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A Mixture Model of Household Retirement Choice *

By

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Abstract

This paper analyzes the labor market participation behavior of the elderly couples when a new option (early retirement) becomes available to the husband. Unlike other studies of household labor supply model, which assume that all the households follow the same decision making structure, we assume there are two types of household, the cooperative type and the non-cooperative type. When facing the choice problem, those belong to the non-cooperative type behave according to a Stackelberg game with male as the leader, while those of the cooperative type follow a simple unitary model. Under this assumption, we formulate a mixture model using the latent class analysis framework. This model explicitly takes account of the unobserved heterogeneity in decision-making structures.

The empirical estimation of the model is based on register data from Statistics Norway. We find that more than half of the households belong to the non-cooperative type. And these households on average have lower education level than those of the cooperative type. Our conjecture is that this may suggest that it is easier for higher education couples to communicate and compromise to reach a efficient solution.

Keywords: household labor supply, retirement, latent class analysis, heterogeneity, econometric models
JEL classification: D10, H55, J26

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1 Introduction

In this paper, we develop and apply a particular empirical modeling framework to analyze the labor market behavior of the elderly couples in Norway, when a new option (early retirement) becomes available to the husband. We focus on the joint behavior of the couple with respect to labor market participation decision, namely for husband 'retire' or 'continue to work' and for wife 'work' or 'not work'. Considering the increasing proportion of elderly persons in the population in most industrialized countries, understanding the elderly labor market behavior is of great importance from a policy point of view.

The central issue in the analysis of family decision-making behavior is that there are two agents involved in the decision process. They may have separate preferences of their own and possible conflicting interests. Things become much more complicated than single-agent choice model, since we need to account for the strategy interaction between the members of the family. One common way to account for the interrelation of this two-agent choice model is to use game theory. As discussed by Bresnahan and Reiss (1990), different solution concepts (and thus the probability assigned to the choice alternatives) can be used to solve the choice problem when we apply different structure to the underlying game. Each different game solution concept reflects the different ‘decision mechanism’ in the family. By ‘decision mechanism’, we mean the interaction structure and relative 'position' between these two members. This is one of the important reasons why there are many different suggestions to model family decision-making in the literature, which is surveyed in Bergstrom (1997). Roughly, the literature can be divided into two strands, the cooperative and non-cooperative approaches. For example, McElroy and Horney (1981) apply a cooperative Nash bargaining solution, while Hiedemann (1998) uses a non-cooperative Stackelberg model with male leadership to model the joint social security acceptance decisions. More recently, Chen and Woolley (2001) tried to tie these two strands together by providing a theoretical analysis of the household demands and intra-household resource allocation using the framework of a non-cooperative Nash-Cournot game as well as a cooperative bargaining game. To deal with the question which is the most proper model in empirical studies, Hernæs, Jia and Strøm (2001) compared the empirical performance of different models in both strands using Norwegian data, and found that the Stackelberg game with male as a leader outperforms the Nash game and the unitary model.

The basic assumption in all of these studies is that the same decision mechanism is employed across the whole sample. However, this may not necessary be the case in real life. We observe that families differ in members' education levels, experiences, level of affection and culture background etc, we may as well suspect that they differ in decision mechanisms. It would be of interest to take these differences into account or at least try to figure out whether the assumption of the same decision mechanism is empirical consistent, when we are modeling the family decision making process. However, very few studies have been concerned with the heterogeneity problem in the family choice models.

Our paper tries to make a first step to analyze this problem by proposing a mixture model to account for the unobserved decision mechanism heterogeneities using the latent class analysis framework. In contrast to the usual assumption that all the families follow just one decision mechanism, we assume that the families can be divided into several types, which are characterized by different decision mechanisms. The families of same type have identical decision mechanism and preference response to the social-economic variables. For each type of families, we derive the choice model according to the decision mechanism. The fact that we could not observe the type of each family directly implies that our model will have a finite mixture framework. This idea is somewhat similar to the preference segmentation model in the marketing literature proposed by Kamakura and Russell (1989). But Kamakura and Russell only dealt with preference difference in a single-agent choice problem and the decision units differ only in the parameter value of the utility functions. The model they derived is essentially a Mixed Logit model with a discrete support.

In our analysis in this paper, we assume that there are two types of household, the cooperative type and the non-cooperative type. The households belonging to the non-cooperative type behave according to a Stackelberg game with male as the leader, while those in the cooperative type follow a simple unitary model.

The empirical analysis is based on register data from Statistics Norway. We basically used the sample of couples in which the husband is qualified to the early retirement program in 1994-1996, and the wife is not qualified. We find that the share of households that belong to the non-cooperative type is around 60%. We also try to assign the observations to the appropriate type, and find that both husbands and wives of the cooperative type have higher education level than those of the non-cooperative type. One possible explanation for this interesting phenomenon is that the higher the education level, the easier the couple communicate and compromise to reach a Pareto optimal solution.

The rest of the paper is organized as follows. In section 2 we derive the model and discuss the method of estimations. In section 3, we consider the econometric specification, while section 4 give a basic description of data sources and the sample used in the analysis. Estimates are given in section 5. Section 6 concludes.

2 Theoretical framework

In this section, we present the theoretical framework that underlies the empirical model to be specified and estimated later.

We consider the problem of analyzing labor market household decision of elderly couples, when a new option (early retirement) becomes available to the husband. The available choices for the husband are either taking the early retirement ($y_m = 1$) or continuing to work ($y_m = 0$), while the wife's choices are not to work ($y_f = 1$) or work ($y_f = 0$). Note that working hours are exogenously determined and that they are not part of the choice alternatives.

As mentioned in the introduction, economists usually deal with this kind of family choice model in two ways, based on the basic assumption whether the family structure is cooperative or non-cooperative.

Traditionally, empirical studies of the household behavior have been routinely based on the 'first generation' economic model that applies a joint unitary optimizing framework. This model is a typical member of the cooperative category. It is often called the 'unitary model' or 'common preference model' in the literature. According to this model, the household is assumed to maximize a joint utility or more precise a household welfare function, which incorporate the preferences of both members, subject to a pooled budget constraints. The most important characteristics of this model as well as of other models of the cooperative type is that these models imply that the family behavior is Pareto optimal, i.e. no family member can be made better off without making another worse off. The unitary model has been a simple and powerful framework for the analysis of household labor supply and consumption. But in the last two decades, there has been an increasing interest to refine the cooperative models by modeling household behavior as the result of a cooperative game, particularly a cooperative Nash bargaining game. See, for example, Bourguignon and Chiappori (1992) for a review. But the empirical modeling along this line is rather difficult. This is mainly because it is almost impossible to identify the preferences and the threat points simultaneously. On the other hand no definite conclusion about which approach is better has been made yet. Kapteyn and Kooreman (1992) argued that more about the players' preferences should be known before one can discriminate between these two kinds of models empirically. So despite of increasing criticisms, the unitary model is still widely used in the empirical analysis of the family decision behaviors, especially in labor supply studies.

Although some consider the use of the non-cooperative game theoretical models in a family context is controversial, those models have raised considerable interests in the literature. Recent example includes Lundberg and Pollak (1994), Kooreman (1994) and Hiedemann (1998). In this framework, both the husband and wife have their own utility functions, and they engage in a non-cooperative game to maximize their own utilities. The great advantage of these models is that the equilibrium is self-enforcing, i.e. nobody will gain by deviate from the equilibrium. So on the contrary to the cooperative models, noncooperative models don't assume that husband and wife can enter binding, costless enforceable agreement, as pointed out by both Lundberg and Pollak (1994) and Chen and Woolley (2001).

Hernæs, Jia and Strøm (2001) recently tries to compare the empirical performance of different decision mechanisms within family using Norwegian data, and concluded that the Stackelberg game with male as a leader outperforms the Nash game and the unitary model. However, it is a quite strong assumption that one model fits every family in the sample. In this paper, instead of making this common assumption that all the households in the sample follow exact the same mechanism, either cooperative or non-cooperative, we would like to make a less restrictive assumption that those two types of family coexist. That is, we assume that the households are composed of two types, cooperative type and non-cooperative type. Both types are characterized by their decision-making patterns and type specific utility function. Thus the traditional setting is just a special case in our framework.

Cooperative type: Households following the unitary model

As we have discussed earlier, although the cooperative game theoretic model generated quite some interests recently, there are still quite a lot problems when we try to apply it for the empirical studies. In the present analysis, we still assume that the households of cooperative type follow the traditional unitary model, where the couple tries to make a choice to maximize their joint utility function

$$U(y_m, y_f) = v(y_m, y_f) + \varepsilon(y_m, y_f) \qquad (1)$$

The probability that alternative (y_m, y_f) is chosen by the decision maker is (we denote this class as class *C*):

$$P_C(y_m, y_f) = \Pr(U(y_m, y_f) = \max_{(y_m^{'}, y_f^{'}) \in S} U(y_m^{'}, y_f^{'})) . \quad (2)$$

where $S = \{(0,0),(0,1),(1,0),(1,1)\}$ is the set of available choices for the household.

Non-cooperative type: households playing a Stackelberg game

For household of this type, we assume that the husband and the wife have his/her own utility function, and both of them maximize their own utilities.

As above, we use a random utility framework, where the utilities are assumed to be composed of the deterministic and random components. We denote the utility for the husband and the wife as $U_m(y_m, y_f)$ and $U_f(y_m, y_f)$ respectively:

$$\begin{aligned} U_m(y_m, y_f) &= v_m(y_m, y_f) + \varepsilon_m(y_m) \\ U_f(y_m, y_f) &= v_f(y_m, y_f) + \varepsilon_f(y_f) \end{aligned} \quad (3)$$

where $(y_m, y_f) =$ (husband's choice, wife's choice) is the choice dummy vector for the household as we have defined. $v_k(.); k = f, m$, are the deterministic components of the utility functions and $\varepsilon_k(.); k = f, m$ are the random parts.

Similar to Bresnahan and Reiss (1990), Kooreman (1994), the choice problem is modeled as a non-cooperative game. The player of the game, husband and wife, can take one of two actions, working or not working. And the pay-off of the game is simply the utility function: $U_k(y_m, y_f)$; k=m,f. The pay-off matrix of the game is given in table 1.

Table 1: The pay-off matrix of the game

Husband	Wife	
	Works, y_f=0	Home, y_f=1
Works, y_m=0	$U_m(0,0)$, $U_f(0,0)$	$U_m(0,1)$, $U_f(0,1)$
Retired, y_m=1	$U_m(1,0)$, $U_f(1,0)$	$U_m(1,1)$, $U_f(1,1)$

As in Hiedemann (1998), We assume that the roles of husband and wife in this game are asymmetric. The husband of the household is assumed to be the leader, while the wife acts as a follower. Then we have a Stackelberg-game with male as a leader. Kooreman (1994) has developed a probability model to specify the probability of choosing each alternative for such a game. We give a detailed discussion of this model in Appendix , and we derive the probability of the couple choosing each alternative y for the case of male as the leader. We then denote this class as class *NC* and write the probabilities to choose the state (y_m, y_f) according to this model as $P_{NC}(y_m, y_f)$.

A mixed model of the household types

Recall that we have assumed that each household belongs to one and only one of these two different types. However, as analysts, we are not able to observe which type the household really is. Estimation conditional on the type membership is not possible. One common way to deal with this incomplete data problem is to assume that the sample in our analysis is a random sample from a mixed population with population share $\alpha_i \; i = C, NC$ ($0 \le \alpha_i \le 1$, and $\alpha_C + \alpha_{NC} = 1$). In other words, we have the probability of belonging to type *i* is the same for every household, namely, $\Pr(\text{household } k \text{ is type } i) = \alpha_i$ for all *k*. Note that these population share parameters also need to be estimated together with the preference parameters.

In last section, we have specified the choice model for each type, i.e. the conditional probability given the household type. Using Bayes's rule, we have

$$\begin{aligned}\Pr(y_m, y_f) &= \frac{\Pr(\text{belongs to type } NC)\Pr_{NC}(y_m, y_f)}{\Pr(\text{belongs to type } NC \mid y_m, y_f)} \\ &= E[\Pr_{type}(y_m, y_f)] \\ &= \alpha_C P_C(y_m, y_f) + \alpha_{NC} P_{NC}(y_m, y_f)\end{aligned} \tag{4}$$

And then the likelihood function follows (in the literature, it is often referred as the incomplete likelihood function).

The probability function has a finite mixture structure. We see immediately that this model takes each component model as special case.

3 The empirical model

In this section, we shall discuss the assumption about the distribution of the unobserved error terms and the functional form of the utility function, which subsequently, enable us to derive the empirical model.

We assume that the deterministic part of the utility function depends on consumption and leisure in both cases. Since we cannot directly observe consumption, disposable income serves as a proxy for it here.

The unitary model:

For the case of the unitary model, we specify the utility function as follows:

$$U(y_m, y_f) = \alpha \ln(C_{y_m,y_f}) + \beta_m \ln(L_{m,y_m}) + \beta_f \ln(L_{f,y_f}) + \beta_c \ln(L_{y_m,y_f}) + \varepsilon_{y_m,y_f} \quad (7)$$

where C_{y_m,y_f} is the joint disposable income. It is equal to annual after-tax income when the husband is in state y_m and the wife is in state y_f. Thus

$C_{y_m,y_f} = r_{m,y_m} + r_{f,y_f} - T(r_{m,y_m}, r_{f,y_f})$; in which r_{m,y_m} is the gross income of the husband when he is in state y_m , and r_{f,y_f} is the gross income of the wife when she is in state y_f, and *T(.)* is the tax function. The unit of tax calculation is the couple, not the individual, which means that the taxes paid by the couple depends on the labor market states of both members of the household. All details of the tax structure have been accounted for when we construct the data. We allow the marginal utility w.r.t joint income to depend on the net wealth of the household, namely, $\alpha = \alpha_0 + \alpha_1(\text{household wealth})$, since we suspect that household with higher wealth will value the disposable income less important, and will be more likely to take out early retirement.

L_{k,y_k} k=F,M is the individual leisure. It is defined as one minus the ratio of hours of work to total annual hours. For instance, for the case when husband choose to continue to work, $L_{m,0} = 1 - (37.5 * 46)/8760$. We expect the health condition of the husband play an important role in the decision. To verify whether this is the case, we let the marginal utility w.r.t male leisure depends on a health indicator of the husband. It is defined as the ratio of sick leave to total working hours in the 15 months prior to early retirement eligibility (AFP-eligibility). We assume that $\beta_m = \beta_{m0} + \beta_{m1}(\text{sick history})$.

In addition, we also included the 'common/joint' leisure term L_{y_m,y_f} in the utility function. It is used to account for the hypothesis that the family of the cooperative type not only derives utility from each member's individual leisure, but also from the leisure they enjoy together. As a direct consequence of this set up, we will see some 'coordination' – they tend to stop working at the same time, as a pattern noticed in Hurd (1997). We suspect this effect is decreasing as the age difference (wife's age subtracted from husband's age) increases. So we specify the marginal utility w.r.t the common leisure as

$\beta_c = \beta_{c0} + \beta_{c1}(\text{agediff}) + \beta_{c2}(\text{agediff})^2$. To cope with the problem that we don't have a detailed time schedule for each of the spouses, we follow the study by Jia (2000) and use $\min(L_{m,y_m}, L_{f,y_f})$ as a proxy for common leisure term in our analysis.

For the error term ε_{ij}, we assume that they are *iid* standard extremely distributed across the choice. Then the model becomes a multinomial logit as following:

$$P_C(y_m, y_f) = \frac{e^{v(y_m, y_f)}}{\sum e^{v(y'_m, y'_f)}}; \qquad (8)$$

The Stackelberg model:

In the case of the game theoretical model the separate utilities of the spouses depend on their own consumption and leisure:

$$\begin{aligned} U_m(y_m, y_f) &= a_m \ln(C^m_{y_m,y_f}) + b_m \ln(L_{m,y_m}) + \varepsilon_m(y_m) \\ U_f(y_m, y_f) &= a_f \ln(C^f_{y_m,y_f}) + b_f \ln(L_{f,y_f}) + \varepsilon_f(y_f) \end{aligned} \qquad (9)$$

However, since we have no direct observation on these values, we assume that there exists an income sharing rule $C^m_{ij} = \mu_m C_{ij}$ and $C^f_{ij} = \mu_f C_{ij}$ where μ_m, μ_f lie between 0 and 1. $\mu_m + \mu_f$ can be greater than 1 due to the existence of the public goods. We assume here that the income sharing rule is determined before the choice problem is presented, so it can be seen as exogenous in our analysis. For different household this sharing rule may be different, i.e. the parameter value μ_m, μ_f can be different across the households.

If we insert the income sharing rules into (9), we have:

$$\begin{aligned} U_m(y_m, y_f) &= \gamma_m + a_m \ln(C_{y_m,y_f}) + b_m \ln(L_{m,y_m}) + \varepsilon_m(y_m) \\ U_f(y_m, y_f) &= \gamma_f + a_f \ln(C_{y_m,y_f}) + b_f \ln(L_{f,y_f}) + \varepsilon_f(y_f) \end{aligned} \qquad (10)$$

Where $\gamma_k = \alpha_k \ln \mu_k$ k=m,f. We see immediately that the income sharing parameters are transformed into the constant term of the utility functions. Recall that in a discrete choice setting it is only the difference in utility that matters, the common factor in utilities of different alternatives is eliminated. So unfortunately given our specification of the utility function form, we are not able to identify the sharing parameters in our analysis.

Similar to the unitary model, we assume that for both husband and wife, the marginal utility w.r.t income depends on the household wealth, i.e. $a_k = a_{k0} + a_{k1}$(household wealth) k=m,f. And for the husband, the marginal utility w.r.t leisure depends on his sick history, i.e. $b_m = b_{m0} + b_{m1}(\text{sick history})$.

For the error terms, we assume that $\varepsilon_m(1), \varepsilon_m(0), \varepsilon_f(1), \varepsilon_f(0)$ is 4-variate normal distributed with zero mean and covariance matrix Ω.

$$\text{where } \Omega = \begin{pmatrix} \frac{\sqrt{2}}{2} & 0 & \frac{\rho}{2} & 0 \\ & \frac{\sqrt{2}}{2} & 0 & \frac{\rho}{2} \\ & & \frac{\sqrt{2}}{2} & 0 \\ & & & \frac{\sqrt{2}}{2} \end{pmatrix}.$$

The motivation is that we wish to allow the possibility of any 'common taste' (correlation between the same choice) between husband and wife. The covariance matrix is specified so that the random variable of interest (as mentioned in section 2) e_m and e_f is bivariate normally distributed with zero mean and covariance matrix $\begin{pmatrix} 1 & \rho \\ & 1 \end{pmatrix}$. Using table 1 in appendix, we can write down the choice probability for each state (y_m, y_f). The detailed formula are also provided in the Appendix.

4 Data

The data is based on register files held by Statistics Norway. It is a yearly-based panel that contains detailed information on labor market behavior and income and other socio-economic variables at individual level for the entire population. We also have information on marriage status, and it allows us to identify the household.

For the present study, we use data from the period 1993-98. During the observation period, 50 per cent of earnings in excess of the basic amount in the public pension system (USD 5 600) when retired were deducted from the pension. With a marginal tax rate on earnings and pension at say 40 per cent, the effective tax rate on earnings was 70 per cent. So disregarding the option of combining earnings and early (partly) retirement in the choices set is not unreasonable.

Starting with eligible persons, we restrict the sample in this study to comprise all married couples in which the husband qualified during the period from 1 October 1994 until 31 December 1996. Since the eligibility age was 64 from 1 October 1993 until 1 October 1997, the couples in the sample then knew at least one year in advance that retirement would become possible, and could plan retirement. We then restrict the data to couples in which the wife did not qualify and in which the wife is between 50 years old and 67 years old. These restrictions are imposed in order to make sure that that the options postulated for the two spouses are reasonable. These restrictions reduce the sample from 12475 couples in which the husbands qualify down to 10008 that fulfill all the criteria.

Because the individual can be observed in one state only, we can observe the gross income of the individual only in that state. In order to model different possible outcomes, we need to impute or simulate the gross income also in those states in which the individual is not observed. f the husband or the wife is observed working in the current period or in the year prior to the date of the husband's eligibility, then working are characterized by their observed earnings and leisure. A justification for this assumption is that at the age of the individuals considered here there is some rigidity in the labor market attachments. If the wife is observed to be out of the labor force the current and the previous period, then working is characterized by predicted earnings based on a log earnings function estimated on earnings data among those women working full time. Detailed regression result is given in Appendix 3. Leisure is predicted as leisure consistent with the working load related to the earnings that are assigned to the women. For the husband, potential pension following eligibility is calculated according to rules applied to his earnings history, which is observed. Details about pension rules are set out in Haugen (2000) [2].

Some descriptive statistics after the imputation are given in Table 1.

Table 1: Descriptive statistics

Variable	Average	Min	Max
Household disposable income, when both are working (100,000 NOK)	3.04	1.14	26.45
Household disposable income, when husband is working but wife is not	1.83	0.51	23.21
Household disposable income, when wife is working but husband is not	2.46	1.11	7.99
Household disposable income, when husband takes early retirement and wife is not working	1.25	0.67	1.65
Wealth (100,000 NOK)	5.82	0	1930.93
Age difference (age of husband – age of wife)	3.6	-3	14
Sick history (proportion of previous 15 months on sick leave)	0.023	0	0.87
No of Observation: 10008			

[2] Some part of this paragraph is from Henaes, Jia and Strom (2001)..

5 Estimation results

The parameters to be estimated include: both husband and wife's utility parameters in the Stackelberg game case, the joint utility function parameter for the unitary model, and the share of each class (component weights) α_i .(since we have the constraint $0 \le \alpha_i \le 1$, a transformation $\alpha_{NC} = 1/(1+\exp(\delta))$ is used in the estimation instead of directly estimating α_{NC}) .

The estimation results for this model are given in table 2. We find out that around 61.4% of the household in the sample behave according to the Stackelberg game with husband as a leader and belong to the non-cooperative type, while the rest behave according to the unitary model and belong to the cooperative type.

Most of the estimates have the expected sign. For the cooperative type, we see the effect of wealth on the marginal utility of disposable income is negative as we expected. This is consistent with our hypothesis. The marginal utility of male leisure increases with sick-history, which suggests that males with not-so-good health condition value their leisure more and they are thus more likely to take out the early retirement. The parameters associated with the common leisure term are all sharply determined. In line with our expectation, the marginal utility w.r.t the common leisure is positive for all relevant age differences, and it decreases as age differences increase up to 12 years. After an age difference of 12 years it begins to climb up very slowly. For the non-cooperative type, we find that unobserved variables affecting the utility levels of the spouses are positively correlated. This can be explained by common taste, either due to why they got married in the first place or it had been formed during the long years of adjustments and compromises from both parties. Hiedemann (1998) reported similar results as well. To our surprise, we find a positive but small wealth effect for the wife. For the husband, similar as was the case for the cooperative type, we find negative wealth effect on marginal utility of disposable income, as well as positive effect of sick-history on marginal utility of leisure.

Table 2: Estimation results for the mixture choice model

Coefficient	Variable	Estimates	Asy t-value
Unitary Models (cooperative type)			
α_0	Income, constant	7.734	18.311
α_1	Income, linear in wealth	-0.024	-3.634
β_{f0}	Female leisure	1.885	1.514
β_{m0}	Male leisure: Constant	1.407	2.574
β_{m1}	Male leisure: Linear in sick history	21.510	21.509
β_{c0}	Common leisure: Constant	27.225	27.008
β_{c1}	Common leisure: Linear in age difference	-47.225	-46.337
β_{c2}	Common leisure: Quadratic term in age difference	20.660	19.806
Stackelberg model (non-cooperative type)			
	Wife's utility function		
α_{f0}	Household disposable income: constant	7.711	22.766
α_{f1}	Household disposable income: linear in wealth	0.062	8.708
β_{f0}	Female leisure	27.603	22.309
	Husband's utility function		
α_{m0}	Household disposable income: constant	0.443	4.578
α_{m1}	Household disposable income: linear in wealth	-0.001	-0.047
β_{m0}	Male leisure: Constant	-1.861	-10.971
β_{m1}	Male leisure: Linear in sick history	6.582	6.546
ρ	Correlation	0.616	18.714
δ	Proxy for Share of the Stackelberg group in population	-0.465	-11.535
α_{NC}	Share of Stackelberg group	0.614	
	Observations	10008	
	Log-likelihood	-11847	
	ρ^2	14.6%	

The assignment of observations to the latent type

To shed some lights on the relationship between family decision-making structure and some social-economical variables, we would like to assign the latent class to each household.

As discussed in McCutcheon (1987), it is frequent that we wish to assign the observations to the appropriate latent class. In our case, it is of interest to compare the value of some social-economic variables such as the education level of each member etc to gain some insights on how these variables affects the behavioral structure of the household.

The only information we can make use of here is the actual choice made and the predicted probabilities assigned to that choice for each class. The method used in the literature is to calculate the conditional probability that household belongs to class *k* (*k=C or NC)* given its actually choice (y_m, y_f):

$$\Pr(\text{belongs to type } k \mid y_m, y_f) = \frac{\alpha_k \Pr_k(y_m, y_f)}{\alpha_C \Pr_C(y_m, y_f) + \alpha_{NC} \Pr_{NC}(y_m, y_f)} \tag{10}$$

And then assign the household to the latent class with the largest conditional probability. It is clear that the assignment is probabilistic, and some error will be involved in this procedure. But since there is no better solution so far, we will use this method in our analysis.

The summary statistics of some variable of interest of these two classes are presented in table 3. We find that compared with the group, which follows the Stackelberg game, the age difference between husband and wife is higher in the unitary group. And more interesting, it seems that both husbands and wives in the unitary group have higher education level than those in the Stackelberg group. One possible explanation will be that the higher the education level is, the better husband and wife can communicate and compromise so that they can adjust their behavior to reach a solution that are beneficial for both of them and Pareto optimal. Of course, we would always keep in mind that the method used to assign the latent class to each household is probabilistic, some errors are involved in the procedure, so the findings here is not precise and we should be careful when we interpret the results.

Table 3: Some summary statistics for the two classes

	Stackelberg		Unitary	
Age difference (age of husband -age of wife)	3.1		4.1	
	Husband	wife	Husband	wife
compulsory education	28.4 %	40.9 %	25.4 %	31.4 %
Secondary	45.6 %	45.9 %	42.6 %	46.3 %
university and college	12.8 %	10.9 %	15.0 %	19.0 %
master degree and above	12.9 %	2.2 %	16.8 %	3.0 %

6 Conclusion

Discussions of heterogeneity in traditional models of labor supply are usually focused on two aspects: either differences in preferences or differences in budget constraints. In this paper, we have introduced an additional source of heterogeneity for two-person households. Specifically, we have assumed that there are two latent types of households, where one type behave according to the unitary model while the other type behave according to the Stackleberg game model. Consequently, the model that corresponds to the data is a mixture of the two conditional models for each type.

We have used this framework to analyze the labor market behavior of elderly couples in Norway, when a new option (early retirement) becomes available to the husband. Under the assumption above, we find that around 60 per cent of the households in our sample behave according to the Stackelberg game. We have also tried to assign the observations to the respective types, and find that both husbands and wives of the cooperative type have higher education level than those of non-cooperative type.

We have demonstrated that it is feasible to estimate endogenously the size of groups that differ regarding the structure of the decision process, and the utility parameters simultaneously using MLE based on our econometric settings. We believe that this framework represents a more realistic setting for analyzing household behavior. Although we have focused on a particular application, the approach developed in this paper can readily be applied to other forms of multi-agents choice settings as well.

One major difficulty with applying this framework is how to specify the latent classes (household types in our analysis). Economic theory and previous empirical evidence don't always provide enough insights on what classes/types should be included in the analysis. And we may not be able to specify a model with all possible classes/groups and wishing to use genera-to-specific approach to find the appropriate model, since over fitting the model will greatly increase the number of parameters to be estimated and may make the already hard estimation impossible sometimes. However, this should not be an argument against to use this framework. We believe it is a fruitful step forward from the single 'naive' homogenous model.

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Appendix. Stackelberg equilibrium

We want to determine the stackelberg equilibrium of the following game described in Table A.1. We assume that the husband behaves as the leader and the wife is the follower. In this case, a state (y_m, y_f) is a Stackelberg equilibrium if and only if:

$$\begin{cases} U_f(y_m, y_f) > U_f(y_m, 1-y_f) \\ U_f(1-y_m, y_f) > U_f(1-y_m, 1-y_f) \end{cases} \text{ and } \quad U_m(y_m, y_f) > U_m(1-y_m, y_f)$$

or

$$\begin{cases} U_f(y_m, y_f) > U_f(y_m, 1-y_f) \\ U_f(1-y_m, y_f) < U_f(1-y_m, 1-y_f) \end{cases} \text{ and } \quad U_m(y_m, y_f) > U_m(1-y_m, 1-y_f)$$

Use this definition and recall that we have the random utility framework

$$U_m(y_m, y_f) = v_m(y_m, y_f) + \varepsilon_m(y_m)$$
$$U_f(y_m, y_f) = v_f(y_m, y_f) + \varepsilon_f(y_f)$$

We can calculate the probability of each state (y_m, y_f) being a Stackelberg Equilibrium.

Take state (1,1) as an example:

It is a Stackelberg equilibrium if and only if:

$$\begin{cases} e_f > v_f(1,0) - v_f(1,1) \\ e_f > v_f(0,0) - v_f(0,1) \end{cases} \text{ and } \quad e_m > v_m(0,1) - v_m(1,1)$$

or

$$\begin{cases} e_f > v_f(1,0) - v_f(1,1) \\ e_f < v_f(0,0) - v_f(0,1) \end{cases} \text{ and } \quad e_m > v_m(0,0) - v_m(1,1)$$

where $e_k = \varepsilon_k(1) - \varepsilon_k(0)$.

It follows immediately that the probability of state (1,1) to be a Stackelberg Equilibrium will equal to

$$\begin{aligned} \Pr(1,1) = & \Pr(e_f > \max(v_f(1,0) - v_f(1,1), v_f(0,0) - v_f(0,1)) \text{ and } e_m > v_m(0,1) - v_m(1,1)) \\ & + \Pr(v_f(1,0) - v_f(1,1) < e_f < v_f(0,0) - v_f(0,1), \text{and } e_m > v_m(0,0) - v_m(1,1)) \end{aligned} \quad (*)$$

It is just simple repetition to calculate the probabilities for other states, and the results is listed in table A.1

Table A.1 Stackelberg equlibrium (SE) (male as leader)

Female utility comparison	Female utility error term requirement	Male utility comparison	Male utility error term requirement	SE
$U_f(1,1)-U_f(1,0)>0$ $U_f(0,1)-U_f(0,0)>0$	$e_f> \max[v_f(1,0)-v_f(1,1), v_f(0,0)-v_f(0,1)]$	$U_m(1,1)-U_m(0,1)>0$	$e_m>v_m(0,0)-v_m(1,0)$	(1,1)
		$U_m(1,1)-U_m(0,1)<0$	$e_m<v_m(0,0)-v_m(1,0)$	(0,1)
$U_f(1,1)-U_f(1,0)>0$ $U_f(0,1)-U_f(0,0)<0$	$v_f(0,0)-v_f(0,1) > e_f > v_f(1,0)-v_f(1,1)$	$U_m(1,1)-U_m(0,0)>0$	$e_m>v_m(0,0)-v_m(1,1)$	(1,1)
		$U_m(1,1)-U_m(0,0)<0$	$e_m<v_m(0,0)-v_m(1,1)$	(0,0)
$U_f(1,1)-U_f(1,0)<0$ $U_f(0,1)-U_f(0,0)>0$	$v_f(1,0)-v_f(1,1) > e_f > v_f(1,0)-v_f(1,1)$	$U_m(1,0)-U_m(0,1)>0$	$e_m>v_m(0,1)-v_m(1,0)$	(1,0)
		$U_m(1,0)-U_m(0,1)<0$	$e_m<v_m(0,1)-v_m(1,0)$	(0,1)
$U_f(1,1)-U_f(1,0)<0$ $U_f(0,1)-U_f(0,0)<0$	$e_f< \min[v_f(0,0)-v_f(0,1), v_f(0,1)-v_f(1,1)]$	$U_m(1,0)-U_m(0,0)>0$	$e_m>v_m(0,0)-v_m(1,0)$	(1,0)
		$U_m(1,0)-U_m(0,0)<0$	$e_m<v_m(0,0)-v_m(1,0)$	(0,0)

However, before we can specify the likelihood function we need to notice that the probability equation (*) involves a max operation, which may cause the likelihood function to be non-differentiable. If the likelihood function is non-differentiable, we will not be able to use gradient-based optimization algorithm such as BFGS, more seriously the MLE estimator will lose its nice asymptotic properties, which make the normal inference inappropriate.

Fortunately, given our empirical setting of the deterministic part of the utility function, we don't have this problem. Once again, we take the state (1,1) as an example:

Using the utility function specification (11), we have

$$\begin{aligned}
&\max(v_f(1,0)-v_f(1,1), v_f(0,0)-v_f(0,1)) \\
&=\alpha_f \max(\ln(C_{10}/C_{11}), \ln(C_{00}/C_{01})) + \beta_f \ln(L_{f0}/L_{f1}) \\
&=\begin{cases} \alpha_f \ln(C_{10}/C_{11}) + \beta_f \ln(L_{f0}/L_{f1}) & \text{if} \quad C_{10}/C_{11} > C_{00}/C_{01} \\ \alpha_f \ln(C_{00}/C_{01}) + \beta_f \ln(L_{f0}/L_{f1}) & \text{other wise} \end{cases}
\end{aligned}$$

We see immediately that for any given household, since the relationship between C_{10}/C_{11} and C_{00}/C_{01} is fixed, the probability doesn't involve any max operation. Thus the likelihood function doesn't have the non-differentiable problem. And we can just proceed as usual.