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Unequal power and the dynamics of rivalry

Halvor Mehlum and Kalle Moene*

Abstract

By incorporating positional dynamics into a conflict model relevant to battlefields and politics, we show that the conditions that induce regime stability can also induce hard conflicts. We show that in contests with incumbent-challenger turnover, i) asymmetric power across groups and positions may magnify conflicts; ii) more severe conflicts can occur with lower turnover of incumbents; iii) power can be self-defeating, as cost advantages can reduce payoffs; and iv) double inequality across positions and groups can maximize the graveness of conflicts and the social waste of resources. The propositions in our paper are contrary to the standard implications of static conflict models.

JEL Codes: C73, D72, D74

Keywords: Contests, political stability, incumbency advantage, conflict and civil war.

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1 Introduction

Although the number of violent conflicts has declined since the end of the Cold War, the remaining conflicts seem, on average, to have become more intense. The number of battle deaths peaked in 1999 at 130,000 deaths, which were largely caused by wars in the Democratic Republic of Congo, Ethiopia and Eritrea, Angola, Chechnya, Sierra Leone and Afghanistan (Cooper et al. 2011). One reason may be that political movements, previously supported by either the East or the West, were left to fend for themselves. Control over domestic resources and territory, and control over the state apparatus became even more important. Hence, incumbency became more critical. A stronger incumbency advantage clearly increased the inequality of power between groups and simultaneously gave groups more to fight for.

Our paper explores how inequality that raises the stakes may intensify conflicts. If our theory of dynamic conflicts is correct, it is antagonism between groups of unequal power rather than between groups of equal power that lead to the most intense conflicts. This contrasts with the standard textbook model that portrays conflicts as a static rent-seeking contest. In such models, a level playing field among the contestants generates the worst confrontations. By exploring a dynamic conflict model with incumbency advantages, this paper argues, in contrast, that unequal power generates the most intense confrontations. We emphasize three aspects.

First, real conflicts are rarely one-shot confrontations. Conflicts typically last several periods, with one group or another having the upper hand. The shares of ceasefires and war terminations that restart as violent conflicts within five years are increasing. They have exceed 40 percent in all periods since the Cold War, and the level has never been higher since World War II (Cooper et al. 2011).

Second, control of the state apparatus and territories alternates. During lasting conflicts, fighting is often terminated and restarted when power changes hands. In the Nicaraguan civil war, for instance, the revolutionary Sandinista National Liberation Front first fought against the Somoza dictatorship in the

1960s and 1970s. After the overthrow of the Somoza regime in 1979, the old Somoza defenders became the challengers as the Contras fighting the Sandinista regime. Generally, after winning an important battle, the challenger may assume power over the main city or even the state apparatus and thus relegate the former incumbents in these positions to challenger status in the continuing fight.

Third, victory in one battle can give the winner an advantageous position in succeeding battles. The strength of this advantage, however, can vary across groups. There are, therefore, not only insider-outsider differences but also differences across potential insiders in their ability to utilize the insider's control maintained by their networks, their ideological orientations, their military connections, and their religious sympathies and antipathies. In Chile, for instance, both Allende and Pinochet held incumbency positions, but only one enjoyed the support of the army. Hence, groups can take advantage of incumbency to varying degrees.

Turkey's recent history is a clear example of lasting conflict with asymmetric power, as Acemoglu (2014) and Meyersson and Rodrik (2014) discussed in a recent exchange in Foreign Affairs. Since its modern foundation in 1923, Turkey has experienced a lasting power struggle between religious traditionalist movements on one side and Atatürk's secular military followers, the Kemalists, on the other. Until recently, the Kemalists have prevailed, partly by resorting to military force to contain the efforts of religious groups to take power. Since 2000, however, the loyalty of the army to the Kemalists have been watered down, and the religious party, the AKP led by Erdoğan, has been allowed to dig in and become the incumbent.[1] Hence, in light of our mechanisms, Turkey has gone through two phases. In phase one, until 2000, the Kemalists had the advantage of loyalty from the army. In phase two, this advantage was gone, and Erdoğan's religious regime enjoy a strong incumbency advantage, not least due to its disrespect of constitutional and human rights.

Below, we explore such long-lasting conflicts with alternating power. Our

[1] The coup attempt in June 2016 was apparently not staged by the Kemalists but by the religious Gülen movement.

main interest is to work out the implications of asymmetric incumbent and challenger power. Thus, we set up a formal model of contests wherein two groups fight over both positions and rents. In any specific period, one of the groups is the incumbent; the other, the challenger. In each fight, the winner becomes next period's incumbent, and the loser becomes next period's challenger.

Our set-up with repeated fighting resembles a *feud*. Yet, we err on the safe side by not focusing on revenge motives. We demonstrate that even when the contestants are forward looking and motivated by material payoffs only, inequality of power is likely to magnify fighting. In any confrontation between the incumbent and the challenger, the existing strengths and actual fighting incentives are decisive no matter how positions have been distributed in earlier periods. Hence, we solve for the Markov perfect Nash equilibrium of this conflict game. We allow for unequal power across groups and for endogenous turnover. Thus, the stakes are endogenously determined. In each period, the prize that a group fights for is the immediate rents plus the value of advantageous positions in future fights. We derive comparative statics of this equilibrium.

The model allows us to address some basic questions where standard static conflict models may be misleading. Can more unequal power stimulate fighting and waste resources? Our model answers "yes", the standard static model answers "no." We offer a more thorough discussion of the effects of asymmetric power in the costs of force and influence across positions and groups. We can, for example, show that social waste of resources can increase as turnover of incumbents declines. By asymmetric power, we refer to power differences that arise whenever one of the contestants enjoys an incumbency advantage that cannot fully be taken advantage of by an opponent who wins. We can then investigate the effects of different types of asymmetry, such as the effects of conflict constellations with a strong ruler facing an even stronger would-be ruler. Do this double inequality of power lead to particularly hard fighting?

Addressing such questions sheds light on some real-life rivalries characterized by unequal power. During Apartheid in South Africa, for instance, inequality of power was considerable. Our approach help us understand why the fighting became so intense and lasted for so long.

In exploring how fighting today may be affected by the power that future incumbents may hold, we extend the literature on conflicts as rent-seeking contests: This literature dates to Trygve Haavelmo (1954), Gordon Tullock (1980), Jack Hirschleifer (1991), Herschel Grossman (1994), Stergios Skaperdas (1992), Kai Konrad and Stergios Skaperdas (1998), Derek Clark and Christian Riis (1998), among others.[2] We are also inspired by Daron Acemoglu and Jim Robinson's work (2001 and 2006) on political transition and elites.

The four papers closest to ours are the contributions by Joan Esteban and Debraj Ray (1999), William Rogerson (1982), Stergios Skaperdas and Constantinos Syropoulos (1996), and Mattias Polborn (1996). Esteban and Ray construct a general model of multi-group conflicts with heterogenous prizes without dynamics. Skaperdas and Syropoulos consider the problem of achieving cooperation when one group's early victory improves the group's position in subsequent periods. Polborn emphasizes how a status quo bias favoring the survival of the incumbent increases the challenger's effort to replace him. Our main contribution to this literature is combining positional dynamics with power asymmetries between groups over time. Rogerson's somewhat overlooked contribution is dynamic. He focuses on insiders and outsiders in a symmetric lobbying game with the same prize and where the winner becomes the insider.[3] To explore power inequality, we need go beyond Rogerson by focusing on asymmetric prizes and costs. We also incorporate a more general contest success function.

The next section sets up and solves our model. Section 3 derives the main propositions of the impact of asymmetric power, while section 4 concludes with some speculations in the light of these results.

[2]For a survey of models of static rent-seeking contests, see the article by Shmuel Nitzan (1994) and the monograph by Kai Konrad (2009).

[3]Johannes Hörner (2004) considers another dynamic aspect in his model of two identical firms about racing to become the technological leader with the highest income and lowest cost of further technological improvements. In his model, the leader may be one, two, or more steps ahead.

2 Contests over rents and power

To set the stage, consider first the well known children's game.

2.1 The King of the Hill

Two children, the king and the challenger, fight each other in several rounds. The current king stands on the top of a mound, "the hill", and the challenger stands below the hill. The child on top presumably enjoys a continuous flow of gratification from being the king, which makes the position attractive.

One child's chance of occupying the hill and becoming the king in the subsequent round is determined by his force relative to that of the other child. Assuming that the children, on a level playing field, are equally strong, the position on the hill is a cost advantage, as the king's fighting effort yields a stronger force. If both exert the same effort, the king remains the king with a probability of greater than one half. The higher the hill, the larger the cost advantage. Normalize the unit cost of effort for the challenger to unity, and denote the unit cost of effort for the king as $C_i < 1$. The higher the hill, the closer C_i is to zero.

Each child wants to maximize his payoff, weighting the expected benefits of increasing the probability of winning against the cost of effort. The first problem is deriving the solution for this strategic interaction. The second problem is deriving the comparative statics of a change in the cost advantage C.

The general insight is that strengthening the incumbency advantage by reducing C has two effects. First, it makes becoming king more difficult, which is a *conflict-dampening* cost effect. Second, it makes the prize associated with becoming king more valuable, which is a *conflict-enhancing* prize effect. In Mehlum and Moene (2006), we analyzed a king of the hill game with symmetric players and a Tullock-type contest success function. We show that the social waste of fighting, measured by the opportunity costs, displays a hump-shaped relationship with the cost advantage. When the incumbency advantage is moderate, the conflict-enhancing prize effect dominates, while when incumbency advantage is strong, the conflict-dampening cost effect dominates. Starting on a level playing field, increasing the height of the hill first intensifies the fight, as becoming king becomes more important due to the possibility of winning subsequent battles. When the hill is very high, however, the conflict dwindles, as the challenger essentially gives up.[4]

Things are not so simple, however, when the two alternating kings differ in their ability to benefit from the incumbency advantage—that is, when power prospects are asymmetric.

2.2 The general case of asymmetric power

There are two groups, denoted a and b, and two states of nature, each characterized by the identity of the incumbent. In the first state, group a is the incumbent, and group b is the challenger. In the other state, group b is the incumbent, and group a the challenger. The incumbent enjoys a cost advantage in fighting, but it may be stronger for one than for the other.—

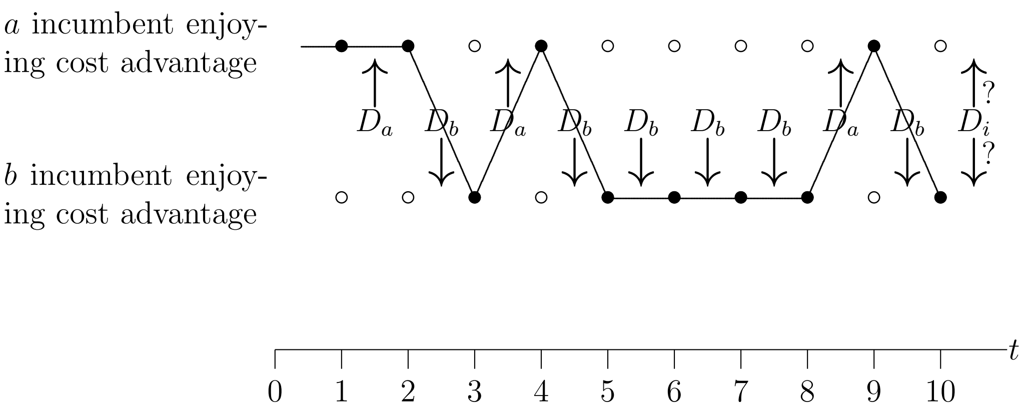
The timing of events in each period t is as follows:

1. Group a and group b, one of which is the incumbent, meet in a simultaneous move contest over who is going to become the incumbent in the next period.
2. The winner of the contest becomes the incumbent and collects the immediate rents of winning, the excess value of implementing the group's favored policy over that of the opponent.
3. The game moves to period $t+1$ (back to stage 1), with the incumbent enjoying a cost advantage.

To define immediate rents of winning D_j, we introduce the convention, used throughout, that capital letters refer to the incumbent position, whereas lower case letter refer to the challenger position. Accordingly, U_a reflects group a's

[4] Mehlum and Moene (2006) show that the incumbency advantage that maximizes social waste is $C_i = \sqrt{(1-\delta)/(1+\delta)} < 1$, where δ is the discount factor. In Figure 4 of this paper, the cost configuration corresponds to point E.

Figure 1: Example time path



evaluation of group a's own incumbency utility, and u_a reflects group a's evaluation of its challenger utility. The immediate rents D_a is the difference between the two; similarly D_b is group b's (ruler) rent[5]

$$D_j = U_j - u_j \quad \text{for} \quad j = a, b \tag{1}$$

One example time path is illustrated in Figure 1, where group a starts out as the incumbent. In period 1, it wins again, it collects D_a and enters period 2 as the incumbent. In period 2, group a loses, and group b collects rents D_b. Consequently, group b enters period 3 as the incumbent, and so on, until period 10, where the illustration stops. Hence, at the start of period $t = 10$, it is not yet clear who will be the winner in period 10 and subsequently enter period 11 as the incumbent.

Turnover and winning probabilities

The probability of winning, the contest success function, depends positively on own effort and negatively on the opponent's effort. Let Y_j be the effort of the incumbent and y_j the effort of the challenger, while C_j and c_j are their unit costs

[5] In a multi-group context, Esteban and Ray (1999) use utility differences like these as an indication of inter-group distance. The value $[U_a - u_a]$ measures the distance from group a to group b, and the value of $[U_b - u_b]$ the distance from group b to group a. The larger these differences, the more antagonism there is between groups and the more polarized the preferences.

of force. The incumbency advantages means that $C_j \leq c_j$. Note that we have made no assumptions regarding the costs of group a relative to those of group b.

The relative force of the incumbent, S_i is a function of effort in the following way

$$S_a = \frac{Y_a/C_a}{Y_a/C_a + y_b/c_b} \tag{2}$$

$$S_b = \frac{Y_b/C_b}{Y_b/C_b + y_a/c_a} \tag{3}$$

while the relative force of the challenger s_j is

$$s_b = 1 - S_a \quad s_a = 1 - S_b \tag{4}$$

The incumbency advantage is represented by more relative force for a given effort. The full expressions of relative force should be written with a subscript for time t, i.e., $S_{a,t}$ and $S_{b,t}$. Whenever there is no chance of misunderstanding, however, we suppress the subscript.

In every period we have

$$S_a + s_b = 1 \text{ and } s_a + S_b = 1 \tag{5}$$

We assume that winning the contest requires that the relative force is larger than a threshold. Analogously to probabilistic voting models, we consider this threshold to be uncertain. The probability that group j wins when it is the incumbent is denoted $\Psi(S_j)$ and is thus dependent only on relative force. In other words, winning chances are homogeneous of degree zero in force. We make three additional assumptions:

i) Force pays: $\Psi'(S_j) > 0$

ii) No force implies a sure loss: $\Psi(0) = 0$

iii) Symmetry: For all S_j, we require that $\Psi(S_j) = 1 - \Psi(1 - S_j)$

iv) Monotonicity: $L(S) \equiv S(1-S)\Psi'(S)$ has an unique maximum for $S = 1/2$

$\Psi(S)$ is a generalization of the standard Tullock contest success function.[6] We use Ψ to demonstrate that our non-standard results hold in a wider class of cases. To be confident that the generalized Ψ does not 'cause' the deviations from the standard results, we impose the mild monotonicity requirement that can be most conveniently stated as in iv).[7]

Assumption iii) above implies $\Psi''(S) = 0$, and hence, $L'(1/2) = 0$. Thus, iv) implies that $S = 1/2$ is in fact the global maximum of L. Assumption iv) can be interpreted in the light of a static conflict model. In the equilibrium of the static model, the share of the prize that is wasted by allocating resources to conflict is equal to $2S(1-S)\Psi'(S)$. Moreover, when both contestants have the same costs, $S = 1/2$ in equilibrium. Hence, applying the contest success function Ψ in static models implies that a level playing field leads to maximal waste of resources. As we shall see below, this is no longer true in the dynamic case.

Payoffs

Being the incumbent in the dynamic case is like holding an asset that generates excess returns. The asset value is the expected present value of the payoff to group $j = a, b$, denoted $V_{j,t}$ when j starts period t as the incumbent and $v_{j,t}$ when it starts period t as the challenger. All payoffs are expressed in real monetary terms, and each group essentially cares about the present value of its material consumption.

We denote the prize obtained from winning $F_{j,t}$, which is made up of the immediate gain D_j of being the ruler plus the valuation of starting out with

[6] Using $\Psi(S_j) = S_j^\gamma / [(1-S_j)^\gamma + S_j^\gamma]$, the probability of winning can be written as

$$\frac{(Y_a/C_a)^\gamma}{(Y_a/C_a)^\gamma + (y_b/c_b)^\gamma}$$

which is the Tullock function (where in most applications $\gamma = 1$, implying $\Psi(S_i) = S_i$). See Skaperdas (1996) for a structured discussion of contest success functions in the n-player case. Our Ψ function satisfies his axioms 1, 2, 3, and 6.

[7] The condition is sufficient for our results but not necessary.

an incumbency advantage in the next period $\delta\left(V_{a,t+1} - v_{a,t+1}\right)$, with δ as the discount factor

$$F_{j,t} = D_j + \delta\left(V_{j,t+1} - v_{j,t+1}\right) \quad \text{for} \quad j = a, b \tag{6}$$

Being in power allows the ruling group to implement its optimal behavior. As long as there is not possible to commit to another behavior, the values of D_a and D_b in (1) can be treated as given.

Now, the expected present value of the payoff to a group consists of i) the discounted value of what the group can guarantee itself by starting the next period as the challenger minus ii) the cost of fighting in the present period plus iii) the prize obtained by winning the battle in the present period multiplied by the probability of winning this battle.

Thus, when group a is the incumbent, the present values $V_{a,t}$ for incumbent a and $v_{b,t}$ for challenger b, can be written as

$$V_{a,t} = \delta v_{a,t+1} - Y_{a,t} + \Psi\left(S_{a,t}\right) F_{a,t} \tag{7}$$

$$v_{b,t} = \delta v_{b,t+1} - y_{b,t} + \left[1 - \Psi\left(S_{a,t}\right)\right] F_{b,t} \tag{8}$$

Symmetrically, when b is the incumbent we have

$$V_{b,t} = \delta v_{b,t+1} - Y_{b,t} + \Psi\left(S_{b,t}\right) F_{b,t} \tag{9}$$

$$v_{a,t} = \delta v_{a,t+1} - y_{a,t} + \left[1 - \Psi\left(S_{b,t}\right)\right] F_{a,t} \tag{10}$$

These equations implicitly define the intertwined interests of the two sides.

2.3 The Equilibrium

We solve for the stationary Markov perfect Nash equilibrium of the model. Along the path, each contestant is forward looking and optimizes its fighting effort in each power constellation. Groups a and b make their choices simultaneously.

With group a as incumbent, it follows from (7) and (8) and from the definition of S_i that the first order conditions for the selected effort, Y_a and y_b, in each period can be expressed as[8]

$$S_a (1 - S_a) F_a \Psi' (S_a) = Y_a \tag{11}$$

$$S_a (1 - S_a) F_b \Psi' (S_a) = y_b \tag{12}$$

Similarly, with b as the incumbent, it follows from (9) and (10) that the first order conditions for each period t with b in power are

$$S_b (1 - S_b) F_b \Psi' (S_b) = Y_b \tag{13}$$

$$S_b (1 - S_b) F_a \Psi' (S_b) = y_a \tag{14}$$

Our strategy for solving the entire model is first to demonstrate that in the solution, all variables are functions of the relative prize F_a/F_b. Once demonstrated, we can show that a solution exists for the relative prize itself and then for all variables of interest.

First, observe that from the first order conditions, we easily obtain $Y_a/y_b = F_a/F_b$ and $Y_b/y_a = F_b/F_a$. Using (2) and (3), it follows that in the Nash equilibrium when a is the incumbent and both (11) and (12) are satisfied, relative force is given by

$$S_a = \frac{F_a/F_b}{F_a/F_b + C_a/c_b} \tag{15}$$

When group b is the incumbent, it follows from (13) and (14) that the relative

[8]The second order conditions are

$$\frac{\partial^2 V_a}{\partial S_a^2} = \frac{F_a S_a^2}{Y_a^2} \Big(\Psi'' (S_a) (1 - S_a)^2 - 2\Psi' (S_a) (1 - S_a) \Big) < 0$$

which implies that

$$\frac{\Psi'' (S_a)}{\Psi' (S_a)} < \frac{2}{1 - S_a}$$

The second order condition for group b is found in an equivalent way

$$-\frac{2}{S_a} < \frac{\Psi'' (S_a)}{\Psi' (S_a)} < \frac{2}{1 - S_a}$$

It is straight forward to check that the monotonicity requirement on $\Psi(\cdot)$ implies that these inequalities hold.

force is

$$S_b = \frac{c_a/C_b}{c_a/C_b + F_a/F_b} \tag{16}$$

Thus, the relative prizes of the two groups F_a/F_b, together with cost of the incumbent relative to that of the challenger, determine the equilibrium force of the two contestants. A contestant with a high stake, a low cost, or both has a high equilibrium relative force.

Next, based on the four first order conditions, we can derive simple expressions of the *prize retention ratios*. The prize retention ratios are the expected returns of a position, over and above the discounted payoff as the challenger relative to the prize F. The retention ratio of group i as incumbent is H_i; for the challenger, h_j.

$$H_i \equiv \frac{V_i - \delta v_i}{F_i} \tag{17}$$

$$h_j \equiv \frac{v_j - \delta v_j}{F_j} \tag{18}$$

These prize retention ratios depend only on the relative prize F_a/F_b. Consider group a. In a given period, the prize it fights for is F_a. How much of this price does a expect to retain? If group a starts as the incumbent, it expects to retain $\Psi(S_a)F_a - Y_a$. If it starts as the challenger, it expects to retain $(1-\Psi(S_b))F_a - y_a$. Dividing by F_a and using (11) and (14), we obtain the fraction of the prize retained H_a as the incumbent and h_a as the challenger. Direct inspection shows that both ratios are determined by the same function $H(\cdot)$ given by

$$H(S) = \Psi(S) - S(1-S)\Psi'(S) \tag{19}$$

with $H'(S) > 0$, $H(0) = 0$ and $H(1) = 1$. One example is provided in Figure 2. For group a, we have

$$H_a = H(S_a), \text{ and } h_a = H(1 - S_b) \tag{20}$$

Similarly, for group b,

$$H_b = H(S_b), \text{ and } h_b = H(1 - S_a) \tag{21}$$

We can use these retention ratios in prize expressions F_a and F_b. Consider group a. A measure of the valuation of incumbency, over and above the immediate rents, is $\delta(H_a - h_a)F_a$, which captures the higher net return associated with a change from being challenger to being incumbent. When adding the immediate rent D_a, we obtain the total prize from winning F_a

$$F_a = D_a + \delta(H_a - h_a)F_a \tag{22}$$

Solving for F_a, and similarly for F_b, we obtain

$$F_j = \frac{D_j}{1 - \delta(H_j - h_j)} \quad \text{for} \quad j = a, b \tag{23}$$

One way to interpret $\delta(H_j - h_j)$ in (23) is as a representation of group j's actual discounting of future rents. To illustrate why $(H_j - h_j > 0)$ enters the discounting, consider the special case where both groups a and b are close to all-powerful as the incumbent and close to powerless as the challenger. Then, in the limit, $H_j = 1$, $h_j = 0$ and $F_j = D_j/(1 - \delta)$. Thus, when victory implies winning the prize forever while defeat implies losing the prize forever, group j evaluates the prize of victory as the ordinary discounted sum of the rent D_j in all future periods.

In another case, none of the groups have an incumbency advantage such that $C_a = c_a$ and $C_b = c_b$. In that case, it follows from (15) and (16) that $S_a = 1 - S_b$, and as a result, $(H_j - h_j) = 0$. In that case, (23) shows that the prize the groups fight for is simply the per period flow utility.

After finding the relative prize F_a/F_b, as we do below, we obtain consistent closed form solutions for the other variables, using (15) and (16), in addition to (11, (12), (13), (14), and (23), all variables only depend on F_a/F_b:

$$V_j = \frac{D_j}{1-\delta(H_j-h_j)} \frac{(1-\delta)H_j+\delta h_j}{1-\delta} \tag{24}$$

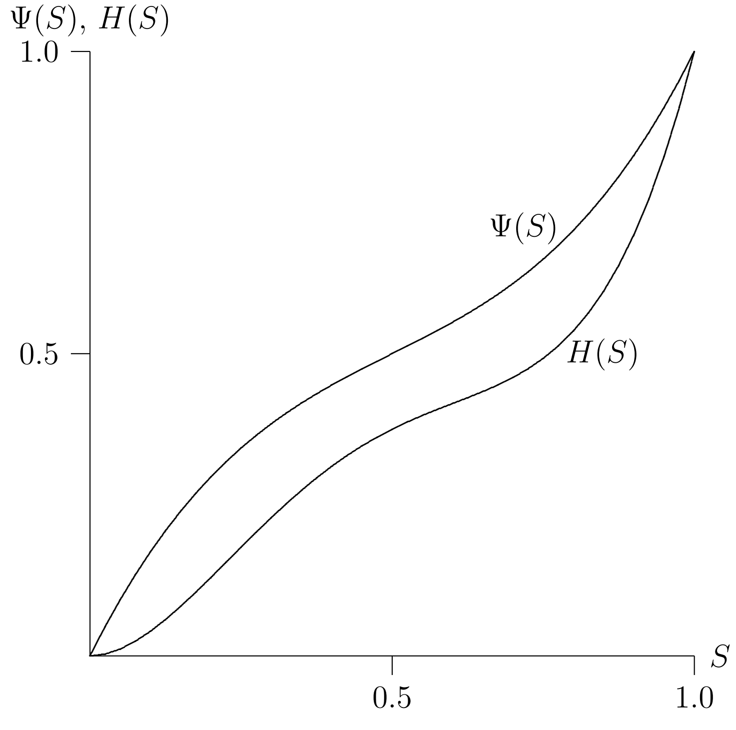
$$v_j = \frac{D_j}{1-\delta(H_j-h_j)} \frac{h_j}{1-\delta} \tag{25}$$

$$Y_j = \frac{D_a}{1-\delta(H_j-h_j)} (\Psi(S_j)-H_j) \tag{26}$$

$$y_a = \frac{D_a}{1-\delta(H_a-h_a)} (1-\Psi(S_b)-h_a) \tag{27}$$

$$y_b = \frac{D_b}{1-\delta(H_b-h_b)} (1-\Psi(S_a)-h_b) \tag{28}$$

Figure 2: Relationship between S, $\Psi(S)$, and $H(S)$



A solution exists

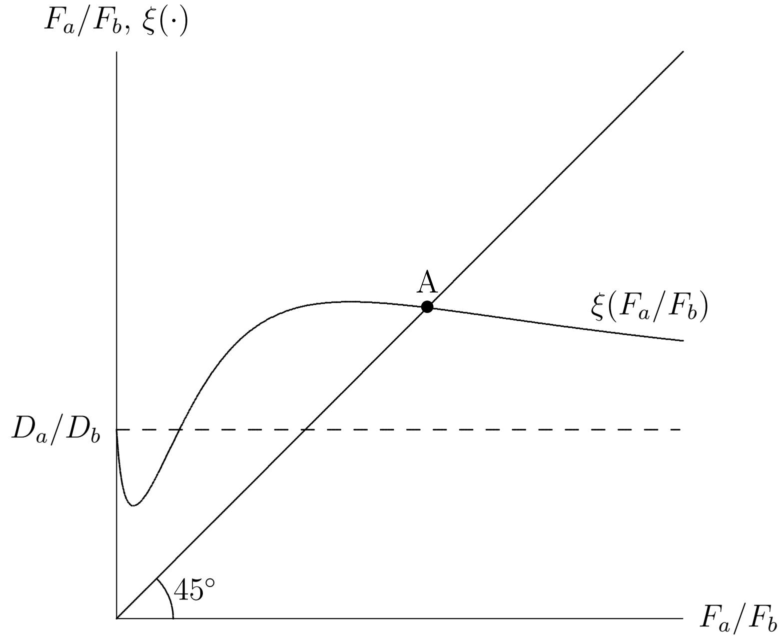
Using (23), (20) and (21), we obtain

$$\frac{F_a}{F_b} = \frac{D_a}{D_b} \frac{1 - \delta \left[H\left(\frac{c_a/C_b}{F_a/F_b + c_a/C_b} \right) - H\left(\frac{C_a/c_b}{F_a/F_b + C_a/c_b} \right) \right]}{1 - \delta \left[H\left(\frac{F_a/F_b}{F_a/F_b + C_a/c_b} \right) - H\left(\frac{F_a/F_b}{F_a/F_b + c_a/C_b} \right) \right]} \equiv \xi(F_a/F_b) \tag{29}$$

It is readily seen that (29) indeed has a fixed point. As F_a/F_b goes to either zero or infinity, the square brackets in the numerator and denominator both go to zero. Therefore, $\xi(F_a/F_b)$ in (29) both starts and ends at the positive and finite value D_a/D_b, and a solution always exist.

Typically, this solution is unique, as illustrated in Figure 3, a necessary and sufficient condition for multiplicity is that $\xi'(F_a/F_b) > 1$ in one fixed point. Such a steep slope is only possible if δ is sufficiently large, and in the following, we assume that it is low enough that the equilibrium is unique.[9]

Figure 3: $\xi(F_a/F_b)$ and the solution for F_a/F_b



[9]In the case $\Psi(S) = S$, $\delta < 0.91$ is a sufficient condition for uniqueness.

3 Unequal power

We now use the solution to explore the role of unequal power between groups and between the challenger and incumbency positions for the same group.

When interpreting the results below, it may be helpful to pay attention to the homogeneity property of the model: Using the four basic equations of (15), (16), and (23) for $j = a, b$, we see that what matters for the choice of conflict effort is the costs of force relative to the prize. A proportional increase in C_a, c_a, or D_a, for instance, has no effect on the equilibrium choice of effort for either group a or group b. Therefore, a k percent increase in costs C_a and c_a has the same effect on effort as a k percent reduction in rents D_a. In the following, we use this homogeneity property to normalize D_a and D_b so that both c_a and c_b can be set to unity.

Thus, our discussion of weak versus strong groups in the following can be interpreted as either a statement about costs or as a statement about rents.

3.1 Incumbency advantage

We start by exploring how an incumbency advantage affects the prize when an advantage emerges in an environment without initial disparities. We show the following result:

Proposition 1 *Compared to the case with equal strength, more unequal power in the form of an incumbency advantage for one (or both) contestant(s) increases the size of the prize on both sides of the conflict.*

Proof. Observe from (15) and (16) that $S_a + S_b \geq 1$ as[10] $C_a C_b \leq 1$ and that $S_a + S_b > 1$ whenever $C_a C_b < 1$. Now, since $H(\cdot)$ is increasing, we have $F_j = D_j$ whenever $C_a C_b = 1$ and $F_j > D_j$ whenever $C_a C_b < 1$. ■

Since it is the *difference* between the valuations of incumbency and challenger positions that matters, the prize gained by winning increases either when the val-

[10] Remember that by normalization, $c_a = c_b = 1$. Without this normalization of challenger costs, the condition would yield $C_a C_b \leq c_a c_b$.

uation of incumbency increases or when the valuation of the challenger position decreases. For instance, let group a obtain an incumbency advantage such that the prize it earns by winning increases. As group a is now expected to remain in power longer once it wins, group b's valuation of being the challenger decreases. Hence, both groups a and b see the prize from winning as increasing as group a gains an incumbency advantage.

While the prize earned by winning increases for the incumbent because the payoff of winning increases, the prize increases for the challenger because the payoff of losing declines. With higher stakes, both sides fight harder to gain an edge for future battles or to prevent the opponent from gaining it. As a result, the amount of resources spoiled in the conflict increases, even though the incumbent who obtains the edge wins the battle more often than the challenger—thus, incumbent turnover decreases. Hence, the conflict-enhancing prize effect dominates the conflict-dampening cost effect.

Absolute incumbency advantage

The property that any incumbency advantage increases the size of prize for both contestants is particularly distinct when the incumbency advantage is very strong. We say that a group has an *absolute incumbency advantage* if its cost of force approaches zero. If a group with an absolute advantage becomes the incumbent, it stays in power forever. Hence, when one, or both, contestants gain an absolute incumbency advantage, we obtain the following result:

Proposition 2 *Compared to the case without an incumbency advantage, the introduction of an absolute incumbency advantage for one contestant increases the prize for both contestants but more so for the contestant who receives the advantage. In the limit case, where both groups have an absolute incumbency advantage, the prizes are* $F_j/(1-\delta)$.

Proof. Both prizes increase since, as shown in proposition 1, $F_j > D_j$ for $j = a, b$ whenever $C_a C_b < 1$. (i) Consider the case where $C_b \to 0$ and $C_a = 1$. It follows from (15) and (16) that $S_b = 1$ and $S_a < 1$ and from (19) that $H(S_b) = 1$ and $H(S_a) < 1$. Using (19) and the symmetry of $\Psi(\cdot)$ in (23), we have

$$\frac{F_a}{D_a} = \frac{1}{1 - \delta[\Psi(S_a) - S_a(1 - S_a)\Psi'(S_a)]} < \frac{1}{1 - \delta[\Psi(S_a) + S_a(1 - S_a)\Psi'(S_a)]} = \frac{F_b}{D_b}$$

ii) Consider the case where $C_a \to 0$ and $C_b \to 0$). It follows from (15) and (16) that $S_a = S_b = 1$, from (19) that $H(S_a) = H(S_b) = 1$, and from (15) and (23) that $F_j = D_j/(1 - \delta)$ for $j = a, b$. ■

As stated, an absolute incumbency advantage, say, to group b implies that a victory for group b leads to an incumbent position that lasts forever. As long as only one group has an absolute incumbency advantage, and as long as this group remains the challenger, fighting is hard, as the stakes are high for both groups. In this case, the present ruler, group a, faces the prospect of losing everything if it is defeated once. The challenger has much to gain and nothing to lose. This, of course, is a recipe for a particularly fierce conflict.

When both groups approach an absolute incumbency advantage, victory in one period implies collecting rents almost for free in all subsequent periods. Hence, the prize is close to the present value of the per period rents.

So far, we have shown that introducing incumbency advantages increases the size of the prize for both contestants in a conflict. The prize for each contestant does not, however, necessarily increase monotonically in the incumbency advantage of the other. Clearly, an increase in the incumbency advantage must always increase the prize for the incumbent who receives the advantage. However, one interesting question is whether the incumbency advantage can reduce the size of the prize for the opponent.

To see that it can, consider the simple symmetric case where $D_a = D_b$, $C_a = C_b = C < 1$, and $\Psi(S) = S$, and thus, where from (19), $H(S) = S^2$. Then, a further reduction in the cost of influence for group a as the incumbent (C_a down from C) would reduce the size of the prize earned by challenger b. In other words, F_b declines if the discount factor δ is sufficiently large. Formally, the condition on δ is:[11]

$$\delta > \frac{1 + 2C + C^2}{3 - 2C - C^2} \quad \{< 1 \text{ if } C < \sqrt{2} - 1\} \tag{30}$$

Therefore, if both contestants initially have a strong incumbency advantage, an even stronger advantage for one group actually decreases the prize for the other.[12]

This result is a combination of two effects. First, when both contestants have strong incumbency advantages, the challenger position is dismal for both. Hence, both v_a and v_b are low and cannot be greatly affected by further reductions in the incumbent fighting costs. Now, if contestant a gains an even stronger incumbency advantage, implying that C_a decreases, contestant a would fight harder as the challenger, reducing group b's valuation of incumbency. If the future matters sufficiently to group a (δ is high), group b's valuation of incumbency would decrease such that the size of the prize F_b also decreases.

The interaction between incumbency advantages and incentives to fight can also introduce ambiguities into a group's appreciation of its own incumbency advantage. This possibility is explored in the next section.

3.2 Self-defeating power

We have seen that incumbency advantages can explain the contestants' greater fighting effort and higher conflict spending. Below, we demonstrate that the challenger may be hurt today from a higher incumbency advantage. Moreover, a current incumbent may be hurt by strengthening its own incumbency advantage.

We define *a weak challenger* as a challenger with a high cost of fighting. Consider the case where group a is the incumbent, and group b is the challenger. Strengthening the incumbency advantage of challenger b represents a threat to incumbent a, who meets the threat with greater resistance. The net outcome could actually be that group b is worse off, as it induces fierce resistance from

[11] This can be shown using (23), (15), (16), (19), (20) and (21) with differentiation with respect to C_a.

[12] As C approaches 0, $\delta > 1/3$ is the condition for (30) to be satisfied.

the present incumbent a. This effect could also make group b worse off even as the incumbent. More precisely,

Proposition 3 *i) For a sufficiently weak group, the prospect of an incumbency advantage reduces the expected payoff of the challenger. ii) For a sufficiently weak and farsighted group, a strengthening of the incumbency advantage reduces the expected payoff of incumbency.*

Proof. See the appendix. ■

The result that power can be self-defeating is in stark contrast to the results of static contests where the returns unambiguously increase when the costs decrease. The intuition is simple enough. When group a is the challenger, it obtains no immediate gain from its incumbency advantage. If the incumbency advantage increases, the probability of becoming the incumbent may decrease so much that there is a net loss. The more surprising possibility is that this mechanism may even be dominant when group a is the incumbent. As a weak incumbent, group a knows that it will play a large share of its future periods as the challenger. Its valuation of incumbency V_a is largely determined by its valuation of the challenger position. Hence, if δ is high (and S_a remains low), an incumbency advantage that reduces v_a may also lower V_a.

This proposition is particularly relevant for weak groups. As previously noted, costs and gains enter symmetrically, implying that a weak group is weak either because it has high costs or because it has little to fight for. Hence, a group that only moderately prefers its own rule over that of the opposition may prefer not to have an incumbency advantage. It may in fact have an incentive to limit its own incumbency advantage – if that were a credible possibility.

Thus far, we have demonstrated how asymmetric power can induce fighting. We now turn to another major focus of the literature: how inequality affects the amount of resources wasted in conflict.

3.3 Social waste

In contrast to the standard static model, maximal waste of resources does not result by leveling of the playing field.[13] From proposition 1, we know that incumbency advantage gives parties more to fight for. If fighting increases as a result, social waste increases with departures from a level playing field. We now consider which deviations lead to the most fighting, distinguishing between immediate graveness within a period and discounted waste for all future periods. As in the rest of the literature, the discussion of graveness and waste is relative to the rents at stake. Obviously, if winning itself means a lot to one of the contestants, as measured by D_j, that group would fight hard and waste a considerable amount of resources in the struggle. We are less interested in this heterogeneity related to the immediate prize from winning. To produce a meaningful metric with which to assess waste and graveness, we need to restrict our attention to the case where the rents at stake are equal for both groups, hence, $D_a = D_b = D$.

Maximal graveness

We can measure the immediate graveness of a conflict as the total resources used for the conflict in a particular period. How does unequal power affect this immediate graveness? Consider a situation where group a is the incumbent. We would like to know which constellation of incumbency costs C_a and C_b maximizes the graveness $Y_a + y_b$. The following proposition provides the answer:

Proposition 4 *The immediate graveness of the conflict is at its maximum when a strong incumbent faces a challenger that obtains an absolute advantage if he wins; that is, $Y_a + y_b$ is maximized for $C_b = 0$ and C_a that is strictly positive but less than $c = 1$.*

Proof. See the appendix. ■

The main lesson of proposition 4 is that double inequality of power can increase the graveness of battle to its highest level. Maximal graveness requires

[13] For a survey of models of static rent-seeking contests, see the article by Shmuel Nitzan (1994) and the monograph by Kai Konrad (2009).

that both sides exhibit a strong willingness to fight. It is easy to understand a high willingness to fight the challenger. That group is fighting for the eventual elimination of the opponent, after which it collects rents for free for all future periods. To obtain maximal graveness, however, the incumbent must also be particularly willing to fight. He must therefore be strong enough to gain a surplus worth fighting for. This is why a certain incumbency advantage induces the incumbent to fight hard to continue to receive the high surplus that strength yields when in power.

At the maximal level of graveness, the resources used for conflict can easily exceed the immediate rent D. In Figure 2, maximal graveness is indicated by point A^*, and we have indicated a contour around A^*, within which the per period waste $(Y_a + y_b)$ is larger than the per period rents (assuming the same rents for both $D_a = D_b = D$). Analogously, point B^* is the cost combination that maximizes graveness with group b as the incumbent.

Maximal waste

To see which cost combinations generate the most waste, we focus on the present value of wasted resources. The question is: Which power disparities within and across states spoil the most resources as measured by their present value?

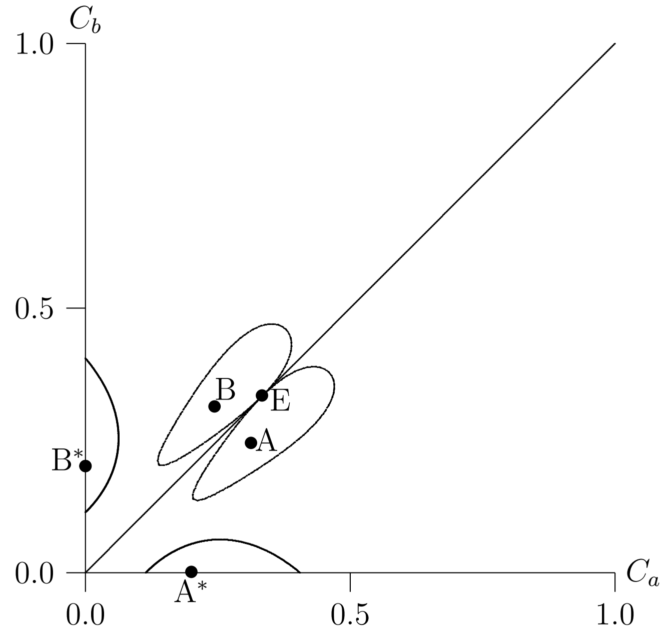
We are interested in the waste ratio, measured as the expected present value of the fighting efforts of the contestants relative to the present value of the rents. The present value of rents is simply $D/(1-\delta)$. Starting in state a, the two contestants are expected to obtain equilibrium payoffs equal to $V_a + v_b$. Combining the two, the waste ratio when group a is the incumbent is ω_a as defined by

$$\omega_a = \frac{\frac{D}{1-\delta} - (V_a + v_b)}{\frac{D}{1-\delta}} \tag{31}$$

We can show the following (when a is the incumbent):

Proposition 5 *The cost combination generating maximal waste has a double*

Figure 4: Maximal waste



inequality; one between the present incumbent and the challenger $C_a < 1$ *and another between the incumbency advantage of the current and would-be incumbents:* $C_b < C_a$.

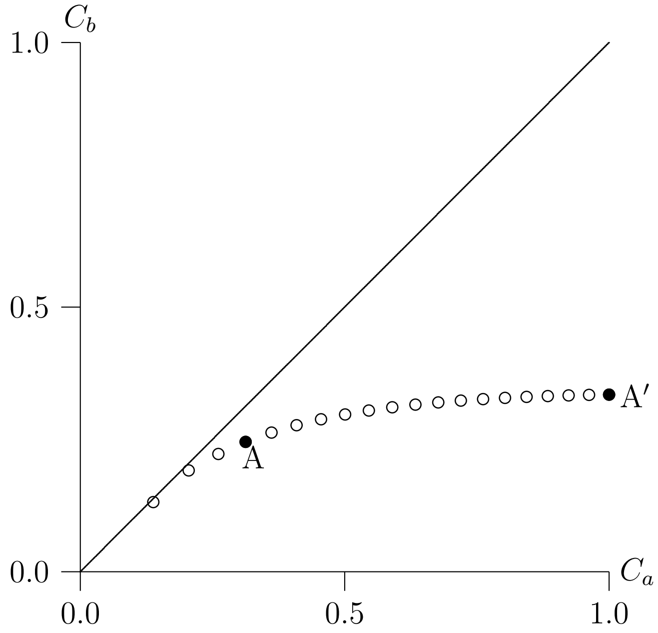
Proof. See the appendix. ■

In other words, maximal waste is obtained when a strong ruler a faces an even stronger would-be ruler b. Both disparities raise the stakes of and increase the present value of resources wasted in the conflict.

Figure 4 summarizes the results regarding maximum graveness and maximum waste. The example (but not the proof) is calculated with $\Psi(S) = S$. With group a as the incumbent, point A is the parameter configuration that maximizes waste, while A* is the point that maximizes graveness. Points B and B* capture similar points with group b as the incumbent. Point E shows the symmetric cost constellation that maximizes waste. The two convex sets tangential to E have equal or larger waste than E, with A and B as the two maxima.

Figure 4 is calculated under the assumption of a quite high discount factor of $\delta = 0.8$. As δ goes to zero, the value of incumbency declines, and the maximum waste points will change. Figure 5 illustrate how the maximal waste point moves

Figure 5: Maximal waste for varying discount factors



as δ approaches zero. It shows that the maximal waste for all δ is found off the diagonal. Maximum waste always requires that the incumbency advantage of the challenger is larger than that of the current ruler. This is true when δ is substantial, as in A, and when δ gets arbitrarily close to zero, as in A′.

The latter feature is striking. As $\delta \to 0$, the model approaches the static model. We know from analyses of the static model that waste is maximized when the groups are equally strong. Hence, in our model, when $\delta = 0$, discounted waste ω_a depends only on the cost configuration in the current state. When group a is the incumbent, $C_a = 1$ maximizes waste, while C_b is irrelevant. However, as Figure 5 illustrates, when δ is marginally positive, maximum waste is found off the diagonal. This property is a general result valid for all Ψ. In Appendix B, we show that that as $\delta \to 0$, the maximum waste configuration approaches $C_a = 1$, $C_b = \frac{2-\Psi'(S_a)}{2+\Psi'(S_a)} < 1$. In the figure where $\Psi(S) = S$ and $\Psi' = 1$, the maximum waste point A′ has coordinates $Ca = 1, C_b = 1/3$.

4 Concluding remarks

We conclude by discussing some archetypal institutional configurations in light of the model. In Figure 6, we emphasize four different incumbency cost distributions between groups a and b (where the challenger's costs of force are normalized to unity).

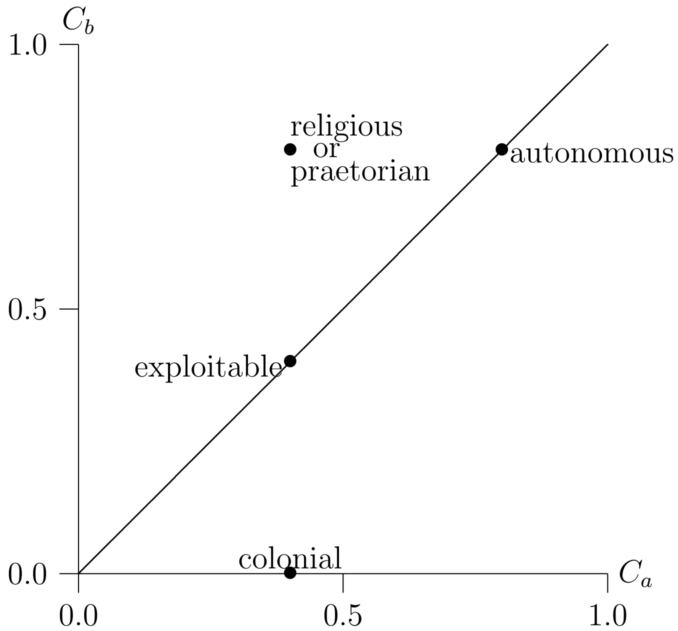
Institutions that we depict as points on the diagonal provide equal incumbency advantages to both groups. There is a difference, however, between institutions that provide a considerable edge – *exploitable* institutions in the figure – and institutions that hardly can be used to strengthen the incumbent – *autonomous* institutions.

In the introduction, we motivated our paper by the violence following the end of the Cold War. A shift from autonomous to exploitable institutions may explain why many civil wars have continued, some at higher intensities. The reason could simply be that after the superpowers left, victory for either side became more important. That is, a stronger incumbency advantage became part of the prize. Countries with two political movements each aligned with a superpower actually experienced a balance of power that resembled that in autonomous institutions, as denoted in the figure. After the Cold War, however, the institutional setting offered greater incumbency advantages that were no longer neutralized by additional superpower support. Hence, after the end of the Cold War, institutions became more exploitable – as denoted in the figure – and fighting became more intense.

If correct, these arguments speak to the debate about the role of weak versus strong states in the stability of divided societies. A state can be weak in the sense that it does not protect the incumbent from easy attacks from challengers – as in the autonomous institutions in Figure 6. Weak states are easier to confront, but the payoffs from victory can be equally modest, as the incumbent is never safe. Hence, the weakness of the state is is not in itself a satisfying explanation why poor countries experience severe conflicts.

A strong state, controlled by one side of a divided society, indicated by the

Figure 6: Institutional archetypes



exploitable institutions in Figure 6, may actually fuel conflicts rather than mitigate them. Control of the state apparatus makes the incumbent group stronger, but a stronger incumbent makes control of the state more valuable, which may lead to a conflict wherein incumbency is long-lasting without deterring challenges from opposing groups. The attractiveness of taking over a strong incumbency dominates the low odds of short-run success.

Off the diagonal in Figure 6, a case labeled *colonial* at the bottom depicts a settler regime a that confronts an independence movement b. The settler regime may be quite strong as an incumbent. However, both groups realize that the independence movement, though weak as a challenger, will remain the incumbent forever if the settler regime is toppled.

Such asymmetry describes the long-lasting conflict between the Apartheid regime (group a) and the black majority (group b) in South Africa. The overwhelming strength of the Apartheid regime was due to the government's access to a ruthless security apparatus. The defenders of black majority rule, a weak challenger, knew that once defeated, the Apartheid regime would never reappear.

The black majority defenders would, in other words, enjoy an absolute advantage if it won. As a consequence, the fight against Apartheid was hard over the many years leading up the release of Nelson Mandela.

Another configuration off the diagonal is called either *religious* or *praetorian*. The religious interpretation we have in mind is an institution based on religious principles giving a religious party, group a, a substantial incumbency advantage, while a secular party, group b, only enjoys a moderate incumbency advantage.

The term *praetorian* is taken from Perlemutter (1977) and indicates a case where group a has particular ties to the army, while group b does not enjoy these sympathies. Hence, in contrast to group a, group b cannot use the same means of violence to preserve its incumbency. An illustrative case is Chile, as discussed in the introduction, with Pinochet representing the praetorian group a.

The recent history of Turkey illustrates a *combination* of praetorian and religious aspects. Until recently, religious groups could not rely on the army. However, the failed coup of July 2016 shows that this has changed; the army is more neutral, while Erdoğan's AKP still has the advantage entailed in religious rule. In a bold interpretation, we can say that Turkey has shifted from a praetorian case, with the Kemalists enjoying the largest incumbency advantage, to religious rule, with the AKP enjoying the largest incumbency advantage.

In addition to the predictions relating to domestic power configurations, our dynamic perspective on conflicts has policy implications for outside interventions. Impartial observers tend to recommend policies and institutions that stabilize political situations by securing the incumbency of the winner. Yet, as we have seen, the stability of a future incumbency can induce fighting both from the current incumbent and the current challenger. In divided societies, well-intentioned efforts to stabilize the future may destabilize the present.

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A Appendix. Proofs

Proposition 3 i) When S_i is sufficiently low, $\partial v_i/\partial C_i > 0$. ii) $\partial V_i/\partial C_i > 0$ as well, if δ is sufficiently high.

Proof. By differentiating (15) and (16) we get

$$\mathrm{d}S_i = S_i \left(1 - S_i\right) \left(\frac{\mathrm{d}F_i}{F_i} - \frac{\mathrm{d}F_j}{F_j} - \frac{\mathrm{d}C_i}{C_i} \right), \ i \neq j$$

By differentiating (23) using (20) and (21), we get

$$\frac{\mathrm{d}F_i}{F_i} = \frac{1}{1 - \delta \left[H\left(S_i\right) - H\left(1 - S_j\right) \right]} \delta \left(h'\left(S_i\right) \mathrm{d}S_i + h'\left(1 - S_j\right) \mathrm{d}S_j \right), \ i \neq j$$

Evaluating at a point without an incumbency advantage (i.e., where $C_i = c_i = 1$ and where, as a result, $S_a + S_b = 1$) yields

$$\frac{\mathrm{d}F_i}{F_i} = -S_a S_b \delta h'\left(S_i\right) \left(\frac{\mathrm{d}C_a}{C_a} + \frac{\mathrm{d}C_b}{C_b} \right) \tag{A.1}$$

$$\mathrm{d}S_i = -S_a^2 S_b^2 \delta (h'\left(S_i\right) - h'\left(1 - S_i\right)) \left(\frac{\mathrm{d}C_a}{C_a} + \frac{\mathrm{d}C_b}{C_b} \right) - S_a S_b \frac{\mathrm{d}C_i}{C_i} \tag{A.2}$$

Consider a marginal incumbency advantage for group a when it has little to fight for (D_a small.) From (17) and (18), in combination with (19), it follows that

$$\frac{\mathrm{d}V_a}{F_a} - \delta \frac{\mathrm{d}v_a}{F_a} = H\left(S_a\right) \frac{\mathrm{d}F_a}{F_a} + h'\left(S_a\right) \mathrm{d}S_a$$

$$\left(1 - \delta\right) \frac{\mathrm{d}v_a}{F_a} = H\left(S_a\right) \frac{\mathrm{d}F_a}{F_a} - h'\left(S_a\right) \mathrm{d}S_b$$

It follows by combining with (A.1) and (A.2) that

$$\mathrm{d}v_a = \frac{\delta}{1 - \delta} h'\left(S_a\right) S_a S_b \left[\left(h'\left(S_b\right) - h'\left(S_a\right) \right) S_a S_b - H\left(S_a\right) \right] \frac{F_a}{C_a} \mathrm{d}C_a \tag{A.3}$$

$$\mathrm{d}V_a = \mathrm{d}v_a - h'\left(S_a\right) S_a S_b \frac{F_a}{C_a} \mathrm{d}C_a \tag{A.4}$$

which shows the effects on the value functions of a marginal incumbency advantage for group a. Proposition 3 relates to the case where a is a weak challenger, implying high costs or little to fight for relative to the other. Both alternatives are captured by considering the case of a small S_a. Using (19), it follows that when S_a is close to zero, (A.3) and (A.4) can be approximated by a first order Taylor expansion around $S_a = 0$:

$$\mathrm{d}v_a \approx \frac{2\delta}{1-\delta} h'(S_a) S_a^2 S_b \Psi'(0) \frac{F_a}{C_a} \mathrm{d}C_a > 0 \tag{A.5}$$

$$\mathrm{d}V_a \approx \frac{1}{1-\delta} S_a S_b h'(S_a) \left[-1 + \delta (2\Psi'(0) + 1)\right] \frac{F_a}{C_a} \mathrm{d}C_a \begin{cases} > 0 & \text{when } \delta \text{ close to 1} \\ < 0 & \text{when } \delta \text{ close to 0} \end{cases} \tag{A.6}$$

(A.5) is the formal proof of the first part of the proposition, regarding a weak group (S_a small) as the challenger. For such a group, an incumbency advantage (C_a down) will reduce the valuation of the challenger position. (A.6) is the formal proof of the second part of the proposition. A weak group, which does not discount much (S_a small and δ large) will value incumbency less if it gains an incumbency advantage. ■

Proposition 4. $(Y_a + y_b)$ is maximized for $C_b = 0$ and $0 < C_a < 1$.
Proof. From (11) and (12), waste with a as incumbent is given as

$$Y_a + y_b = S_a(1 - S_a)\Psi'(S_a)(F_a + F_b) \tag{A.7}$$

S_a does not depend on C_b, and direct inspection of (23) and (19, 20, 21) shows that for any C_a, both F_a and F_b are maximized for $S_b = 1$, which obtains when $C_b = 0$.

The final step is to rule out that either $C_a = 1$ or $C_a = 0$ can maximize waste.

When $C_b = 0$ and $D_a = D_b = D$, we have from (15) and (23) that

$$S_a = \frac{F_a/C_a}{F_a/C_a + F_b} \tag{A.8}$$

$$F_a = \frac{D}{1 - \delta[\Psi(S_a) - S_a(1 - S_a)\Psi'(S_a)]} \tag{A.9}$$

$$F_b = \frac{D}{1 - \delta[\Psi(S_a) + S_a(1 - S_a)\Psi'(S_a)]} \tag{A.10}$$

From the proof of proposition 2, we know that when $C_a = 1$ and $C_b = 0$, $F_b > F_a$ and $S_a < 1/2$. Hence, if C_a is reduced from 1, S_a increases. From the Ψ axioms, we know that $\Psi(S_a)$ and $H(S_a) = [\Psi(S_a) - S_a(1 - S_a)\Psi'(S_a)]$ are both increasing in S_a. Moreover, $S_a(1 - S_a)\Psi'(S_a)$ is increasing with S_a when $S_a < 1/2$. It follows that both F_a and F_b increase as S_a increases. It follows that $Y_a + y_b$ unambiguously increases when C_a is reduced from 1; hence, $C_a = 1$ cannot maximize waste.

We know that both F_a and F_b are positive and finite. Then, it follows that waste approaches zero as S_a approaches one for C_a close to zero; hence, $C_a = 0$ cannot maximize waste.

■

Proposition 5. ω_a is maximized for $0 < C_b < C_a < 1$

Proof. We first define short-run waste. The short-run waste, ω_i^*, is the waste ratio in a particular period. It is state dependent and is measured by the sum of effort in a period divided by the per period rents.

$$\omega_a^* = \frac{Y_a + y_b}{D} \qquad \omega_b^* = \frac{Y_b + y_a}{D} \tag{A.11}$$

The discounted waste, ω_a and ω_b, is the discounted valuation of the waste. It is the state-dependent weighted average of the short-run waste ratios.[14]

$$\omega_a = \theta_a \omega_a^* + (1 - \theta_a)\, \omega_b^* \tag{A.14}$$

$$\omega_b = \theta_b \omega_b^* + (1 - \theta_b)\, \omega_a^* \tag{A.15}$$

The discounting weights follow from the stationary distribution of the Markov process such that

$$\theta_a = \frac{1 - \delta\Psi(S_b)}{1 + \delta - \delta\Psi(S_a) - \delta\Psi(S_b)} \tag{A.16}$$

$$\theta_b = \frac{1 - \delta\Psi(S_a)}{1 + \delta - \delta\Psi(S_a) - \delta\Psi(S_b)} \tag{A.17}$$

In order to prove proposition 5, we proceed in steps. First, we use implicit differentiation of (A.14) and (A.14) using (A.11), (A.16), and (A.17) to find the effect on ω_a by changes in C_a and C_b starting at the point where $C_a = C_b$. We have that

$$\mathrm{d}\omega_a = \frac{A+B}{2}\mathrm{d}C_a + \frac{A-B}{2}\mathrm{d}C_b \tag{A.18}$$

$$A \equiv 2F/DS^2 \left((2S-1)\psi' - S(1-S)\psi'' - F/D\delta S(1-S)2\psi'^2\right) \tag{A.19}$$

$$B \equiv \frac{2F/DS^2((2S-1)\psi' - S(1-S)\psi'')(1-\delta)}{[1 - 2\delta F/DS(1-S)(H'(S) - H'(1-S))](1 + \delta - 2\delta P_a)} \tag{A.20}$$

For $C_a = C_b$, the effect on ω_a is given by A. Increasing C_a and decreasing C_b, the effect is given by B. If $A = 0$, then $B > 0$.

Constrained by $C_a = C_b$, the maximum level of ω_a is obtained for a value S such that $A = 0$. Starting from such an extreme point and reducing C_b, the level of ω_a increases according to (A.18). Hence, the point where $C_a = C_b$ cannot be the unconstrained maximum.

[14] The weights follow when solving the following two equations

$$\omega_a = (1-\delta)\omega_a^* + \delta(P_a\omega_a + (1-P_a)\omega_b) \tag{A.12}$$

$$\omega_b = (1-\delta)\omega_b^* + \delta(P_b\omega_b + (1-P_b)\omega_a) \tag{A.13}$$

with respect to ω_a and ω_b

To rule out that $C_a < C_b$ maximizes ω_a, observe that $C_a < C_b$ implies $S_a > S_b > 1/2$ and, thus, a cost configuration where $\omega_b^* > \omega_a^*$ and $\theta_b > \theta_a$. At this point, we therefore have $\omega_b > \omega_a$. Consequently, $C_a < C_b$ cannot maximize waste (with a as the incumbent), since replacing a with b as the incumbent would increase the waste ratio, and the maximum waste must be obtained for $C_a > C_b$.

■

B Waste for discount factors close to zero

As $\delta \to 0$ with a as incumbent, the maximum waste point approaches a point where $C_a = 1$ and $C_b < 1$.

Proof. In order to determine how the maximum waste loci for ω_i and ω_i^* move as $\delta \to 0$, we need to consider the limit where $\delta > 0$.

We consider the case where group a is the incumbent and set $C_a = 1$. As a first order approximation, we formulate the model as a two-period model. When $\delta = \epsilon \approx 0$, this is a good approximation, as period 3 would only influence period 1 with factor ϵ^2. Starting from period 2, we know from (11), (12), (15) and (16) that when $F_{a,2} = F_{b,2} = D$, then

$$S_{a,2} = \frac{1}{1 + C_a} \tag{B.1}$$

$$S_{b,2} = \frac{1}{1 + C_b} \tag{B.2}$$

$$Y_{a,2} = y_{b,2} = S_{a,2}(1 - S_{a,2})D\Psi'(S_{a,2}) \tag{B.3}$$

$$Y_{b,2} = y_{a,2} = S_{b,2}(1 - S_{b,2})D\Psi'(S_{b,2}) \tag{B.4}$$

With group a as the incumbent in period 1. The corresponding period 1 relations are as follows

$$S_{a,1} = \frac{F_{a,1}}{F_{a,1} + F_{b,1}} \tag{B.5}$$

$$Y_{a,1} = S_{a,2}(1 - S_{a,2})F_{a,1}\Psi'(S_{a,2}) \tag{B.6}$$

$$y_{b,1} = S_{a,2}(1 - S_{a,2})F_{b,1}\Psi'(S_{a,2}) \tag{B.7}$$

Using (6), it follows that

$$F_{a,1} = D + \delta\left[(\Psi(S_{a,2})D - Y_{a,2}) - ((1 - \Psi(S_{b,2})D - y_{a,2})\right] \tag{B.8}$$

$$F_{b,1} = D + \delta\left[(\Psi(S_{b,2})D - Y_{b,2}) - ((1 - \Psi(S_{a,2})D - y_{b,2})\right] \tag{B.9}$$

where the square brackets capture the value of starting as the incumbent.

This system of equations can be solved by setting all higher order occurrences of δ to zero ($\delta^2 = \delta^3 = 0$). We can then show that

$$\frac{\partial \omega_a}{\partial C_b} = ((\Psi'(S_{a,1}) + 2)C_b - (2 - \Psi'(S_{a,1})))[\ldots] \tag{B.10}$$

where $[\ldots]$ is strictly negative. Therefore, as $\delta \to 0$, w_a has its maximum for

$$C_a = 1 \quad \text{and} \quad C_b = \frac{2 - \Psi'(S_{a,1})}{2 + \Psi'(S_{a,1})} < 1 \tag{B.11}$$

■