

MEMORANDUM

No 08/2003

THE TALE OF TWO RESEARCH COMMUNITIES: THE DIFFUSION OF RESEARCH ON PRODUCTIVE EFFICIENCY

By

Finn R. Førsund and Nikias Sarafoglou

ISSN: 0801-1117

Department of Economics
University of Oslo

This series is published by the
University of Oslo
Department of Economics

P. O.Box 1095 Blindern
 N-0317 OSLO Norway
 Telephone: + 47 22855127
 Fax: + 47 22855035
 Internet: <http://www.oekonomi.uio.no/>
 e-mail: econdep@econ.uio.no

In co-operation with
**The Frisch Centre for Economic
 Research**

Gaustadalleén 21
 N-0371 OSLO Norway
 Telephone: +47 22 95 88 20
 Fax: +47 22 95 88 25
 Internet: <http://www.frisch.uio.no/>
 e-mail: frisch@frisch.uio.no

List of the last 10 Memoranda:

No 07	Taryn Ann Galloway and Rolf Aaberge Assimilation Effects on Poverty Among Immigrants in Norway. 26 pp.
No 06	Geir B. Asheim Green national accounting with a changing population. 23 pp.
No 05	Geir B. Asheim and Ylva Søvik The semantics of preference-based belief operators. 30 pp. Rolf
No 04	Zhiyang Jia A Mixture Model of Household Retirement Choice. 21 pp.
No 03	Erling Eide Optimal Provision of Public Goods with Rank Dependent Expected tility. 21 pp.
No 02	Hilde C. Bjørnland Estimating the equilibrium real exchange rate in Venezuela. pp.
No 01	Svenn-Erik Mamelund Can the Spanish Influenza pandemic of 1918 explain the baby-boom of 1920 in neutral Norway?. 33 pp.
No 36	Elin Halvorsen A Cohort Analysis of Household Saving in Norway. 39 pp.
No 35	V. Bhaskar and Steinar Holden Wage Differentiation via Subsidised General Training. 24 pp.
No 34	Cathrine Hagem and Ottar Mæstad Market power in the market for greenhouse gas emissions permits – the interplay with the fossil fuel markets. 21pp.

A complete list of this memo-series is available in a PDF® format at:
<http://www.oekonomi.uio.no/memo/>

**THE TALE OF TWO RESEARCH COMMUNITIES:
THE DIFFUSION OF RESEARCH ON
PRODUCTIVE EFFICIENCY***

by

Finn R. Førsund[‡]

Department of Economics, University of Oslo, Norway

and

Nikias Sarafoglou

Department of Economics

Mid-Sweden University, Sweden

Abstract. The field of theoretical and applied efficiency analysis is pursued both by economists and people from operational research and management science. Each group tends to cite a different paper as the seminal one. Recent availability of electronically accessible databases of journal articles makes studies of the diffusion of papers through citations possible. The conventional wisdom that the seminal paper within economics lay dormant for two decades, and that efficiency studies only got rolling after the operational research paper appeared, is shown to be wrong. Citation peaks have been found to be typically five to seven years, with a long tailing off. Both seminal papers followed quite different diffusion patterns. Research strands inspired by the seminal paper within economics are identified and followed by citation analysis. In recent years a weak trend toward convergence of the two camps into a common network for efficiency and productivity analyses is documented.

Keywords: Bibliometric methods, Farrell efficiency measures, DEA.

JEL classification: B21, D24

* The paper is a further development of a paper presented at the IFORS conference, Athens, 1999. We are indebted to William W. Cooper, Harry Burley, Sverre Kittelsen, Arnold Reisman and Edwardo Rhodes for valuable inputs and comments.

[‡] Corresponding author, address: Box 1095, 0317, Norway. E-mail: f.r.forsund@econ.uio.no

1. Background

In bibliometrics a paper becomes classical when the number of citations exceeds 500. In the research field of efficiency and productivity of production activities there are two papers, published with a span of 20 years difference, that are now classical: “The measurement of productive efficiency” by Michael James Farrell (1957), having 903 citations in the web-based Social Science Citation Index (SSCI) (ISI Web of Knowledge) by February 2003 and ”Measuring the efficiency of decision making units” by Abraham Charnes, William Wager Cooper and Edwardo Rhodes (1978) (CCR), having 989 (this paper will also have many citations in the SCI). The number of citations does not necessarily measure the ”true” value of journal contributions, but papers achieving classical status may have advanced new paradigms, or at least have been very influential.

The two papers are rooted in two different research environments, economics and operational research or management science (OR/MS). Our concern is that there seems to be too little knowledge among OR/MS researchers of the contributions of Farrell and the research inspired by him in the 20 years before CCR was published¹, and too little ”cross-camp” contacts afterwards, resulting in unnecessary parallel research efforts. The existence now of databases of citations based on the bulk of published journal papers like the SSCI makes it possible to track the diffusion of Farrell and CCR and reveal citation patterns for evidence of cross-camp contacts. The specific purpose of this paper is to document - using bibliometric methods - both the broader activity of efficiency analysis of the period between Farrell (1957) and CCR (1978) and the continuing influence of Farrell (1957) after the publication of CCR (1978). The diffusion of Farrell and CCR through citations over time and distribution through journals will be used to give an impression of the scope of diffusion. Special emphasis will be put on the diffusion of Farrell up to the publication of CCR, and the nature of the diffusion of the two papers will be revealed. To show the extent and development of cross-camp contacts we will make use of joint citation analysis.

¹ Førsund and Sarafoglou (2002) tried to redress this apparent imbalance in the conventional opinion among OR/MS researchers as to the origins of DEA analysis. The focus was on how Farrell’s ideas were followed up, up to and including the contribution by Charnes, Cooper and Rhodes. The focus in the present paper is on bibliometric aspects of diffusion of the two seminal papers up to the present time. The presentation of Farrell’s main contributions is overlapping.

To set the following citation analysis in the right perspective one should be aware of the fact that about 2/3 of journal papers never earn any citations except self citations. Impact factors of journals, i.e. expected number of citations in other journals per journal article per year, and half-life, i.e. estimate of number of years a paper in a journal will be quoted, varies a lot with journals. Citation peaks have been found to be typically five to seven years, with a long tailing off (see e.g. Price (1976) and Johnson (1997) for studies of diffusion distributions). However, the bulk of the citations of the two papers we are investigating are from the 90s, underlining the observation made by Johnson (1997) that "old capital" should not be depreciated too quickly when measuring influence.

With papers of such large impact there is the danger now that their advancement of knowledge has become intrinsic. It is referred to as "obliteration by incorporation" in Zuckerman (1987, p. 331). The key concepts are used without reference, becoming canonical knowledge or household expressions, like *Walras law*, or the *Cobb – Douglas* production function² (Johnson, 1997). One could, of course, ask whether it is necessary to refer to seminal classical papers if we only want to signal the use of some concepts or techniques, such as Farrell efficiency measures or the DEA model. However, we take the temerity to suggest that quite a number of papers while just referring to the classics now, would have benefited even more had the authors studied the cited papers. There is the well-known danger of citations following "success breeds success" (Price, 1976), with little real knowledge accumulation signified by the citations.

A word of caution is necessary as to the quality of the SSCI database. It does not encompass all relevant journals, and mistakes and oversights may exist, further complicated by the trivial reason that authors occasionally misspell their references.

In Section 2 we will motivate the need for our study by demonstrating lack of sufficient recognition in many OR/MS papers of Farrell and his legacy. In section 3 the Farrell contributions are reviewed. Section 4 portrays the pattern of diffusion of Farrell over time and also across journals based on citations prior to the publication of CCR. For a comparison the diffusion of citations of CCR for its first decade is also studied in a similar way. The total development of citations up to now is shown

² This mathematical function was actually introduced to economics by Knut Wicksell (1893) (Sandelin, 1976).

for Farrell and CCR and also for some seminal papers representing milestones in the development of the field of efficiency studies. In Section 5 some explanations for the different patterns of diffusion are offered. In Section 6 the pattern of joint references is investigated to analyse the contacts between the camps, the change in interrelations, and also differences in diffusion across types of journals is illustrated. Concluding remarks are offered in Section 7.

2. The neglect by the OR/MS camp

There is a special relationship between economics and operations research. According to Mirowski (1999) the rise of operations research had an important influence on the development of post (second world) war economics. A core concept in OR developed during the war was the production function appearing in neoclassical economics (see Mirowski, p. 692). When outsiders (engineers, physicists, computer scientists) “invade” a field it is often the case that the “trespassers [are] oblivious of their predecessors.” (Mirowski, p. 692).

The way the origin of efficiency analysis of production units is regarded within the two research communities may be an example of just this phenomenon. Among OR/MS researchers the general opinion is that the DEA model or analysis was introduced by CCR (1978). Referring to answers from 25 researchers about the most influential DEA publications, Seiford (1996, p. 104) states:³

A few researchers listed Farrell (1957) among the ten most influential papers, while one *more accurately* [our Italics] characterised it as one of the ten least influential!

Gattoufi, Oral and Reisman (2003a, footnote 1) expressed the same sentiment recently:

There is a widespread agreement that Charnes et al. (1978) represents the “official birth” of DEA.

This point of view also prevails in the International Journal of Production Economics (IJPE), which we will use as an example for an in depth investigation. IJPE is No. 10 on the list of 490 journals publishing most DEA papers up to August 2001 with 32 DEA publications, according to the

³ See Sarafoglou (1997) for a bibliometric scrutiny of Seiford’s “top ten” list.

definition (not explicitly stated) of DEA publications used in Reisman, Oral and Gattoufi (2003), Table 3.

The detailed results are shown in Table 1. The 19 DEA papers (defined as entering “DEA” or “envelopment among the key words) we found published in IJPE between 1995 (a special issue on DEA appeared in this volume) and 2001 are classified as to whether they refer to Farrell (1957) and/or CCR-1978, and whether Farrell or CCR is cited as the originator of the DEA model. No one refers to Farrell as the originator of DEA, and only six (1/3) refer to Farrell at all, although all papers are concerned with calculating or using Farrell efficiency measures in some way or other. All but four of the 19 papers refer to CCR, and cite it as the originator of DEA. Of the four papers, one takes the knowledge of DEA as given (Ritchie and Rowcroft, 1996), one is concerned more about the frontier technology as such (Tulkens and van den Eeckhaut, 1995), one considers only papers from “own circle,” although a DEA model is set up (Färe, Grosskopf and Roos, 1995), and one (Lovell, 1995) uses FDH on a macro level (though the Farrell efficiency concept is utilised without reference). Without actually going through all the DEA papers (over 1800) reported in Gattoufi, Oral and Reisman (2003a) we have the feeling that a similar pattern as revealed by Table 1 for IJPE is representative for OR/MS journals.

The DEA papers sampled in IJPE are remarkable in the sense that among the 35 authors we have top ranked DEA writers; No. 1 William Wager Cooper, No. 3 Rolf Färe, No. 5 Shawna Grosskopf, No. 7 C. A. Knox Lovell, No. 8 Rajiv D. Banker, No. 14 Robert M. Thrall, and No. 19 Russell G. Thompson, according to the ranking of “Top twenty” DEA authors per August 2001 in Gattoufi et al. (2002). Within the OR/MS camp neither Cooper nor Banker refers to Farrell. More surprisingly, this is also the case for the economists, Färe, Grosskopf, Lovell and Tulkens. However, they do not refer to CCR either! It is only Thompson and Thrall from the OR/MS camp among the top twenty authors that refer to Farrell, but not as an originator.

In the special issue of International Journal of Production Economics, mentioned above, an interesting review of production function theory and application, covering major scientific concepts created and used, appeared (Grubbström, 1995). However, efficiency analyses were not covered. This paper can therefore also be seen as filling this void.

3. The contributions of Farrell

In order to be able to evaluate our claim that Farrell (1957) may be considered as the originator of the DEA approach, we will give a brief review of Farrell's contributions. The fundamental assumption of Farrell was the possibility of inefficient operations, immediately pointing to a *frontier production function* concept as the benchmark, as opposed to a notion of *average performance* underlying most of the previous econometric literature on the production function. Inspired by the activity analysis of Koopmans (1951)⁴, he worked with a piecewise linear frontier function technology. Inspired by Debreu (1951), analysing efficient use of resources at a macro level using a "coefficient of resource use," he introduced - at the micro level - a radial contraction/expansion connecting inefficient observed points for production units with (unobserved) reference points on the production frontier as the basis for the measure of *technical efficiency*.⁵ Two more measures were defined, the *allocative*, or price efficiency measure showing the inefficiency due to choice of input mix only at existing input prices, and overall efficiency as the product of technical and allocative efficiency. The decomposition of overall efficiency follows from the fundamental duality between cost- and production functions.

The original illustration of the Farrell efficiency measures is shown in Figure 1. The definitions are:

Technical efficiency: inputs needed at best practice to produce observed outputs relative to observed input quantities, keeping observed input ratios; OQ/OP .

Allocative- or Price efficiency: costs of producing observed output at observed factor prices, assuming technical efficiency, relative to minimised costs at the frontier; OR/OQ .

Overall efficiency: costs of producing observed output if both technical efficiency and price efficiency are assumed relative to observed costs; $OR/OP = (OQ/OP) (OR/OQ)$.

⁴ Farrell knew activity analysis and Koopmans well from a visit at Cowles Commission 1951-1952, see Farrell (1954).

⁵ Some authors refer to this efficiency measure as the Farrell-Debreu measure. However, as pointed out in Førsund and Sarafoglou (2002) we prefer to attach only Farrell's name to the measure. An additional argument to the ones put forward in Førsund and Sarafoglou is that in the case of a general production function, one has to specify the orientation of the efficiency measure, i.e. input- or output orientation. Debreu was only concerned with the saving of resources, while Farrell mentioned both orientations.

In the choice of a production frontier benchmark Farrell adopts a most practical approach, starting with engineering considerations and ending up with recommending observed best practice. He used a *piecewise linear* envelopment of the data as the most pessimistic specification of the frontier, in the sense of the function being as close to the observations “as possible,” as illustrated in Figure 2 (the original figure).

Concerning the calculation of the efficiency measures Farrell set up a system of linear equations that in principle would yield the efficiency scores also for multiple outputs. However, his empirical application to US agricultural farms on a state level used only a single output, and four inputs. As far as we know his solution algorithm for the multiple output case has never been tried, because in the discussions of Farrell’s paper, A. J. Hoffman (1957) made the very crucial intervention that the newly developed algorithm for solving linear programmes (LP) could be used. This LP idea was implemented in Farrell and Fieldhouse (1962), but the application was still to the single output case, although it indicated how a general multiple output case could be set up.

Agricultural economists at Berkeley picked up Farrell’s approach. At a symposium in 1966, the group staged a “Farrell revival” workshop. Four contributions were published in the symposium volume in 1967. In it Boles (1967) restated and interpreted the LP formulation in Farrell and Fieldhouse (1962), and gave concrete indications for how a multiple output case could be formulated. He completed this task in a working paper (Boles, 1971), which never made it into a journal paper. As far as we know, it has only been referred to once in a journal article (Hanoch and Rothschild, 1972). As pointed out in Førsund and Sarafoglou (2002) the LP formulation in Boles (1971) is identical to what was later to become known as the “CCR DEA model”.

The case for being a little more careful when referring to the origins of the DEA approach should now have been established. But let there be no misunderstanding: CCR was fully aware of Farrell’s contributions and comments at length on the inspiration from Farrell and offers extensive interpretations of Farrell, and points out the improvements introduced by CCR.⁶ The only oversight

⁶ The improvements are documented in Førsund and Sarafoglou (2002) at length. Suffice it to add here that CCR formulated directly the LP model for multiple outputs, and in general set up the LP model in such a way that it become mathematically precise and easily accessible, including discussion of the duals (but this is also the case in Boles (1971), although much more limited).

is the contributions of the Berkeley agricultural economists, and especially Boles (1967, 1971). It is the *followers* of Charnes and Cooper in the MS/OR camp that tend to overlook Farrell. One reason may be that the expression “DEA” was coined in CCR (or more precisely, in the Ph.D. dissertation of E. Rhodes from the same year).

Within what we can identify as the *Farrell approach* to analyses of productive efficiency there were some distinct lines of theoretical and empirical developments to follow for later researchers in the field. These include:

- i) Extending the definitions of efficiency
- ii) Extending the specification of the frontier production function
- iii) Extending the estimation techniques of the frontier function and/or the efficiency measures
- iv) Extending the computational techniques to estimate the frontier function and/or the efficiency measures
- v) Extending the techniques for presentation of efficiency results

A very important task not entered in the list above is the motivation of the use of the Farrell approach in real life applications. The reason why is not that this aspect is not of utmost importance, but that this task should not necessarily be left to academic researchers.

Farrell (1957) presented ideas on all the points above. It is our assertion that the CCR (1978) contributions all fall within the list above, thus being firmly within the Farrell approach to analyses of productive efficiency. We will return to the list to evaluate the contribution after Farrell (1957) in the next section.

4. The diffusion patterns

The diffusion of Farrell (1957) prior to CCR (1978)

There seems to be a widespread misunderstanding, not only in the OR/MS community, that Farrell was forgotten until CCR was published. A typical attitude is expressed by the following quotation:

Farrell's contribution was itself ignored for more than two decades. It was rediscovered by Charnes, Cooper, and Rhodes (1978), who referred to the mathematical-programming method of measuring technical efficiency as data envelopment analysis (DEA), an appellation that seems to have stuck. Their paper has led to a flood of papers applying DEA, most of them in management-science/operations-research journals.”

(Russell, 1998, p. 28)

On the contrary, compared with an average paper, Farrell received quite a widespread attention *prior* to the publication of CCR. The number of citations per year, the authors, and the journals where the citations appeared, can be seen from Table 2. Economists both from Europe and U.S. made early citations of Farrell. The first citation came in 1959, and was not very enthusiastic, as revealed by the evaluation:

Mr. Farrell's work has the virtue of making a start – though perhaps a false start – where previously little had been done.

Hall and Winston (1959, p. 85)

The next reference was in 1960 by Hicks, and then in 1961 by Ruist.⁷ Within the first five years⁸, Farrell received four citations, including one self-citation, distributed on three journals. The next five-year period brought the total number of citations up to nine, distributed on seven journals. The journals having citations the first 10 years reflect the heterogeneity of the diffusion, and they have on average high “impact factors” like *Economic Journal*, *Econometrica*⁹ and *Review of Economics and Statistics*.

Note that an early citation came in 1964 by an OR scientist (Amey), and was, quite interestingly, a consequence of 1963 being officially designated as a National Productivity Year in England. The conference theme of the National Conference of the Operational Research Society was “Productivity Criteria: Their use and abuse”. Amey (1964) was read at the meeting. However, this exposure did not create any measurable reaction from the OR community.

The group of researchers was heterogeneous both regarding nationality and research profile, ranging from econometricians, agricultural economists and mainstream economists to management- and OR scientists. Concerning the latter group it should be remarked that there are only two citations, in 1964 and in 1976. By and large the OR/MS community missed out on Farrell before CCR.

⁷ Erik Ruist (1960) referred to Farrell in a booklet in Swedish covering the same ground as Ruist (1961).

⁸ Remember that it takes five to seven years for an average paper's citations to peak.

⁹ Note that in the survey of Walters (1963) there is only a passive reference to Farrell (1957).

In the first decade the only development of the Farrell approach was by Farrell himself, in Farrell and Fieldhouse (1962). Here the constant returns to scale assumption used in Farrell (1957) was generalised within a single output framework, and a linear programming (LP) format adopted, following the advice by Hoffman in the Discussion (1957). A mainframe programme was developed by Boles (1967, 1971) and used by Seitz (1970, 1971).

The next decade saw quite another level of activity regarding citations. The reason is the development of methods for estimating the frontier function introduced by Farrell (1957).¹⁰ The first paper out was Aigner and Chu (1968). The non-parametric approach of Farrell was dropped, but the programming format was kept for the calculation of the parameters of the Cobb – Douglas frontier function (and the efficiency score). The key feature of the formulation of the estimation problem was a *one-sided deviation* from the frontier function. This was handled by inequality constraints on each observation expressing the production function on a logarithmic form. An influential follow-up was Timmer (1971) with a potential link to Charnes and Coopers' concept of chance constraints by introducing a possibility to overshoot the frontier.¹¹

Afriat (1972) was the next milestone. He elaborated further ideas from Farrell (1957) and what came up during the discussion of Farrell's paper. A statistical framework was formulated for finding Maximum likelihood estimators for the parameters of frontier functions, leaving the pure programming format. But Afriat (1972) also contributed within the non-parametric framework of piecewise linear frontier functions by formulating the model with variable returns to scale. This was later launched as the BCC (1984) model by the OR/MS community, without reference to Afriat.

A "crowning piece" concerning statistical estimation of parametric frontier functions was the composed error approach, allowing overshooting the frontier due to "white noise," but keeping the one-sided error term capturing inefficiency, published independently at the same time by Aigner, Lovell and Schmidt (1977) and Meeusen and Broeck (1977).¹²

¹⁰ Notice, however, that Førsund and Sarafoglou (2002) expose a clear lack of appreciation of the ideas in Farrell (1957), and especially the ideas generated by the published discussion of Farrell's paper.

¹¹ However, his so-called probabilistic approach boiled down to dropping enough frontier observations until a given percentage overshoot was reached.

¹² Meeusen and Broeck (1977) were soon "driven out" of the citation market, illustrating the "Matthew effect", as pointed out in Førsund and Sarafoglou (2002).

Developments of the efficiency measures themselves were performed in Førsund (1971). These elaborated on the concept of the efficiency frontier in input coefficient space (single output) in a setting of a general production function, exhibiting the “Regular Ultra Passum Law” of Frisch (1965). Førsund and Hjalmarsson (1974), (1979) introduced the input- and the output-oriented efficiency measures and scale efficiency measures needed for the general case of a variable returns to scale (VRS) production frontier. Färe (1975) pointed out the equality between Shephard’s concept of distance function and Farrell’s measure of technical efficiency.¹³

The total number of citations of Farrell (1957) for the twenty-year period is 46, distributed over 27 journals. There is a fairly even spread on journals, top journals like *Journal of Political Economy* and *American Economic Review* having three citations each, the same as a general journal like *Swedish Journal of Economics* (later *Scandinavian Journal of Economics*). It is interesting to note that the first methodological advance came in *American Economic Review*, then several developments in *International Economic Review*, and first in the last year a methodological contribution appeared in a specialised econometrics journal (*Journal of Econometrics*). As pointed out earlier the agricultural economist category was the only group trying to develop the non-parametric approach of Farrell. The citations in agricultural journals numbers eight distributed on three journals, two of them with potentially high impact, thus constituting the most marked sector diffusion of Farrell. Otherwise there is no marked applied profile judging from the involved journals.¹⁴

The diffusion of Charnes, Cooper and Rhodes (1978) the first decade

The CCR (1978) paper had a more rapid diffusion than Farrell (1957), as shown in Table 3. During the first five years there are 22 references distributed on 14 journals. However, there is a marked presence of self-citations, 8 (36 per cent), but this is not so much “advertising” as further development and applications of the method by the originators and their associates. CCR achieved almost as much in the first four years as Farrell did in ten as regards citations and journal diffusion. During the remaining years of the first decade the citations of CCR and diffusion over journals increased rapidly all years but two, ending up with a total of 73 citations distributed on 37 journals.

¹³ This contribution of Färe (1975) was overlooked in Førsund and Sarafoglou (2002), but they are in good company since Färe and Lovell (1978) also overlooked the paper.

¹⁴ It would be interesting to classify all citing papers as suggested in Reisman (1992) using the taxonomy of Gattoufi, Oral and Reisman (2003b), Gattoufi et al. (2003). However, that is beyond the scope of this paper.

Notice that the spread on journals is higher with a factor of 1.4 for one decade compared with what Farrell had for two. The activity of the originators is revealed for the whole decade by the 18 papers written and co-authored by Charnes and Cooper with self-citations. The second most important source for CCR citations in the first decade is Banker with nine citations. Other early Ph.D. students are also contributing in the first decade.

As to type of journal CCR (1978) is clearly diffused among OR/MS journals. Only six journals are typical economics journals (and one applied economics journal in Danish in addition). The first economics citations came in 1983 (Färe & Grosskopf, and Lovell & Sickles), coinciding with the first citation by economists in an OR/MS journal (Färe and Grosskopf). The only paper by Charnes and Cooper and associates in an economics journal appeared in 1985 in *Journal of Econometrics*. *Management Science* and *European Journal of Operational Research* have the highest number of their accumulated citations with 14 and 8 respectively, but otherwise the citations are evenly distributed. Judging from the titles of journals CCR in the first decade was probably cited in many applied papers especially in the fields of education and health.¹⁵

Concerning theoretical developments in the first decade it may be reasonable to consider the choices opening up for applications. In the OR/MS community it is customary to name the model approach that is followed, using the names of the authors of seminal papers. The two basic ones are the “CCR model” and the “BCC model,” the constant returns to scale model being developed in CCR (1978) and the extension to variable returns to scale model in Banker, Charnes and Cooper (1984). The appropriateness of the usage of the CCR model has already been dealt with. It is interesting to note that the special form of a piecewise linear model set up in Banker, Charnes and Cooper was actually introduced already in Afriat (1972) (although for the case of a single output). Moreover, it was empirically implemented in the case of multiple outputs in Färe, Grosskopf and Logan (1983). References to these, and especially the last, are almost completely overlooked in the OR/MS literature. We may also add that a key concept, when using variable returns to scale frontier functions, is *optimal scale*. This concept was introduced in the economics production theory literature already in the thirties by Frisch (1965), termed *Technically Optimal Scale*, but

¹⁵ Footnote 14 also applies here.

overlooked in Banker (1984) and BCC and in the OR/MS literature since, exclusively referring to the Banker concept of MPSS (most productive scale size).¹⁶

A third significant development of the DEA model was inspired by a real life problem. The standard CCR model did not seem to offer practical solutions to the problem of locating a high-energy physics laboratory in Texas. Russell G. Thompson, Robert M. Thrall and associates (1986) increased the realism in applications by introducing bounds on the shadow prices (called “multipliers” by the OR community) associated with the output- and input constraints of the LP problem on “envelopment” form.¹⁷ They introduced the concept of “assurance region” and this concept is later also called the “cone-ratio” approach. Within the first decade the approach was followed up in Dyson and Thanassoulis (1988).

Among innovations of current importance for DEA users we may mention the introduction of discretionary variables (Banker and Morey, 1986a), and categorical variables (Banker and Morey, 1986b).

The introduction of bounds on multipliers is a good example of the research philosophy among many people developing DEA, especially (the late) Abraham Charnes and William W. Cooper: *application-driven* theoretical development. Theoretical developments are inspired by real life problems (see Cooper (2002) for a historic account of this research philosophy).

The diffusion of Farrell (1957) and CCR (1978) up to year 2000

The diffusion of the central papers is set out in Table 4. To guide the choice of additional papers we could use the classification of schools offered in Thompson and Thrall (1993, pp.13-14):

Farrell’s seminal paper was followed by a relatively large numbers of refinements and extensions, which may be broadly classified into three schools of thought (where the initial contributor’s name provides a convenient reference here).

Thompson and Thrall (1993) differ from the typical OR/MS researcher in pointing to the large

¹⁶ We should add that Frisch operated with a single output, while Banker dealt with multiple outputs. Rajiv D. Banker is now well aware of the contribution of Frisch, but this does not seem to be the case for the standard OR/MS researcher.

¹⁷ It has become customary to name the LP problem where the efficiency score is the objective function for the envelopment form, and the dual LP problem for the “multiplier” form.

numbers of follow-ups. However, as to the alleged initiators of the schools of thought they should have been more accurate. Their three schools are i) The Afriat School, ii) The Charnes School and iii) The Shephard School. The first school covers the econometricians' parametric estimation approach we have identified with Aigner and Chu (1968), Afriat (1972) and Aigner, Lovell and Schmidt (1977). The second school should obviously also have Cooper's name associated with it, while the last one may more accurately be termed the axiomatic production theory school, where Rolf Färe has been the most prolific contributor.¹⁸ Shephard never addressed inefficiency as such. We will show the yearly developments of the three papers above belonging to the "Afriat School," and then from the "Charnes School" we will include Banker, Charnes and Cooper (1984). We have not chosen any paper from the "Shephard School," or papers representing the developments in the 90's from the two first schools. This is not because they are not important, but the main models used in applied research appeared before the last decade. (Recent developments will be mentioned below.)

Although the diffusion of citations was much more rapid for CCR than Farrell, we see from Figure 3 that for the same time period there is a remarkable similarity of development. Up to 1994 the citations of Farrell actually outnumber those of CCR, with the reverse occurring afterwards. The role of special issues may be important for explaining the local peaks in the development. Farrell citations certainly obtained a boost from the special issue of *Journal of Econometrics* in 1980. Its next special issue on efficiency topics was in 1990, and now we see the effect of also inviting DEA researchers to contribute. Both Farrell and CCR citations dramatically increased. One factor behind the significant increase in citations in the 90's is the appearance of *Journal of Productivity Analysis* (JPA) from 1989. According to Reisman, Oral and Gattoufi (2003) it has the second highest number of DEA papers for the period 1978-2001, about half the number of *European Journal of Operational Research* (EJOR) being No. 1, but about double the number in *Management Science*, being No. 5. Moreover, according to Gattoufi, Oral and Reisman (2003b) JPA has the highest what they call "DEA – footprint" – 46 per cent of all articles published are DEA papers, as opposed to 6 per cent for EJOR and 3 percent for *Management Science*. Both papers had a peak citation in

¹⁸ Somewhat surprisingly, mainstream economists doing efficiency and productivity research based on Farrell's many applied insights, are left out.

1996. After that year the citations fall back somewhat, and with parallel movements for both, but with CCR receiving consistently more citations.

The first contribution on methods for computing parametric frontier functions, Aigner and Chu (1968), received an attention paralleling Farrell's paper at that time, but was made "technologically redundant" by the contribution of Aigner, Lovell and Schmidt (1977) nine years later. The redundancy clearly shows up in the five years with zero citations in the first half of the 80s, and the low number of citations afterwards, compared with citations of Aigner, Lovell and Schmidt. Most of the Aigner and Chu citations after the publication of Aigner, Lovell and Schmidt now occur together with citations of the latter. The relative low citations of Afriat (a total accumulated of 89 compared with 431 for Aigner, Lovell and Schmidt for year 2000), in spite of his importance, may reflect that the paper was rather long and not easily accessible, and philosophical rather than providing a ready-made method to apply. It appeared rather unedited. Aigner, Lovell and Schmidt on the other hand was more accessible, and presented a method that could readily be applied by researchers and econometrically inclined Ph.D. students.

The development of citations of Banker, Charnes and Cooper (1984) picked up from a slow start to a level of on the average 10 from four to nine years after publication, and then to 21 in 1993, and 62 in 1996, mainly due to a special issue that year. It is one of the most quoted papers in DEA, as seen in Table 4 from the total accumulated count of 413 in 2000. The innovative paper of Thompson, Thrall et al. (1986) has a much lower impact with an accumulated count of 58 in 2000.

Following the Farrell approach

Referring to the taxonomy of developments of the Farrell approach to efficiency studies mentioned earlier, the most dominating follow-up has been the developments of the method for estimation of the production frontier. The classification of the nature of the piecewise linear frontier was offered in Grosskopf (1986), and almost all researchers have since used one of the possibilities identified there, i.e. constant returns to scale (CRS), variable returns to scale (VRS), or non-increasing returns to scale (NIRS). Within the axiomatic approach there has been work on establishing an axiomatic basis for efficiency measures (Färe and Lovell, 1978), and alternatives to the radial Farrell measures have been introduced, but they have not caught on. Although the programming approach has

provided papers giving theoretical insights into the formal structures of the programming problems and solution algorithms, and introduced the additive and multiplicative models, the foundations laid in the first decade are still the dominating ones for applied analyses (see Cooper, Seiford and Tone (2000) for the latest survey of developments).

The theoretical developments of both the parametric approach and the programming approach continued after the periods covered in detail in Table 2 and 3. We will not go into details, but just mention some highlights. Within the econometric approach a panel data model was introduced in Cornwell, Schmidt and Sickles (1990) to model change in the efficiency distribution over time. Battese and Coelli (1992, 1995) introduced panel data model formulations that are the standards today (cf. the survey of parametric frontier models in Kumbhakar and Lovell, 2000). To overcome the shortcoming within parametric frontier models of using a single output, parametric formulations of the efficiency score function (distance function), allowing multiple outputs, have been introduced (see e.g. Coelli and Perleman, 2000).

One reason why the programming approach has not been much developed within economics may be the lack of statistical measures for the quality of estimators. Indeed, the programming approach was often characterised as deterministic. Of course, a programming format is also a way of estimating the unknown frontier production function. The problem has been lack of statistical test procedures for choosing the scale nature of the function, or for studying choice of variables, and the difficulties of finding the properties of estimators for efficiency scores. But Banker (1993) has done a start, followed by Simar (1996), Kittelsen (1998) and Simar and Wilson (1998, 2000, 2002). The latter papers have introduced bootstrapping to provide confidence intervals for the efficiency scores. However, the approach is still under development and is so far quite computer intensive.

A problem with estimation of frontier production functions is that utilisation of constraints derived from modelling of behaviour is not so straightforward as is the case for average functions where derived conditions from profit maximisation or cost minimisation have been used with success (Hotelling's lemma and Shephard's lemma). The problem is the lack of any good theory for inefficient behaviour. One possibility is that all units in fact optimise correctly, but that they face different constraints. The logical step is then to find the constraints. Techniques embodied in the

capital equipment may be such a constraint, but then we have the problem that only a few units - at the extreme only one - exhibit the frontier technology, and thus the units cannot be treated symmetrically as in the usual approach of estimating parametric stochastic frontiers functions (see Førsund, 1985-86).

Seitz (1967,1971) pioneered an approach followed within the non-parametric programming approach, dating back to Nerlove (1965) within a parametric approach. Variables not specified in the model when calculating efficiency scores, are utilised in a second stage regression as explanatory variables for the efficiency scores. This may be dubious from a theoretical point of view, but is followed in a number of practical applications. Banker (1999) gives theoretical conditions that may support the practice.

Principal – agent situations may give a theoretical rationale for rational inefficient behaviour. If the agent has more information about his production technology than the principal does, it may be profitable not to reveal the full extent of his information in order to collect the information rent. Bogetoft (1994, 2000) and Dalen (1996) have constructed situations where the principal utilises efficiency scores to reward/punish agents in order to induce incentives to be efficient.

5. Explanations of the different diffusion patterns

Main factors influencing citations apart from the quality of the research contribution are reputation of journal, reputation of author, number of close followers; colleagues, “cadres of protégés” (Zuckerman, 1987, p. 332), Ph.D. students, and extent of network (“invisible college”).

Journal outlets

The journal Farrell published in was well known in England, but not so in U.S., where the journal was not included in the social science databases. He himself had a good reputation, although not a prolific journal contributor, but he had no group of Ph.D. students around him (although he was in touch with Seitz and Sitorus when he visited Berkeley in 1965). The Cambridge location should not be a disadvantage from a network point of view. It may be of significance that he did not himself

participate so much in the development of his ideas. The first and only self-citation - in general a possible sign of advertising - came quite naturally in a follow-up paper five years later on scale issues of the frontier function (Farrell and Fieldhouse, 1962).

CCR had a better start. They were older (in their 60s as to Farrell being 31 years old) and well established as world authorities within the field of programming, with a better reputation as to publishing record than Farrell (he had six journal publications prior to the 1957 one, see the bibliography in Fisher, 1976), and were influential within a sizeable group of Ph.D. students, as documented below. But their outlet, *European Journal of Operations Research*, was new and not so well known. They participated very actively in the development of their own ideas. Table 3 reveals that of the 73 citations the first decade 18 are self-citations, or almost 25 per cent.

The impact of Ph.D. students

Thesis advising is one channel for asserting influence and promoting own research, and securing citations through successful followers (a vehicle for “promoting your own genes,” to borrow an analogy from biology). Here the profile of Farrell, and Charnes and Cooper are quite different. Farrell had a few collaborators, but no Ph.D. student in the productive efficiency field with a significant “take off” of his own.¹⁹ Charnes and Cooper, on the other hand, had a sizeable group of followers. The entry “Thesis adviser” available in the UMI Dissertation Abstracts (University Microfilms International (UMI)) available on the web from 1987 reveals that Charnes had been thesis adviser for 13 between 1987 and 2000, and Cooper 10 (with some formally overlapping), but they both initiated and assisted many more dissertations at University of Texas. Some of their first students within DEA are among the leading contributors to the DEA field today, like Banker, Schinnar, Seiford, Sueyoshi and Golany.²⁰ Using the database on for the period 1976 – 2000 we see from Table 5 that DEA and production frontier themes are about equal until the 90’s. Then there is an explosion in DEA-related dissertations and a marked decline in the number of frontier

¹⁹ In fact, since Farrell did not have a Ph.D. himself, he would not be a main supervisor of Ph.D. students in Cambridge, although he advised some students in econometrics broadly defined. He had a heavy teaching load and had to cope with a polio paralysis after 1957. Notice that the idea of an English economist doing a Ph.D. in economics at that time was not a familiar (or even welcome) culture outside the science disciplines (private communication from Harry Burley).

²⁰ Morey, Bessent and Lewin were not formal students, but learned about DEA from Charnes and Cooper (private communication from W.W. Cooper).

production function dissertations. It must be underlined that this database is mainly covering U.S. and not Europe.

Foreign Ph.D. students of Charnes and Cooper DEA continued research in their home countries on returning home, especially in Asia. In the Nordic countries followers of the production theory school of Frisch and Johansen (see Førsund (1999) for a survey), cited Farrell more than CCR. Researchers located at business schools, like Warwick in England dominated the growth in citations in Europe of CCR, cf. the early citations in Thanassoulis et al. (1987) and Dyson and Thanassoulis (1988).

Ease of application

Farrell struggled to compute the efficiency measures for his illustration, consisting of only 48 units and five variables. Employing one of the earliest electronic computers in Europe, the ESDAC computer²¹ in Downing Street, up to 60 hours for one run is reported (Farrell (1957), p.265). (ESDAC had probably less computing capacity than a modest pocket calculator of today!). In contrast, Charnes and Cooper and associates could offer easily accessible computer LP codes to be run on speedy mainframes.²² The first commercial software for PC's, IDEAS, appeared in 1989. This is probably an important factor behind the success of DEA in the 90's. The econometric follow-ups of Farrell's ideas the first two decades involved a lot more difficult computational work. Although improvements in calculation techniques benefited the following up of both the parametric and non-parametric approaches, the threshold is probably higher for applying econometrics packages than LP- ones.

Networks

The extent and type of networks are obviously important for the diffusion of new ideas. Followers of Farrell and CCR started out operating within different networks or invisible colleges, which we can call the OR/MS- and the economics networks. The OR/MS community had several national- and

²¹ Electronic Delay Storage Automatic Calculator (EDSAC) is claimed to be the world's first fully operational computer, and was inaugurated 6 May 1949. In 1958 it was replaced by EDSAC 2, which was used in Farrell and Fieldhouse (1962). See Brown, Houthakker and Prais (1953) for an account.

²² The computing work for the empirical application in the dissertation of Edwardo Rhodes was done by applying a standard LP routine the required number of times (private communication from Edwardo Rhodes).

international conferences where ideas were presented and exchanged. In the late 90ies electronic networks were established. Special issues devoted partially or fully to efficiency and/or DEA have had a significant impact on citations, especially for CCR because close associates often edited special issues. Among the special issues we have European Journal of Operations Research 1993, 1995, 1997, Annals of Operations Research 1985, 1996, 1997, Journal of Productivity Analysis 1992, 1996, 1997, Computers and Operational Research 1997, Journal of Banking and Finance 1993, and Journal of Econometrics, 1980, 1990.

Farrell did not so actively create a network as Charnes and Cooper, but through being well known among Oxford and Cambridge economists his ideas had possibilities of being spread. He also visited Cowles Commission in 1951-1952 and established contacts with Debreu and Koopmans, as well as meeting with Charnes and Cooper at Carnegie Mellon. His activity approach to efficiency measurement was inspired by this exposure, and the Cowles Commission group should have known his 1957 paper. Farrell also visited Berkeley in the 60ies and met with agricultural economists there. As stated previously this group organised a special workshop on Farrell's approach in 1966. The diffusion through agricultural economists can be seen in Table 2.

Leif Johansen of Oslo University made Farrell's efficiency paper known in Scandinavia. He had spent a year in Cambridge 1959 –1960 and brought home a reprint of the Farrell paper. We see the Nordic influence in Table 2.²³

External events and fads

Another factor to consider is the role of fads in economics (see e.g. Bronfenbrenner, 1966). To improve productivity is always on the agenda, but in the late 70s the first energy crisis had been there, with the subsequent need for restructuring out of energy intensive activities. A general mistrust in the trade of macroeconomist was evident, and a resulting increased interest in microeconomics. The need for tools to study productivity at the micro level was brought home by the stagnation in macro-economic productivity, as revealed by National Accounts, following in the wake of the 1974 and 1979 oil crises. But when evaluating the increase in absolute numbers of citations, one should be

²³ The Swedish economist, Bo Carlsson, knew the Farrell paper through his Ph.D. adviser Timmer, an agricultural economist and early contributor to the development of parametric frontier estimation, at Stanford.

aware of the general explosion in the number of journal articles in the 90ies due to new journals. To measure relative importance of research topics a deflator is needed, for example total number of papers.

6. The convergence of the invisible colleges

As a natural consequence of competition among different research groups some frictions are evident in the literature, and were observed at conferences, and unfair selection felt practised in refereeing processes. After a period of some frictions at ORSA/TIMS meetings and informal workshops organised by Ali Dogramaci at Rutgers University from early 80-ties, a series of events at the end of the decade lead to the different networks establishing extensive cross-links. A “peace conference” was organised at Chapel Hill in 1988 with proceedings published in *Journal of Econometrics* 1990. Charnes and Cooper at Austin arranged a broadly based conference as to participants from the two “camps” in 1989. The conference volume was out as a book in 1994 (Charnes et al., 1994). The *Journal of Productivity Analysis* was started in 1989, springing out of the enthusiastic work over many years of Ali Dogramaci at Rutgers University, arranging informal workshops on efficiency and productivity, documented in several books. Also the electronic network PARN, started in 1992, should be mentioned.

One way of measuring interaction and convergence between the camps is to inspect combinations of joint citations. The pattern of cross-references is set out in Table 6. The number of papers citing both Farrell and CCR has clearly increased over time. But as a share of the total citations of the two papers the share of the CCR citations have been around 1/3, except for 1990 when the share was 55 percent, partly due to the special issue of *Journal of Econometrics*. The joint citations as a share of the Farrell citations has increased from 22 percent in 1986 to the upper 30ies in the 90ies, and ends up on almost 40 percent in year 2000. It is only on a slightly higher level at the end of the decade than on average through the decade, so the convergence effect is rather weak measured by joint citations.

The two OR/MS papers CCR and BCC have a very high overlap. We see that in 1986 the share of co-citation relative to the BCC citations is $\frac{3}{4}$, and varying between 100 and 78 per cent for the other years. But the overlap between the econometric papers is rather weak. The idea of an “Afriat School” corresponds poorly with the overlap of citations, having shares of 50 per cent for 1986, but only between 20 and 25 per cent for the other years. There is a markedly higher overlap between Aigner and Chu and Aigner, Lovell and Schmidt with 50 per cent for the two first years, but then almost 90 per cent for 1995, and falling back to 25 per cent for year 2000. The joint citations between the three main papers within the “Afriat School” are very poor indeed, and adding Farrell to the comparison makes the joint citations going further down to almost zero. For the whole period 1986 – 2000 there is only one paper that cites all six papers.

Another angle on the convergence process is provided in Table 7, where citations of Farrell and CCR are distributed on type of journal. The group “Operations research” consists of journals with “Operations research” in the title, and the group “Business” consists of journals with “Management” in the title. Economics journals include Journal of Productivity Analysis and Applied Economics, but the typical sector journals are excluded. The different groups are obviously using their traditional outlets in 1991, while the pattern is changed in 1996, in the sense that especially economists but also the OR/MS community do significantly more citing of both Farrell and CCR together.

7. Concluding remarks

We started out the paper by postulating that OR/MS researchers have tended to overlook the contributions of Farrell (1957). It may be of interest to note that the importance of Farrell (1957) was not really seen by his contemporary colleagues, even 20 years later. Both in the obituary in *The Times* 1975 and in Fisher (1976) a paper on how increasing returns to scale or non-convexities in general can be reconciled with competitive equilibrium (Farrell, 1959) is viewed as the most important contribution (the paper has 20 citations per February 2003 compared with over 900 for Farrell 1957). By citation analysis we have shown that Farrell (1957) is now a classical paper, as is Charnes, Cooper and Rhodes (1978). The latter paper has been the leading one in generating a spectacular interest in the Farrell approach to efficiency analysis based on the programming

approach in the 90ies. But as clearly demonstrated, we should not forget the development of the econometric approach in the years between the publications of the two seminal papers. Indeed, one of the econometric papers is also a citation classic now with over 500 citations (ca 560 per February 2003), and the other OR/MS paper is almost there (ca. 490 citations per February 2003).

Some puzzles remain. It should not be overlooked that the efforts of agricultural economists to improve the Farrell estimation method for non-parametric frontiers already in 1966, by employing a LP model that was to become the centrepiece of the DEA method introduced in CCR, failed almost completely to become acknowledged. The journal outlet was one factor, and the low journal article productivity of the group another. Only one member of the group, Seitz, had a publication record afterwards that could make some impact. We will try to address this puzzle in future works.

The recent theoretical contributions to developing statistical measures for quality of estimates flash a warning signal as to the overuse of the programming approach. A disturbingly high number of applied studies operate with rather few degrees of freedom.

An underdeveloped area according to the taxonomy list in Section 3 is presentation of results. Farrell showed isoquants and efficiency score diagrams. Not much has happened since. The efficiency profession must pay more attention to result presentations in order to obtain the role for efficiency analysis it deserves.

A final reflection on the nature of Farrell's contribution is in place. Already Hall and Winsten (1959, p. 85) noted:

However, his paper still suffers from a central weakness: that he does not analyse the concept of efficiency.

There have been contributions as to the nature of the Farrell efficiency measures, cf. efforts by the axiomatic "Shephard School", and of general economists, but not really contributions as to the nature of inefficiency as such.

In the Discussion Mr Colin Clark expressed:

I think we will agree, having heard this paper, that Mr. Farrell has already reached some interesting and successful results, and has come nearer than any previous investigator to a

true measure of agricultural efficiency, which figures in their turn will send the economists further down the road hunting for the social and other factors which lie behind them.

(Discussion on Mr. Farrell's Paper, p.282)²⁴

However, the hunts down the road have not been too successful. Let this be a challenge to future research.

References

Afriat, S. (1972): "Efficiency estimation of production functions," *International Economic Review* 13(3), 568-598.

Aigner, D. J. and S.F. Chu (1968): "On estimating the industry production function," *American Economic Review* 58, 226-239.

Aigner, D. J., C.A.K. Lovell and P. Schmidt (1977): "Formulation and estimation of stochastic frontier production function models," *Journal of Econometrics* 6(1), 21-37.

Amey, L. R. (1964): "The allocation and utilization of resources," *Operational Research Quarterly* 15(2), 87-100.

Banker, R. D. (1984): "Estimating most productive scale size using data envelopment analysis," *European Journal of Operational Research* 17, 35-44.

Banker, R. D. (1993): "Maximum likelihood, consistency and data envelopment analysis: a statistical foundation," *Management Science* 39(10), 1265-1273.

Banker, R. D. (1999): "Evaluating contextual variables affecting productivity using Data Envelopment Analysis", paper presented at the Sixth European Workshop on Efficiency and Productivity Analysis, Copenhagen October 29 – 31, 1999.

Banker, R. D. and H. Chang (1995): "A simulation study of hypothesis tests for differences in efficiencies," *International Journal of Production Economics* 39, 37-54.

Banker, R. D. and R. C. Morey (1986a): "Efficiency analysis for exogenously fixed inputs and outputs," *Operations Research* 34(4), 513-521.

Banker, R. D. and R. C. Morey (1986b): "The use of categorical variables in Data Envelopment Analysis", *Management Science* 32(12), 1613-1627.

²⁴ Also quoted in Burley (1994).

Banker, R. D., A. Charnes, and W. W. Cooper (1984): "Some models for estimating technical and scale inefficiencies," *Management Science* 39, 1261-1264.

Battese, G. E. and T. J. Coelli (1992): "Frontier production functions, technical efficiency and panel data: with application to paddy farmers in India," *Journal of Productivity Analysis* 3, 153-169.

Battese, G. E. and T. J. Coelli (1995): "A model for technical inefficiency effects in a stochastic frontier production function for panel data," *Empirical Economics* 20, 325-332.

Bogetoft, P. (1994): "Incentive Efficient Production Frontiers: An Agency Perspective on DEA." *Management Science* 40, 959-968.

Bogetoft, P. (2000): "DEA and Activity Planning under Asymmetric Information." *Journal of Productivity Analysis* 13, 7-48.

Boles, J. N. (1967): "Efficiency squared—efficient computation of efficiency indexes," *Western Farm Economic Association, Proceedings 1966*, Pullman, Washington, 137-142.

Boles, J. N. (1971): "The 1130 Farrell Efficiency System – Multiple Products, Multiple Factors", Giannini Foundation of Agricultural Economics, February 1971.

Bronfenbrenner, M. (1966): "Trends, cycles, and fads in economic writing," *American Economic Review, Papers and Proceedings*, LVI (May), 538- 552.

Brown, J. A. C., H. S. Houthakker and S. J. Prais (1953): "Electronic Computation in Economic Statistics," *Journal of the American Statistical Association* 48(263), 414-428.

Burley, H.T. (1994): "Farrell, Data Envelopment Analysis and Leibenstein on efficiency measurement" in Vaughn, K.I. (ed.), *Perspectives on the History of Economic Thought*, Vol. 10. Brookfield: Elgar, 256-265.

Carlsson, B. (1972): "The measurement of efficiency in production: an application to Swedish manufacturing industries," *Swedish Journal of Economics* 74, 468-485.

Charnes, A., W.W. Cooper and E. Rhodes (1978): "Measuring the efficiency of decision making units," *European Journal of Operations Research* 2, 429-444.

Charnes, A., W.W. Cooper, B. Golany, L. Seiford and J. Stutz (1985): "Foundations of data envelopment analysis for Pareto – Koopmans efficient empirical production functions," *Journal of Econometrics* 30(1/2), 91-107.

Charnes A, W.W. Cooper, A.Y. Lewin, and L. M. Seiford (1994): *Data Envelopment Analysis*. Boston: Kluwer.

Clark, C. (1957): "Discussion on Mr. Farrell's Paper." *Journal of the Royal Statistical Society, Series A*, 120 (III), 282.

Coelli, T. and S. Perleman (2000): "Technical efficiency of European railways: a distance function approach," *Applied Economics* 32 (15), 1967-1976.

Cooper, W.W. (2002): "Abraham Charnes and W.W. Cooper (et al.): a brief history of a long collaboration in developing industrial uses of linear programming," *Operations Research* 50(1), 35-41.

Cooper, W. W., L. M. Seiford and K. Tone (2000): *Data envelopment analysis: a comprehensive text with models, applications, references and DEA-solver software*, Dordrecht/Boston/London: Kluwer Academic publishers.

Cornwell, C., P. Schmidt and R. C. Sickles (1990): "Production frontiers with cross-sectional and time-series variation in efficiency levels," *Journal of Econometrics* 46, 185-200.

Dalen, D.M. (1996): "Strategic Responses to Frontier-Based Budget Allocation: Implications for Bureaucratic Slack," *Journal of Productivity Analysis* 7(1), 29-40.

Debreu, G. (1951): "The coefficient of resource utilization," *Econometrica* 19, 14-22.

Dyson, R. G. and E. Thannassoulis (1988): "Reducing weight flexibility in data envelopment analysis," *Journal of the Operational Research Society* 39, 563-576.

Farrell, M. J. (1954): "An application of activity analysis to the theory of the firm," *Econometrica* 22, 291- 302.

Farrell, M. J. (1957): "The measurement of productive efficiency," *Journal of the Royal Statistical Society, Series A*, 120 (III), 253-281.

Farrell, M. J. (1959): "The convexity assumption in the theory of competitive markets", *Journal of Political Economy* 67 (4, August), 377-391.

Farrell, M.J. and M. Fieldhouse (1962): "Estimating efficient production functions under increasing returns to scale," *Journal of the Royal Statistical Society* 125, 252-267.

Fisher, M. R. (1976): "The economic contribution of Michael James Farrell," *Review of Economic Studies* XLIII (3), 371-382.

Frisch, R. (1965): *Theory of production*, Dordrecht: D. Reidel.

Färe, R. (1975): "Efficiency and the production function," *Zeitschrift für Nationalökonomie* 35, 317-324.

Färe, R. and S. Grosskopf (1983a): "Measuring congestion in production," *Zeitschrift für Nationalökonomie*, 43(3), 257-271.

Färe, R. and S. Grosskopf (1983b): "Measuring Output Efficiency," *European Journal of Operational Research* 13, 173-179.

Färe, R. and C. A. K. Lovell (1978): "Measuring the Technical Efficiency." *Journal of Economic Theory* 19(1), 150-162.

Färe, R., S. Grosskopf, and J. Logan (1983): "The relative efficiency of Illinois electric utilities." *Resources and Energy* 5, 349-367.

Färe, R., S. Grosskopf and P. Roos (1995): "Productivity and quality changes in Swedish pharmacies," *International Journal of Production Economics* 39, 137-147

Førsund, F.R. (1971): "A note on the technically optimal scale in inhomogeneous production functions," *Swedish Journal of Economics* 73, 225-240.

Førsund, F.R. (1985-86): "Comment on frontier production functions," *Econometric Reviews*, 4, 329-334.

Førsund, F.R. (1999): "On the contribution of Ragnar Frisch to production theory," *Rivista Internazionale di Scienze Economiche e Commerciali (International Review of Economics and Business)* XLV, 1-34.

Førsund, F. R. and L. Hjalmarsson (1974): "On the measurement of the productive efficiency," *Swedish Journal of Economics* 76, 141-154.

Førsund, F.R. and L. Hjalmarsson (1979): "Generalised Farrell measures of efficiency: an application to milk processing in Swedish dairy plants," *Economic Journal* 89, 294-315.

Førsund, F.R. and N. Sarafoglou (2002): "On the origins of Data Envelopment Analysis", *Journal of Productivity Analysis* 17, 23-40.

Gattoufi, S., M. Oral and A. Reisman (2003a): "Data envelopment analysis literature: a bibliography update (1996–2001)," Working Paper SUGSM – 02-08, forthcoming in *Journal of Socio-Economic Planning Sciences*.

Gattoufi, S., M. Oral and A. Reisman (2003b): "Content analysis of data envelopment analysis literature and comparison with other fields of OR/MS for relevance to applications," Working Paper SUGSM – 02-22, Sabanci University Istanbul.

Gattoufi, S., M. Oral and A. Reisman (2003c): "A taxonomy for data envelopment analysis," Working Paper SUGSM – 02-15, Sabanci University Istanbul, forthcoming in *Journal of Socio-Economic Planning Sciences*.

Gattoufi, S., M. Oral, A. Kumar and A. Reisman et al. (2003): "Epistemology of data envelopment analysis and comparison with other fields of OR/MS for relevance to applications," Working Paper SUGSM – 02-11, Sabanci University, Istanbul, forthcoming in *Journal of Socio-Economic Planning Sciences*.

Grosskopf, S. (1986): "The role of the reference technology in measuring productive efficiency", *Economic Journal* 96, 499-513.

Grubbström, R. W. (1995): "Modelling production opportunities - an historical overview," *International Journal of Production Economics* 41(1-3), 1-14.

Hall, M. and C. Winsten (1959): "The ambiguous notion of efficiency," *Economic Journal* LXIX (March), 71-86.

Hanoch, G. and M. Rothschild, (1972): "Testing the assumptions of production theory: a nonparametric approach," *Journal of Political Economy* 80(2), 256-275.

Hicks, J. R. (1960): "Linear theory," *The Economic Journal* 70(280), 671-709.

Hoffman, A. J. (1957): "Discussion on Mr. Farrell's Paper." *Journal of the Royal Statistical Society, Series A*, 120 (III), 284.

ISI Web of Knowledge: <http://www.isinet.com>

Johnson, D. (1997): "Getting noticed in economics: the determinants of academic citations," *The American Economist* 41(1), 43-52.

JSTOR: <http://www.jstor.org>

Kumbhakar, S.C. and C.A.K. Lovell (2000): *Stochastic Frontier Analysis*, Cambridge University Press.

Kittelsen, S. A. C. (1998): "Using data envelopment analysis to measure production efficiency in the public sector," *Økonomiske Doktoravhandlinger*, Universitetet i Oslo.

Koopmans, T. C. (1951): *Activity Analysis of Production and Allocation*, New York: Wiley.

Leibenstein, H. (1973): "Competition and X-efficiency: Reply," *Journal of Political Economy* 81(3), 765-777.

Lovell, C. A. K. (1995): "Measuring the macroeconomic performance of the Taiwanese economy," *International Journal of Production Economics* 39, 165-178.

Lovell, C. A. K. and R. C. Sickles (1983): "Testing efficiency hypotheses in joint production – a parametric approach," *Review of Economics and Statistics* 65(1), 51-58.

Meeusen, W. and J. van den Broeck (1977): "Efficiency estimation from Cobb-Douglas production functions with composed errors," *International Economic Review* 18, 435-444.

Mirowski, P. (1999): "Cyborg agonistes: economics meet operations research in mid-century," *Social Studies of Science* 29(5), 685-718.

Nerlove, M. (1965): *Estimation and Identification of Cobb – Douglas Production Functions*, Amsterdam: North-Holland Publishing Company.

PARN: www.busieco.ou.dk/PARN/

Price, D. de S. (1976): "A general theory of bibliometric and other cumulative advantage processes," *Journal of the American Society for Information Science* 27, 292-306.

Reisman, A. (1992): *Management science knowledge: its creation, generalization and consolidation*, Westport CT: Quorum Books Publishing Company.

Reisman, A., M. Oral and S. Gattoufi (2003): "Absolutely positively operations research: 50 years of contributions by William Wager Cooper," Working Paper SUGSM –02-10, Sabanci University Istanbul.

Ritchie, P. C. and J. E. Rowcroft (1996): "Choice of metric in the measurement of relative productive efficiency," *International Journal of Production Economics* 46, 433-439

Ruist, E. (1960): *Industriföretagets Produktionseffektivitet. Några mätningmetoder*, IUI, Stockholm.

Ruist, E. (1961): "Production efficiency of the industrial firm. Some methods of measurement," *Productivity Measurement Review*, OECD, Paris.

Russell, R. R. (1998): "Distance functions in consumer and producer theory," in R. Färe, S. Grosskopf and R.R. Russell (eds.). *Index Numbers: Essays in Honour of Sten Malmquist*, Boston/London/Dordrecht: Kluwer Academic Publishers, 7-90.

Sarafoglou, N. (1997): "The most influential DEA publications: a comment on Seiford," *Journal of Productivity Analysis* 9(3), 279-281.

Sandelin, B. (1976): "On the origin of the Cobb – Douglas production function," *Economic History* XIX, 117-123.

Seiford, L. M. (1996): "Data Envelopment Analysis: the evolution of the state of the art (1978-1995)," *Journal of Productivity Analysis* 7 (2/3), 99-137.

Seitz, W. D. (1967): "Efficiency measures for steam-electric generating plants," *Western Farm Economic Association, Proceedings 1966*, Pullman, Washington, 143-151.

Seitz, W. D. (1970): "The measurement of efficiency relative to a frontier production function," *American Journal of Agricultural Economics* 52, 505-511.

Seitz, W. D. (1971): "Productive efficiency in the steam-electric generating industry," *Journal of Political Economy* 79, 878-886.

Shephard, R.W. (1953): *Cost and production functions*, Princeton University Press, Princeton.

Simar, L. (1996): "Aspects of statistical analysis in DEA-type frontier models," *Journal of Productivity Analysis* 7 (2/3), 177-185.

Simar, L. and P. W. Wilson (1998): "Sensitivity analysis of efficiency scores: how to bootstrap in nonparametric frontier models," *Management Science* 44(11), 49-61.

Simar, L. and P. W. Wilson (2000): "Statistical inference in nonparametric frontier models: the state of the art," *Journal of Productivity Analysis* 13, 49-78.

Simar, L. and P. W. Wilson (2002): "Non-parametric tests of returns to scale," *European Journal of Operational Research* 139, 115-132.

Social Science Citation Index: An international multidisciplinary index to the literature of social, behavioral and related sciences, Institute for Scientific Information, Philadelphia, (1961-).

SSCI CD-ROM: Social Science Citation Index (1981-1992).

SSCI Online: Social SciSearch (1972-).

Thanassoulis, E., R.G. Dyson, and M.J. Foster (1987): "Relative efficiency assessments using Data Envelopment Analysis - an application to data on rates departments," *Journal of the Operational Research Society* 38(5), 397-411.

Thompson, R.G. and R.M. Thrall (1993): "Need for MS/OR in public policymaking," in Rhodes, E.L. (ed.), *Public policy applications of management science, Applications of Management Science* 7, 3-21, Greenwich: JAJ Press.

Thompson, R. G., F. D. Singleton, Jr., R.M. Thrall and B. A. Smith (1986): "Comparative site evaluation for locating a high-energy physics lab in Texas," *Interfaces* 16, 35-49.

Timmer, C. P. (1971): "Using a probabilistic frontier production function to measure technical efficiency," *Journal of Political Economy* 79, 776-794.

Tulkens, H. and P. van den Eeckhaut (1995): "Non-frontier measures of efficiency, progress and regress for time series data," *International Journal of Production Economics* 39, 83-97.

UMI-Dissertations: www.lib.umi.com/dissertations/

Wicksell, K. (1893): *Über Wert, Kapital und Rente*, Jena.

Walters, A. A. (1963): "Production and cost functions: An econometric survey," *Econometrica* 31(1-2), 1-66.

Zuckerman, H. (1987): "Citation analysis and the complex problem of intellectual influence," *Scientometrics* 12(5-6), 329-338.

Tables and figures to Førsund and Sarafoglou: THE TALE OF TWO RESEARCH COMMUNITIES:
THE DIFFUSION OF RESEARCH ON PRODUCTIVE EFFICIENCY

Table 1. References to Farrell (1957) and Charnes, Cooper and Rhodes (1978) in DEA articles in International Journal of Production Economics 1995 – 2001

Table 2. Journals with Farrell (1957) citations 1957-1977

Table 3. Journals with Charnes, Cooper and Rhodes (1978) citations 1978-1988

Table 4. Diffusion of Farrell, CCR and main contributions 1957-2000

Table 5. Ph.D. dissertations with DEA and frontier production function as search words

Table 6. Pattern of cross-references

Table 7. Distribution of citations of Farrell and CCR on SSCI journals

*Figure 1. The definition of Farrell's technical efficiency measures.
(Note that y and x are input coefficients.)*

*Figure 2. The piecewise linear production frontier (unit isoquant).
(Note that y and x are input coefficients.)*

Figure 3. The development of Farrell and CCR citations

Table 1. References to Farrell (1957) and Charnes, Cooper and Rhodes (1978) in DEA articles in International Journal of Production Economics 1995 – 2001

DEA article	Citing Farrell	Farrell original	Citing CCR	CCR original
Banker and Chang (1995)	No	No	Yes	Yes
Bogetoft (1995)	No	No	Yes	Yes
Färe, Grosskopf and Roos (1995)	No	No	No	No
Gong and Sun (1995)	Yes	No	Yes	Yes
Lovell (1995)	No	No	No	No
Olesen. (1995)	No	No	Yes	Yes
Olesen and Petersen (1995)	No	No	Yes	Yes
Thompson, Dharmapala and Thrall (1995)	Yes	No	Yes	Yes
Tulkens and van den Eeckhaut (1995)	No	No	No	No
Hartman and Storbeck (1996)	No	No	Yes	Yes
Ritchie and Rowcroft (1996)	Yes	No	No	No
Chen (1997)	No	No	Yes	Yes
Chandra, Cooper et al. (1998)	No	No	Yes	Yes
Talluri, Baker and Sarkis (1999)	No	No	Yes	Yes
Park and Lesourd (2000)	Yes	No	Yes	Yes
Talluri and Yoon (2000)	No	No	Yes	Yes
Homburg (2001)	No	No	Yes	Yes
Uri (2001)	Yes	No	Yes	Yes
Murillo-Zamorano and Vega-Cerevera (2001)	Yes	No	Yes	Yes

Table 2. Journals with Farrell (1957) citations 1957-1977

Year	Authors	Journal	No. of citations	Cum. No. of citations	Cum. No. of journals
1959	Hall & Winsten*	Economic Journal	1	1	1
1960	Hicks*	Economic Journal	1	2	1
1961	Ruist*	Productivity Measurement Review	1	3	2
1962	Farrell & Fieldhouse	Journal of Royal Statistical Society-B	1	4	3
1963	Walters	Econometrica	1	5	4
1964	Amey	Operational Research Quarterly	1	6	5
1967	Glague	Review of Economics and Statistics	1		
	Boles, Bressler*	Western Farm Economics			
		Association Proceedings	2	9	7
1968	Aigner & Chu	American Economic Review	1		
	Clemhout*	Review of Economics and Statistics	1		
	Clemhout*	Review of Economic Studies	1	12	9
1970	Seitz	American Journal of Agricultural Economics	1		
	Bessell	Journal of Agricultural Economics	1		
	Nadiri	Journal of Economic Literature	1		
	Nabb	Journal of Royal Statistical Society-D	1	16	12
1971	Lau	American Economic Review	1		
	Seitz, Timmer	Journal of Political Economy	2		
	Førsund	Swedish Journal of Economics	1	20	14
1972	Afriat	International Economic Review	1		
	Hanoch & Rothschild	Journal of Political Economy	1		
	Carlsson	Swedish Journal of Economics	1	23	15
1973	Yotopoulos	American Economic Review	1		
	Teague	Applied Economics	1		
	Kalish	Journal of Industrial Economics	1		
	Leibenstein	Journal of Political Economy	1	27	18
1974	Muller, Lin	American Journal of Agricultural Economics	2		
	Caves	Economica	1		
	Levin	Public Financial Quarterly	1		
	Førsund & Hjalmarsson	Swedish Journal of Economics	1	32	20
1975	O'Connor	American Journal of Agricultural Economics	1		
	Keating	Review of Social Economy	1		
	Färe	Zeitschrift für Nationalökonomie	1	35	22
1976	Araj	American Journal of Agricultural Economics	1		
	Ahmed	Economic and Social Review	1		
	Weston	European Journal of Marketing	1		
	Toda	Review of Economics and Statistics	1		
	Goldstein & Ehrenberg	Southern Economic Journal	1	40	25
1977	Shapiro	Economic Development and Cultural Change	1		
	Todd	Economic and Social Review	1		
	Meeusen & Broeck	International Economic Review	2		
	Førsund & Jansen				
	Aigner & Lovell & Schmidt	Journal of Econometrics	1	46	27

*) Not in SSCI

Sources: SSCI ("In print", CD-ROM, WEB-based), JSTOR

Table 3. Journals with Charnes, Cooper and Rhodes (1978) citations 1978-1988

Year	Authors	Journal	No. of cit.	Cum per year	Cum. No. of journ.
1979	CCR	European Journal of Operational Research	1	1	1
1980	Charnes & Cooper (CC)	Accounting, Organizations and Society	1		
	Bessent & Bessent	Education Administration Quarterly	1		
	Banker	European Journal of Operational Research	1		
	Morey & Mccann	Management Science	1	5	5
1981	Schaible	European Journal of Operational Research	1		
	Banker & CCR, CCR	Management Science	2	8	5
1982	Sengupta	International Journal of Systems Science	1		
	Bessent et al.	Management Science	1		
	CC & Schinnar, Lewin et al.	Omega	2		
	Lindsay	Review of Educational Research	1		
	CC et al.	Socio-economic Planning Sciences	1	14	9
1983	Bessent & CC et al.	Educational Administration Quarterly	1		
	Färe & Grosskopf	European Journal of Operational Research	1		
	Nunamaker	Health Services Research	1		
	Bessent et al.	Management Science	1		
	CC et al.	Operations Research Letters	1		
	Parks	Policy Studies Journal	1		
	Lovell & Sickles	Review of Economics and Statistics	1		
	Färe & Grosskopf	Zeitschrift für Nationalökonomie	1	22	14
1984	CC, Banker,	European Journal of Operational Research	3		
	Bessent et al.	Interfaces	1		
	Banker & CC, Byrnes et al.	Management Science	2		
	Sherman	Medical Care	1		
	Sherman	Sloan Management Review	1	30	17
1985	Danilin & Lovell et al.	Economica	1		
	Capettini & Dittman & Morey	Journal of Accounting and Public Policy	1		
	CC et al.	Journal of Econometrics	1		
	CC et al.	Journal of Marketing	1		
	Nunamaker	Managerial and Decision Economics	1		
	Morey & Capettini & Dittman	Policy Sciences	1	36	23
1986	Grosskopf	Economic Journal	1		
	Stern	Education and Urban Society	1		
	Baxter et al.	Energy Economics	1		
	CC & Sueyoshi	European Journal of Operational Research	1		
	Thompson & Thrall et al.	Interfaces	1		
	Lewin & Minton, Banker & Maindiratta, B & Morey, B et al.	Management Science	4		
	Pedersen	Nationaløkonomisk Tidsskrift	1		
	Banker & Morey	Operations Research	1		
	CC & Thrall	Operations Research Letters	2		
	Macmillan	Papers of Regional Science Association	1		
	Miller	Systems Research	1	51	30
1987	Macmillan	Environment and Planning A	1		
	Thanassoulis et al., Bowlin	Journal of Operational Research Society	2		
	Smith & Mayston	Omega	1		
	Jesson & Mayston & Smith	Oxford Review of Education	1	56	32
1988	Glover et al.	Decision Science	1		
	Banker & Maindiratta	Econometrica	1		
	Thompson et al.	Interfaces	1		
	Kamakura	Journal of Consumer Research	1		
	Golany, Dyson & Thanassoulis	Journal of Operational Research Society	2		
	Färe et al.	Journal of Public Economics	1		

Sengupta, Ahn&CC	Managerial and Decision Economics	2		
Sengupta	Management Decision Economics	1		
Golany, Kamakura	Management Science	2		
Bessent et al.	Operations Research	1		
Woodhouse&Goldstein	Oxford Review of Education	1		
Ray, Ahn&CC (2)	Socio-Economic Planning Science	3	73	37

Sources: SSCI ("In print", CD-ROM, WEB-based), JSTOR

Table 4. Diffusion of Farrell, CCR and main contributions 1957-2000

Year	Farrell	Aigner &Chu	Afriat	Aigner& Lovell& Schmidt	CCR	BCC
1957	0					
8	0					
9	1					
1960	1					
1	1					
2	1					
3	1					
4	2					
1965	0					
6	0					
7	3					
8	2	0				
9	0	0				
1970	4	4				
1	4	4				
2	2	1	1			
3	4	1	0			
4	5	4	1			
5	5	3	0			
6	4	3	0			
7	5	5	2	1		
8	8	5	1	4	0	
9	11	7	4	5	1	
1980	17	6	5	9	4	
1	8	0	2	3	3	
2	10	0	2	9	6	
3	13	0	2	6	8	
4	11	0	1	7	8	0
5	7	0	2	3	6	2
6	23	8	4	11	15	4
7	12	1	2	8	5	1
8	21	6	5	11	16	11
9	17	4	4	15	14	7
1990	30	4	10	13	20	12
1	23	5	6	17	22	10
2	26	4	4	18	27	8
3	35	6	8	23	32	21
4	36	6	10	28	39	15
5	37	9	8	32	44	35
6	77	14	7	44	97	62
7	75	7	15	34	79	48
8	63	10	4	34	83	53
9	67	9	7	43	94	64
2000	66	8	5	52	79	39
Total	754	146	122	431	717	413

Sources: SSCI ("In print", CD-ROM, WEB-based), JSTOR

Table 5. Ph.D. dissertations with DEA and frontier production function as search words

Year	DEA	Frontier production function
1976	0	1
7	0	1
8	1	0
9	0	1
1980	1*	2
1	3	1
2	0	2
3	5	1
4	2	5
5	5	2
6	8	2
7	5	3
8	7	1
9	1	9
1990	10	3
1	9	6
2	8	2
3	10	8
4	9	8
5	10	8
6	10	11
7	22	4
8	22	8
9	14	5
2000	26	3
Total	187	97

Source: Dissertations Abstracts UMI (CD-ROM, and WEB-based)

* Banker's 1980 dissertation cannot be found in the database using DEA as a search word because it is occurring neither in the title nor abstract.

Table 6. Pattern of cross-references

Cited papers, combinations	Citations 1986	Citations 1990	Citations 1995	Citations 2000
(1) Farrell	23	30	37	66
(2) CCR	15	20	44	79
(3) Aigner&Chu	8	4	9	8
(4) Afriat	4	10	8	5
(5) Aigner et al.	11	13	32	52
(6) Banker&CC	3	6	7	9
Intersection (1), (2)	5	11	13	26
Intersection (2), (6)	3	6	6	7
Intersection (4), (5)	2	2	2	1
Intersection (3), (5)	4	2	8	2
Intersection (3), (4), (5)	1	0	2	0
Intersection (1), (3), (4), (5)	1	0	1	0

Source: SSCI (WEB-based)

Table 7. Distribution of citations of Farrell and CCR on SSCI journals

	Operations research			Economics			Business		
	86/90*	91	96	86/90	91	96	86/90	91	96
Farrell	1.6	2	12	12.8	14	50	2.8	0	6
CCR	3.4	11	46	5.2	2	35	4.4	6	13
Total	5	13	58	18	16	85	7.2	6	19

* Yearly averages

Source: SSCI (WEB-based)

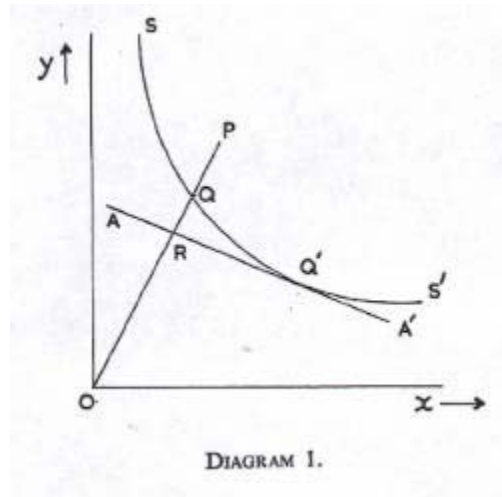


Figure 1. The definition of Farrell's technical efficiency measures.
(Note that y and x are input coefficients.)

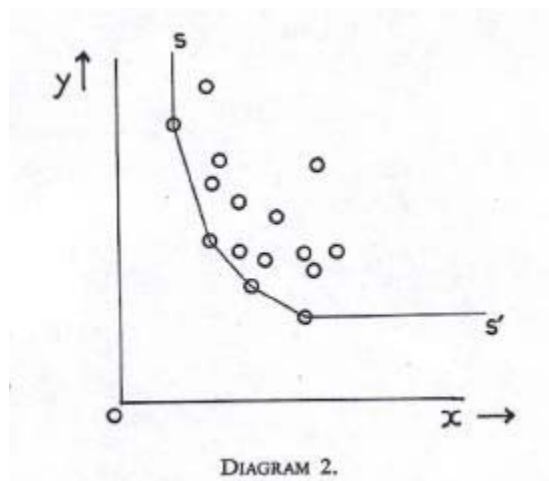


Figure 2. The piecewise linear production frontier (unit isoquant). (Note that y and x are input coefficients.)

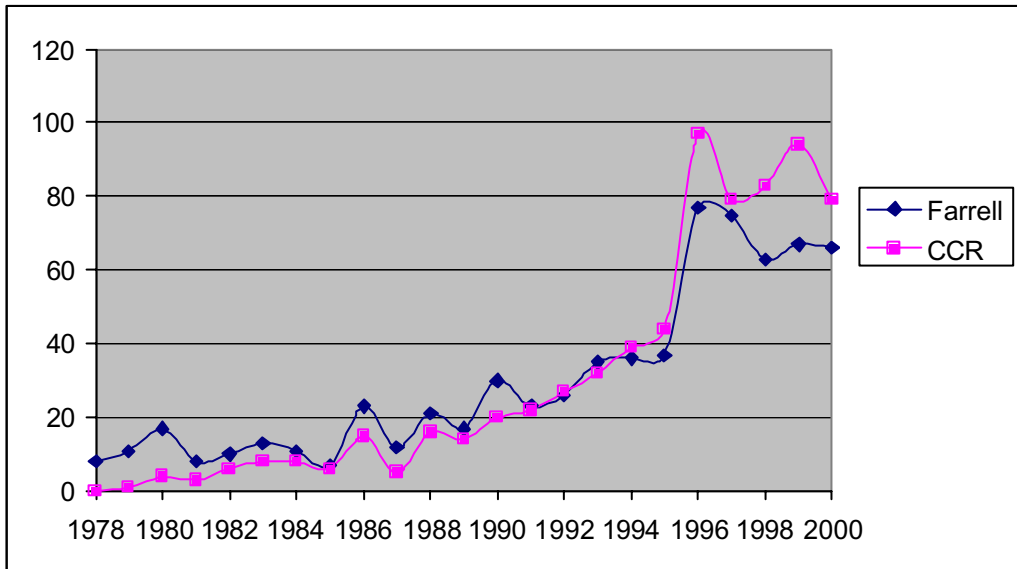


Figure 3. The development of Farrell and CCR citations