrium selection in games.* MIT Press, Cambridge.

Harstad, B. (2012). Climate contracts: A game of emissions, investments, negotiations, and renegotiations. *The Review of Economic Studies*, 79(4):1527–1557.

Hjermann, D. Ø., Stenseth, N. C., and Ottersen, G. (2004). Competition among fishermen and fish causes the collapse of Barents Sea capelin. *Proceedings of the National Academy of Sciences*, 101:11679–11684.

Hoel, M. (1978). Resource extraction, uncertainty, and learning. *The Bell Journal of Economics*, 9(2):642–645.

Kemp, M. (1976). How to eat a cake of unknown size. In Kemp, M., editor, *Three Topics in the Theory of International Trade*, pages 297–308. North-Holland, Amsterdam.

Kim, Y. (1996). Equilibrium selection in n-person coordination games. *Games and Economic Behavior*, 15(2):203–227.

Kossioris, G., Plexousakis, M., Xepapadeas, A., de Zeeuw, A., and Mäler, K.-G. (2008). Feedback nash equilibria for non-linear differential games in pollution control. *Journal of Economic Dynamics and Control*, 32(4):1312–1331.

Koulovatianos, C. (2015). Strategic exploitation of a common-property resource under rational learning about its reproduction. *Dynamic Games and Applications*, 5(1):94–119.

Lemoine, D. and Traeger, C. (2014). Watch your step: Optimal policy in a tipping climate. *American Economic Journal: Economic Policy,*, forthcoming:1–47.

Lenton, T. M., Held, H., Kriegler, E., Hall, J. W., Lucht, W., Rahmstorf, S., and Schellnhuber, H. J. (2008). Tipping elements in the earth's climate system. *Proceedings of the National Academy of Sciences*, 105(6):1786–1793.

Levhari, D. and Mirman, L. J. (1980). The Great Fish War: An Example Using a Dynamic Cournot-Nash Solution. *The Bell Journal of Economics*, 11(1):322–334.

Miller, K. A., Munro, G. R., Sumaila, U. R., and Cheung, W. W. L. (2013). Governing marine fisheries in a changing climate: A game-theoretic perspective. *Canadian Journal of Agricultural Economics*, 61(2):309–334.

Mitra, T. and Roy, S. (2006). Optimal exploitation of renewable resources under uncertainty and the extinction of species. *Economic Theory*, 28(1):1–23.

Nævdal, E. (2001). Optimal regulation of eutrophying lakes, fjords, and rivers in the presence of threshold effects. *American Journal of Agricultural Economics*, 83(4):972–984.

Nordhaus, W. (2015). Climate clubs: Overcoming free-riding in international climate policy. *American Economic Review*, 105(4):1339–1370.

Polasky, S., Zeeuw, A. d., and Wagener, F. (2011). Optimal management with potential regime shifts. *Journal of Environmental Economics and Management*, 62(2):229 – 240.

Rob, R. (1991). Learning and capacity expansion under demand uncertainty. *The Review of Economic Studies*, 58(4):655–675.

Sakamoto, H. (2014). A dynamic common-property resource problem with potential regime shifts. Discussion Paper E-12-012, Graduate School of Economics, Kyoto University.

Scheffer, M., Bascompte, J., Brock, W. A., Brovkin, V., Carpenter, S. R., Dakos, V., Held, H., van Nes, E. H., Rietkerk, M., and Sugihara, G. (2009). Early-warning signals for critical transitions. *Nature*, 461(7260):53–59.

Scheffer, M., Carpenter, S., Foley, J. A., Folke, C., and Walker, B. (2001). Catastrophic shifts in ecosystems. *Nature*, 413(6856):591–596.

Tavoni, A., Dannenberg, A., Kallis, G., and Löschel, A. (2011). Inequality, communication, and the avoidance of disastrous climate change in a public goods game. *Proceedings of the National Academy of Sciences*, 108(29):11825–11829.

Tsur, Y. and Zemel, A. (1994). Endangered species and natural resource exploitation: Extinction vs. coexistence. *Natural Resource Modeling*, 8:389–413.

Tsur, Y. and Zemel, A. (1995). Uncertainty and irreversibility in groundwater resource management. *Journal of Environmental Economics and Management*, 29(2):149 – 161.

van Damme, E. (1989). Renegotiation-proof equilibria in repeated prisoners' dilemma. *Journal of Economic Theory*, 47(1):206–217.

Appendix

A.1 Proof of Proposition 1

Recall that Proposition 1 states that *when the location of the threshold is known with certainty, then there exists, for every combination of N, T, and R, a value of $\bar{\beta}$ such that the first-best can be sustained as a Nash-equilibrium when $\beta \geq \bar{\beta}$. The larger is N, or the closer T is to 0, the larger has to be β.*

For a given N, first note that $\frac{\mathrm{d}\bar{\beta}}{\mathrm{d}T} = -\left([\frac{u'}{N} \cdot u + u \cdot u' \frac{N-1}{N}]/[u^2]\right) < 0$ so that the players need to be the less patient the more valuable it is to stay below the threshold (i.e. as T grows). Second, note that for $T \to 0$, the last term of (4) approaches 0 so that the right-hand-side of (4) approaches 1. But since it approaches 1 from below, we can always find some value of β that could still sustain the first-best. Finally, note that as $N \to \infty$, $u(T/N) \to 0$. Again, $\bar{\beta}$ approaches 1 from below, allowing to find some value of β that could still sustain the first-best.

A.2 Proof of Proposition 2

Recall that Proposition 2 states that *the socially optimal total use of the resource is either s_0 for all t or $s_0 + \delta^*(s_0)$ for $t = 0$ and, if the resource has not collapsed, s_1 for all $t \geq 1$.*

The proof consist of two parts: First, I show that there exists some values s^* at which it is not optimal to further expand the set of safe consumption possibilities. In the second part, I show that for any $s \neq s^*$ it is better to make one step and then stay than to make two steps and then stay.

(1) There exists some values s^* at which it is optimal to stay. At least one such a value exists because at $s = R$ there is no other choice but to remain standing. Furthermore, values of $s^* < R$ exist when the survival function $L_s(\delta)$ is sufficiently small close to R. The intuition is that the marginal benefit from staying is strictly increasing with s, but the marginal benefit from making a small step is then decreasing close to R: While the potential reward is small (since the value function is bounded above by $u(R)/(1 - \beta)$), the likelihood of triggering the regime shift becomes high. When it is known that there is a catastrophic threshold on $[0, R]$, we have $L_s(\delta) \to 0$ as $\delta \to R - s$ and there will always be values of $s^* < R$. To show this more formally, compare the value from staying at some value of s close to R, that is $s = R - \varepsilon$, to the value of making a step towards R so that one stays at $R - \delta$ (with $\delta \in (0, \varepsilon]$). I claim that for some ε, we have:

$$\frac{u(R-\varepsilon)}{1-\beta} \geq u(R-\delta) + \beta L_s(\delta)\frac{u(R-\delta)}{1-\beta} \tag{A-1}$$

Clearly, $\lim_{\varepsilon \to 0}\left[\frac{u(R-\varepsilon)}{1-\beta}\right] = \frac{u(R)}{1-\beta}$ but since $\delta \in (0, \varepsilon]$ and $L_s(\delta) \to 0$ as $\delta \to R - s$, we have that $\lim_{\varepsilon \to 0}\left[u(R-\delta) + \beta L_s(\delta)\frac{u(R-\delta)}{1-\beta}\right] = u(R) < \frac{u(R)}{1-\beta}$.

(2) For any $s \neq s^*$ it is better to make any one step and then stay than to make two steps and then stay. Take any s and find δ^* as defined in equation (8) above. I then show that at some $\tilde{s}$ below s (with $\tilde{s} = s - \delta_1$) the payoff from choosing $\delta_2 = \delta_1 + \delta^*$ (i.e. taking one step) exceeds the payoff from first taking one step from $\tilde{s}$ to s and then taking the second step δ^*:

$$\begin{aligned} &u(\tilde{s}+\delta_1+\delta^*) + \beta L_{\tilde{s}}(\delta_1+\delta^*)\frac{u(\tilde{s}+\delta_1+\delta^*)}{1-\beta} \\ &\qquad \geq u(\tilde{s}+\delta_1) + \beta L_{\tilde{s}}(\delta_1)\Big(u(\tilde{s}+\delta_1+\delta^*) + \beta L_{\tilde{s}+\delta_1}(\delta^*)\frac{u(\tilde{s}+\delta_1+\delta^*)}{1-\beta}\Big) \end{aligned} \tag{A-2}$$

The important thing to note at this stage is that: $L_{\tilde{s}}(\delta_1)L_{\tilde{s}+\delta_1}(\delta^*) = \frac{L(\tilde{s}+\delta_1)}{L(\tilde{s})}\frac{L(\tilde{s}+\delta_1+\delta^*)}{L(\tilde{s}+\delta_1)} = \frac{L(\tilde{s}+\delta_1+\delta^*)}{L(\tilde{s})} =$

$L_{\tilde{s}}(\delta_1 + \delta^*)$. Hence, (A-2) can, upon inserting $\tilde{s} = s - \delta_1$, be written as:

$$u(s+\delta^*) + \beta \frac{L(s+\delta^*)}{L(s-\delta_1)} \frac{u(s+\delta^*)}{1-\beta} \geq u(s) + \beta \frac{L(s)}{L(s-\delta_1)} u(s+\delta^*) + \beta^2 \frac{L(s+\delta^*)}{L(s-\delta_1)} \frac{u(s+\delta^*)}{1-\beta}$$

$$\Leftrightarrow$$

$$u(s+\delta^*) - \beta \frac{L(s)}{L(s-\delta_1)} u(s+\delta^*) + \beta(1-\beta) \frac{L(s+\delta^*)}{L(s-\delta_1)} \frac{u(s+\delta^*)}{1-\beta} \geq u(s)$$

$$\Leftrightarrow$$

$$\left[1 + \beta \frac{L(s+\delta^*) - L(s)}{L(s-\delta_1)}\right] u(s+\delta^*) \geq u(s) \qquad \text{(A-2')}$$

Now by the definition of δ^* we know that:

$$u(s+\delta^*) + \beta L_s(\delta^*) \frac{u(s+\delta^*)}{1-\beta} > \frac{u(s)}{1-\beta}$$

$$\Leftrightarrow$$

$$(1-\beta) u(s+\delta^*) + \beta L_s(\delta^*) u(s+\delta^*) > u(s)$$

$$\Leftrightarrow$$

$$\left[1 - \beta + \beta \frac{L(s+\delta^*)}{L(s)}\right] u(s+\delta^*) > u(s)$$

$$\Leftrightarrow$$

$$\left[1 + \beta \frac{L(s+\delta^*) - L(s)}{L(s)}\right] u(s+\delta^*) > u(s) \qquad \text{(A-3)}$$

Since $L'(s) < 0$, we know that $0 > \frac{L(s+\delta^*)-L(s)}{L(s-\delta_1)} > \frac{L(s+\delta^*)-L(s)}{L(s)}$. Therefore, combining (A-2') and (A-3) establishes the claim and completes the proof:

$$\left[1 + \beta \frac{L(s+\delta^*) - L(s)}{L(s-\delta_1)}\right] u(s+\delta^*) > \left[1 + \beta \frac{L(s+\delta^*) - L(s)}{L(s)}\right] u(s+\delta^*) > u(s)$$

A.3 Proof of Proposition 3.

Proposition 3 states that δ^* is decreasing in s.

Recall that the payoff from starting at s, taking a step δ^*, and remaining standing thereafter is given by:

$$\varphi(\delta^*; s) = u\left(s+\delta^*\right) + \beta L_s(\delta^*) \frac{u\left(s+\delta^*\right)}{1-\beta} \qquad (6)$$

Accordingly the equilibrium choice of a positive expansion $\delta^*(s)$ is the solution of $\varphi'(\delta^*; s) = 0$ where φ' is given by (7):

$$\varphi'(\delta^*; s) = u'\left(s+\delta^*\right) + \frac{\beta}{1-\beta}\left[L_s'(\delta^*) u\left(s+\delta^*\right) + L_s(\delta^*) u'\left(s+\delta^*\right)\right] \qquad (7)$$

with the second-order condition:

$$\varphi''(\delta^*; s) = u'' + \frac{\beta}{1-\beta}\Big(L_s''(\delta^*) u + 2L_s'(\delta^*) u' + L_s(\delta^*) u''\Big) < 0 \qquad \text{(A-4)}$$

To show that δ^* is declining in s, I need to show that:

$$\frac{\mathrm{d}\delta^*}{\mathrm{d}s} = -\frac{\partial\big[\varphi'(\delta^*; s)\big]/\partial s}{\partial\big[\varphi'(\delta^*; s)\big]/\partial \delta^*} < 0$$

Since the denominator is negative when the second-order condition is satisfied, we have that $\frac{\mathrm{d}\delta^*}{\mathrm{d}s} < 0$ when $\frac{\partial[\varphi'(\delta^*;s)]}{\partial s} < 0$, so that the condition to check is (A-5):

$$\frac{\partial[\varphi'(\delta^*;s)]}{\partial s} = u'' + \frac{\beta}{1-\beta}\left(\frac{\partial L_s'(\delta^*)}{\partial s}u + L_s'(\delta^*)u' + \frac{\partial L_s(\delta^*)}{\partial s}u' + L_s(\delta^*)u''\right) < 0 \tag{A-5}$$

Noting the similarity of (A-5) to the second-order condition (A-4), and realizing that (A-4) can be decomposed into a part A and a part B and that (A-5) can be decomposed into a part A and a part C, a sufficient condition for (A-5) to be satisfied is that $B > C$.

$$\underbrace{\left[u'' + \frac{\beta}{1-\beta}\Big(L_s'(\delta^*)u' + L_s(\delta^*)u''\Big)\right]}_{A} + \underbrace{\frac{\beta}{1-\beta}\Big(L_s''(\delta^*)u + L_s'(\delta^*)u'\Big)}_{B} < 0 \tag{A-4'}$$

$$\underbrace{\left[u'' + \frac{\beta}{1-\beta}\Big(L_s'(\delta^*)u' + L_s(\delta^*)u''\Big)\right]}_{A} + \underbrace{\frac{\beta}{1-\beta}\left(\frac{\partial L_s'(\delta^*)}{\partial s}u + \frac{\partial L_s(\delta^*)}{\partial s}u'\right)}_{C} < 0 \tag{A-5'}$$

In order to show that $L_s''(\delta)u + L_s'(\delta)u' > \frac{\partial L_s'(\delta)}{\partial s}u + \frac{\partial L_s(\delta)}{\partial s}u'$, I use the first-order condition for an interior solution from (7) to write u' in terms of u:

$$u' = \frac{-L_s'(\delta)}{\frac{1-\beta}{\beta} + \frac{1}{N}L_s(\delta)}u$$

Upon inserting and canceling u, I need to show that:

$$L_s''(\delta) + L_s'(\delta)\left[\frac{-L_s'(\delta)}{\frac{1-\beta}{\beta} + L_s(\delta)}\right] > \frac{\partial L_s'(\delta)}{\partial s} + \frac{\partial L_s(\delta)}{\partial s}\left[\frac{-L_s'(\delta)}{\frac{1-\beta}{\beta} + L_s(\delta)}\right] \tag{A-6}$$

Recall that $L_s(\delta) = \frac{L(s+\delta)}{L(s)}$ and hence:

$$L_s'(\delta) = \frac{L'(s+\delta)}{L(s)} \qquad \frac{\partial L_s(\delta)}{\partial s} = \frac{L'(s+\delta)L(s) - L(s+\delta)L'(s)}{[L(s)]^2}$$

$$L_s''(\delta) = \frac{L''(s+\delta)}{L(s)} \qquad \frac{\partial L_s'(\delta)}{\partial s} = \frac{L''(s+\delta)L(s) - L'(s+\delta)L'(s)}{[L(s)]^2}$$

Tedious but straightforward calculations then show that (A-6) is indeed satisfied.

$$\underbrace{\left[N\frac{1-\beta}{\beta} + \frac{L(s+\delta)}{L(s)}\right]}_{a}\frac{L''(s+\delta)}{L(s)} - \left[\frac{L'(s+\delta)}{L(s)}\right]^2 > \underbrace{\left[N\frac{1-\beta}{\beta} + \frac{L(s+\delta)}{L(s)}\right]}_{a}\frac{\partial L_s'(\delta)}{\partial s} - \frac{\partial L_s(\delta)}{\partial s}L_s'(\delta)$$

$\Leftrightarrow$

$$a\frac{L''(s+\delta)}{L(s)} - \left[\frac{L'(s+\delta)}{L(s)}\right]^2 > a\frac{L''(s+\delta)L(s) - L'(s+\delta)L'(s)}{[L(s)]^2} - \frac{\partial L_s(\delta)}{\partial s}L_s'(\delta)$$

$\Leftrightarrow$

$$aL(s) > L(s+\delta)$$

$\Leftrightarrow$

$$\left[N\frac{1-\beta}{\beta} + \frac{L(s+\delta)}{L(s)}\right]L(s) > L(s+\delta)$$

$\Leftrightarrow$

$$N\frac{1-\beta}{\beta}L(s) > 0$$

A.4 Proof of Proposition 4.

Recall that Proposition 4 states that *for* $s_0 \geq \bar{s}^{nc}$ *coordination to stay at* s_0 *can be supported as a Nash equilibrium. For* $s_0 < \bar{s}^{nc}$ *taking one step and then staying at* $s_1 = s_0 + \delta^{nc}$ *can be supported as a Nash equilibrium.*

First note that if it is a Nash equilibrium to stay at some s in any one period, it will be a Nash equilibrium to stay at that s in all subsequent periods. Again, there will be some s^{nc} at which staying is a Nash equilibrium, because at least at $s = R$, there is no other choice. But parallel to the argument in Proposition 2, there will also be some $s^{nc} < R$ when s close enough to R and $L_s(\delta)$ becomes sufficiently small. Also here, there will always be values of $s^{nc} < R$ when it is known that there is a catastrophic threshold on $[0, R]$. Suppose all other players stay at $s = R - \varepsilon$, then for ε small, the value from staying at $s = R - \varepsilon$ is at least as large as the value of making a step towards R so that the updated value is $R - \delta$ (with $\delta \in (0, \varepsilon]$):

$$\frac{u\left(\frac{R-\varepsilon}{N}\right)}{1-\beta} \geq u\left(\frac{R-\varepsilon}{N} + \delta\right) + \beta L_s(\delta)\frac{u\left(\frac{R-\delta}{N}\right)}{1-\beta} \tag{A-7}$$

Parallel to the social optimum we have $\lim_{\varepsilon\to 0}\left[\frac{u((\frac{R-\varepsilon}{N})}{1-\beta}\right] = \frac{u(R/N)}{1-\beta}$. Again, since $\delta \in (0, \varepsilon]$ and $L_s(\delta) \to 0$ as $\delta \to R - s$ we have $\lim_{\varepsilon\to 0}\left[u\left(\frac{R-\varepsilon}{N} + \delta\right) + \beta L_s(\delta)\frac{u(\frac{R-\delta}{N})}{1-\beta}\right] = u(R/N) < \frac{u(R/N)}{1-\beta}$.

Now, as there is some s^{nc} at which staying is a Nash equilibrium, there will be a last step at which this value is reached. Take some value s at which staying is not a Nash equilibrium. Suppose the strategy of the opponents is to take some step $\delta_1^{-i} < \delta^{nc}(s)$ and then some step $\delta_2^{-i*}(s + \delta_1^{-i} + \delta_1^i)$. The following calculations show that the best-reply from player i is to take only one step δ_1^{i*}. Hence $\delta_2^{-i*}(s + \delta_1^{-i} + \delta_1^i) = 0$ and the equilibrium will be to reach a value at which staying is a Nash equilibrium in one step.

For player i the payoff from making one step $\delta_1^{i*} = s^{nc} - s_0 - \delta_1^{-i}$ exceeds the payoff from making two steps $\delta_1^i < s^{nc} - s_0 - \delta_1^{-i}$ and $\delta_2^{i*} = s^{nc} - s_1 - \delta_2^{-i*}$ when:

$$\begin{aligned} &u\left(\frac{s_0}{N} + \delta_1^{i*}\right) + \frac{\beta}{1-\beta}L_{s_0}(\delta_1^{i*} + \delta_1^{-i})u\left(\frac{s^{nc}}{N}\right) \\ &\quad\geq u\left(\frac{s_0}{N} + \delta_1^i\right) + \beta L_{s_0}(\delta_1^i + \delta_1^{-i})\left[u\left(\frac{s_1}{N} + \delta_2^{i*}\right) + \frac{\beta}{1-\beta}L_{s_1}(\delta_2^{i*})u\left(\frac{s^{nc}}{N}\right)\right] \end{aligned} \tag{A-8}$$

As for the coordinated case, $L_{s_0}(s_1 - s_0)L_{s_1}(s^{nc} - s_1) = L_{s_0}(s^{nc} - s_0)$ so that (A-8) implies:

$$u\left(\frac{s_0}{N} + \delta_1^{i*}\right) - u\left(\frac{s_0}{N} + \delta_1^i\right) \geq \beta\left[\frac{L(s_1)}{L(s_0)}u\left(\frac{s_1}{N} + \delta_2^{i*}\right) - \frac{L(s^{nc})}{L(s_0)}u\left(\frac{s^{nc}}{N}\right)\right] \tag{A-9}$$

For clarity, write this inequality as $A - a \geq B - b$. This inequality holds because both $A > B$ and $a < b$. To see that $A > B$ note that u is an increasing and concave function so that $u(\frac{s_0}{N} + \delta_1^{i*}) > u(\frac{s_1}{N} + \delta_2^{i*})$ when $\frac{s_0}{N} + \delta_1^{i*} > \frac{s_1}{N} + \delta_2^{i*}$. Inserting $\delta_1^{i*} = s^{nc} - s_0 - \delta_1^{-i}$, $\delta_2^{i*} = s^{nc} - s_1 - \delta_2^{-i*}$ and $s_1 = s_0 + \delta_1^i + \delta_1^{-i}$ in this inequality simplifies to $(N-1)(\delta_1^i + \delta_1^{-i}) > 0$, which is true. By the same argument, $a < b$ when $\frac{s_0}{N} + \delta_1^i < \frac{s^{nc}}{N}$. Re-write this as $N\delta_1^i < s^{nc} - s_0$. This inequality holds because it is implied by the definition that $\delta_1^i < s^{nc} - s_0 - \delta_1^{-i}$ and $\delta_1^{-i} < \delta^{nc}(s)$.

Recall that the best-reply function $g(\delta^{-i}, s)$ in equation (12b) is therefore defined by the interior solution to the first-order-condition of maximizing $\phi(\delta^i; \delta^{-i}, s)$:

$$\begin{aligned} &\phi'(\delta^i; \delta^{-i}, s) = u'\left(\frac{s}{N} + \delta^i + \delta^{-i}\right) \\ &\quad + \frac{\beta}{1-\beta}\left[L_s'(\delta^i + \delta^{-i})u\left(\frac{s}{N} + \delta^i + \delta^{-i}\right) + \frac{1}{N}L_s(\delta^i + \delta^{-i})u'\left(\frac{s}{N} + \delta^i + \delta^{-i}\right)\right] \end{aligned}$$

For a symmetric step size $\delta^{-i} = (N-1)\delta^i$, we have:

$$\phi'(\delta^{nc}; s) = u'\left(\frac{s}{N} + \delta^{nc}\right) + \frac{\beta}{1-\beta}\left[L_s'(N\delta^{nc})u\left(\frac{s+\delta^{nc}}{N}\right) + \frac{1}{N}L_s(N\delta^{nc})u'\left(\frac{s+\delta^{nc}}{N}\right)\right]$$

The value of $\underline{s}^{nc}$ is defined by $\delta^{nc} = \frac{R-s}{N}$, which is the largest value of s at which equation (13) does not yet have an interior solution but $\phi'(\delta, s) > 0$ for all $\delta \in [0, R-s)$. Similarly, the value of $\overline{s}^{nc}$ is defined by $\delta^{nc} = 0$, which is the smallest value of s at which equation (13) no longer has an interior solution but $\phi'(\delta, s) < 0$ for all $\delta \in (0, R-s]$

A.5 Proof of Proposition 5.

Let me repeat the comparative statics results here:

(a) The boundaries $\underline{s}^{nc}$ and $\overline{s}^{nc}$, and aggregate extraction for $s \in [\underline{s}^{nc}, \overline{s}^{nc}]$, decrease with β.

(b) An increase in N leads to more aggressive extraction when $\frac{N}{N+1} > u'(\frac{R}{N})/u'(\frac{R}{N+1})$.

(c) The more unlikely the regime shift (in terms of a first-degree stochastic dominance), the larger the range where a cautious Nash-equilibrium exists.

(d) The higher the maximum potential reward R, the larger the range where a cautious Nash-equilibrium exists.

First, as $\phi' = 0$ implicitly defines a monotonically decreasing function $\delta^{nc}(s)$ on $[\underline{s}^{nc}, \overline{s}^{nc}]$ (which can be shown by replacing $\delta^*(s)$ with $N\delta^{nc}(s)$ in the proof of Proposition 3) and $\delta^{nc}(s)$ is bounded above by $R-s$ and below by 0, an increase in δ^{nc} will also lead to an increase in $\underline{s}^{nc}$ and $\overline{s}^{nc}$ respectively.

(a) To prove the proposition's part with respect to β it is thus sufficient to analyze $\frac{\mathrm{d}\phi'}{\mathrm{d}\beta}$. We have $\frac{\mathrm{d}\phi'}{\mathrm{d}\beta} = \frac{2\beta[...]-\beta}{(1-\beta)^2}$, where the term in the squared brackets [...] is term in the squared brackets of equation (13). We know that this term must be negative for an interior solution because $u' > 0$. Therefore: $\frac{\mathrm{d}\phi'}{\mathrm{d}\beta} = \frac{2\beta[...]-\beta}{(1-\beta)^2} < 0$.

(b) I now turn to the effect of increasing N. To provide a sufficient condition for when an increase in N decreases the range where there is a cautious equilibrium, and therefore increases aggregate expansion, I make the following argument: $\underline{s}^{nc}$, the largest value at which immediate extirpation is the only Nash equilibrium becomes larger when adding another player and $\frac{N}{N+1} > u'(\frac{R}{N})/u'(\frac{R}{N+1})$. For a given number of players N we have at a given $\underline{s}^{nc} = \hat{s}$ that $\phi'(\frac{R-s}{N}; \hat{s}) = 0$ and I show that $\phi'(\frac{R-s}{N+1}; \hat{s}) > 0$ when $\frac{N}{N+1} > \frac{u'(\frac{R}{N})}{u'(\frac{R}{N+1})}$:

$$\phi'(\tfrac{R-s}{N+1}; \hat{s}) - \phi'(\tfrac{R-s}{N}; \hat{s}) > 0$$

$$\Leftrightarrow$$

$$u'(\tfrac{R}{N+1}) - u'(\tfrac{R}{N}) + \frac{\beta}{1-\beta}\left[\left(u(\tfrac{R}{N+1}) - u(\tfrac{R}{N})\right)L_{\hat{s}}' + L_{\hat{s}}\left(\frac{1}{N+1}u'(\tfrac{R}{N+1}) - \frac{1}{N}u'(\tfrac{R}{N})\right)\right] > 0$$

The first part of the last line is positive due to concavity of u, the first term in the squared bracket is positive since $L_s' < 0$ and $u(\frac{R}{N}) > u'(\frac{R}{N+1})$, and the last term in the squared bracket is positive whenever $\frac{N}{N+1} > \frac{u'(\frac{R}{N})}{u'(\frac{R}{N+1})}$.

(c) Consider the equation (13) at $s = \underline{s}^{nc}$:

$$\phi'\left(\frac{R-\underline{s}^{nc}}{N}; \underline{s}^{nc}\right) = u'\left(\frac{R}{N}\right) + \frac{\beta}{1-\beta}\left[L_s'(R-\underline{s}^{nc})u\left(\frac{R}{N}\right) + \frac{1}{N}L_s(R-\underline{s}^{nc})u'\left(\frac{R}{N}\right)\right] = 0$$

Now when the regime shift is more likely, we have $\tilde{L} < L$ and $\tilde{L}' < L'$. The term in the squared brackets above will therefore be smaller in absolute terms. As it is negative, it must mean that:

$$\tilde{\phi}'\left(\frac{R-\underline{s}^{nc}}{N};\underline{s}^{nc}\right) = u'\left(\frac{R}{N}\right) + \frac{\beta}{1-\beta}\left[\tilde{L}_s'(R-\underline{s}^{nc})u\left(\frac{R}{N}\right) + \frac{1}{N}\tilde{L}_s(R-\underline{s}^{nc})u'\left(\frac{R}{N}\right)\right] > 0$$

so that the range of values at which a cautious equilibrium exists is larger.

(d) Finally, to see the effect of an increase in R, note that this does not impact equation (13) directly, but it does have an effect on the first value $\underline{s}^{nc}$: As the diagonal line defining the upper bound of $\delta \in [0, R-s]$ shifts outwards, and $\delta^{nc}(s)$ is a downward sloping function steeper than $R-s$, the first value at which it is not optimal to extirpate the resource must be smaller.

A.6 Proof of Proposition 6

Recall that Proposition 6 states that *Also when crossing the threshold at time t triggers the regime shift at some (potentially uncertain) time $\tau > t$, it is still socially and individually rational to experiment – if at all – in the first period only.*

The key is to realize that yesterday's decisions are exogenous today. This means that threat of a regime shift can be modeled as an exogenous hazard rate: Let h_t be the probability that the regime shift, triggered by events earlier than and including time t, occurs at time t (conditional on not having occurred prior to t, of course). The player's problem in this situation can be formulated as:

$$V^i(s,\Delta^{-i}) = \max_{\delta^i \in [0,R-s]} \left\{u(s+\delta^i) + (1-h_t)\beta L_s(\delta^i+\delta^{-i})V^i(s+\delta,\Delta^{-i})\right\} \tag{A-10}$$

The structure of (A-10) is identical to the one in equation (9), only the effective discount factor decreases by $(1-h_t)$. Thus, the learning dynamics are unchanged.

A.7 Proof of Proposition 7

Proposition 7 states that *a known threshold is crossed in the first period, or never.*

I argue by contradiction. Suppose there is a Nash equilibrium where the threshold is not crossed up to some period t and the resource (prematurely) exhausted at period t. This implies:

$$u(R_t - \frac{N-1}{N}T) > u(\frac{T}{N}) + \beta V_t(R_t - T) \tag{A-11}$$

The gain from preempting all other players is given by:

$$u(R_{t-1} - \frac{N-1}{N}T) - \left[u(\frac{T}{N}) + \beta u(\frac{R_{t-1}-T}{N})\right] \tag{A-12}$$

Equation (A-12) is positive if $u(R_{t-1} - \frac{N-1}{N}T) > u(\frac{T}{N}) + \beta u(\frac{R_{t-1}-T}{N}) = u(\frac{T}{N}) + \beta u(\frac{R_t}{N})$. Now clearly $u(R_{t-1} - \frac{N-1}{N}T) > u(R_t - \frac{N-1}{N}T)$ (as $R_{t-1} > R_t$) and because $V_t(R_t - T) > u(\frac{R_t}{N})$ (otherwise exhaustion at period t would have been optimal), we know that:

$$u(R_{t-1} - \frac{N-1}{N}T) > u(R_t - \frac{N-1}{N}T) > u(\frac{T}{N}) + \beta V_t(R_t - T) > u(\frac{T}{N}) + \beta u(\frac{R_{t-1}-T}{N}) \tag{A-13}$$

Thus, it cannot have been an equilibrium to conserve part of the resource stock up to period t.

A.8 Proof of Proposition 8

Recall that Proposition 8 states that *in the game described by* (17) *when T is unknown, there exists, in addition to the aggressive equilibrium in which the resource is exhausted in the initial period, a pareto dominant equilibrium in which experimentation – if at all – is undertaken in the first period only and $s_1 = s_0 + \delta^{nc}(s_0)$ is an upper bound on aggregate extraction for the remainder of the game.*

The existence of the aggressive equilibrium is self-evident. The pareto superiority of the cautious equilibrium is also obvious as – by assumption – there was a pareto-dominant equilibrium with several periods of extraction in a world without the threshold. I now show that experimentation in the second period of the game is not individually (and socially) optimal:

For a given safe value s_1 and a given stock of the remaining resource R_1 in the second period, the value of the game for player i, when all other players share the extraction of the safe amount s_1 equally, is given by:

$$V_2(R_1, s_1) = \max_{\delta^i} \left\{ u(\tfrac{s_1}{N} + \delta^i) + \beta L_{s_1}(\delta^i) V_3\left(R_1 - s_1 - \delta^i\right) \right\} \tag{A-14}$$

The first-order condition for an (interior) expansion of the set of safe values is given by:

$$u'(\tfrac{s_1}{N} + \delta^i) = -\beta L'_{s_1}(\delta^i) V_3\left(R_1 - s_1 - \delta^i\right) + \beta L_{s_1}(\delta^i) V'_3\left(R_1 - s_1 - \delta^i\right) \tag{A-15}$$

Suppose that the first derivative of the objective function is declining in s in a neighborhood of δ^{i*} (this is shown below). Then for large s the right-hand side of (A-15) is larger than the left-hand side (hence there is no interior solution). Accordingly, the last value of s at which no expansion can be coordinated upon in the second period of the game is defined by:

$$u'(\tfrac{s_1}{N}) = \beta V'_3\left(R_1 - s_1\right) - \beta L'_{s_1}(\delta^i) V_3\left(R_1 - s_1\right) \tag{A-16}$$

Now in the first period, the corresponding value function for player i (again presuming that all other players remain at s_0) is:

$$V_1(R_0, s_0) = \max_{\delta^i} \left\{ u(\tfrac{s_0}{N} + \delta^i) + \beta L_{s_0}(\delta^i) V_2(R_0 - s_0 - \delta^i, s_0 + \delta^i) \right\} \tag{A-17}$$

The condition for the last value of s at which no expansion can be coordinated upon is given by:

$$u'(\tfrac{s_0}{N}) = \frac{\beta}{N} \frac{\left[V_2^i\left(R_0 - s_0 - \delta^i, s_0 + \delta^i\right)\right]}{\partial s_0} - \beta L'_{s_0}(\delta^i) V_2^i\left(R_0 - s_0 - \delta^i, s + \delta^i\right) \tag{A-18}$$

Now consider the value of s at which (A-18) holds. At this level of s, equation (A-16) will not hold (but the right-hand side will be larger than the left-hand side) as $\frac{\partial[V_2^i]}{\partial s} > u'$ and $V_2\left(R_0 - s_0 - \delta^i, s + \delta^i\right) > V_3\left(R_1 - s_0 - \delta^i\right)$. This means that experimentation in the second period is no longer optimal but it is still optimal in the first period. As φ is a continuous function of s, the same holds also for depletion (the other corner solution).

What remains to be shown is that the first derivative of the objective function is declining in s in a neighborhood of the optimal expansion δ^{i*}. To this end, denote the derivative of the objective function by φ. We then have (omitting all sub and superscripts to avoid clutter):

$$\begin{aligned} \frac{\partial \varphi}{\partial s} &= \frac{1}{N} u'' + \beta \left(\frac{L''(s+\delta)L(s) - L'(s+\delta)L'(s)}{[L(s)]^2} u - \frac{1}{N} \frac{L'(s+\delta)}{L(s)} u' \right) \\ &\quad - \frac{\beta}{N} \left(\frac{L'(s+\delta)L(s) - L(s+\delta)L'(s)}{[L(s)]^2} u' + \frac{L(s+\delta)}{L(s)} u'' \right) \end{aligned} \tag{A-19}$$

Note that the second-order condition for a maximum at δ^{i*} requires:

$$\begin{aligned} \frac{\partial \varphi}{\partial \delta} &= \frac{1}{N} u'' + \beta \left(\frac{L''(s+\delta)}{L(s)} u - \frac{1}{N} \frac{L'(s+\delta)}{L(s)} u' \right) \\ &\quad - \frac{\beta}{N} \left(\frac{L'(s+\delta)L(s) - L(s+\delta)L'(s)}{[L(s)]^2} u' + \frac{L(s+\delta)}{L(s)} u'' \right) \end{aligned} \tag{A-20}$$

Equation (A-19) and (A-20) are similar. In fact, $\frac{\partial \varphi}{\partial s} = \frac{\partial \varphi}{\partial \delta} - \frac{L'(s+\delta)L'(s)}{[L(s)]^2}$. The term $\frac{L'(s+\delta)L'(s)}{[L(s)]^2}$ is positive as $L'(s+\delta) < 0$ and $L'(s) < 0$. Hence equation (A-19) is negative when the second-order condition holds.

Thus experimentation in the second period is not optimal, and *a forterori* not in any later period.