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Optimal Provision of Public Goods with Rank Dependent Expected Utility*

by

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October 2002

Abstract

In this paper the theory of rank-dependent expected utility (RDEU) is substituted for the theory of expected utility (EU) in a model of optimal provision of public goods. The substitution generalizes the Samuelson rule, previously modified to include deadweight loss and tax evasion loss.

Keywords: Tax evasion, optimal taxation, rank dependent expected utility,

JEL Classification: D81, H21, H26.

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1 Introduction

The main purpose of this paper is to analyse to what extent the assumption of rank dependent expected utility changes previous results concerning the optimal amount of public goods. The Samuelson rule of allocating resources to the production of public and private goods has been generalized in various ways. In particular, a study Usher (1986) includes deadweight loss and tax evasion loss. Substituting in Usher's study the assumption of rank dependent expected utility (RDEU) for his (common) assumption of expected utility (EU), the present study submits a further generalisation of the Samuelson rule, which gives a necessary condition of optimal welfare.

There are good reasons for applying the RDEU model.[1] The expected utility model, still dominant in economic analysis of uncertainty, has been seriously challenged in a number of studies, and it is arguable that the RDEU model is the best alternative among the non-expected utility models.[2] In particular, the RDEU model is notable for sharp comparative statics results.[3]

A particular puzzle in the literature of tax evasion also calls for an alternative to the common EU theory. According to Allingham and Sandmo's (1972) portfolio choice approach to income tax evasion[4], a risk-averse taxpayer, with a von Neumann-Morgenstern utility function, will under-report his income whenever the expected gain minus the expected punishment of evasion is positive. Intuition as well as empirical evidence seems to contradict this conclusion. For the more common types of tax evasion the sanctions in many countries consist of fines less (or not much higher) than the amount evaded, whereas the probabilities of tax returns being audited are of the order of a few percent. In general, expected utility maximisers would therefore be tax evaders, a result that is not supported by empirical evidence, – quite a few seem to comply or to evade less than what the EU theory predicts. Some explanations of why people are more law abiding than perhaps expected are related to social norms, stigma, or moral sentiments. The

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[1] The model is also known as anticipated utility (AU), expected utility with rank-dependent preferences (EURDP), the μ-θmodel, the dual theory of choice under uncertainty, and rank-dependent utility (RDU). The many names indicate that authors dealing with different problems have come up with essentially the same model.
[2] See e.g. Weber and Kirsner (1997).
[3] The rather sharp comparative statics results are related to the fact that the RDEU model is separable in wealth and probability. See Eide (1995 and 2001) for various comparative statistics result in the RDEU model.
[4] For summaries of the literature on tax evasion see Franzoni (2000) or Eide (2000).

RDEU theory offers a competing or additional explanation, cf Bernasconi (1998), and Eide (2001).

In section 2 some of the relevant properties of the RDEU theory are presented. The formal structure of a RDEU model of tax evasion is given in section 3. In section 4 RDEU is substituted for EU in a model of optimal taxation developed by Usher (1986).

2 RDEU vs EU

The flourishing field of generalised expected utility theory has provided explanations of several phenomena that appear as paradoxes within the theory of expected utility.[5] Several of these phenomena seem to be related to prpperty of the expected utility model that the marginal utility of wealth and attitude towards risk are merged. This amalgamation makes the EU model particularly simple and tractable, but at the same time hampers a more profound study of the individual's attitude towards uncertainty. The EU concept of risk aversion is partly a property of attitudes to wealth, and not of attitudes to risk per se. By accepting the von Neumann-Morgenstern utility function, and at the same time allowing for transformations of probabilities, the RDEU model generalises the EU model.

Intuition and empirical studies suggest that people have rather vague knowledge about the risk of being punished for tax evasion.[6] This vagueness can be represented by the concept of *ambiguity*. Ellsberg (1961) provided an early demonstration of the importance of ambiguity in decision making, and he showed that uncertainty is not totally captured by the concept of probability. Ambiguity is an intermediate state between ignorance, in which no distributions can be ruled out, and risk, in which all but one distribution is ruled out. Ambiguity results from the decision maker having limited or vague information and knowledge of the process generating outcomes.[7] In a situation of ignorance the decision maker has no information concerning the likelihood of potential outcomes. In a situation of risk the decision maker has objective or subjective probabilities of given outcomes. Empirical evidence indicates that people distinguish

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[5] Quiggin (1993, 37-49) gives an overview of the many challenges to EU theory, i.a. the Allais paradox, the Ellsberg paradox, preference reversal, insurance and gambling jointly, and difficulties in empirical constructions of utility functions. He also (49-32) discusses, and challenges, some attempts to solve the various problems by introducing weighting of probabilities, in particular the prospect theory. Attempts to model this phenomenon by just transforming probabilities have not been successful (63).

[6] See Elffers et al. (1987), Hessing et al. (1992), and Sheffrin and Triest (1992).

between risk and ambiguity.[8] Studies show that ambiguity aversion and risk aversion are not (highly) correlated, a correlation one would expect if they were just different designations of the same phenomena.[9] Both ambiguity avoidance and ambiguity-seeking behaviour have been found in laboratory experiments.[10]

In studies of ambiguity, i.e. in studies where objective probabilities are absent, Gilboa (1987) and Schmeidler (1989) have (seemingly independently) come up with the RDEU model. In these studies, the decision weights are interpreted as non-additive subjective probabilities. In the standard RDEU model developed by Quiggin (1993) objective probabilities are assumed known. These probabilities are then transformed by non-additive decision weights.[11]

It is worth noticing that Allais (1988) in his axiomatisation of the main ideas in his 1953 article comes up with the RDEU model. Discussing the independent works of Quiggin (1982), Yaari (1987) and Segal (1987) he states: "It is *very significant* that, starting from entirely different premises, all three authors have been led to a *mathematical formulation* that is analogous to my own" (emphasis in original).

Alm (1988) found that increased uncertainty about the risk of punishment had a substantial impact on a number of taxpayers' decisions, including investing in tax shelters, receiving compensation in wage or non-wage forms, spending on tax deductible items, and under-reporting one's income. Beck and Jung (1989) concluded that the effects of uncertainty on taxpayer compliance can differ depending on risk-taking attitudes, the likelihood of audit and the magnitude of penalties. When the magnitude of penalties and the perceived likelihood of audit are high, increasing uncertainty increases compliance regardless whether taxpayers are risk-averse or risk-neutral. However, when audit probabilities and penalty rates are low (and closer to the values that would be expected to occur naturally), risk-neutral taxpayers are shown to have incentives to

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[7] Ellsberg (1961, 657) defined ambiguity as "a quality depending on the amount, type, reliability and "unanimity" of information, giving rise to one's degree of "confidence" in an estimate of relative "likelihoods" of future events.
[8] Camerer (1999).
[9] Cohen, Jaffray, and Said (1985), and Schoemaker (1982).
[10] See i.a. Becker and Brownson (1964), and Einhorn and Hogarth (1986). Surveying several experimental studies Edwards (1992, p. 5) makes the following comment: "Currim and Sarin […] compared experimental subjects' assessed expected utility models with their prospect theory, weighted utility, and lottery dependent utility models; and Daniels and Keller […] assessed expected utility and lottery dependent models. Overall, expected utility did about as well as generalised utility models in predicting choices on a hold-out sample of paired comparison choices, even when the problems were structured to induce expected utility property violations. However, the potential for improved predictive performance by generalised expected utility models may still be achieved. For example, Daniels and Keller […] have explored a choice-based assessment mechanism in which lottery dependent expected utility appears to perform better than expected utility. Also, Shafir et al. […] proposed an advantage model of choice that outperformed two special cases of expected utility".
[11] Quiggin argues that the difference in interpretation could be seen as only a difference in authors' tastes.

reduce compliance. For risk-averse taxpayers, the effects of increasing uncertainty depend on the degree of risk aversion.[12] Substituting RDEU for EU in tax evasion models makes it possible to more systematically study the effect of the uncertainty revealed in studies like those of Alm and of Beck and Jung.

Among the host of non-expected utility models with different preference functionals that have been proposed in order to tackle various theoretical and empirical problems raised in studies of individual behaviour under uncertainty, the RDEU model is chosen also for various other reasons.[13] According to Quiggin (1993, p. 72) the model (see section 3 below) is the only possible generalisation of the EU theory that is separable in outcomes and probabilities, and in which the requirements of first stochastic dominance, transistivity and continuity are satisfied.[14] Separability makes the model simple, and is crucial to some of the sharp comparative statics results of this theory. It also performs quite well in experiments where various utility theories have been compared.[15]

Whereas risk aversion in the EU theory corresponds to a simple condition on the utility function, the RDEU model implies a fundamental distinction between attitudes to probabilities and attitudes to outcomes, cfr. Quiggin (1993, p. 76):

> First there is outcome risk aversion, associated with the idea that marginal utility of wealth is declining. This is the standard notion of risk aversion from EU theory defined by concavity of the utility function. Second, there are attitudes specific to probability preferences.[16] An obvious ground for risk aversion in probability weighting arises for people characterized by pessimism, that is, those who adopt a set of decision weights that yields an expected value for a transformed risky

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[12] See Sawyers (1990) for a survey of results in this field.

[13]Different types of models have been developed to explain and predict behaviour under ambiguity. These include (i) models based on the idea of anchor probability (Einhorn and Hogarth (1986)), models which represent ambiguity as a second order probability (Marshak (1975) and Bernasconi (1997)), and models in which the probability of events is not additive (Gilboa and Schmeidler (1989)). The present paper is an example of the latter. Prospect theory has also been developed in order to accommodate such characteristics, but this theory, at variance with ambiguity theory, concerns gambles with well-defined probabilities.

[14] E.g. the theory of Kahneman and Tversky (1979) violates dominance requirements.

[15] Hey and Orme (1994) conclude such an investigation thus (p. 1321): "Expected utility theory (and its special case risk neutrality) emerges from this analysis fairly intact. For possibly 39% of the subjects … EU appears to fit no worse than any of the other models … For the 61% of the subjects, one or more of the eight "top-level" functionals … fits significantly better in statistical terms, though often the economic significance is not all that great. Of the eight "top-level" functionals it would appear that the two rank dependent functionals and the quadratic utility model emerge as strongest contenders (with the Quiggin weighting function having a modest lead over its power weighting rival)." Harless and Carmerer (1994), using as much as 23 data sets, conclude, however, that our choice of preference functionals must depend on the researchers' preference for fit and parsimony. No functional is to be preferred on both accounts.

prospect lower than the mathematical expectation. This yields a natural generalization of the basic definition or risk aversion to the RDEU model.

3 The RDEU model

In presenting the rank dependent expected utility model I follow Quiggin (1993, p. 57) and his notation. Let **x** be a vector of n outcomes with the probability vector **p**, and $U(\mathbf{x})$ a primitive utility function. The characteristic feature of this model is a probability weighting function q:[0,1]→[0,1], which is applied, not to the probabilities of individual events, but to the cumulative distribution function $F(\mathbf{x})$. The RDEU functional to be maximised is

$$V(\{(x_1, x_2, ..., x_n); (P_1, P_2, ..., P_n)\}) = \sum_i U(x_i) h_i(P_{1,} P_2, ..., P_n),$$

where

$$h_i(P_{1,} P_2, ..., P_n) = q(\sum_{j=1}^{i} P_j) = q(F(x_i)) - q(F(x_{i-1})).$$

In the case of two outcomes (punished or not punished), we have

$$h_1(P_{1,} P_2) = q(F(x_1)) - q(F(x_0)) = q(P_1), \qquad (1)$$

$$h_2(P_{1,} P_2) = q(F(x_2)) - q(F(x_1)) = q(1) - q(P_1) = 1 - q(P_1). \qquad (2)$$

That is, q defines the weight on the worst outcome (unsuccessful evasion) and 1-q defines the weight on the better outcome (successful evasion). A pessimist will typically behave as if $q(P_1) > P_1$, i.e. that the worst outcome is overweighted. Vague information about P might produce a similar effect.

[16] It is arguable that the term 'risk aversion' is more properly applied to preferences over probabilities than to preferences over outcomes. Like Quiggin, we find it inopportune not to characterise the utility function by the well established risk concepts.

4 Substitution of RDEU for EU

Optimal taxation in which the cost of tax evasion is taken into consideration, has been studied by several authors.[17] In this section rank dependent expected utility is substituted for expected utility in Usher's (1986) model.

In Usher's model people choose the number of hours of work per day and the amount of income declared to maximise utility with due regard to the cost of punishment. The government chooses the quantity of public goods, the income tax rate, and expenditure to deter tax evasion so as to maximise the utility of the representative taxpayer.

Consider an economy with H identical people whose utility functions are $U(Z, L, G, T)$, where Z is the amount of private goods consumed per head, L is the number of hours of labour supplied per head, G is the supply of public goods in total, and T is the severity of punishment.[18] Labour is the only factor of production, and hours of labour is chosen as a numeraire. Consequently, the wage rate is 1, and we can think of L as total income per head, not all of which is declared. The representative consumer who, confronted with a proportional income tax at a rate θ, must choose how much income to declare, X. The amount of undeclared income is $E = L - X$, and the amount of tax evaded is θE. The taxpayer can prevent detection at a cost $C = C(E, D)$,[19] where D is public expenditure to decrease tax evasion.[20] We assume $C_E > 0$, $C_{EE} > 0$, $C_D > 0$, and $C(0, D) = 0$. The government chooses $T(E)$, the appropriate severity of punishment, corresponding to any given amount of tax evasion. For convenience we assume $T(E) = fT$, where the parameter f is the severity of punishment per dollar of tax evasion, respectively. The government's costs of preventing evasion is a function of the severity of the sanction, $m(T)$. For convenience we assume $m(T) = mT$, where the parameter m is the marginal cost of punishment.

Instead of assuming maximisation of expected utility, we now assume that each individual maximises the RDEU functional

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[17] See Franzoni (2000) for a survey.

[18] When the sanction is a fine the utility function becomes $U(Z - T, L, G)$ because income and fines are both expressed in money terms.

[19] The private costs of evasion includes (i) the labour of hiding taxable objects from the tax collecter, (ii) loss of income in switching from more taxed, more profitable endevours to less taxed, less profitable endevours, (iii) expenditures on litigation, and (iv) the risk of punishment for tax evasion.

$$V = U(Z, L, G, T)q(P) + U(Z, L, G, 0)(1 - q(P)) \quad (6)$$

where the probability P of detecting tax evasion is a function of the amount of undeclared income (E), the expenditure by the tax evader to escape detection (C), and the public expenditure to decrease evasion (D), $P = P(E, C, D)$. The probability of detection is transformed by the weighting function $q = q(P)$.

Each taxpayer is assumed to maximise (6) subject to the individual budget constraint

$$hZ + C = L(1 - \theta) + \theta E\,, \quad (7)$$

where h is the number of hours of labour required to produce a unit of the private good.

The production possibility frontier of the economy as a whole is

$$M = H[hZ + C + mfEP] + h_G G - HL = 0 \quad (8)$$

where h_G is the number of hours of labour required to produce a unit of the public good. The first two terms in the bracket are the total production costs of private consumption goods and the consumers' detection prevention costs, respectively. The third term represents the government's costs in detecting evasion. Its interpretation depends on the type sanction, whether it is a fine (a transfer), or eventually a prison sentence. The last term two terms of (8) are the production costs of public goods and the total income of the economy, respectively.

In order to determine the optimal values of the policy variables θ, G, D, and f the procedure is as follows (see appendix 2):

(i) By maximising the utility functional subject to the individual budget constraint, one obtains for the representative consumer the optimal values of Z, L, and E, and C as functions of the policy variables θ, G, D, and f, i.e. $Z = Z(\theta, G, D, f)$, etc.

(ii) These functions are employed to express both the production possibility constraint and the utility functional in terms of the same policy variables. The production

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[20] In order to stick to the same notation as in Eide (2001), and elsewhere in this paper, we have substituted X, Z, P, h and θ for Usher's Y, X, π, P, and t, respectively.

possibility constraint then takes care of the taxpayers' propensity to evade tax and to substitute taxable labour to non taxable leisure.

(iii) The government chooses θ, G, D, and f to maximise the representative utility functional, given the functions of Z, L, C, and E, and subject to the constraint $M = 0$.

(iv) A modified Samuelson rule for choosing the optimal supply of public goods is derived from the first order conditions of the government's optimising problem. Maximisation of (6) subject to (7) gives

$$H\frac{rd\exp[U_G]}{rd\exp[U_Z]} = S[\frac{h_G}{h} - \frac{H\theta X_G}{h} - \frac{Hfm}{h}q_P\frac{dP}{dG}] \tag{9}$$

where $rdexp[U_G]$ means rank dependent expectation of U_G, etc., and

$$S = \frac{1}{1 + \frac{\theta X_\theta}{X} - \frac{fmP}{\theta X}q_P\varepsilon_{P\theta}} \tag{10}$$

and

$$\varepsilon_{P\theta} = \frac{dP}{d\theta}\frac{\theta}{P}.$$

In order to interpret this result, we take as point of departure the original Samuelson (1954) rule of optimal taxation, which in our notation can be stated as

$$H\frac{U_G}{U_Z} = \frac{h_G}{h}, \tag{9a}$$

which says that the sum of the marginal rates of substitution between public and private goods is equal to the rate of transformation between these goods, (or, equivalently, that the sum of the marginal benefits of public goods HhU_G equals marginal costs $h_G U_Z$).

The Samuelson rule has been generalized by Usher (1986) in a model where tax evasion is foolproof. (We may think of a society with punishments that scares everybody from cheating.) Usher's model results in the following modified Samuelson rule of optimal taxation:

$$H\frac{U_G}{U_Z} = S[\frac{h_G}{h} - \frac{H\theta X_G}{h}] \tag{9b}$$

where

$$S = \frac{1}{1+\frac{\theta X_\theta}{X}}. \tag{10b}$$

Here, the last term in the denominator of (10b), $\frac{\theta X_\theta}{X}$, is the elasticity of declared income with respect to the tax rate. The denominator as a whole may be interpreted as the elasticity of tax revenue with respect to the tax rate. The tax revenue is θX. Its derivative with respect to θ is $X + \theta X_\theta$, and its elasticity with respect to θ is equal to the denominator. If the declared income were invariant with respect to the tax rate, we would get S=1, as implied by (9a). If the primitive utility function exhibits decreasing absolute risk aversion, it has been shown that $\frac{\theta X_\theta}{X} > 0$, and consequently S >1.[21] Comparing (9b) with (9a) we see that S >1 increases the rate of substitution. Tax evasion increases the cost of public goods, and consequently private goods are substituted for public goods.

The sign of the second term of the bracket of (9b) depends on the sign of X_G, i.e. on how an increase in the amount of the public good affects reported income. We do not, however, have a priori information about the sign of X_G.

The model by Usher that we finally will compare our own model with assumes an individual utility function similar to (6) where P is substituted for $q(P)$. The model ends up with the following modified Samuelson rule:

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[21] See Eide (2001) for the effects of changes in the tax rate within a RDEU version of the Allingham-Sandmo (1972) type of tax evasion model.

$$H\frac{\exp[U_G]}{\exp[U_Z]} = S[\frac{h_G}{h} - \frac{H\theta X_G}{h} - \frac{Hfm}{h}\frac{dP}{dG}] \quad (9c)$$

where

$$S = \frac{1}{1 + \frac{\theta X_\theta}{X} - \frac{fmP}{\theta X}\varepsilon_{P\theta}} \quad (10c)$$

and

$$\varepsilon_{P\theta} = \frac{dP}{d\theta}\frac{\theta}{P}.$$

The elasticity $\varepsilon_{P\theta}$ in the new term of (10c) represents the ultimate effect of an increase in the tax rate, through the intermediary of *E* and *C*, on the probability of detection. If $\varepsilon_{P\theta} > 0$, the new term in the denominator of (10c) leads to a higher value of *S*, which implies an increase in the tax evasion loss.

On the left hand side of (9c) we now have a rate of substitution between expectations. The reason is that utilities now depend on whether or not the consumer is punished. Usher notes that there is no basis for predicting the sign of the last term in the bracket of (9c).

Our relations (9) and (10) differ from the corresponding equations in Usher (1986) (equations 9c and 10c above) in three respects. First, on the left hand side of (9) the rates of substitution are between *rank dependent* expected utilities instead of between expected utilities as in Usher's (9c). Second, the last term of the bracket is in our model multiplied by $q_P = \frac{dq(P)}{dP}$, which is assumed to be positive. Depending on the sign of q_P, and thereby on the sign of the last term of the bracket in (9c)), rank dependence may increase or decrease the effect upon S of this term. As in Usher's model, there is, however, no basis for predicting the sign of that term. Third, the last term of the denominator of (10) includes q_P. Thus the sign of the term does not change as a consequence of rank dependence.

As one would expect, the assumption of rank dependence has some effect on the value of the "risk terms" of the modified Samuelson rule.

5 Conclusion

The simple Samuelson rule is elegant and instructive as a textbook result, showing how resources should be allocated among private and public goods in order to obtain maximum welfare. When people, as in Usher (1986), pay or evade taxes according to what maximises their expected utility, the rule becomes more complicated. When both tax rates and sanctions for tax evasion are taken into account, individual behaviour produces a combination of deadweight loss and tax evasion loss that results in an intricate formula containing terms that cannot be signed a priori.

Many laboratory experiments suggest that people do not maximise expected utility. Among the many non-expected utility models, the rank dependent utility model is theoretically simple and seems to be a strong empirical contender to the expected utility model. We have compared our model of optimal provision of public goods based on rank dependent expected utility with the expected utility model by Usher (1986). Since expected utility is a special case of rank dependent expected utility, our rule of optimal provision of public goods is a generalisation of the Usher version of the Samuelson rule. The following differences occur: (i) The marginal rate of substitution is between the rank dependent expected utilities of the public and private goods instead of between their expected utilities. (ii) The "risk term" modifying the marginal rate of transformation between private and public goods has been modified by the derivative of the weighting function (the probability transformation function). (iii) The "risk term" of the tax evasion relation (S) has been modified by the same derivative.

These differences show that the generalisation comes at a price. The new terms cannot be signed a priori, unless additional assumptions are introduced. As a practical tool our rule (as well as that of Usher) requires substantial empirical knowledge in order to quantify the various terms of the formula. Perhaps our rule mirrors some of the complexity of life?

Appendix[22]

(i) The taxpayer chooses *Z*, *L*, *E*, and *C* to maximise the rank dependent expected utility

$$V = q(P)U(Z, L, G, fE) + (1 - q(P))U(Z, L, G, 0)$$

subject to the budget constraint

$$hZ + C - L(1-\theta) - \theta E = 0$$

where $P = P(D, C, E)$. For convenience, we define $\hat{U} = U(Z, L, G, fE)$ and $\bar{U} = U(Z, L, G, 0)$. By assumption, the punishment *fE* is proportional to the amount of income concealed.

The first order conditions are[23]

$$q\hat{U}_Z + (1-q)\bar{U}_Z - \alpha h = 0$$

$$q\hat{U}_L + (1-q)\bar{U}_L + \alpha(1-\theta) = 0$$

$$q\hat{U}_{fE} f + q_P P_E(\hat{U} - \bar{U}) + \alpha\theta = 0$$

$$q_P P_C(\hat{U} - \bar{U}) - \alpha = 0$$

where $q_P = \dfrac{dq(P)}{dP}$.

From the first order conditions and the budget constraint one may, in principle, derive the taxpayer's "demand functions" *Z = Z(θ, G, D, f), L = L(θ, G, D, f), E = E(θ, G, D, f), and C = C(θ, G, D, f).* We substitute these functions into the budget constraint and then find the derivatives of this constraint with respect to the government's variables:

[22] This appendix is to a large extent identical to a part of the appendix in Usher (1986), except in particular for the substitution of rank dependent expected utility for expected utility.

$$hZ_\theta + C_\theta - L_\theta(1-\theta) + (L-E) - \theta E_\theta = 0$$

$$hZ_G + C_G - L_G(1-\theta) - \theta E_G = 0$$

$$hZ_D + C_D - L_D(1-\theta) - \theta E_D = 0$$

$$hZ_f + C_f - L_f(1-\theta) - \theta E_f = 0$$

where Z_θ is the partial derivative of Z with respect to θ in the function $Z = Z(\theta, G, D, f)$, etc. These functions are then used below in the analysis of the government's utility maximization.

(ii) Substituting the "demand functions" into the utility functional and the budget constraint of the government, one may derive:

$$V^* = V^*(\theta, G, D, f)$$

(iii) The government chooses the variables at its control to maximise the same utility functional as the taxpayer, subject to the production possibility constraint

$$M = HhZ + HC + HfmP(D, C, E) + h_G G - LH = 0$$

where the taxpayer's variables Z, L, E, and C are themselves functions of the government's variables.[24]

The government equates the first derivatives of the objective function V^* to the first derivatives of the constraints. The derivative of the objective function with respect to θ is

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[23] We assume that functions are differentiable to the relevant order.
[24] In order to concentrate on the effect of assuming rank dependent expected utility, I will not here discuss Usher's definition of the cost to society of preventing evasion (the term *HfmP* in our notation).

$$\partial V^* / \partial \theta = q(\hat{U}_Z X_\theta + \hat{U}_L L_\theta + \hat{U}_{fE} f E_\theta) + (1-q)(\hat{U}_Z X_\theta + \hat{U}_L L_\theta) + (\hat{U} - \bar{U}) q_P (P_C C_\theta + P_E E_\theta)$$

$$= [q\hat{U}_Z + (1-q)\bar{U}_Z] Z_\theta + [q(\hat{U}_L + (1-q)\bar{U}_L] L_\theta$$

$$+ [q\hat{U}_{fE} f + q_P P_E (\hat{U} - \bar{U})] E_\theta + [q_P P_C (\hat{U} - \bar{U})] C_\theta$$

$$= \alpha h Z_\theta - \alpha(1-\theta) L_\theta - \alpha\theta E_\theta + \alpha C_\theta$$

$$= \alpha[h Z_\theta - (1-\theta) L_\theta - \theta E_\theta + C_\theta]$$

$$= -\alpha(L - E)$$

By a similar series of substitutions from first order conditions and derivatives of the budget constraint, it may be shown that

$$\frac{\partial V^*}{\partial G} = q\hat{U}_G + (1-q)\bar{U}_G = rdE[U_G]$$

$$\frac{\partial V^*}{\partial D} = (\hat{U} - \bar{U}) q_P P_D$$

$$\frac{\partial V^*}{\partial f} = Eq\hat{U}_{fE}$$

where *rdexp* means the rank dependent expectation.

The derivative of the production possibility constraint with respect to θ is

$$\frac{\partial M}{\partial \theta} = H[hZ_\theta + C_\theta + fmq_P \frac{dP}{d\theta} - L_\theta] = -H[(L-E) + \theta(L_\theta - E_\theta) - fmq_P \frac{dP}{d\theta}]$$

where

$$\frac{\partial P}{\partial \theta} = P_C C_\theta + P_E E_\theta$$

Similarly,

$$\frac{\partial M}{\partial G} = h_G - H[\theta(L_G - E_G) - fmq_P \frac{\partial P}{\partial G}]$$

$$\frac{\partial M}{\partial D} = -H[\theta(L_D - E_D) - fmq_P \frac{\partial P}{\partial D}]$$

$$\frac{\partial M}{\partial f} = -H[\theta(L_f - E_f) - fmq_P \frac{\partial P}{\partial f}]$$

(iv) Finally, equating first derivatives of the utility functional V^* with the first derivative of the constraint M, we obtain

$$\frac{rd\exp[U_G]}{-\alpha X} = \frac{h_G - H\theta X_G - Hfmq_P \frac{dP}{dG}}{-H\left[X + \theta X_\theta - fmq_P \frac{dP}{d\theta}\right]}, \text{ or } \frac{\frac{\partial V^*}{\partial G}}{\frac{\partial V^*}{\partial \theta}} = \frac{\frac{\partial M}{\partial G}}{\frac{\partial M}{\partial \theta}}$$

From the first order condition of individual maximisation we have $\alpha h = rd\exp[U_Z]$. Substituting into $\frac{\partial V^*}{\partial \theta} = -\alpha X$ we obtain $\frac{\partial V^*}{\partial \theta} = -\frac{rd\exp[U_Z]}{h}$, and the modified Samuelson rule becomes

$$H\frac{rd\exp[U_G]}{rd\exp[U_Z]} = S\left[\frac{h_G}{h} - \frac{H\theta X_G}{h} + \frac{Hfm}{h} q_P \frac{dP}{dG}\right]$$

where

$$S = \frac{1}{1 + \frac{\theta X_\theta}{X} - \frac{fmP}{\theta X} q_P \varepsilon_{P\theta}}$$

and

$$\varepsilon_{P\theta} = \frac{dP}{d\theta}\frac{\theta}{P}.$$

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