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Another model of sales. Price discrimination in a horizontally differentiated duopoly market.

Halvor Mehlum *

Abstract

Using a model of horizontal differentiation where a variety dimension is added to Hotelling's (1929) "linear city" duopoly model, I show that even when costs and demand are symmetric, price discrimination may be an equilibrium phenomenon. In the model each customer have a preferred variety and a preferred firm. They have perfect information about all prices and may be induced to switch variety and firm given a sufficient price difference. Price discrimination equilibrium exists when a sufficient fraction of consumers are elastic both with respect to variety and firm.

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Keywords: Duopoly, price discrimination

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1 Introduction

Some markets are characterized by having a limited number of firms competing against each other with similar product lines, where each element in the line are preferred by different consumers. One example is two airlines that both sell apex and business tickets on the same route. Another is Coca Cola Company and PepsiCo that both sell diet and regular cokes.

When setting the price of a high quality business fare and a low quality apex fare an airline needs both to take into account the loss as customers switch to the other airline but also the cannibalization effect as brand-loyal customers switch from business variety to the apex variety. Such concerns in the pricing of vertically differentiated goods have been studies in several influential works.[1]

This article concerns a situation *without* vertical differentiation but where consumers, for idiosyncratic reasons, in addition to preferences for a firm, prefer one variety of the good over the other. One example may be the purchase of gasoline on Monday or on Tuesday. The Monday variety is not of a better or worse quality but some consumers happens to need/prefer gasoline on Tuesday others on Monday. In its pricing decision a gasoline station (a firm) needs to consider the possibility of losing customers to another gas station but also cannibalization as firm loyal customers switch from one day to the other. E.g they switch from the Monday variety to the Tuesday variety. It has been documented that many areas exhibit synchronous pricing pattern of gasoline over the week, e.g Noel (2007). The question in this article is whether such non-uniform pricing, of a priori similar goods, may be explained as a Nash equilibrium between the firms.

Several articles, e.g. Katz (1984) and Canoy, M., & Peitz (1997) have found that in a vertically differentiated environment a duopoly equilibrium may entail margin differences between low and high qualities if the high quality consumers also are more firm loyal. My question is whether similar margin based price differences may arise in an horizontally differentiated environment where different consumers have different tastes with regards to the varieties.

[1] For instance Mussa & Rosen (1978), Sing & Vives (1984), Katz (1984), Stole (1995), Johnson & Myatt (2006).

I analyze a situation where consumers have preferences over two dimensions of a good, firm and variety. Following Hotelling (1929) two firms located at fixed locations sell goods in two varieties. The question is as follows: When unit costs are identical and constant and the demand towards varieties and firms are identical are equilibrium price discrimination still possible? Is it possible to explain margin differences in this environment in a one-shot Bertrand equilibrium? In order to put this starting point in perspective it's informative to contrast it to another literature discussing margin differences namely the economics of sales. e.g. Shilony (77), Varian (80) and Salop and Stiglitz (82). Varian writes in his article "A model of sales" (p.652)

> In this article, I explicitly address the question of sales equilibria. The model may be regarded as a combination of the Salop- Stiglitz and the Shilony models described above. As in the Salop-Stiglitz model, it will be assumed that there are informed and uninformed consumers. As in the Shilony model, I will allow for the possibility of randomized pricing strategies by stores. I will be interested in characterizing the equilibrium behavior in such markets.

My approach differs from Varian by considering a case where there rather than *informed/uninformed* consumers are *price sensitive/insensitive* consumers. Rather than *randomized* pricing strategies there are *full information* pricing.

I find that in a symmetric environment a price discrimination equilibrium exists when a sufficient fraction of those consumers who are are close to indifferent with respect to variety also are close to indifferent with respect to firm.

2 The model

The model is a generalization of Hotelling (1929) but where a variety dimension is added to the linear city. Also the preferences and transport cost of consumers are generalized.

There is a market with two firms L and R each producing two varieties of a differentiated good. The good space is $\mathbb{R}^2$ where the first dimension is the firm dimension indicated by $f = l, r$ (left and right) while the second dimension is the variety dimension indicated by $t = u, d$ (up and down). The location of the two goods produced by L is fixed at $(f, t) \in \{(l, d), (l, u)\}$ while the location of the two goods produced by R is fixed at $(f, t) \in \{(r, d), (r, u)\}$. By normalization the four goods spans out a rectangle centered on the origin where $-l = r > 0$ and $-d = u > 0$.

There is a unit mass of consumers each consuming a homogeneous good m, with price normalized to unity, and where each consume exactly one unit of one of the differentiated goods. The consumer has individual preferences for the differentiated good. Following Economides (1986), a consumer with ideal combination (x, y) has utility function

$$U_{xy}[f, t, m] = m + V_{xy} - \alpha(|x - f| + |y - t|) \tag{1}$$

where where V_{xy} is the utility associated with consuming the ideal variety while α captures the transport cost associated with any deviation between the actually consumed good ft relative to the ideal xy. Consumers are price takers, they considers all four goods' prices p_{ft} and buy the one among the four differentiated goods that minimizes the sum of transport cost and prize.[2] A consumer located at (x, y) will choose to consume the good ft that minimizes the sum of price and transport,

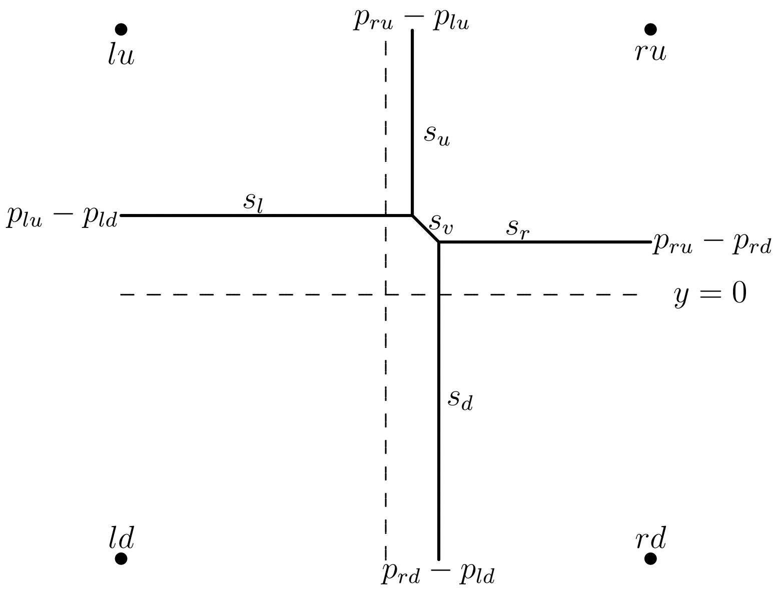
$$ft = \underset{ft}{\text{argmin}}[p_{ft} + |f - x| + |t - y|] \tag{2}$$

The distribution of consumers tastes G is assumed to be symmetric both around $x = 0$ and around $y = 0$. G has domain $[X_l, X_r]. \times [Y_d, Y_u]$ The symmetry requirement[3]

[2]I implicitly assume that V_{xy} is so high that all consumers get positive addition to utility by consuming one unit of the differentiated good. Moreover, as V_{xy} is not fixed across individuals, the formulation does not imply that the consumers with the highest transportation cost has the lowest utility.

[3]Note that there is generally no rotation symmetry (which would have implied $G(x, y) = G(y, x)$).

Figure 1: Catchment area



implies that

$$G(x,y) = G(-x,y) = G(-x,-y) = G(x,-y) \text{ for all } (x,y) \tag{3}$$

$$\text{while} \quad X_l = -X_r \quad Y_d = -Y_u \tag{4}$$

For given prices the minimization procedure by all consumers defines a catchment area for each good ft.[4] The catchment areas are illustrated in Figure 1. Here the solid line defines the borders of the catchment areas, while the dashed lines shows the $x = 0$ and $y = 0$ axes. The borders of the catchment areas consists of five segment. The two segments s_u and s_d are the *external* margins while the two segments s_l and s_r are the *internal* margins. The external margins traces consumers that, given the prices, are indifferent between the two firms but not indifferent between varieties. The internal margins traces consumers that, given the prices, are indifferent between the two varieties but not between firms. The position of the external and the internal margins are given by the price difference between the goods on either side of the border.[5] It follows from the geometry of the problem that, with four different prices,

[4] A similar generalization of transport cost is done in Neven (1986), while similar divison of the market appears in a vertically differentiated context in Barron et al. (2000). Note that the linear transport cost is not restrictive. With the general G, G may also capture nonlinearities in transport costs.

[5] In the particular example in Figure 1, p_{ld} is the lowest, p_{rd} is the second lowest, while p_{ru} is

two regions positioned diagonally vis a vis each other will also share a border, as is the case between ru and ld. This fifth margin is the *diagonal* margin, labeled s_v.

The fractions of consumers within each catchment area is defined as the integral of G over the area.

$$x_{ft} = \left| \int_{y=Y_t}^{p_{fu}-p_{fd}} \int_{x=X_f}^{p_{ft}-p_{(-f)t}} G(x,y)\, \mathrm{d}x\, \mathrm{d}y \right| - \iint_{\bigtriangledown} G(x,y)\, \mathrm{d}x\, \mathrm{d}y \tag{5}$$

where $-f$, by assumption, implies the complement of f. The last term captures the truncation of the catchment area in the case of a diagonal margin, where $\bigtriangledown$ indicates the triangle defined by vertices $y = (p_{fu} - p_{fd})$, $x = (p_{rt} - p_{lt})$, and s_v.

The number of consumers on each margin is defined as the integral of G over the segments.

$$g_i = \int_{s_i} G(x,y)\, \mathrm{d}\ell \quad i = d, u, l, r, v \tag{6}$$

where ℓ is a auxiliary variable used as short hand to indicate the movement along the segments.

The number of consumers in each catchment area x_{ft} determines the sales of each firm/variety while the number of marginal customers on the border of the catchment areas g_i determine the marginal gains and losses associated with changes to the prices, either via cannibalization or via consumers lost to the competing firm. The margins g_i are therefore crucial in the firms profit maximization problem.

2.1 Prices and profits

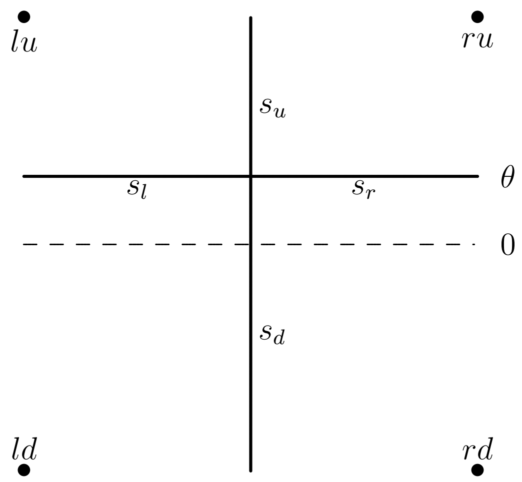
The sales for firm f is x_{fu} and x_{fd}. The corresponding profits π_f are therefore

$$\pi_f = p_{fd} x_{fd} + p_{fu} x_{fu} \tag{7}$$

When setting prices the two firms engage in a strategic interaction with the other firm. Using one-shot noncooperative equilibrium as the solution concept and assum-

the highest.

Figure 2: Symmetric market shares with price discrimination.



ing Bertrand competition, the first order conditions for firm f for optimum is

$$\frac{\partial \pi_f}{\partial p_{fu}} = x_{fu} + p_{fu}\frac{\partial x_{fu}}{\partial p_{fu}} + p_{fd}\frac{\partial x_{fd}}{\partial p_{fd}} = 0 \tag{8}$$

$$\frac{\partial \pi_f}{\partial p_{fd}} = x_{fd} + p_{fu}\frac{\partial x_{fu}}{\partial p_{fd}} + p_{fd}\frac{\partial x_{fd}}{\partial p_{fu}} = 0 \tag{9}$$

In the following I only examine pure strategy symmetric Nash equilibria. That is where the price of each variety is the same across the two firms ($p_{ld} = p_{rd}$ and $p_{lu} = p_{ru}$). In such an equilibrium the external margins will be aligned and the internal margins will be aligned, and there will be no diagonal margin. In an equilibrium with price discrimination $p_{fd} \neq p_{fu}$. The extent of price discrimination is measured by θ

$$\theta \equiv p_{lu} - p_{ld} = p_{ru} - p_{rd}, \tag{10}$$

If $\theta > 0$ the u-variety consumers pay the highest price and are discriminated against. If $\theta < 0$ the d-variety consumers are discriminated against. The price difference θ also determines the catchment areas and the four margins all meet in the point $(0, \theta)$. One example is given in Figure 2. The internal margins are both positioned at the height θ meeting at $x = 0$, while the external margins are positioned at $x = 0$ meeting at height θ. In a symmetric case the margin densities can all be written as functions of θ alone.

$$g_e(y_1, y_2) \equiv \int_{y=y_1}^{y_2} G(0, y)\, \mathrm{d}y \tag{11}$$

$$g_d(\theta) \equiv g_e(Y_d, \theta) \tag{12}$$

$$g_u(\theta) \equiv g_e(\theta, Y_u) \tag{13}$$

$$g_l(\theta) \equiv \int_{x=X_l}^{0} G(x, \theta)\, \mathrm{d}x \tag{14}$$

$$g_r(\theta) \equiv \int_{x=0}^{X_r} G(x, \theta)\, \mathrm{d}x \tag{15}$$

where the function $g_e(y_1, y_2)$ is introduced as a short hand for the density over the external margin for any segment $[y_1, y_2]$. It follows from the symmetry that

$$g_d(\theta) = g_u(-\theta) \text{ and } g_l(\theta) = g_r(\theta) \tag{16}$$

while

$$g_d(\theta) + g_u(\theta) = g_e(Y_d, Y_u) = g_T \tag{17}$$

Here g_T is the total number of consumers on the external margins in a symmetric equilibrium. This number is the integral of the density over the entire vertical axis.

In a symmetric equilibrium also the market shares can be written as functions of θ alone

$$x_{ft} = x_t(\theta) \equiv \left| \int_{y=Y_t}^{\theta} \int_{x=X_f}^{0} G(x, y)\, \mathrm{d}x\, \mathrm{d}y \right| \tag{18}$$

where, again as a result of symmetry

$$x_d(\theta) = x_u(-\theta) = \frac{1}{2} - x_u(\theta) \tag{19}$$

Using the notation above the first order conditions for firm f can be solved with respect to the prices, as functions of the price differential θ to yield

$$p_{fu} = q(\theta, Y_u)\left(1 - \frac{g_f(\theta)\theta}{x_u(\theta)}\right) \tag{20}$$

$$p_{fd} = q(Y_d, \theta)\left(1 + \frac{g_f(\theta)\theta}{x_d(\theta)}\right) \tag{21}$$

$$\text{where} \quad q(y_1, y_2) \equiv \frac{x_d(y_2) - x_d(y_1)}{g_d(y_2) - g_d(y_1)} \quad , \quad y_2 \geq y_1 \tag{22}$$

Here $q(\theta_1, \theta_2)$ is the Hotelling price that would be the equilibrium price in a strictly segmented market occupied by the subset of consumers with $y \in [\theta_1, \theta_2]$ and where firms are restricted to set a single price in that segment. In the following this price is labeled the *segmented price*.

For later use also observe that $q(y_1, y_3)$ can be written as the harmonic mean of the segmented price in two sub segments, one for $y \in [y_1, y_2\rangle$ and one for $y \in [y_2, y_3]$

$$q(y_1, y_3) = \frac{1}{\frac{1}{q(y_1,y_2)}(1-w) + \frac{1}{q(y_2,y_3)}w} \quad \text{where} \quad w = \frac{x_d(y_3) - x_d(y_2)}{x_d(y_3) - x_d(y_1)} \tag{23}$$

the weights, w and $1-w$, are the relative number of consumers in each sub segment.

Consider now the price discrimination equilibrium with $\theta > 0$ then (20) gives the equilibrium price, p_{fu}, as the segmented market price $q(\theta, Y_u)$ adjusted downward due to the incentive to limit cannibalization. That is, compared to the segmented market price the price is lower, so as to prevent the low price variety to cannibalize the high price customers across internal margin g_f. The expression (21) is similar but now with the low segmented price $q(Y_d, \theta)$ being adjusted upwards, again to limit cannibalization. In both conditions the magnitude of the adjustment depends positively on the number of consumers on the internal margin g_f

With the first order conditions, (20) and (21), an immediate candidate for an equilibrium is the one without discrimination where $\theta = 0$ and where all prices are equal $p_{fu} = p_{fd}$. In such an equilibrium the prices satisfy

$$p_{fu} = q(0, Y_u) = p_0 \tag{24}$$

$$p_{fd} = q(Y_d, 0) = p_0 \tag{25}$$

$$\text{where} \quad p_0 = \frac{1}{2g_T} = q(Y_d, Y_u) \tag{26}$$

Thus there always exist a uniform p_0 that satisfies all four first order conditions. The last equality shows that p_0 coincides with the segmented market price derived above when the entire market is treated as a segment. This equality follows formally from (38). It also follows directly from the symmetry of G.

Whether or not there also exist price discrimination solutions and whether or not price discrimination is profitable depends on the actual shape of G. Of particular importance is how the competitive pressure for consumers choosing the high price variety compares to the average consumer. The following proposition address these issues:

Proposition 1. *Price discrimination equilibria are more profitable that uniform pricing equilibria. Price discrimination equilibria exist if and only if the consumers choosing the high price variety represents sufficiently low competitive pressure compared to the average consumer.*

Proof. The second part follows when combining (20) and (21) assuming $\theta > 0$. It then follows that a price discrimination equilibrium has to satisfy

$$\left(1 + \frac{g_f(\theta)}{x_d(\theta)}\theta\right) q(Y_d, \theta) = \left(1 - \frac{g_f(\theta)}{x_u(\theta)}\theta\right) q(\theta, Y_u) - \theta \tag{27}$$

If $q(\theta, Y_u)$ is less than or equal to $q(Y_d, \theta)$ there will be no solution except $\theta = 0$. There will, however, be a $\theta > 0$ that satisfies the equation if $q(\theta, Y_u)$ is sufficiently much larger than $q(Y_d, \theta)$. The argument is as follows: Consider a G^0 where $q(\theta, Y_u) = q(Y_d, \theta)$ for all θ. Fix a θ strictly positive that is sufficiently small so that the right hand side of (27) is positive. Then consider a different distributions G. It is always possible to find a G with a higher but finite $q(\theta, Y_u)$ and a lower but positive $q(Y_d, \theta)$, that otherwise shares all the characteristic properties of G^0

appearing in (27) and that satisfies the equation (27).

The first part follows from the profits in the uniform pricing equilibrium

$$\pi_0 = [x_{fu}(0) + x_{fd}(0)]\, p_0 = \frac{1}{4g_T} \tag{28}$$

Adding the two first order conditions (8) and (9) and using the definition of θ and g_T and utilizing $x_d(\theta) + x_d(\theta) = 1/2$, yields

$$1/2 = g_u(\theta)\theta + p_{fd}g_T \tag{29}$$

A similar rewriting can be done to the profit relationship (7)

$$\pi = x_u(\theta)\theta_f + p_{fd}/2 \tag{30}$$

Combining these two, eliminating p_{fd} and combining with (28), yields

$$\pi = \pi_0 + \theta g_u(\theta)\left(q(\theta, Y_u) - p_0\right) \tag{31}$$

which proves the first part of the proposition. □

The proposition is the main insight of the article, if the competitive pressure across the the two firms is sufficiently low for consumers with strong variety preference price discrimination equilibria are possible. Moreover, in such an equilibrium profits are higher that in a uniform pricing equilibrium. Therefore in a situation where a price discrimination exist, both firms are best of coordinating on price discrimination.

3 Two illustrations

3.1 Discrete types in the variety preference dimension

The first illustration is based on a somewhat degenerate case. The variety preference, the y variable, is assumed to be discretely distributed and takes three distinct values, each with density 1/3.[6] Conditional on y, the firm preference x variable, is assumed to be censored normally distributed.[7] The three types with regards variety preferences are: the d-type prefers variety d with $y = -1$ and who has standard deviation σ_x^d in the x-direction, the u-type prefers variety u has $y = 1$ and has standard deviation σ_x^u in the x-direction. In between is the b-type that prefers no variety over the other thus has $y = 0$ and has standard deviation σ_x^b in the x-direction. Here symmetry requires that $\sigma_x^d = \sigma_x^u$.

If the firms where competing over each of these groups separately it follows from the definition of the normal distribution that in such a solution[8]

$$p^j = \sqrt{\frac{\pi}{2}}\sigma_x^j, \quad j = u, d, b \tag{32}$$

Thus in the case of a normal, there is a linear relationship between the standard deviation in the x-direction and a uniform equilibrium price. The reason is that an increase in the standard deviation lowers the semi-elasticity of demand one to one. Now if the b-type, being exactly indifferent between variety, tends to be indifferent also when it comes to firm preferences it follows that $\sigma_x^b < \sigma_x^u = \sigma_x^d$ and therefore that $p^b < p^u = p^d$. Moreover, given that the three distinct types has size 1/3 each, the segmented price as defined above will be

$$q(y_1, y_2) = \begin{cases} p^d, & \text{when } y_1 < -1 \text{ and } -1 < y_2 < 0 \\ \frac{1}{\frac{1}{2p^d} + \frac{1}{2p^b}}, & \text{when } y_1 < -1 \text{ and } 0 < y_2 < 1 \\ \frac{1}{\frac{2}{3p^d} + \frac{1}{3p^b}}, & \text{when } y_1 < -1 \text{ and } 1 < y_2 \end{cases} \tag{33}$$

where it is used that $p^d = p^u$ and where the segmented price for other $[y_1, y_2]$ intervals follows from the symmetry properties of $q(y_1, y_2)$.

When the unconditional distribution of y is made up of the three discrete points $\{-1, 0, 1\}$ the internal margin g_f will, when it is defined, be zero.[9] Inserting in (27)

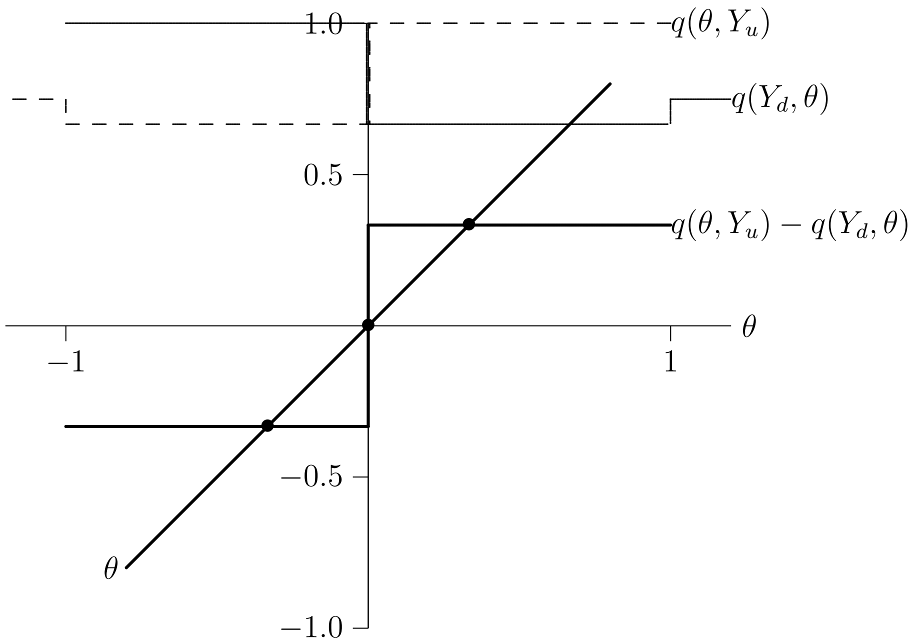
[6] Such a division into distinct types has been used in the vertical differentiation context. For instance by Katz (1984), Stole (1995) and Dai et al (2014).

[7] Censored to satisfy the assumption about limited domain.

[8] Here it is assumed that the distribution of x is censored so that x has domain sufficiently limited so as to assure the existence of a symmetric solution.

[9] The cannibalization concern reflected in the first order condition will take the form of inequality

Figure 3: Equilibria with three types of variety preference



yields

$$\theta = q(\theta, Y_u) - q(Y_d, \theta) \tag{34}$$

The solution of (34) for the case where $p^u = p^d = 1, \quad p^b = 1/2$ is illustrated in Figure 3. The figure shows that there indeed are three equilibria. One with uniform pricing, $\theta = 0$ and two with price discrimination. The price discrimination equilibria are mirror images of each other with $\theta = -1/3$ and $1/3$ respectively. In the uniform equilibrium the price is $q(Y_d, Y_u) = 3/4$. In the price discrimination equilibium with $\theta = 1/3$, where the u-type is discriminated against the low price is $q(Y_d, \theta) = 2/3$ while the high price is $q(\theta, Y_u) = 1$. When $\theta = -1/3$ these prices are shifted. Moreover, the arithmetic average of prices is larger in discrimination equilibrium compared to the uniform equilibrium, 3/4 vs 7/9.[10] Therefore profits are larger in the discrimination equilibrium. In summary: Compared to the uniform pricing equilibrium, price discrimination yields higher profits. It also implies lower prices for the majority that is pooled with the b-type. The high price is paid by the

constraints at each of the values $\{-1, 0, 1\}$. That is incentive compatibility constraints that has to be satisfied in order for different consumer types to actually choose differently priced goods.

[10]The calculation is as follows: $7/9 = 1/3 \times 1 + 2/3 \times 1/3$.

type (d-type or u-type) that is not pooled with the b-type.

3.2 A mix of two normally distributed populations.

Without resorting to a discrete distribution a convenient way to have parametric control over the distribution G is to consider a mix of densities. Consider the case where G^A is a density made up of a population of A-consumers who are normally distributed[11] centered over the origin, with standard deviation σ_x^A in the x-direction and σ_y^A in the y-direction.As in the previous example, it follows the from the definition of the normal distribution that the equilibrium uniform price[12] for population A is

$$p^A = \sqrt{\frac{\pi}{2}}\sigma_x^A \tag{35}$$

Consider now another population, the B-consumers, with G^B normally distributed with $\sigma_x^B < \sigma_x^A$ and with $\sigma_y^B < \sigma_y^A$. The equilibrium uniform price for population B is

$$p^B = \sqrt{\frac{\pi}{2}}\sigma_x^B < p^A \tag{36}$$

If now these populations were mixed in equal proportions the resulting expression for the uniform price in the mixed distribution, p_0^{AB}, would be

$$p_0^{AB} = \frac{\frac{1}{2}}{\frac{1}{2}\frac{1}{\sigma_x^A\sqrt{2\pi}} + \frac{1}{2}\frac{1}{\sigma_x^B\sqrt{2\pi}}} = \frac{1}{\frac{1}{2}\frac{1}{p_0^A} + \frac{1}{2}\frac{1}{p_0^B}} \tag{37}$$

Here the fist equality follows from the definition of the pdf of the normal distribution. The second equality follows directly from the definitions of p_0^A and p_0^B. Again the harmonic mean yields the price when mixing two populations of consumers. This

[11] Also here censored.
[12] in fact the only equilibrium price.

result echoes the result in (38) and a generalization of (37) yields

$$q^{AB}(y_1, y_2) = \frac{1}{\frac{w_A}{w_A+w_B}\frac{1}{p^A} + \frac{w_B}{w_A+w_B}\frac{1}{p^B}} \tag{38}$$

$$\text{where} \quad w_J = \int_{x=X_l}^{0} \int_{y=y_1}^{y_2} G^J(x,y)dxdy, \quad J = A, B \tag{39}$$

The weights, w_A and w_B, reflects the fraction of population members within each sub segment. Thus, the segmented price within an interval (y_1, y_2) is the harmonic mean of the segmented price of each population weighted by the relative number of population members within the interval. These weights will be determined by the σ_Y-s. The distribution with the highest σ_Y will be most spread out and will have the smallest weight when y is close to zero and the highest weight when y is large in absolute value.Hence as $\sigma_y^B < \sigma_y^A$ then when calculating $q^{AB}(Y_l, \theta)$ and $q^{AB}(\theta, Y_u)$ population A will have a larger weight when the distance between $Y_t, \quad t = u, d$ and θ is small.

An equilibrium has to satisfy (27) which can be rewritten as

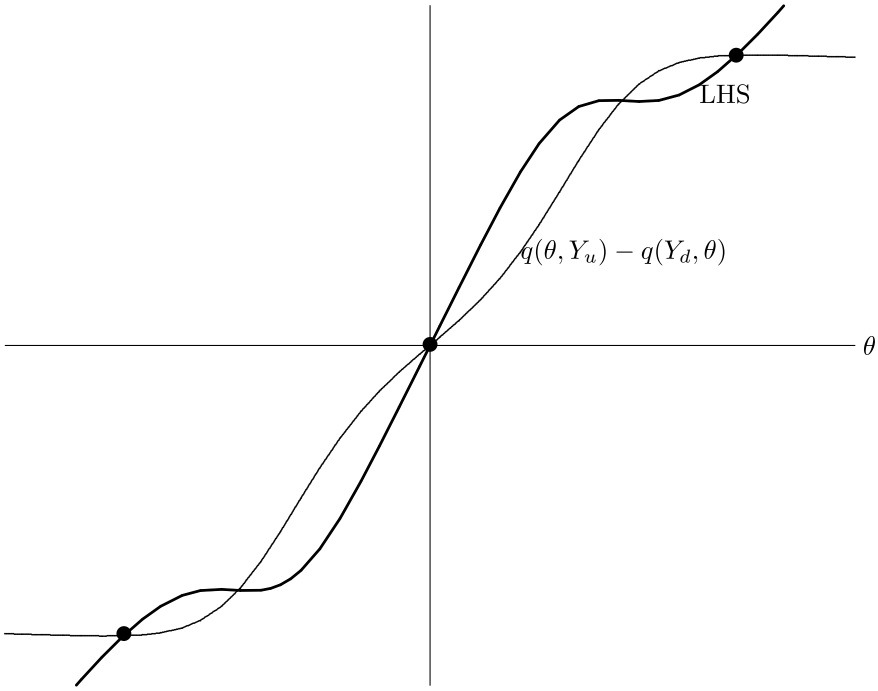
$$\theta\left(1 + \frac{g_f(\theta)}{g_u(\theta)} + \frac{g_f(\theta)}{g_d(\theta)}\right) = q(\theta, Y_u) - q(Y_d, \theta) \tag{40}$$

where compared to (34) the left hand side contains the cannibalization concern reflected by g_f. One parametrized example is given in Figure 4. As in the the previous example $q(\theta, Y_u) - q(Y_d, \theta)$ displays a stepwise shape. The stepwise shape follows from the shifting weights of the two populations as given in (38). Compared to Figure 3 it is more smooth resulting from a G-distribution that is continuous in both dimensions. Also the left hand side (LHS) shows a more involved pattern, most importantly due g_f being non zero. However, the general picture displays the same result as in the first illustration. A stepwise curve intersecting with a upward sloping curve leading to the existence of price discrimination equilibria. [13]

Again, the figure confirms the insights of the Proposition 1. Introducing a population of consumers who are generally less loyal both to firm *and* to variety (popula-

[13] Detailed inspection around the equilibrium points reveals that the intersections marked by bullets satisfy the requirement for optimal pricing for each firm conditioned on the pricing of the other. The non marked intersections are saddle points.

Figure 4: Equilibria with a mix of two normal



tion B) will increase the competitive pressure. The downward pressure on prices is dampened, however, in a price discrimination equilibrium. In such an equilibrium, consumers who are largely indifferent both in the variety and firm dimension (with B consumers overrepresented) are attracted to the low price variety. As the consumers still preferring the high price (with A consumers overrepresented) on average also are more firm loyal the high price can be sustained also in a competitive duopoly environment.

4 Conclusion

Using a model of horizontal differentiation where a variety dimension is added to Hotelling's (1929) "linear city", I show that even when costs and demand are symmetric, price discrimination may be an equilibrium phenomenon.

The model was presented in a general firm/variety plane. The results may be translated into a several concrete settings. Car models: two door hard top versus four doors sedan. Soft drink: diet versus sugar. In the introduction sales was one motivation. In a translation making the model relevant for sales, say of a hand bag, it has to be assumed that time is finite consisting of two periods {summer, winter}[14], and that the firms {Hermes, Chanel} in a one-shot game set prices for their bags for both periods. Then the model shows that a price discrimination equilibrium may arise where both firms set a lower prise in one of the periods. By doing that all consumers who are indifferent between summer and winter will chose the low price period. If these consumers are also the consumers who are indifferent between firms, then the consumers left in the high price period will largely be firm loyal consumers who represent a modest competitive pressure. Hence, the high price in this period can be sustained.

In the case of the gasoline market the logic can be formulated as follows (still strictly speaking in a two-day one shot pricing environment). Gasoline customers who do not follow the gauge may have limited choice of where {Chevron, Mobil} and when {Monday, Tuesday}to fill their tank. The competitive pressure represented by these customers is therefore moderate. The competitive pressure increase, however, if there also are some customers, who follow the gauge and fill their tank at the cheapest place at the cheapest day. In such an environment the model shows that the stations may without any collusion coordinate on an equilibrium with either Monday or Tuesday as the cheap day. With Tuesday as the cheap day all the price sensitive buyers will be served on Tuesday. As a result the competitive pressure is less on Monday and prices can be higher.

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[14]The varieties are "new bag for the summer" and "new bag for the winter"

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