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**Optimal bailout during currency and financial crises: A sequential game analysis**

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# Optimal bailout during currency and financial crises: A sequential game analysis*

By

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**ABSTRACT**

We present a model that illustrates the close relationship between the possibility of a currency crisis and the amount of private-sector debt within a four-stage sequential game framework. In the first stage, the government announces its exchange rate policy, and all agents in the economy receive probabilistic information about a future shock that will occur in the last stage. This shock will affect unemployment and net returns on private sector investment. The private sector in stage 2 forms expectations about the future exchange rate and engages in risky investments. In stage 3, the government faces costs due to expectations of future devaluation and private-sector debt, anticipating the stochastic shock that will occur in stage 4 and may or may not find it optimal to pre-emptively abandon its fixed exchange rate policy. The government can commit already in stage 1 to bailing out part of the private sector's outstanding debt if a bad shock occurs or wait until stage 4 to give an optimal bailout. A commitment to bailing out provides a reconciliation of the multiple equilibria that result from self-fulfilling expectations. Moreover, the government may sometimes avert currency crises by committing to bailing out.

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## 1. Introduction

In light of the events that have occurred in East Asia and Latin America since 1997, many questions have been raised in about the causes of currency and financial crises. There is already a growing literature on both types of crisis. This study will mainly focus on the following two questions:

i) Can bailouts, a) effectively ameliorate the effects that costly liquidation associated with financial crisis can have on the fundamentals, and b) avoid the currency crises that usually follow a panic in the financial sector.

ii) Can financial and/or currency crises be caused by inconsistent and unsustainable macroeconomic problems, short-term domestic foreign currency debt, and/or self-fulfilling expectations.

It has been argued that a safety net may increase the moral hazard incentives for excessive risk-taking on the part of banks (Schwartz (1998) and Bordo and Schwartz (2000). It could also arise moral hazard on behalf of the claim-holders that will not exert a sufficient effort in closely monitoring the institutions, as they know that in case of default, they will be bailed out, as Calomiris (1999) argues when analysing the crisis of Mexico, Korea and Thailand. A regulatory and supervisory system is therefore also necessary to reduce excessive risk-taking in the financial system (Mishkin (1999)).

In developed countries, the domestic central bank can control the supply of domestic currency without losing much of its credibility. In addition to a sound national currency, a history of price stability plays a very important role in increasing the incentive for counterparts to enter into contracts of sufficient duration. Under these conditions, it is easier for the financial sector to raise capital with debt denominated in domestic currency. In emerging economies, however, the situation is quite different. These economies are usually characterised by highly variable inflation causing the policymakers to have credibility

problems. Thus, an expansionary monetary policy to promote recovery from a financial crisis is likely to lead to higher inflation and depreciation of the national currency. Consequently, the international financial sector has very few incentives to sign contracts denominated in the domestic currency and foreign lending, to a large extent, usually takes the form of foreign currency loans. Thus, central banks in these economies lose control over foreign claims on the economy because they are denominated in a foreign currency. We argue here that in such cases, a lender-of-last-resort (LOLR), being the domestic central bank and/or the IMF, may be useful for shoring up balance sheets in order to ameliorate the effects of a financial crisis, should it occur. Freixas (2001) indicates that the existence of a LOLR is typically justified by the possibility of a systemic financial crisis, defined as a crisis where the standard mechanisms and financial channels stop working, thus impairing the well-functioning of the payment side of economic transactions. Mishkin (1995) and Freixas, Parigi and Rochet (1998) find that there could be cases where bailing out banks may be efficient. Most people would agree that if a LOLR guarantees at least some of the outstanding stock of liabilities, such guarantee should be of a short-run type, and for it to be effective, it has to be implemented very quickly. Less intervention will be subsequently required, once market participants realise that some necessary liquidity has been injected into the system, thus decreasing their uncertainty.[1] Hopefully, this is what the IMF had in mind when it provided Brazil with liquidity on August 8, 2002. That is to breaking the panic and market turbulence that Brazil was experiencing, in spite of having sound fundamentals. Brazil may now have the opportunity to service its debt to dilute uncertainty about future economic perspectives while the private sector may also be in a better position to obtain capital funding from both national

[1] On the second weekend of July 2000, finance ministers from the G-7 agreed a radical plan of reform for the lending facilities of the IMF. This is intended to improve the IMF's effectiveness in tackling financial crises, while extracting it from long-term lending that could equally well be provided by the private sector. It was agreed to have to cut the cost of the Contingent Credit Line –a short-term facility intended for well-run countries affected by financial contagion that has so far not been used at all. The IMF hopes that more countries will be encouraged to use it to prevent crises developing and spreading. (*Financial Times*, Monday July 10th, 2000).

and international markets at reasonable terms. A new collapse of the Brasilian exchange rate can then be avoided (Williamson (2002).

There may be a clear consensus on the necessary intervention of the LOLR at the macroeconomic level, providing liquidity to the institutions when the market liquidity dries up. However, the role of such interventions at the microeconomic level (to specific institutions) is still much of a debate and not final conclusions have been established yet. One can agree that there has been a great development in technology and financial environment such as emergence of repo markets and of real time gross payment systems facilitating monetary transfer from one institution to another, deposit insurance, higher banking competition, and strict regulation of solvency capital. The relevant literature then sustains that in these days there is no need for a LOLR at the microeconomic level. Still, even if this may be reasonable, it would be harder to argue that such policy is credible. Time consistency problems are likely to arise. Is it then credible to commit to no-intervention at all when ex-post maybe be optimal to avoid sistemic risk and all the costs that this involves? (Freixas (2001)).

A main result of this paper is that the government will generally have incentives to pre-commit to making such bailouts. Ex-ante commitments to optimal bailouts are "good" in that they facilitate the achievement of a more efficient unique equilibrium (with small devaluation or none at all) and lesser detrimental of the fundamentals that could otherwise occur due to potential financial crisis.

With respect to the issue of the mechanisms of the twin crisis, Allen and Gale (2000) argue that although banking crises typically precede currency crises, the common cause of both is usually a fall in asset values due to a recession or a weak economy. They find it difficult to explain the selection of one exchange rate equilibrium or another by "sunspots", i.e. random events (shifting beliefs that are self-fulfilling) unrelated to the real economy. Recent empirical

work has also attempted to study empirically the most likely causes of currency crises and their relationship with credit problems. Studies by the IMF (1998) and Kaminsky and Reinhart (1999) linking banking and currency crises, find that most currency crises are preceded by a build-up of private-sector debt, usually after a boom. Glick and Hutchison (1999) study the empirical relationship between currency crises and banking crises over the period 1975-97, and find a substantial correlation that is highest for the East Asian economies. Eichengreen and Rose (1998) and Rossi (1999) reach similar conclusions for subsets of developing countries. The consensus appears to be that, at least in some cases, financial crises may have "caused" exchange rate crises; in many cases, however, the twin phenomena are likely to be symptoms of underlying weaknesses of the economy, which may be manifested in various ways.[2]

The purpose of this paper is to study the interrelationship between exchange rate crises and debt crises. The model here shows that crises may be both belief-driven and fundamentals-based attacks. The model is presented as a four-stage sequential game in which the players are the government and the private sector. By private sector we mean both firms and financial institutions. As Tirole (2002) discusses, firms and financial institutions satisfy their liquidity (what he calls *inside liquidity*) needs by selling the securities that they usually hold in other firms and financial institutions. Holmstrøm and Tirole (2001) find this *inside liquidity* to be sufficient for the private sector's needs as long as i) the shock faced by private sector entities are independent, and ii) inside liquidity is properly allocated within the private sector. Otherwise *outside liquidity* will be needed and this is where the LOLR will have a function. Our analysis is based on the assumption that there are instead aggregate shocks and that the condition (ii) is not satisfied, as is usually the case in emerging economies and as it is

[2] See also Miller (1996), who considers the opposite type of causation, from a currency attack to a banking crisis.

empirically observable for most of us. *The private sector* must then search for outside insurance.[3]

The sequence of our game is as follows. Information about the probabilistic distribution of a forthcoming (good or bad) shock is given in the *first stage*. This shock will be realised in the fourth stage and will affect private-sector debt and unemployment. In this *first stage*, the government announces that it will pursue a fixed exchange rate regime and it may or may not already at this stage commit to an optimal bailout only if a bad shock occurs.[4] It will also incur certain (fixed, constant and not further explained) costs when the fixed exchange rate regime is abandoned. In the *second stage*, private-sector agents form expectations about the future exchange rate, triggering speculative attacks in the market, and in addition undertake risky investment activities. The private sector borrows in foreign currency to produces a good for export in the international market. Thus, revenues are obtained in foreign currency: If a bad shock occurs, the net returns of investment become negative and a devaluation will make it even more negative. *In stage 3*, the government may react by either retaining or abandoning the peg. It should be noted that defending the exchange rate regime against speculative attacks during this stage implies costs in the interim period for the government and the economy in the form of adverse shifts in the economic fundamentals. The latter is a result of having expectations that a bad shock will occur, and of the government's policy to defend the exchange rate regime against a speculative attack at this stage. In *stage 3*, the regime may be abandoned after the onset of the speculative attack, before it does further significant damage to the economy. This outcome, where a self-fulfilling speculative attack leads necessarily to devaluation, resembles the Krugman (1979) first-generation model, where a deterioration of

[3] We do not here emphasise on whether such outside insurance should be created by the existence of domestic outside liquidity or foreign outside liquidity. For our purpose, we do not distinguish between those two. See Tirole (2002) for further discussion.

[4] This policy does not need to be interpreted as strictly as a fixed exchange rate regime, but it can be more realistically thought of as having an exchange rate target.

the fundamentals (in addition to the market's anticipation of a financial crisis) plays an important role in triggering crises. In the *final stage 4,* the shock realises and the government decides on two things: the optimal bailout, if it has not committed already in stage 1; and whether or not to abandon the fixed-rate regime (given that the fixed rate has been retained up to this stage). Decisions at this stage depend on the state of the economy.

We show that when *no commitment* has been made to a specific bailout, the model may yield multiple (2 or 3) equilibria. One of the equilibria involves always retaining the fixed peg *when the exchange rate regime is not expected to be abandoned* and there are no speculative attacks, even in the event of a bad shock and cumulative investment losses. *When devaluation is expected*, two more equilibria are possible. One involves abandoning the peg only in the worst state, and the last one that can occur in stage 3 involves always abandoning the peg. The last equilibrium is most likely reached when there is a sufficiently high prior probability that a bad shock to debt and unemployment will occur and/or if shocks are quite serious when they do occur.

It turns out, however, that a *commitment* to a bailout (*only if a bad shock occurs*) in stage 1, expressly announced before the market forms expectations, has very important implications for the outcome of this sequential game. Specifically, we show that it is possible to obtain a unique equilibrium with not devaluation or smaller devaluation than in the case when there is no commitment, in spite of possibilities of cumulative investment losses. Committing to bail out guarantees a less drastic impact of the outstanding debt on the economic fundamentals. The private sector takes this into account when forming expectations: A commitment will cause private sector debt to have more limited adverse effects on the fundamentals. Therefore, devaluation (or large devaluation) is regarded by the private sector as less likely to take place.

Thus, ex-ante commitments to optimal bailouts serve as a strategic device for the government since they facilitate the implementation of a more efficient unique equilibrium

(without devaluation or a small devaluation) when the government has a potential problem of credibility for its exchange rate policy. Such an equilibrium is not a sunspot equilibrium, like the three multiple equilibria mentioned above, when there was no ex-ante commitment to bailing out.

The paper is organised as follows: The next section highlights some of the many contributions to the literature on currency and financial crises. Section 3 presents an overview of the stages of the sequential game; section 4 presents the government's problem, while section 5 presents the representative investor's problem. Section 6 aims to show the optimal solutions in stage 4, both with and without speculative attacks and for both favourable and unfavourable state of the economy. Section 7 shows the possible optimal decisions that can be taken in stage 3, while section 8 presents the government's problem when it decides to commit to a bailout at stage 1. Section 9 presents numerical solutions to the non-commitment and commitment cases and finally, section 10 presents a conclusion.

## 2. Background to the literature

As noted, this paper proposes a new theoretical approach to several different strands of literature, in particular to the "first-" and "second-generation" models of currency crises. It also presents certain characteristics of both models. Henderson and Salant (1978), Krugman (1979) and Flood and Garber (1984) represent the "first-generation" models, in which a devaluation entails no reputational cost but a currency crisis arises as a necessary consequence of adverse fundamentals. Our model also presents an "endogenous-policy" model of currency crises such as the ones pioneered by Obstfeld (1986, 1994, 1996). The government rationally chooses – on the basis of their assessment of costs and benefits in terms of social welfare - whether or not to maintain a fixed exchange rate regime. A crisis is driven by self-validating shifts in expectations where multiple equilibria are possible.

There is one important feature of the model in this paper that makes it different from the "second-generation" models. In these "second-generation" models, private sector expectations are assumed to be formed at the "ex ante" stage, while the government's decision to retain or abandon the fixed exchange rate regime is made "ex post", after the realisation of some stochastic variable. Our approach, in contrast to these models, is one in which the government's own possible actions at the "ex ante" stage are also considered, which they ought to be if "ex ante" and "ex post" are separated in real time.

Morris and Shin (1998 a,b) have extended the basic results of Obstfeld's (1995) model to the case with less than common beliefs about the actual game played. By assuming that agents in the economy may not have common knowledge of the underlying fundamentals, they show that such uncertainty about the beliefs of others regarding the fundamentals may yield a single course of action leading to uniqueness of equilibrium. Extensions of the present model incorporating uncertainty about beliefs should, however, be pursued in future work.

A vast literature has already attempted to explain the ERM 1992 crisis, the 1994 crisis in Mexico, or the recent (1997) financial and currency crisis in Southeast Asia,[5] and analyses such phenomena by considering the simultaneous modelling of financial and currency crisis. Chang and Velasco (1998) consider the interaction between bank fragility, exchange rate and monetary regimes on the basis of the Diamond and Dybvig (1983) model of bank runs. They find that different exchange rate and monetary regimes induce different real consumption allocations and imply different degrees of financial fragility. They find that there may be a close relationship between financial and currency crises, in that maintaining a fixed exchange rate peg and stabilizing the banking sector may become mutually incompatible objectives. In their set-up, there are multiple equilibria, with the crisis brought on by a pure shift in expectations. Allen and Gale (2000) take a step further from the approach of Chang and

[5] An early example of contribution to this literature was Velasco (1987).

Velasco (1998) and argue that it is problematic to obtain multiple equilibria because the selection between the good and the bad equilibrium is not modelled. They consider both the cases where countries can issue debt denominated in their own domestic currency (e.g. advanced industrial economies) and where they ought to issue bonds denominated in the foreign reserve currency (e.g. emerging economies). The latter occurs because the country lacks financial discipline and the lenders are then not willing to buy bonds denominated in the domestic currency. They show that when there is a combination of an appropriate exchange rate policy and borrowing and lending by banks in the international capital market, there will be optimal risk sharing, and inefficient liquidation of investment leading to bankruptcy can be prevented as well. However, this will be possible if and only if the debt is denominated in the domestic currency. In emerging economies, the authorities cannot adjust foreign claims on the domestic economy because the debt is in foreign currency. It is here that an international institution can play an important role; by providing liquidity to prevent inefficient liquidation of banks. The present paper does not consider the issue of optimal risk sharing, but suggests that bailouts provided by the government (in a industrialised economy) or a lender of last resort such as the IMF (in a emerging economy), could provide the necessary liquidity and prevent insolvency. This is important in a world where there are strong financial links between countries but each individual country has very few incentives to provide itself with liquidity, as Allen and Gale (2000) also suggest.

Burnside et al. (2000) also study the connection between banking crises and currency crises. In their study, government guarantees to repay bank's foreign loans are contingent on devaluation occurring. This causes the investors to expose themselves to exchange rate risk and to declare insolvency when devaluation occurs. Such a guarantee scheme introduces the possibility of self-fulfilling currency crises, because the private sector is given incentives to expect devaluation, given that the government will finance banks' insolvency with bailouts.

By bailing banks out, the government validates the expectations of the private sector. Chari and Kehoe (2000) take another approach by modelling investors as having herd-like behaviour. In their paper, investors have to choose between investing in a risky project in the emerging economy or in a safe project in their domestic economy. In each period a signal about the profitability of the risky project reaches the economy and is privately observed by one of the investors. Investors observe the aggregate amount of investment in each period and optimally decide whether to invest or to wait for more information. Waiting longer is costly. If the signals lead investors to be sufficiently optimistic, they choose to forgo the opportunity to acquire information and they all immediately invest in the emerging economy. If investors become sufficiently pessimistic they all invest in their home economy and capital flows to the emerging economy dry up completely. This is called herd behaviour.

Aghion et al. (2000) present a model that analyses what they define as "triple" crises, which involve currency, banking and output crises where multiple equilibria in the foreign exchange market are possible. In their dynamic model, prices are rigid in the short run and currency depreciation leads to an increase in the foreign currency debt repayment obligations of the firms that will consequently have reduced profits. Lower profits will in turn mean lower net worth, leading to less investment and lower output in the next period.

To our knowledge, this paper presents the first sequential-game model integrating the issues of exchange rate crisis and financial crisis. As such, it represents an advance, but we nevertheless recognise some weaknesses in the model that should be remedied in future work. First and foremost perhaps, the concept of private-sector debt is introduced in a somewhat rudimentary manner, simply as a factor contributing to a worsening of the unemployment problem. In particular, we do not consider the actual process of debt formation in the private sector. Second, we have no theory as to how and why the private sector may expect a

devaluation and decide to attack the domestic currency. Last, we assume that the cost to the government of leaving the fixed-rate policy is a constant.

## 3. Stages of the sequential-game theoretical model

The four stages are represented by figure 1.

### Stage 1

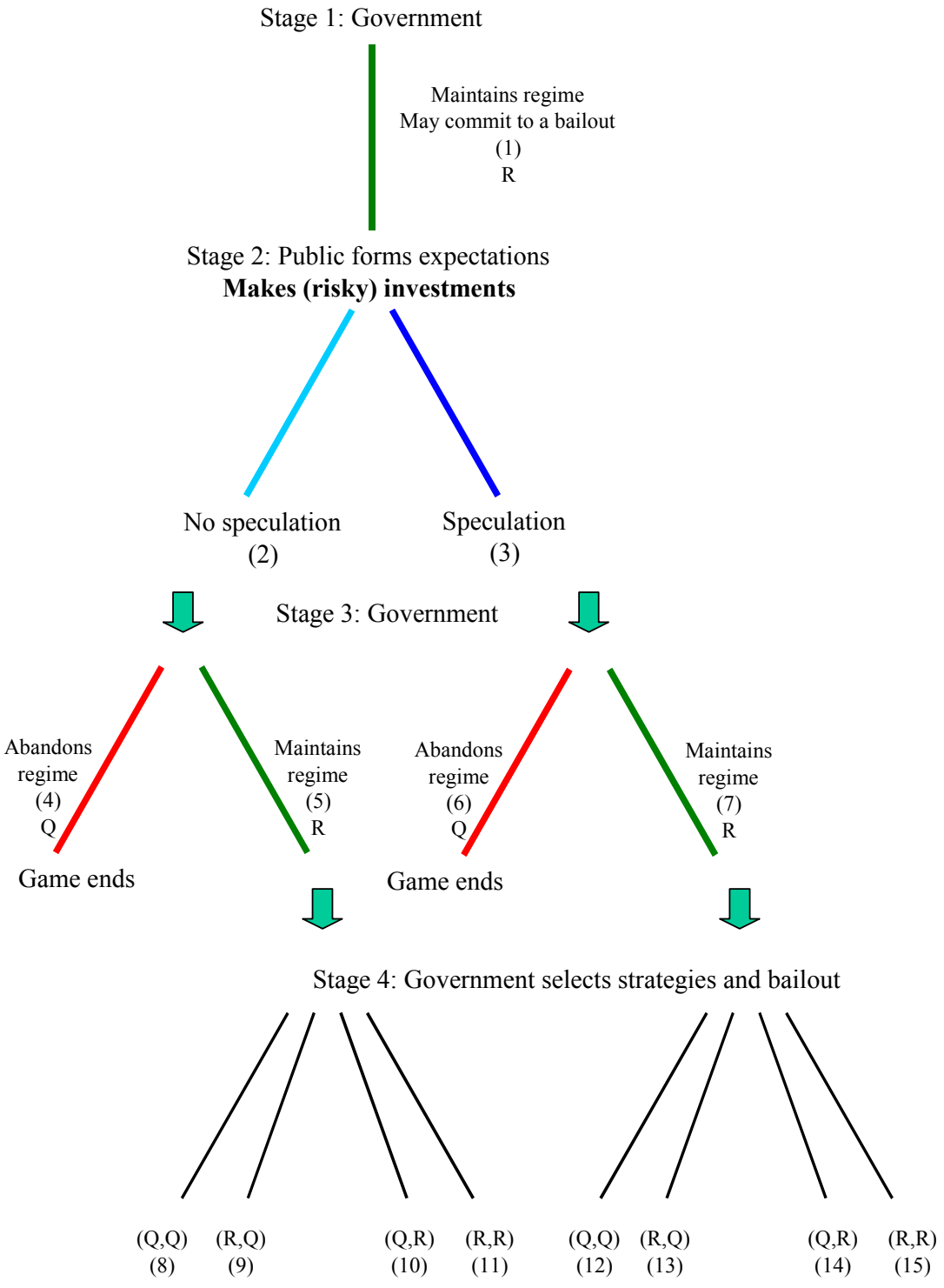
All the agents in the economy receive probabilistic information about a future shock that will occur in the last stage, affecting unemployment and the net returns on investments made by the private sector. There will be a bad shock, $s_1$, or a good shock, $s_2$, which will occur with probability $\pi$ and $(1-\pi)$, respectively. We assume that $s_2$ takes the value of zero. That is, if $s_2$ occurs, this shock will have no effect on the economic fundamentals. At this stage, the government announces that it will be pursuing a fixed exchange rate regime and may or may not commit to bailing out part of the outstanding debt of the private sector.

### Stage 2

If it is assumed that the regime is maintained until stage 4:

- The private sector forms expectations about the exchange rate that will be set by the government in stage 4 after the shock has occurred.
- The private sector makes (risky) investments to produce a good for export in the international market and incurs in costs of investment that result from borrowing in foreign currency. The final net return to investment depends on the shock and on the exchange rate determined in stage 4. The private sector uses these final net returns to consume in the national market. The lower such net returns are, the lower the consumption.

**Figure 1: Game tree describing possible strategies in the sequential game**




Q represents a decision to abandon the regime; R represents a decision to maintain the regime. (m,n), m=R or Q if shock of type one, $s_1$, occurs; and n=R or Q if shock of type two, $s_2$, occurs.

**Stage 3**

Only the government moves. It defends the fixed exchange regime against any speculative attack, or it may give up this regime already at this stage. We find that when there is no speculative attack, the equilibrium strategy of the government involves retaining the fixed exchange rate regime. However, as soon as speculative attacks start, the government incurs certain costs that may lead it to abandon the regime already at this stage. These costs take the form of first, expected adverse shifts in unemployment because of changes in the interest rate. The government uses interest rates to defend the exchange rate regime when there are speculative attacks. The other additional costs are the expected loss of credibility (given that devaluation may occur), and the expected net loss of investment, which could always worsen in the event of devaluation and an adverse shock. If the exchange rate is abandoned at some point in this stage, the government will still face these costs until it decides to abandon the regime during this stage.

**Stage 4**

A stochastic shock, either $s_1$ or $s_2$, occurs and affects employment and the net returns on investment. Also at this stage, the government decides on the value of the exchange rate and the proportion of the private liabilities of outstanding stock that will be bailed out if $s_1$ occurs and no ex-ante commitment to bailing out has been made in stage 1.[6]

A few additional introductory remarks are in order. First, we assume that whenever the government abandons the fixed exchange rate, we say that the game ”ends”; this applies to the game of setting the exchange rate, which is left entirely to competitive market forces from then on. Even in such cases, however, the government may still need to make decisions about

[6] See Freixas (2000) for a description of the different ways of providing liquidity guarantees and managing crises.

bailouts in stage 4. We will come back to this below. Secondly, in stage 4 there are 8 possible end nodes for the game (provided that the game progresses to that stage). Few of these can be reached. We note that a rational expectations equilibrium can be compatible only with end nodes (11) (corresponding to maintaining the fixed rate in both states) and (14) (giving up the fixed rate in state 1, but not in state 2). Thirdly, the structure of payoffs in the game is such that end node (12) can also be counted out. The reason is that if the strategy combination (Q,Q) were relevant in stage 4, the government would always anticipate this and quit preemptively already in stage 3, thus avoiding the costs of a speculative attack in stage 4 (i.e., one actually ends up in node 6). We will in the following disregard possible cases with the strategy combinations (Q,Q) and (R,Q) and assume that the fixed rate will never be abandoned in stage 4 when a ”good” shock occurs in that stage.

**4. The private sector's problem**

The decisions of the private sector plays a most important role in the model when the government decides to make an ex-ante commitment to bailout at stage one, that is, before the market forms expectations. As explained above, it is basically for the purpose of influencing these expectations that the government may want to make commitments on bailouts. It is important to keep in mind that when the private sector form expectations about the exchange rate (deciding whether to speculate or not) in stage 2, it also decides on the level of risky investment it will engage in. Therefore, influencing expectations also implies influencing the level of risky investment that the private sector will make.

K will denote private-sector investment and $R_i(K)$ the net gross return on these investments, in the final stage, 4, when the shock i ($s_i$, i=1,2) or state i is realised (i=1, 2). We make the following assumptions about the net gross return functions:

When *$s_1$ occurs*: $R_1(K) < 0$; $R''_1(K)<0$ in the bad state 1.

When *$s_2$ occurs*: $R_2(K) \geq 0$; $R''_2(K)<0$ in the good state 2.

Investments are risky and use to produce a good for export in the international market. $R_1(K)$ is negative as a result of the occurrence of a bad shock. This shock can be interpreted as in the bad state due to self-fulfilling panics, bank runs, higher borrowing costs (either abroad or in the national market), and/or difficulties in selling abroad (i.e. the rest of the world is in recession).

The private sector maximises expected net return with respect to the amount invested, taking into consideration that a fraction $\phi$ of the net losses will be bailed out by the government *only* in state 1. The expected net return function, NR, is as follows:

$$E[NR] = \pi (1-\phi(s_1))(x(s_1)/\bar{x}) [R_1(K)] + (1-\pi)(x(s_2)/\bar{x}) [R_2(K)], \quad (1)$$

where:

- $x(s_i)$ is the (log) exchange rate (number of domestic currency units per unit of foreign currency).
- $\bar{x}$ can be interpreted either as the (log) fixed exchange rate level or the long-run equilibrium level that the government wishes to attain at every t. We normalised to be equal one.
- $\phi(s_1)$ takes values between 0 and 1 and also $\phi(s_2)$ equal zero. The absolute value of $[\phi(s_1) (x(s_1)/\bar{x}) R_1(K)]$ is the total bailout in state 1. $(1-\phi(s_1))(x(s_1)/\bar{x})R_1(K)$ is then the net loss of the private sector. Note that the realisation of a bad shock reduces the investor's expected return (1).

Maximising (1) with respect to K yields the following first-order condition:

$$\frac{R_1'(K)}{R_2'(K)} = -\frac{(1-\pi)x(s_2)}{\pi(1-\phi)x(s_1)} \quad (2)$$

In (2), the left-hand side is the ratio between the marginal net return on investment in the bad state and the good state. For (2) to hold true, the marginal net return on a given

investment should be positive in one of the states (state 2) while negative in the other state (state 1) at the optimal solution for the representative agent of the private sector. K will depend on the exchange rates and the bailout. We derive the following partial derivatives:

$$\frac{\partial K}{\partial \phi} = \frac{\pi R_1'(K)x(s_1)}{(1-\pi)R_2''(K)x(s_2) + \pi(1-\phi)R_1''(K)x(s_1)} > 0 \text{ ; and} \tag{3}$$

$$\frac{\partial K}{\partial x(s_1)} = \frac{-\pi(R_1'(K))(1-\phi)}{(1-\pi)R_2''(K)x(s_2) + \pi(1-\phi)R_1''(K)x(s_1)} < 0 \text{ ;} \tag{4}$$

if the marginal net return on a given investment in state 1 ($R'_1$) is negative. (3) indicates that the anticipation of a greater fraction of bailouts, $\phi$, in the "bad" future state (1), all else being equal, thus leads to higher investment more, and consequently to more debt, in stage 2. From (4), an increase in $x(s_1)$, i.e. a stage 4 devaluation which is rationally anticipated by the private sector in stage 2, makes them incur less debt. (3) then indicates that the private sector would have a greater incentive to prefer high levels of risky investments, such as K, the larger the bailout. Thus, the possibility of a safety net evidently causes a moral hazard problem. However, since the government will only provide a bailout in the case that a "bad" shock occurs, this may cause the private sector to choose a lower K than they would otherwise would because of the uncertainty as to what stage of the economy will prevail in stage 4. Note also that when devaluation is expected, there will be less risky investments. We note that we are not modelling how different types of investment may be chosen, e.g. in terms of their riskiness, in response, for example, to $\phi$.

## 5. The government's problem

We assume that the government does not like unemployment, high bailouts and any outstanding debt. These variables will enter the government's loss function.

The government expected loss function when the fixed exchange rate is retained until stage 3 is the following:[7]

$$V_3(s_i, Ex, x(s_i), \phi(s_i)) = a_1(u_3 - u_n)^2 + a_2 E(u_4 - u_n)^2 - E\phi(s_i)(x(s_i)/\bar{x})R_i(K) + EC \quad (5)$$

The loss function in stage 4 is:

$$V_4(s_i, Ex, x(s_i), \phi(s_i) = a_2(u_4 - u_n)^2 - \phi(s_i)(x(s_i)/\bar{x})R_i(K) + \lambda C \quad (6)$$

- The shock of type 2, $s_2$, always equal zero.
- All parameters are greater than zero.
- E represents the conditional expectations operator.
- C represents the costs of abandoning the fixed exchange regime, for example through loss of credibility. On the other hand, $\lambda$ is a dummy variable that takes the value of one when the fixed-rate policy is abandoned, and zero otherwise.
- $u_j$ is the actual rate of unemployment in stage j (j=3 or 4).
- $u_n$ is its natural rate level.

We assume that the unemployment rate in state i in stage 4 is determined as follows:

$$u_4(s_i) \equiv u(x(s_i)) = -\alpha_1[x(s_i) - \bar{x}] + u_n + \alpha_2 r + s_i - \alpha_3[(1 - \phi(s_1))(x(s_1)/\bar{x})R_i(K)]; \quad (7)$$

In (7), the first term implies that currency devaluation ex-post in stage 4 reduces unemployment. Private-sector debt problems however have negative consequences for employment in state 1 because debts lead to insolvency and bankruptcies and consequently job losses. Any unemployment gap can however be influenced by the bailout. Moreover, the government may use high interest rates to stave off a speculative attack or eliminate devaluation expectations, but increases in interest rates may slow down economic activity and therefore yield higher unemployment rates. Thus, the following relationship is assumed:

[7] Note that we use still $\bar{x}$ even though it is assumed to be equal 1.

$$r = g(Ex - \bar{x}); \tag{8}$$

where g is a positive constant. Ex- $\bar{x}$ represents the magnitude of the speculative attack/ devaluation expectations that the government may try to fight against in stages 3 and 4 by increasing interest rates.

Finally, note that the government loss function in the third stage (5) includes the costs of excess unemployment originated by the speculative attacks at that stage plus the expected ones in stage 4. (5) also includes the expected costs of any outstanding debt and of bailing out.

**6. Stage 4**

**6.1 There are <u>no devaluation expectations (or speculative attacks)</u> (Ex= $\bar{x}$ ) and this conditions the level of risky investment, say $\widetilde{K}$ , chosen also by the private sector. The government makes <u>no commitment to a specific bailout</u> before stage 4.**

**Proposition 1:** *A rational expectations equilibrium can only be compatible with the end node (11), in figure 1. This node corresponds to the decision of maintaining the fixed exchange rate when either shock takes place. Node (11) implies that:*

$$x(s_1) = \bar{x};$$
$$x(s_2) = \bar{x}$$

**Proof:**

i) Nodes (8), (9) and (10) cannot be possible equilibria. If the government is certain to abandon the regime when either shock occurs, the private sector will always anticipate this and will always expect devaluation and speculate.

ii) When there are not devaluation expectations, the end nodes (8) and (9) in figure 1 must be counted out: It will not be optimal to devalue in state 2. Note that in state 2, by assumption, the shock and the outstanding debt are zero so that no bailout will be

necessary and no unemployment gap will exist. The government loss as a result of devaluing will be larger than if it does not do so because:

The loss associated with maintaining a fixed exchange rate ($x(s_2)=\bar{x}$) is zero:

$$V_4(s_2, Ex=\bar{x}, x(s_2)=\bar{x})=0, \tag{9}$$

while the loss when devaluing in state 2 will be C:

$$V_4(s_2, Ex=\bar{x}, x(s_2)>\bar{x})=C. \tag{10}$$

iii) If there are not devaluation expectations and the loss of credibility, C, is sufficiently large, node (10) will never be reached: It will not be optimal to devalue when a bad shock occurs. This conclusion is obtained by comparing the following loss functions: [8]

$$V_4(s_1, Ex=\bar{x}, x(s_1)=\bar{x}, \tilde{\phi}, \tilde{K}) = a_2[s_1 - \alpha_3(1-\tilde{\phi})R_1(\tilde{K})]^2 - \tilde{\phi}R_1(\tilde{K}), \text{ and} \tag{11}$$

$$\begin{aligned} V_4(s_1, Ex=\bar{x}, \hat{x}(s_1)>\bar{x}, \hat{\phi}, \tilde{K}, C) = a_2[&-\alpha_1(\hat{x}(s_1)-\bar{x}) + s_1 \\ &- \alpha_3(1-\hat{\phi})(\hat{x}(s_i)/\bar{x})R_1(\tilde{K})]^2 - \hat{\phi}(\hat{x}(s_i)/\bar{x})R_1(\tilde{K}) + C \end{aligned} \tag{12}$$

Therefore, the only possible reachable node is then (11) where it is optimal for the government not to devalue in either state. The optimal proportion of debt that will be bailed out in the bad state is found out by minimising the government's loss function (11) and it is:

$$\tilde{\phi} = 1 - \frac{2a_2 s_1 \alpha_3 - 1}{2a_2 \alpha_3^2 R_1(\tilde{K})} \tag{13}$$

Note first that there will not be any bailout if:

a) $2a_2 s_1 \alpha_3 < 1$, and

b) $R(K_1) \geq \dfrac{2a_2 s_1 \alpha_3 - 1}{2a_2 \alpha_3^2}$

[8] If the government decides instead to devalue in state 1 (say sets an exchange rate equal to $\hat{x}(s_1) > \bar{x}$), it takes as given $\tilde{K}$ which the private sector's investment decision when it does not expect a devaluation. Under such conditions, the optimal proportion of debt that will be bailed out, say ($\hat{\phi}$) is obtained by minimising (12) with respect to $\phi$. See appendix.

**6.2 There are <u>devaluation expectations (or speculative attacks)</u> (Ex>$\bar{x}$) and this conditions the level of risky investment, say $\hat{K}$, that the private sector will choose. The government makes <u>no commitment to a specific bailout</u> before stage 4.**

**Proposition 2:** *When there are speculative attacks, a rational expectations equilibrium is compatible only with end node (14), in figure 1, which corresponds to abandoning the fixed rate if a "bad" shock takes place but maintaining it if a "good" shock occurs. Node (14) implies that:*

$$x(s_1) > \bar{x};$$
$$x(s_2) = \bar{x}$$

**Proof:**

i) Node (12) in figure 1 will never be reached because if the regime is abandoned in both states, the government will find it optimal to abandon the regime already in stage 3, reaching instead node (6). In such case, the costs of defending the exchange rate regime in stage 4 will be avoided.

ii) When a <u>"good" shock</u> (state 2) occurs and <u>there are speculative attacks</u>, it will never be optimal to devalue.[9] The structure of payoffs in the game is such that end nodes (12) and (13)) in figure 1 must be also counted out because when $x(s_2)=\bar{x}$, the loss resulting from keeping the fixed exchange rate is:

$$V_4(s_2, Ex>\bar{x}, x(s_2)=\bar{x}) = a_2 [\alpha_2 r]^2 \quad (14)$$

while the loss when devaluing will be:

$$V_4(s_2, Ex>\bar{x}, x(s_2)>\bar{x}, C) = a_2 [-\alpha_1(x_2-\bar{x}) + \alpha_2 r]^2 + C. \quad (15)$$

[9] Note that by assumption, the shock and the outstanding debt are zero in state 2, making a bailout unnecessary, however, there will be an unemployment gap caused solely by the devaluation expectations.

It becomes evident that devaluing will be more costly for the government than not doing so.

iii) Node (15) can never be reached either, because if the government is certain to maintain the regime in both states, the private sector will not have devaluation expectations and any incentive to speculate.

Note also that if the government considers not devaluing, it will take $\hat{K}$ (the investment level chosen when the private sector decides to speculate in the foreign exchange market) as given but it will need to determine another bailout, say $\breve{\phi}$, by minimising the following loss function:

$$V_4(s_1, Ex = \bar{x}, x(s_1) = \bar{x}, \breve{\phi}, \hat{K}) = a_2[s_1 - \alpha_3(1-\breve{\phi})R_1(\hat{K})]^2 - \breve{\phi}R_1(\hat{K}) \quad (16)$$

Note however that (17) equals (11), for the following: If the private sector expects that the government will never devalue, it will not speculate and chose a risky investment equal to $\tilde{K}$ and not $\hat{K}$. Consequently, the optimal bailout will again be $\tilde{\phi}$ ($\breve{\phi}$ must be then equal to $\tilde{\phi}$). The government will rationally never maintain the fixed exchange rate regime in the event of speculative attacks and a "bad" shock.

Thus, the only possible reachable node is then (14). There will be also some unemployment gap because of the shock, the outstanding debt and the speculative attacks (see (7)). Note that expectations in the foreign exchange market are:

$$Ex = \pi x_1 + (1-\pi)x_2 \quad (17a)$$

From propositions 1 and 2 we know that the government sets $x_2 = \bar{x}$ so that:

$$Ex - \bar{x} = \pi(x_1 - \bar{x}) \quad (17b)$$

In contrast to the case where there are devaluation expectations, equation (8) will now come into play. Equation (8) represents the adverse effect on fundamentals caused by the

speculative attack. The government raises domestic interest rates to stave off speculation but this policy shifts unemployment up. Unemployment may of course decrease in proportion to the size of the bailout. The optimal bailout is obtained by minimising:

$$V_4(s_1, Ex > \bar{x}, x(s_1) > \bar{x}, \hat{\phi}, \hat{K}, C) = a_2[-\alpha_1(\hat{x}(s_1) - \bar{x}) + \alpha_2 g\pi(\hat{x}(s_1) - \bar{x}) + s_1 - \alpha_3(1-\hat{\phi})(\hat{x}(s_i)/\bar{x})R_1(\hat{K})]^2 - \hat{\phi}(\hat{x}(s_i)/\bar{x})R_1(\hat{K}) + C \quad (18)$$

Assuming $\bar{x}$ =1, the optimal bailout when the government devalues in state 1 is:

$$\hat{\phi} = 1 - \frac{2a_2\alpha_3 s_1 - 1 + 2a_2\alpha_3(\hat{x}(s_1) - 1)(\alpha_2 g\pi - \alpha_1)}{2a_2\alpha_3^2 \hat{x}(s_1)R_1(\hat{K})} \quad (19)$$

Note that there will not be any bailout if:

a) $2a_2\alpha_3 s_1 < 1 - \left[2a_2\alpha_3(\hat{x}(s_1) - 1)(\alpha_2 g\pi - \alpha_1)\right]$ and

b) $R_1(\hat{K}) \le \frac{2a_2\alpha_3 s_1 - 1 + \left[2a_2\alpha_3(\hat{x}(s_1) - 1)(\alpha_2 g\pi - \alpha_1)\right]}{2a_2\alpha_3^2 \hat{x}(s_1)}$

The optimal $\hat{x}(s_1)$ will be then:

$$\hat{x}(s_1) = \frac{\hat{\phi}R_1(\hat{K}) + 2a_2\{\alpha_2 g\pi - \alpha_1 - \alpha_3(1-\hat{\phi})R_1(\hat{K})\}(\alpha_2 g\pi - \alpha_1)}{2a_2[\alpha_2 g\pi - \alpha_1 - \alpha_3(1-\hat{\phi})R_1(\hat{K})]^2} + \frac{2a_2 s_1\{\alpha_2 g\pi - \alpha_1 - \alpha_3(1-\hat{\phi})R_1(\hat{K})\}}{2a_2[\alpha_2 g\pi - \alpha_1 - \alpha_3(1-\hat{\phi})R_1(\hat{K})]^2} \quad (20)$$

## 6. Stage 3

Proposition 3 indicates that when there are no speculative attacks, it will never be optimal to abandon the regime in stage 3. The government might as well wait until stage 4 to make decisions about the exchange rate and bailouts. Different issue is though when Ex> $\bar{x}$ .

Consider the case where the government has made no commitment to a specific bailout before stage 4. At this third stage, by rational expectations, the private sector knows that if in stage 4 a "bad" shock occurs, there will be given a bailout $\hat{\phi}$ (equation (19)), and that the

exchange rate will one equal (20). Given this, agents of the private sector will choose an amount of risky investments equal to $\hat{K}$ .

The loss function in stage 3 as described in (5) will take the following functional form after using (7) and assuming, $u_3 - \bar{u} = \alpha_2 g(Ex - \bar{x})$:

$$V_3(s_i, Ex > \bar{x}, x(s_i) > \bar{x}, \hat{\phi}, \hat{K}) = a_1(\alpha_2 g)^2 (Ex - \bar{x})^2 + a_2 \{\pi[u_4(s_1) - u_n]^2 + (1-\pi)[u_4(s_2) - u_n]^2\} - \pi\hat{\phi}(x(s_1)/\bar{x})R_1(\hat{K}) + \pi C; \quad (21)$$

Now, when speculative attacks have been initiated, the government can make either of the following decisions:

i) To abandon the fixed exchange regime right after the private sector's expectations have formed, that is at the very beginning of stage 3.

ii) To maintain the exchange rate regime, bearing all the costs of defending it, and wait until stage 4 to make a decision on the exchange rate. Such a decision implies that the government may consider it worthwhile to wait until the shock to the economy occurs and to abandon the regime only in the bad state, i.e. $x(s_1) > \bar{x}$ and $x(s_2) = \bar{x}$ .

iii) To stave off speculative attacks during stage 3 and face the costs of defending the exchange rate regime. If in the interim, the government anticipates that the costs of continuing to defend the regime during stage 3, will be unaffordable, it is possible that the exchange rate regime may be abandoned regardless of whether a "bad" or "good" shock is expected to occur in stage 4, i.e. $x(s_1) > \bar{x}$ and $x(s_2) > \bar{x}$ .

In order to make the optimal decision, it is necessary to evaluate the loss functions resulting from each of the above alternatives and compare them with the one from maintaining the exchange rate in stage 4 where devaluation occurs only in the bad state.

If the government chooses (i), it will incur a cost equal to C. On the other side, if the government decides to defend the exchange rate regime in stage 3 against speculative attacks

and maintain it until stage 4 only in the good state (i.e.chooses (ii)), its loss function (21) after using (17) becomes:

$$V_3(Ex > \bar{x}, \hat{x}(s_1) > \bar{x}, x(s_2) = \bar{x}, \hat{\phi}, \hat{K}) = a_2\pi[(-\alpha_1(\hat{x}(s_1) - \bar{x}) + \alpha_2 g\pi(\hat{x}(s_1) - \bar{x}) + s_1 -$$

$$\alpha_3(1-\hat{\phi})(\hat{x}(s_1)/\bar{x})R_1(K)]^2 + [a_1 + a_2(1-\pi)]\pi^2(\alpha_2 g)^2(\hat{x}(s_1) - \bar{x})^2 \qquad (22)$$

$$-\pi\hat{\phi}(\hat{x}(s_1)/\bar{x})R_1(K) + \pi C$$

Note that one of the differences between (22) and (18) (the government's loss function in the fourth stage) is that (22) now includes both the shifts during stage 3, and the expected shifts in stage 4, in the fundamentals (unemployment). Importantly, the latter can happen due to expectations of devaluation, net loss of investment and abandonment of the regime in stage 4.

Finally, if the government will abandon the regime in stage 4 in either state (i.e. $x(s_1) > \bar{x}$ and $x(s_2) > \bar{x}$, that is, it chooses (iii)), (21) will be written as:[10]

$$V_3(Ex > \bar{x}, \hat{x}(s_1) > \bar{x}, x(s_2) > \bar{x}, \hat{\phi}, \hat{K}) = a_2\{\pi[-\alpha_1(\hat{x}(s_1) - \bar{x}) + \alpha_2 g\pi(\hat{x}(s_1) - \bar{x}) + s_1 -$$

$$\alpha_3(1-\phi)(\hat{x}(s_1)/\bar{x})R_1(K)]^2 + (1-\pi)[-\alpha_1(x(s_2) - \bar{x}) + \alpha_2 g(Ex - \bar{x})]^2\} + \qquad (23)$$

$$a_1(\alpha_2 g)^2(Ex - \bar{x})^2 - \pi\hat{\phi}(\hat{x}(s_1)/\bar{x})R_1(K) + \pi C$$

**Proposition 3:** *Assume that the government has the alternative of maintaining the exchange rate regime until stage 4 when it will devalue only if a "bad" shock occurs; it has made either no commitment to a specific bailout before stage 4; and there are devaluation expectations. It will never be optimal to abandon the exchange rate regime at the very start of stage 3 if the probability that a bad shock will occur, $\pi$, is very small.*

[10] Note that we now do not use (17) but rather $Ex = \pi x(s_1) + (1-\pi)x(s_2)$ since it is expected rationally that $x(s_2) > \bar{x}$ .

**Proof:** If the government abandons the regime just after the end of stage 2, it will incur in a cost equal to C, the loss of credibility. Waiting until stage 4 and devaluing only in the bad state implies that the government will incur in costs equal πC and other costs that are also multiplied by π. Thus, even if the costs of maintaining the regime until stage 4 are significant, it will not be optimal to abandon the exchange rate regime if the probability that the economy will end up in the bad state, π, is very small.

**Proposition 4:** *Assume that the government has the alternative of maintaining the exchange rate regime until stage 4 but it will abandon it if a "bad" shock occurs; it has made no commitment to a specific bailout before stage 4; and there are devaluation expectations. The government will then stave off the speculative attacks during stage 3 and wait until stage 4 to make decisions on the exchange rate and bailout if the following condition is fulfilled:*

$$a_1(\alpha_2 g)^2 \pi^2 (\hat{x}(s_1) - \bar{x})^2 > (1-\pi)\{V_4[s_1, Ex > \bar{x}, x(s_1) > \bar{x}, x(s_2) = \bar{x}, \hat{\phi}, \hat{K}, C] \qquad (24)$$

Note that the expression on the left-hand side represents the costs of staving off speculative attacks in terms of higher unemployment during stage 3. The expression in the right-hand side is the probability that a "good shock" will occur times the total cost of maintaining the exchange rate regime until stage 4 when there are devaluation expectations that trigger speculative attacks.[11]

**Proof.** (24) is obtained by comparing (18) and (22).

**Proposition 5:** *Assume that the government has the alternative of abandoning the exchange rate regime in stage 4 either when a "bad" shock or a "good" occurs; it has made no commitment to a specific bailout before stage 4; and there are devaluation expectations. It*

[11] Notice that: $a_1(\alpha_2 g)^2 \pi^2 (\hat{x}(s_1) - \bar{x})^2 = a_1\{\alpha_2 g(Ex - \bar{x})\}^2 = a_1(u_3 - u_n)^2$

*will be optimal for the government to abandon the exchange rate regime already in stage 3 if the following condition is fulfilled:*

$$a_1(\alpha_2 g)^2\pi^2(\hat{x}(s_1)-\bar{x})^2 < (1-\pi)\{V_4[s_1, Ex>\bar{x}, \hat{x}(s_1)>\bar{x}, x(s_2)=\bar{x}, \hat{\phi}, \hat{K}, C] + a_2[-\alpha_1(x(s_2)-\bar{x})+\alpha_2 g(Ex-\bar{x})]^2\} \quad (25)$$

In contrast to (24), the right-hand side now also includes the costs, in terms of higher unemployment, for planing to devalue also in both states.[12]

**Proof.**

(25) is obtained by comparing (18) and (23).

## 8. Bailout commitments

When expectations for devaluation are born, non-commitment to any optimal bailout before stage 4 has been made, and if it is too costly to devalue already in stage 3, it will be optimal to devalue in stage 4 only in the bad state (proposition 2). In this section we analyse a situation in which the government can commit to a certain bailout before the market forms expectations, that is at stage 1. There are two questions that we would like to answer here: What would then be the optimal bailout that the government can commit to before the private sector forms expectations, in order to avoid both the multiple equilibria outcome and a large devaluation or any devaluation at stage 4? What will be the size of the bailout that the government could optimally commit to at stage 1 in comparison to the bailout given at stage 4 that is only contingent on the state of the economy?

If a commitment to an optimal bailout were to be made, it should be to avoid a large deterioration of the economic fundamentals that could arise as a consequence of a financial crisis. Furthermore and as we found here, it is likely to be the case that better perspectives on

[12] Notice that: $(1-\pi)a_2\{\alpha_1(x(s_2)-\bar{x})+\alpha_2 g(Ex-\bar{x})\}^2 = (1-\pi)a_2\{u_4(s_2)-\bar{u}\}^2$, since $\hat{x}(s_2)>\bar{x}$.

the economic fundamentals will make the public to expect rationally that any or a large devaluation become unnecessary. The government in this case could reach *at best* node (11) in figure 1 (see proposition 1) if the private sector expects no devaluation at all. Obviously, committing to an optimal bailout in stage 1 will be most important if great expectations of devaluation were imminent and consequently a large devaluation were likely to occur in the bad state at stage 4. By committing to a certain bailout, the government may avoid the costs for staving off the speculative attacks during stages 3 and 4, and those associated with large devaluation or the abandon of the fixed exchange rate regime in stage 4 in the bad state.

When committing, the government will find the optimal bailout at stage 1 by minimising a loss function (say $V_1$) that includes costs of maintaining the exchange rate regime during stage 3, the expected costs of keeping the regime until stage 4, and of devaluing in case a bad shock occurs. Having this established, the loss function will be:

$$V_1 = V_1(Ex > \bar{x}, \hat{x}(s_1) > \bar{x}, x(s_2) = \bar{x}, \phi, K) \tag{26}$$

The functional form of (26) needs to be like (22), thus, the optimal bailout (say $\phi^{COMM}$) that the government can commit in stage 1 can be obtained from the following expression:

$$\frac{dV_3}{d\phi} + \frac{dV_3}{dK}\left[\frac{\partial K}{\partial \phi} + \frac{\partial K}{\partial \hat{x}(s_1)} * \frac{d\hat{x}(s_1)}{d\phi}\right] = 0, \tag{27}$$

where:

$$\frac{dV_3}{d\phi} = \frac{\partial V_3(Ex > \bar{x}, \hat{x}(s_1) > \bar{x}, x(s_2) = \bar{x}, \phi, K)}{\partial \hat{x}_1(Ex > \bar{x})} * \frac{d\hat{x}(s_1)}{d\phi}$$

$$+ \frac{\partial V_3(Ex > \bar{x}, \hat{x}(s_1) > \bar{x}, x(s_2) = \bar{x}, \phi, K)}{\partial \phi},$$

and:

$$\frac{dV_3}{dK} = \pi(x(s_1)/\bar{x})(R_1')\{2a_2\alpha_3(1-\phi)[(-\alpha_1(x(s_1)-\bar{x}) + \alpha_2 g\pi(x(s_1)-\bar{x}) + s_1 -$$

$$\alpha_3(1-\phi)(x(s_1)/\bar{x})R_1(K)] - \phi\}$$

$\delta K/\delta\phi$ (the moral hazard effect) and $\delta K/\delta \hat{x}(s_1)$ can be obtained from (3) and (4) while $d\hat{x}(s_1)/d\phi$ in turn will be calculated from (20).

It is not possible to get an explicit solution for $\phi^{COMM}$ from (27). However, by assuming an explicit functional form for the returns to investment ($R_i(K)$, i=1,2) and assigning reasonable parameter values, the size of $\phi^{COMM}$ can be found by solving (27) numerically. We assume that:

$$R_i(K) = \gamma_i K - \delta_i K^2; \qquad \text{i=1,2}$$

The following parameter values are assumed in the numerical simulations presented in the next section:

$\bar{x}$ =1, $a_1=a_2=0.6$, $\alpha_1=2$, $\alpha_2=4$, $\alpha_3=1$, g=0.7, p=0.5, $s_1=0.25$, $\gamma_1=0.2$, $\delta_1=0.8$, $\gamma_2=2$, $\delta_2=0.08$.

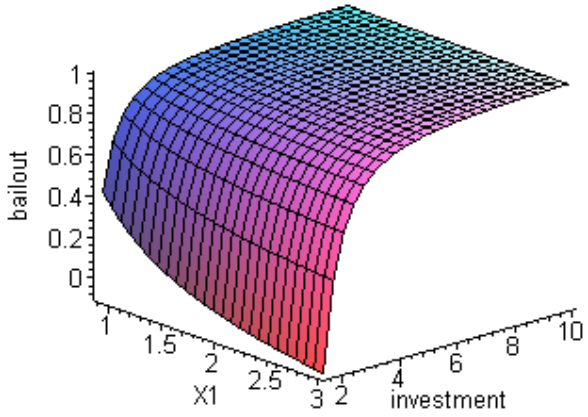
These values were the most appropriate to obtain reasonable values for the proportion of outstanding debt that the government will bail out, that is values of $0\leq\phi\leq1$, and obviously to obtain positive values for the exchange rate and investment.

**9. Numerical simulations**

We shall first illustrate graphically the solutions for stage 4 when no commitments on bailout have been made before stage 4 and the government devalues optimally in the most adverse state. Recall again that without commitments and when the private sector expects devaluation triggering speculative attacks, the government determines the equilibrium exchange rate and bailout at the fourth stage from (20) and (19), respectively, taken as given the private sector's decisions on investment.

The following figure 2 shows different levels of optimal bailout, $\hat{\phi}$, *for given values* of the exchange rate level, $x(s_1)$, and the amount of investment (K) made by the representative agent of the private sector when a "bad" shock occurs (relation (19)).

Figure 2. Optimal bailout for given exchange rate and investment




This implies that, with non-commitment and for given investment level, the larger the devaluation, the smaller the bailout ($\hat{\phi}$) the government will give to the private sector. It also the case that for given exchange rate level ($x_1$), the larger K the larger $\hat{\phi}$.

Simulations were done to check the robustness of the solutions. Changes in the parameter values did cause changes in the figure above. We found that for a given depreciation of the exchange rate a higher bailout will be offered when:

i) The probability that the economy will be in the bad state becomes larger.

ii) The effect of the investment losses on unemployment becomes larger.

iii) The effect of the debt problems on unemployment increases.

iv) The size of the "bad" shock is larger.

v) Depreciation has greater effect on unemployment.

Figure 3 shows different levels of optimal exchange rate levels, ($\hat{x}(s_1)$), *for given values* of $\phi$ and K when a "bad" shock occurs (relation (20)). For levels of bailout smaller than 50% and the larger K, the larger the devaluation needs to be. Otherwise, if the proportion of outstanding debt that will be bailed out is sufficiently large, the devaluation needs not to be that large when the level of investment increases.

Figure 3. Optimal exchange rate for given bailout and investment

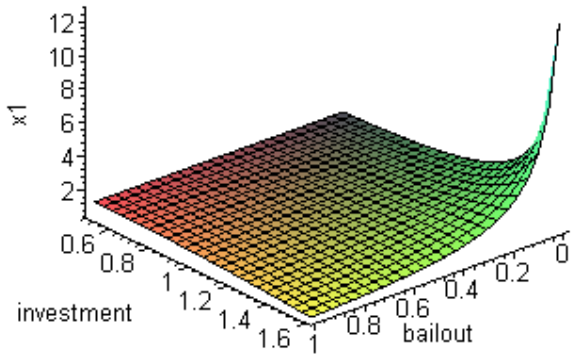



Table 1 below will present the numerical results on the equilibrium levels of the exchange rate, investment and bailouts both when the government does not commit (before stage 4) and when it commit (in stage 1) to bail out. It is important to note than in the *no-commitment case*, the government takes as given investment but the private sector however decides on the investment level taking into account the optimal level of no-committed bailout and exchange rate (that is (19) and (20)).[13] This implies that investment decisions are endogenous. Thus, the solution of $\hat{\varphi}$, $\hat{x}(s_1)$, and investment ($\hat{K}$) (in table 1) are obtained by solving (2), (19) and (20) simultaneously. Note that such solution does not depend on the values of $\delta K/\delta\phi$ and $\delta K/\delta \hat{x}(s_1)$. In the *commitment case* however, $\phi^{COMM}$, obtained by solving (27), depends on those values. This implies that we take into account moral hazard effects of bailing out when solving for the commitment case. More specifically, moral hazard effects are not just endogenised (as in the non-commitment case) but they can be affected by the bailout policy. The latter was not possible without commitments.

Thus, when the government decides *to commit to an optimal bailout* ($\phi^{COMM}$) in stage 1, before it does so it needs to form expectations about i) the private sector's returns to

[13] The private sector uses forward induction when making optimal decisions.

investment which depend on the probabilistic distribution of the shock; and ii) the effect of such commitment on the private sector's decisions regarding not only investment, but also importantly on the private sector's expectations regarding the future exchange rate. With no commitment, as propositions 1 and 2 indicate, government decisions on the exchange rate and bailout in stage 4 are made after the private sector has formed expectations on the exchange rate and decided on investment levels. The private sector's decision are then taken as given as pointed out above.

We can now summarise our numerical results shown in table 1 in some Propositions. Let me just point out that most of the simulations are made for the different values of $\gamma_2$, $s_1$, $\alpha_1$, $\alpha_2$, and $\alpha_3$, while holding the other parameters unchanged. Changes in these parameters will also induce changes in the effects of the different government policies regarding bailout and exchange rate on the economic fundamentals such as unemployment and production, and vice verse.[14]

**Table 1**

| | | $\hat{\phi}$ | $\hat{x}$ | $\hat{K}$ |
|---|---|---|---|---|
| **NON-COMMITMENT CASE** | **Initial parameter values** | 0.3298 | 1.5235 | 1.6739 |
| | **$\gamma_2$=1.5** | 0.4633 | 1.12 | 1.6739 |
| | **$s_1$=0.3** | 0.3567 | 1.5871 | 1.6739 |
| | | | | |
| | | $\phi^{COMM}$ | $x^{COMM}$ | $K^{COMM}$ |
| **COMMITMENT CASE** | **Initial parameter values** | 0.2544 | 1.25 | 1.7138 |
| | **$\gamma_2$=1.5** | 0.3486 | 0.941 | 1.7378 |
| | **$s_1$=0.3** | 0.2952 | 1.311 | 1.7205 |

**Proposition 6:** *When the fixed exchange rate regime is expected to be abandoned in state 1 but not in state 2, the optimal bailout under commitment is smaller than the optimal bailout with non-commitment when speculative attacks in the foreign exchange market are expected to take place.*

[14] Not all the results are presented in table 1. The other results are available upon request.

**Proof:** The comparison between the 2nd and 7th rows in table 1 shows the proposition. Without commitments, the equilibrium value for $\hat{\phi}$ equals 0.33 while the equilibrium value for $\hat{x}(s_1)$ is 1.52. The private sector consequently calculate the values of $\hat{\phi}$ and $\hat{x}(s_1)$ and finds that it is optimal to have a $\hat{K}$ equal to 1.67.

Using the same parameter values, in the commitment case, the equilibrium values for the committed bailout ($\phi^{COMM}$) is now 0.25, and for $x^{COMM}$ is 1.25. The private sector takes this into account and will optimally decide that the optimal level of investment, $K^{COMM}$ equal to 1.71. Commitments to a specific bailout may cause moral hazard in the sense that it will induce large level of risky investments, K, and this is taken into consideration by the government.

**Proposition 7:** *When the government commits to a specific optimal bailout at stage 1, it will need to devalue less or simply not to devalue. A commitment to bailing out can ameliorate the financial crisis and prevent a currency crisis.*

**Proof**: Lower devaluation decreases private sector net losses in spite of receiving lower bailout when there is commitment. There will be consequently a smaller deterioration of the fundamentals and as result it becomes not necessary to have a large devaluation or to devalue at all. The private sector takes this into account when forming expectations about the exchange rate: devaluation is less likely to occur. With other parameter values one could obtain no devaluation as seen by comparing the 3rd and 8th rows.

If the government now would like to avoid this expected 50% devaluation (from 2nd row and assuming $\bar{x}=1$) (with no devaluation expectations there will not be a need for commitment), it will have to commit (ex-ante) to an optimal bailout ($\phi^{COMM}$) equal 25% in which case it would only need to devalue 25%, as we mentioned above.

By the time the private sector form expectations, they realise that with commitments, it will never be optimal for the government to devalue that much or not devalue at all. In either case,

we obtain one single equilibrium, either node (11) or node (14). Node (11) is though less likely to be reached if devaluation has important effects on unemployment (i.e. $\alpha_1$ increases) and the larger the difference between the returns in the bad and the good states (i.e. $\gamma_2$ increases).

Other results can be summarised as follows:

- The government will commit to a larger bailout and will also devalue more than otherwise if: i) unemployment response to government decisions to devalue becomes larger (i.e. $\alpha_1$>2.2), and ii) unemployment response to the private sector expectations of devaluation becomes smaller (i.e.$\alpha_2$<4). If a currency is overvalued we find that if the improvement in the macroeconomic fundamentals is larger when a devaluation occurs, the government will have incentive to devalue more than otherwise. It will then be necessary for the government to commit to a larger bailout such as the private sector's financial status does not deteriorate further as a result of a larger devaluation. Not to do so will have negative effects on the fundamentals. Also, if the fundamentals become less responsive to larger devaluation expectations, the government will devalue more. There will be more incentive to self-fulfill expectations.[15]
- If there are larger negative effects of the outstanding private sector debt on the macroeconomic fundamentals, the government may find it optimal to devalue more and commit to a larger bailout. $\phi^{COMM}$ will have to be larger because it is too costly for the government in terms of unemployment to have larger (due to the devaluation) remaining outstanding debt from the private sector.

[15] We find this not to be the case when there is non-commitment. There the government will depreciate less.

- There are moral hazard effects caused by the government's commitment to guarantees such as bailouts, except in the case in which devaluation are very large. Thus, the private sector's investment decisions will be affected both by bailouts and by the exchanger rate.[16]
- If the difference between the returns to investment in the two states become smaller (or that $\delta_2$ becomes larger and/or $\gamma_2$ becomes smaller), we find that the optimal level of investment will be lower, the committed bailout will be higher and there will be higher depreciation. Such results are an indication that when the diversification is not that great, and all projects are equally risky, the level of capital investment will tend to be lower. Since there will be always large investment losses, even in the good state, the government will have to commit to a larger bailout to ameliorate the effect of such losses on the fundamentals. It will need to devalue more (due to larger deterioration in the fundamentals) but not necessarily as much as when there is no commitment. Having fewer possibilities of diversification, the private sector prevents their large investment losses by investing less.

## 10. Conclusions

We have studied a model for analysing the interrelationships between exchange rate crises and debt crises. It is shown that crises can be characterised by both belief-driven and fundamentals-driven attacks. The model is presented as a four-stage sequential game where the players are the government and the private sector. Information about the probabilistic distribution of a forthcoming (good or bad) shock is given in the first stage. This shock will be realised in the fourth stage and will affect private-sector returns to investment and unemployment. The government may or may not wish to commit to a bailout for a specific

[16] Recall again that there is potential moral hazard effect of bailing out on the amount of risky investment (relation (3)) but also that higher depreciation decreases the amount of risky investment (relation (4)).

amount of private debt before the market forms expectations, in stage 1. In stage 2, the private sector forms expectations and makes investment decisions. In stage 3, the government may react by either retaining or leaving the peg, depending on the costs that staving off speculative attacks at this stage and the expected costs that the government will incur in stage 4 if the peg is maintained. The regime may then be abandoned after the onset of the speculative attack, before it does further significant damage to the economy. This outcome, where self-fulfilling expectations of a speculative attack lead necessarily to devaluation, resembles the Krugman (1979) first-generation model, where a deterioration of the fundamentals (in addition to the market's anticipation of a financial crisis) plays such an important role in triggering a crisis. In the final stage, 4, the shock occurs and the government decides on the optimal bailout, if it has not committed already in stage 1, and whether or not to abandon the fixed-rate regime (given that the fixed rate has been retained up to this stage). Decisions are here contingent on the prevailing state of the economy.

It turns out that when there is no commitment at the early stage of the game, the model yields multiple equilibria. However with an ex-ante commitment, the model is likely to yield a unique equilibrium where no devaluation occurs or a smaller (than with non-commitment) devaluation. Moreover, the proportion of private sector debt that will be bailed out will be smaller than the proportion (non-committed) that is contingent on the state of the economy. It then seems that just the *commitment* to reduce the private sector's outstanding debt that could otherwise have severe impact on the economic fundamentals will make devaluation, or large devaluation, unnecessary. The private sector takes this into account when forming expectations and will find that to expect large devaluation cannot be rational. The government will then avoid a currency crisis by committing ex-ante to bailing out part of the private sector's debt. Thus, ex-ante commitment to optimal bailouts serves as a strategic device for the government when the government has a potential problem of credibility in its exchange

rate policy, since it facilitates the implementation of a more efficient unique equilibrium (without devaluation or with smaller devaluation). Such an equilibrium is not a "sunspot" equilibrium. This result becomes unambigous even after the moral hazard effects of bailing out are taken into consideration. In the model, the government measures the moral hazard effect of this type of guarantee when it decides on the optimal bailout that it can commit to before the private sector forms expectations, and decides on the level of risky investments. Another result is that it is never optimal for the government to allow devaluation to take place when there are no speculative attacks even if a bad shock occurs and the private sector suffers investment losses. We also find that with no ex-ante commitment to a specific bailout before stage 4, a larger bailout will always be necessary when there are speculative attacks than when no attacks occur.

Otherwise, the greater the shock in the bad state, the stronger the government's aversion to unemployment and the greater the probability that the economy will end up in the bad state, the higher the equilibrium bailout level chosen by the government. This holds true whether the government commits ex-ante to a specific bailout or not.

This paper has some weaknesses that need to be taken up in future work. First, private-sector debt is introduced in a somewhat rudimentary manner, simply as a factor contributing to an exacerbation of the unemployment problem. In particular, we do not consider the actual process of debt formation in the private sector. Nor do we model how different types of investment may be chosen, e.g. in terms of their riskiness, in response to the amount of (committed or non-committed) bailout that the government will give. Nevertheless, the model presented here, in which the moral hazard effects of bailing out are balanced against expectational effects, has not been considered before in the relevant literature or more specifically a theoretical relationship between currency and financial crisis.

Finally, in countries where the liquidity supply is low or simply difficult for their own governments to provide because of lack of credibility, a lender of last resort such as the IMF may play an important role in providing such liquidity and preventing inefficient liquidation. Without such support, countries may find it difficult to sustain their exchange rate regime. If the currency is allowed to depreciate in an emergency, lenders may have more incentive to withdraw as soon as posible their money from such countries without losing anything, leaving behind insolvent firms. A further analysis of such issues is also an important topic for future work.

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**APPENDIX**

An optimal bailout ( $\hat{\phi}$ ) is obtained by minimising the loss function (12) with respect to ϕ:

$$V_4(s_1, Ex = \bar{x}, \hat{x}(s_1) > \bar{x}, \hat{\phi}, \tilde{K}, C) = a_2[-\alpha_3(\hat{x}(s_1) - \bar{x}) + s_1 \\ -\alpha_3(1-\hat{\phi})(\hat{x}(s_i)/\bar{x})R_1(\tilde{K})]^2 - \hat{\phi}(\hat{x}(s_i)/\bar{x})R_1(\tilde{K}) + C \tag{A1}$$

The optimal ϕ will be:

$$\hat{\phi} = 1 - \frac{2a_2\alpha_3 s_1 - 1 - \left[2a_2\alpha_3\alpha_1(\hat{x}(s_1) - \bar{x})\right]}{2a_2\alpha_3^2(\hat{x}(s_1)/\bar{x})R_1(\tilde{K})} \tag{A2}$$

The optimal devaluation $\tilde{x}(s_1)$ will be:

$$\hat{x}(s_1) = \frac{2a_2\alpha_1^2\bar{x} + 2a_2\alpha_1\bar{x}(1-\hat{\phi})R_1(\tilde{K}) + 2a_2 s_1\alpha_3(1-\hat{\phi})R_1(\tilde{K})}{2a_2[-\alpha_1 - (1-\hat{\phi})R_1(\tilde{K})]^2} + \frac{\phi R_1(\tilde{K}) + 2a_2 s_1\alpha_3}{2a_2[-\alpha_1 - (1-\phi)R_1(\tilde{K})]^2} \tag{A3}$$

Note also here that there will not be a bailout if:

i) $2a_2\alpha_3 s_1 < \left[1 + 2a_2\alpha_3\alpha_1(\hat{x}(s_1) - \bar{x})\right]$ and

ii) $R_1(\tilde{K}) \leq \dfrac{2a_2\alpha_3 s_1 - \left[1 + 2a_2\alpha_3\alpha_1(\hat{x}(s_1) - \bar{x})\right]}{2a_2\alpha_3^2(\hat{x}(s_1)/\bar{x})}$