The Malmquist productivity index

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1. Introduction

When studying productivity change for units within a sector a basic question is what part of the change can be accounted for by the unit becoming more efficient within its own technological potential, and what part is accounted for by the change of potential technological possibilities. An old article by Maywald (1957) studies the change in energy productivity of thermal power plants based on an international data set and focuses on the development of the distance between average and best practice. This distance was also suggested by Farrell (1957) as a measure of structural efficiency. In Førsund and Hjalmarsson (1974, 1987) a detailed analysis of the dynamics of structural change based on average and best practice within a vintage framework is offered. New technology can only be introduced by investments. Existing units are "inefficient" in the sense that capital equipment is outdated. There are two aspects of technical change that are of interest to measure: Shift in the frontier function and shift in the short-run industry function (Johansen, 1972) at efficiency levels over the range from best practice to worst practice. Following Salter (1960), technical change of a parametric frontier function is measured by the shift's impact on costs at optimal scale (Førsund and Hjalmarsson, 1979a), and technical change (limited to variable inputs) of the non-parametric short-run function is measured by unit cost changes at constant prices (Førsund and Hjalmarsson, 1983). The average catching up with the best or structural efficiency following Farrell, is measured by the average unit's relative efficiency in the frontier function case, and by comparing unit costs improvements at best and average practice in the case of the short-run function.

2. The decomposition of productivity change

When loosening the strict vintage assumption of embodied technical change only the notion of improving efficiency within a unit's own potential becomes relevant. A paper often referred to when no vintage structure is assumed is Nishimizu and Page (1982). The popular figure presented there is the following:

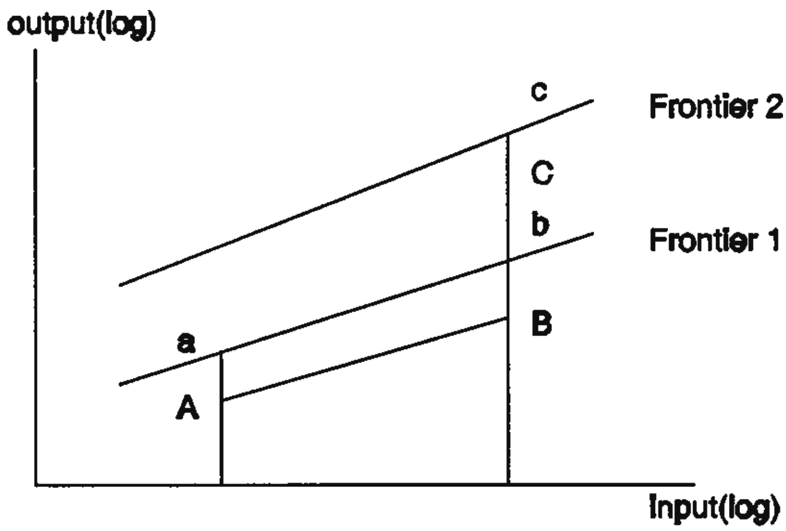

Figure 1. The Nishimizu-Page productivity decomposition

The basic observations for a unit is point A in period t_1 and point C in period t_2. All the variables are measured in logs. Constant returns to scale is assumed, and the output level at point B corresponding to using input levels observed in period t_2 with period t_1 technology is introduced. Constant returns imply that the productivity level observed in period t_1 at A is the same as the level at B.

The productivity change is measured by the distance BC: The observed output (in logs) is C. Employing period t_2 input levels within period t_1 technology yields output at B denoted by $y_{t1}(x_{t2})$ (the relative distance to the frontier is the same as observed in period t_1). This change is decomposed into change in the frontier itself and relative change in distance from the frontier:

$$cC + BC = bc + bB,$$

$$BC = bc + (bB - cC), \tag{1}$$

where bc is the change of the frontier itself and (bB - cC) the relative change in distance to the frontier. Moving closer to the frontier, bB > cC, improves productivity.

The line distances in (1) can be replaced with output levels, remembering that all the variables are measured in logs:

$$\ln y_{t_2} - \ln y_{t_1}(x_{t_2}) = \ln y_{t_2}(f_{t_2}) - \ln y_{t_2}(f_{t_1}) + [\ln y_{t_2}(f_{t_1}) - \ln y_{t_1}(x_{t_2}) - (\ln y_{t_2}(f_{t_2}) - \ln y_{t_2})] \tag{2}$$

where $y_i(f_j)$ = output in period i employing inputs x_{t2} within frontier technology of period j, $i = t_2$, $j = t_1, t_2$. Taking antilogs yields the multiplicative decomposition of productivity change:

$$TFP = \frac{y_{t_2}}{y_{t_1}(x_{t_2})} = \frac{y_{t_2}(f_{t_2})}{y_{t_2}(f_{t_1})} \cdot \frac{\dfrac{y_{t_2}(f_{t_1})}{y_{t_1}(x_{t_2})}}{\dfrac{y_{t_2}(f_{t_2})}{y_{t_2}}} = \frac{y_{t_2}(f_{t_2})}{y_{t_2}(f_{t_1})} \cdot \frac{\dfrac{y_{t_2}}{y_{t_2}(f_{t_2})}}{\dfrac{y_{t_1}(x_{t_2})}{y_{t_2}(f_{t_1})}} \tag{3}$$

The term after the first equality sign shows that the TFP measure is the relative factor productivity for the two observations measured in year two. By transforming period one observation to period two the input level x_{t2} becomes a common factor. The first term after the second equality sign is the ratio between the frontier outputs of the two technologies when employing period two inputs. The ratio measures the relative shift in frontier technology. The second term has the ratio of frontier output using period one technology and "observed" period one output adjusted to period two input level in the nominator, and the ratio between frontier output using period two technology and observed period two output in the denominator. The term measures the relative movement towards the frontiers. If TFP > 1 then period two observation is more productive than period one. If the term measuring the shift in the frontier is greater than one technology shift contributes positively to TFP. If the second term is greater than one, then period two observation is closer to its frontier than period one observation is to its frontier, and the contribution to TFP is positive. We shall return to the rearrangement of the movement to the frontier term after the third equality sign in (3) in the sequel.

3. Farrell efficiency measures

Although N&P refer to the seminal Farrell (1957) paper in their introduction, they fail to point out the direct relationship with their TFP measure and its decomposition and Farrell's measures of efficiency. Following the generalisation in Førsund and Hjalmarsson, (1974, 1979b) from constant returns to scale functions to general production functions, Farrell's measure of output increasing efficiency for a unit is defined as the ratio between obseved output on potential output employing observed amounts of inputs within frontier technology.

Referring to the last expression in (3) and the last term we have that the nominator is the ratio between observed output and frontier output in period two employing observed inputs. This is the direct definition of the Farrell measure:

$$E_{22} = \frac{y_{t_2}}{y_{t_2}(f_{t_2})} \tag{4}$$

The first subscript refers to the period of the technology and the second to the observation. The corresponding measure for period t_1 involves "observed" output in that period and the potential at the frontier utilising the same period t_2 amounts of inputs:

$$E_{11} = \frac{y_{t_1}(x_{t_2})}{y_{t_2}(f_{t_1})} \tag{5}$$

The "average catching up with best practice" is expressed by the ratio of these output increasing efficiency measures. Both measures are between zero and one, and $E_{22} > E_{11}$ means that the unit is closer to its frontier in period two than in period one, and hence efficiency is improved relatively.

The first term in the last expression of (3) is the ratio of frontier outputs of the two technologies employing the same amounts of inputs. This relative distance between frontiers can also be interpreted in terms of Farrell output increasing efficiencies. Measuring the efficiency of the observation in period t_2 relative to period t_1 frontier technology yields the efficiency measure:

$$E_{12} = \frac{y_{t_2}}{y_{t_2}(f_{t_1})} \tag{6}$$

Since the potential output when employing observed inputs is obtained within a different technology than the one belonging to the period of observation, the efficiency measure may be greater than one.

Combining the latter measure with the measure for the period t_2 for the same period observation the relative change measure can be expressed by E_{12}/E_{22}. We see from (4) and (6) that y_{t2} cancels out and that the ratio measures the relative distance between the frontiers in period t_2. Expressed in terms of Farrell efficiency measures the Nishimizu and Page productivity index (3) can be written:

$$TFP = \frac{E_{12}}{E_{22}} \cdot \frac{E_{22}}{E_{11}} = \frac{E_{12}}{E_{11}} \tag{7}$$

The interpretation of the last expression is the productivity of the year 2 observation relative to year one observation when the common factor making a comparison meaningful is to evaluate both observations within the same frontier technology for year one.

Notice that all measurements refer to period t_2. This is possible due to the transformation of the observation in period t_1 to period t_2 keeping the same efficiency level. However, this procedure can be questioned. What is general about the procedure above, and what depends on the special assumptions?

N&P assume constant returns. This is unnecessary to achieve a decomposition, but an assumption of keeping relative efficiency is needed to be able to transform period t_1 observation to period t_2.

This is done by Perelman and Pestieau (1988). However, they seem to have overlooked two problems. Firstly, N&P performs the additive decomposition in logarithmic form. Decomposing in the same fashion with input and output measured in natural units yields a different decomposition[1]. Secondly, total factor productivity is not measured conventionally by BC except when assuming constant returns. The scale effect when moving from period t_1 to period t_2 even with constant relative efficiency distorts BC as a TFP measure.

We will proceed to show that the productivity measure (7) with its decomposition is in fact quite general provided we leave the narrow TFP interpretation and a more stringent approach is taken.

First we need to generalise the Farrell measures to multi output frontier technology. The transformation functions (with standard properties) descibing frontier technologies are:

$$F_i(y_i,x_i) = 0 \ , \ i = 1,2 \qquad (8)$$

where y is the output vector and x the input vector. We assume that no unit can perform better than shown by the frontier technology $F_i(.)$, i.e. a deterministic frontier (Førsund et al.,1980) is adopted.

When working with general production functions Farrell measures can either be defined in an input saving direction keeping observed outputs constant, or in an output increasing direction keeping observed inputs constant, as shown in Førsund and Hjalmarsson (1974,1979b). The definition of the Farrell input saving measure is based on comparing potential inputs required at frontier technology for producing observed outputs when keeping factor proportions as observed. As presented above Farrell's measure of output increasing efficiency for a unit is defined as the ratio between obseved output and potential output employing observed amounts of inputs within frontier technology. When applying the

[1] There is also a misprint when decomposing; bB and cC should change places in the last paragraph on p. 436(or maybe this is due to Perelman and Pestieau interchanging B and C in their figure).

principle of measuring observed outputs to potential outputs within frontier technology multiple outputs are dealt with by keeping observed ratios between outputs constant.

For input saving measures we need to find the minimal input vector, $x^*(F_{ij})$, keeping observed factor ratios, sufficient to produce the observed output vector y_j when employed within the frontier technology $F_i(.)$, i.e. writing F_{ij} in parantheses means that x^* is found from solving $F_i(y_j,x^*) = 0$, $i,j = 1,2$. The Farrell input saving efficiency measure based on frontier technology for period i, for a unit observed in period j, can then be defined:

Definition 1.

The Farrell input saving efficiency measure, E_{ij}^1, for unit j with frontier technology, $F_i(.) = 0$ as reference is:

$$E_{ij}^1 = \frac{x_s^*(F_{ij})}{x_{js}} = Min_{\alpha_i} \{\alpha_i : F_i(y_j,\alpha_i x_j) = 0\} \ , \ i,j = 1,2 \tag{9}$$

The superscript, 1, indicates that we have an input-saving measure. The subscript s in the second expression indicates an arbitrarily chosen input no. s. When i = j the measure must be between zero and one. When i # j the measure may be greater than one if the observation is outside the other period frontier technology.

The second expression shows the direct connection between the Farrell measure and the Shephard (1953) concept of input distance function. The Shephard definition is the inverse of our definition of Farrell input saving measure.

Farrell output increasing efficiency measure is defined analogously. In our context the production relation $F_i(.)$ is the frontier, observed outputs are y_j and maximal frontier outputs $y^*(F_{ij})$ are found by solving $F_i(y^*,x_j) = 0$, where the proportions between the outputs in the vector y^* are the same as observed for y_j. Taking out one component, k, of the output vectors the Farrell efficiency measure is:

Definition 2.

The Farrell output increasing efficiency measure, E_{ij}^2, for unit j with frontier technology, $F_i(.) = 0$ as reference is:

$$E_{ij}^2 = \frac{y_{jk}}{y_k^*(F_{ij})} = Min_{\beta_i} \{\beta_i : F_i(\frac{1}{\beta_i} y_j, x_j) = 0\} \; , \; i,j = 1,2 \tag{10}$$

The superscript 2 on E indicates output increasing measure. When i = j the efficiency measures are between zero and one, but they can be greater than one when i # j.

The last expression is the Shephard output distance function.

4. The Malmquist productivity index

N&P refer to Malmquist (1953) and an working paper version of Caves et al.(1982a),but without exploring the potential within their framework. Malmquist (1953), within a consumer context, introduced the notion of **proportional scaling** needed for year 2 observed quantities for a consumer generating the same utility level as observed in year 1. The proportional change factor was the quantity index. Notice that no prices are used as weights, but that the technology (utility function) has to be known.

Caves et al.(1982a) developed the Malmquist idea[2] to a productivity index proper, assuming possibly different production functions when comparing two units (e.g. the same unit at two different points in time). They make use of the Shephard concept of distance functions without noticing the direct connection with Farrell efficiency measures.[3]

[2] Caves et al. claim that Moorsteen (1961) independently had the same idea, but this seems not too well founded. Moorsteen is concerned with the interpretation of cardinal **price weigthed** indices of economies, and measures relative efficiency by calculating the ratio of output and input indices, choosing which feasible output and input mixes to keep as reference. Keeping the same mix implies, of course, proportional changes.

[3] The connection has been pointed out by Fare et al. (1985), and follows directly from the definitions of distance functions and Farrell efficiency measures.

For a start Caves et al. distinguish between output-based and input-based productivity indices. This corresponds to input saving and output increasing Farrell measures. The productivity index is based on binary comparisons. Conventionally the comparison will be for the same unit at two different points in time, but in general any two units can be considered. The units will be called unit 1 and unit 2. Only quantities are involved, and at least one technology has to be known. As a convention we will compare unit no. 2 with unit no. 1, i.e. expressions involving unit no. 2 will be in the nominator and expressions involving unit 1 will be in the denominator. The idea of the Malmquist firm 1 input-based productivity index for two units denoted 1 and 2, is to find the minimal proportional input scaling for unit no. 2 such that the scaled input vector for firm 2 and the firm 2 output vector are just on the production surface of firm 1. The definition of the Malmquist firm 1 output-based productivity index is to find the minimal proportional output scaling of unit no. 2 such that the scaled output vector and the input vector of unit no. 2 are just on the production surface of unit no. 1.

Caves et al.(1982a) assumed the units to operate on their production functions, i.e. to be efficient. Fare et al.(1989) extended the Malmquist index approach to inefficient observations like the set-up in Nishimizu and Page (1982). The extension of the Caves et al. definitions is quite straightforward, substituting "frontier technology" for "technology". The definitions above then have to take into consideration that unit no. 1 is no longer efficient. In order to make a meaningful comparison **both** units must be adjusted to the frontier technology in question.

When defining the Malmquist productivity index the definitions of the Farrell indices can be utilised directly.

Definition 3.

The Malmquist input-based productivity index, M_i^1, with frontier technology, $F_i(.) = 0$ as reference is:

$$M_i^1 = \frac{E_{i2}^1}{E_{i1}^1} = \frac{Min_{\alpha_i}\{\alpha_i : F_i(y_2, \alpha_i x_2) = 0\}}{Min_{\alpha_i}\{\alpha_i : F_i(y_1, \alpha_i x_1) = 0\}} \quad , \ i = 1,2 \tag{11}$$

Using the frontier technology relevant for unit no. 1, i.e. period 1 frontier technology, as reference the nominator shows the proportional adjustment of the observed input vector of unit no. 2 in order to be on the frontier function with observed outputs, and the denominator shows the proportional adjustment of the observed input vector of unit no. 1 for observed outputs to be on the same frontier function. The latter measure is always between zero and one, while the former measure may be greater than one. If $M_i^1 > 1$, then unit no. 2 is more productive than unit no. 1. This holds irrespective of which technology is the reference.

When using the frontier technology associated with unit no. 2 as a reference we first adjust unit no. 2 to the frontier technology in the nominator, and then the proportional scaling necessary in unit no. 1's input vector to be on the frontier is entered in the denominator.

Our definition of the firm i Malmquist output-based productivity index follows analogously:

Definition 4.

The Malmquist output-based productivity index, M_i^2, with frontier technology $F_i(.) = 0$ as reference is:

$$M_i^2 = \frac{E_{i2}^2}{E_{i1}^2} = \frac{Min_{\beta_i}\ \{\beta_i\colon F_i(\frac{1}{\beta_i}y_2,x_2) = 0\}}{Min_{\beta_i}\ \{\beta_i\colon F_i(\frac{1}{\beta_i}y_1,x_1) = 0\}} \quad ,i = 1,2 \tag{12}$$

If $M_i^2 > 1$, then unit 2 is more productive than unit 1, evaluated using period i technology.

The impact of the scale properties of the frontier functions on the Malmquist indices are not revealed explicitly in the definitions. Applying a result from Førsund and Hjalmarsson (1987,Ch.3) for the single output case in our multiple output setting we have the following relationship between the Farrell efficiency measures and scale properties:

$$\frac{\ln E_{ij}^2}{\ln E_{ij}^1} = \overline{\varepsilon_{ij}}\ ,\ i,j = 1,2 \tag{13}$$

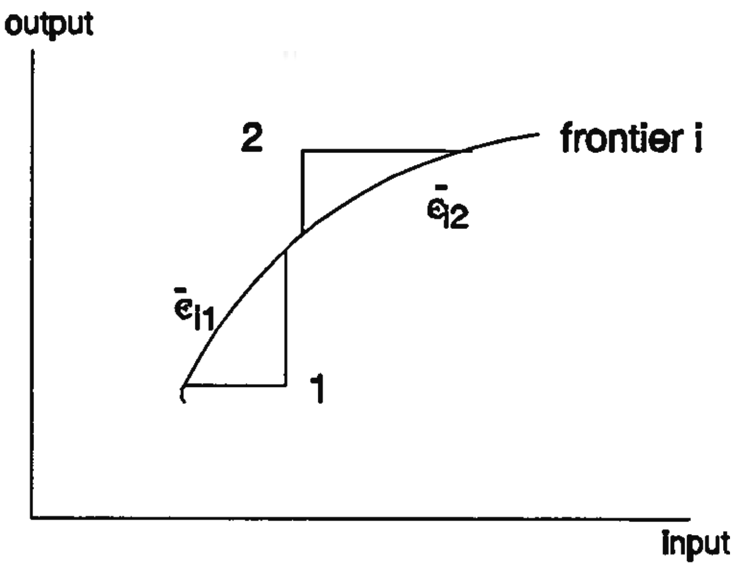

Figure 2. Average scale elasticities defined by observations 1 and 2

where $\bar{\varepsilon}$ is the average of the scale elasticity over the part of the production function spanned by the input saving and output increasing measures. An illustration in the case of one output - one input is provided in figure 2. Taking the ratio of the Malmquist indices in logarithmic form we have:

$$\frac{\ln M_i^2}{\ln M_i^1} = \frac{\ln(\frac{E_{i2}^2}{E_{i1}^2})}{\ln(\frac{E_{i2}^1}{E_{i1}^1})} = \frac{\ln E_{i2}^2 - \ln E_{i1}^2}{\ln E_{i2}^1 - \ln E_{i1}^1} = \frac{\frac{1}{\ln E_{i1}^1}\overline{\varepsilon_{i2}} - \frac{1}{\ln E_{i2}^1}\overline{\varepsilon_{i1}}}{\frac{1}{\ln E_{i1}^1} - \frac{1}{\ln E_{i2}^1}} \tag{14}$$

The ratio between the Malmquist output based and input based indices in logarithmic form is equal to a weighted average of the average scale elasticities defined for the input saving and output increasing Farrell measures for unit 2 and 1 with expressions in input saving efficiency measures for both observations as weights. When i = 1 we have that $\ln E_{11}^1 < 0$, and when i = 2 we have $\ln E_{22}^1 < 0$. If the two average scale elasticities are equal, we see that the ratio of the Malmquist indices on log form is equal to this common value. When both frontier functions exhibit constant returns to scale the Malmquist indices are equal. If we have no inefficiency, i.e. both observations are on their respective frontiers, the ratio of Malmquist indices is equal to the average scale elasticity evaluated over the part of the frontier defined by the observation with the opposite index than the technology.[4]

The Malmquist productivity index can be multiplicatively decomposed into two parts showing the catching up and the pure technology shift as for the Nishimizu and Page (1982) index shown in section 3. This decomposition is also shown in Fare et al.(1989). Following eq.(7) we have:

Proposition 1.
The Malmquist productivity index, M_i^s, can be multiplicatively decomposed into two parts:

$$M_i^s = MC^s \cdot MF_i^s \quad ,s,i = 1,2 \tag{15}$$

where MC^s is the catching up effect and MF_i^s is the frontier distance effect:

$$\begin{aligned} MC^s &= \frac{E_{22}^s}{E_{11}^s} \quad , s=1,2 \\ MF_i^s &= \frac{E_{1j}^s}{E_{2j}^s} \quad , s,i,j=1,2,i\neq j \end{aligned} \tag{16}$$

The decomposition of the Malmquist productivity index in terms of the Farrell efficiency indices can generally be written:

[4] In Caves et al.(1982a) it is stated that "output and input productivity indexes differ from each other by a factor that reflects the returns to scale" (pp.1401-1402), and Caves et al.(1981) is referred to. However, as we have seen the relationship is not so straigtforward as in the case of a continuous production framework there and assuming that the firms are efficient. When assuming that the firms operate on the frontier function we get a similar relationship between the log of Malmquist indices and the average scale elasticity as in Caves et al.(1981) between output based and input based productivity indices. But since we have two different technologies the average scale elasicity concept has to be used. The scale elasticity is not evaluated at a frontier point in our case as in Caves et al.(1981) since year j observation is not on frontier i.

$$M_i^s = \frac{E_{i2}^s}{E_{i1}^s} = \frac{E_{i2}^s}{E_{jj}^s} \cdot \frac{E_{jj}^s}{E_{i1}^s} \quad , \; s,i,j = 1,2 \; , \; i \neq j \tag{17}$$

The first expression after the last equality sign is the distance between the frontiers when the technology index is 1, and the catching up term when the technology index is 2, and vice versa for the second term. Notice that we need to know one frontier technology, no. i, to calculate the productivity index, but knowledge about two frontiers, no. i and j, to perform the decomposition.

The output based and the input based index decompose in the same way. The only difference is whether we measure parallel to the input or output plane. The interpretation of the terms is the same as in section 3: The ratio of "own" efficiency measures, E_{22}^s/E_{11}^s, represents the catching up effect of relative movement towards the frontier. The relative distance between the frontiers at point of observation j is measured by the ratio E_{1j}^s/E_{2j}^s. The distance between the frontiers is measured at the observation with the opposite index number than the technology. If both observations are on their frontiers (the Caves et al.(1982a) case) the catching up measure equals one and the Malmquist index is a pure relative frontier distance measure.

In the definitions 3-4 and Proposition 1 only two periods are considered. In a more general setting of a cross section - time series data set the question arises how to adapt the definitions. One obvious way is to do calculations on successive pair of years. The reference technology then changes. One preferable property with an index over a longer period of time is that it is possible to chain it, i.e. that the index obeys the "circular relation" of Frisch(1936). Inspecting eq.(17) we see that the Malmquist index does not chain, and as to the decomposition, neither does the frontier index, but the catching up index does chain.

When having more time periods than two the definitions 3-4 and Proposition 1 can be generalised in the sense that the index, i, for the reference technology does not have to be any of the two observation indices. However, the interpretation of the decomposition of the index is then no longer so intuitively appealing.

When keeping the technology index fixed and going through pairwise observation years we see from eq.(17) that the Malmquist index does chain, but neither of the decomposed parts. Since the interpretations of them do not come through well anyway, this may not be a drawback. If, for instance, the reference technology is the first year frontier and we are measuring the productivity change between the last two years, the catching up effect does only contain one "own" efficiency measure, e.g. the efficiency for the last year. The other efficiency term relates the next to last year observation to the frontier technology of the first year. The frontier distance index makes more sense. It measures the distance between one of the last two years and the reference frontier measured at the corresponding observation point.

In Fare et al.(1989), Fare et al.(1990) the Malmquist productivity index is defined as the geometric mean of the two indices one get changing the reference technology:

$$M^{s} = \sqrt{M_1^s M_2^s} = \frac{E_{22}^s}{E_{11}^s}\sqrt{\frac{E_{11}^s}{E_{21}^s}\cdot\frac{E_{12}^s}{E_{22}^s}} \quad ,s=1,2 \tag{18}$$

The catching up index remains the same as in (17), but the frontier index is the geometric mean of the distances between the frontiers at both observations. Notice that using (18) as definition requires knowledge of both frontier technologies to calculate the index, whereas our definitions 3 or 4 require only one.

Caves et al.(1982a) is referred to when introducing (18). However, Caves et al. introduce the geometric mean in order to show the connection between the Malmquist and the Tørnqvist indices in the case of the distance functions being of the translog form. We will therefore maintain the definitions 3 and 4 as the proper definitions of the Malmquist productivity index. The chaining problem is not adressed in Fare et al. and successive change of reference technology is adapted. Inspecting eq.(18) we see that the catching up index does chain, but neither the total index nor the frontier index.

Taking the geometric mean may be utilised in the context of general cross section - time series data, however. A fixed reference technology creates the usual problems when moving away from the base year (see Berg et al, 1991, for an application to Norwegian banks). One way of overcoming this problem and at the same time preserving the circular relation is to use two frontiers, for instance the first and the last year of the data set, as technology reference, and take the geometric mean when calculating the Malmquist index between two intermediate years, t_1 and t_2[5]:

$$M_{0T}^{s}(t_1,t_2) = \sqrt{M_0^{s}(t_1,t_2)\cdot M_T^{s}(t_1,t_2)} \quad ,s=1,2 \tag{19}$$

The first year used as reference is dated 0 and the last T. The circular relation for this index is preserved But when using eq.(19) the circular relation does not hold neither for the catching up nor the frontier distance indices.

5. Concluding remarks

The most common approach when measuring productivity change from discrete data is to weigh together inputs and outputs by price weights. Appealing to neo-classical economic adjustment of the firms this approach can be consistent with underlying general, unknown production functions. However, the assumption of all firms being efficient is often counter to the very motivation for mesasuring productivity at the firm or micro unit level. At this real level inefficiency is the rule. The Malmquist productivity index therefore offers a way to measure productivity for micro units in a environment of inefficiency. The index is only based on quantity variables, but knowledge about at least one frontier production function is required. The Malmquist index and its parts can be expressed exclusively in terms of Farrell efficiency measures, so the need for production function knowledge boils down to being able to calculate these measures. Working with non-parametric frontiers these are most easily established by directly calculating Farrell measures. The opinion in Caves et al.(1982a) that .. "without knowledge of the parameters (of the production function) neither (Malmquist) index can be computed. Thus the empirical usefulness of the Malmquist indexes is limited.", is rather too pessimistic.

[5] This idea is due to Sigbjørn Atle Berg.

References

Berg, S.A., F.R.Førsund and E.S.Jansen (1991): "Malmquist productivity growth during the deregulation of Norwegian banking 1980-1989", **Working paper**, Bank of Norway.

Caves, D.W., L.R. Christensen and J. A. Swanson (1981): "Productivity growth, scale Economies, and capacity utilization in U.S. railroads, 1955-1974", **American Economic Review**, 71, 994-1002.

Caves, D.,W., L.R. Christensen, and W.E.Diewert (1982a): "The economic theory of index numbers and the measurement of input, output, and productivity", **Econometrica**, 50, 1393-1414.

Caves, D.,W., L.R. Christensen and W.E. Diewert (1982b): "Multilateral comparisons of output, input, and productivity using superlative index numbers", **Economic Journal**, 92, 73-86.

Farrell, M. J. (1957): "The measurement of productive efficiency", **Journal of the Royal Statistical Society, Series A**, 120 (III), 253-281.

Frisch, R.(1936):"Annual survey of general economic theory: the problem of index numbers", **Econometrica**, IV, 1-38.

Fare, R., S. Grosskopf, and C.A.K. Lovell (1985): **The measurement of efficiency of production**, Kluwer-Nijhoff Publishing, Boston, MA.

Fare, R., S. Grosskopf,B. Lindgren and P. Roos (1989): "Productivity developments in Swedish hospitals", **Mimeo.**

Fare, R., S. Grosskopf, S. Yaisawarng, S.K. Li and Z. Wang (1990): " Productivity growth in Illinois electric utilities", **Resources and Energy**, forthcoming.

Førsund, F.R. and L. Hjalmarsson (1974): "On the measurement of productive efficiency", **Swedish Journal of Economics**, 76, 141-154.

Førsund, F.R. and L. Hjalmarsson (1979a): "Frontier production functions and technical progress: Astudy of general milk processing in Swedish dairy plants", **Econometrica**, 47, 883-900.

Førsund, F.R. and L. Hjalmarsson (1979b): "Generalised Farrell measures of efficiency: An application to milk processing in Swedish dairy plants", **Economic Journal**, 89, 294-315.

Førsund, F.R. and L. Hjalmarsson (1983): "Technical progress and structural change in the Swedish cement industry 1955-1979", **Econometrica**, 51, 1449-1467.

Førsund, F.R. and L. Hjalmarsson (1987): **Analyses of industrial structure: A putty-clay approach**, The Industrial Institute for Economic and Social Research, Stockholm: Almqvist and Wiksell International.

Førsund, F.R., C.A.K. Lovell and P. Schmidt (1980): "A survey of frontier production functions and their relationship to efficiency measurement", **Journal of Econometrics**, 13, 5-25.

Johansen, L.(1972): **Production Functions**, Amsterdam: North- Holland.

Malmquist, S.(1953): "Index numbers and indifference surfaces", **Trabajos de Estatistica**, 4, 209-242.

Maywald, K.(1957): "The best and the average in productivity studies and in long-term forecasting", **The Productivity Measurement Review**, 9, 37-49.

Moorsteen, R.H.(1961): "On measuring productive potential and relative efficiency", **Quarterly Journal of Economics**, 75, 451-467.

Nishimizu, M. and J.M. Page, Jr. (1982): "Total factor productivity growth, technological progress and technical efficiency change: Dimensions of productivity change in Yugoslavia, 1965-78", **Economic Journal**, 92, 920-936.

Salter, W.E.G. (1960): **Productivity and technical change**, Cambridge: Cambridge University Press.

Shephard, R.W.(1953): **Cost and Production Functions**, Princeton: Princeton University Press.

Perelman, S. and P.Pestieau (1988): "Technical performance in public enterprises: A comparative study of railways and postal services", **European Economic Review**, 32, 432-441.

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