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Risk, Return, and the Optimal Design of a Pension System

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Abstract: Conventional wisdom suggests that the ideal pension system should include a small PAYGO but a large fully funded component. This is partly justified on the grounds of higher return to the individual pensioner and greater capital accumulation for the economy. Adapting the standard OLG model to include portfolio finance this paper argues that, provided pensioners are risk averse, it is the PAYGO component that may need to be more heavily weighted. Individual welfare is enhanced, while capital accumulation effects need not be adverse. But since the resulting system is more vulnerable to demographic ageing, a solution is needed that preserves its welfare enhancing benefits.

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Pension issues have taken front stage in many countries and their resolution is proving difficult. A contentious issue concerns the optimal design of a multi-pillar pension system. While many pensioners apparently prefer a PAYGO system with adequate guaranteed benefits, the scale of the fiscal problems now being encountered has led to the widespread advocacy of fully funded, individual accounts systems. Conventional wisdom, as embodied in say the World Bank's report of 1994 entitled "Averting the Old Age Crisis", is that an ideal system should rest on three pillars. The first pillar should consist of a mandatory, preferably small, defined benefit, pay-as-you-go (PAYGO) system; the second and main pillar should be a mandatory, defined contribution, fully funded, individual accounts scheme; while the third pillar should involve a voluntary savings scheme to provide additional insurance as needed by the individual.[1]

This influential recommendation is based on a balancing of several considerations such as the social security component's rate of return on the contributions paid in compared to other alternatives, the need to promote capital accumulation, the scope for maintaining the financial viability of the pension system in the face of aging related demographic shocks, catering to individual insurance requirements, and providing efficient income redistribution. On the surface it is an attractive mix for industrial countries, whose traditional PAYGO systems are experiencing financial distress as a consequence of the aging phenomenon. Even though these countries are likely to experience significant financial costs in implementing it, chiefly from honouring pension liabilities of existing systems, some argue that on balance they would be better off insofar as there is more capital accumulation and alleged labour market distortions are reduced.[2] The new design is also proposed for many developing and transition economies whose pension systems are either rudimentary or have broken down, and for whom the need to accumulate capital may be paramount. Not surprisingly, many countries around the world have begun to implement the recommended pension system design.

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[1] For an informative presentation and critique of the World Bank's proposals see Orszag and Stiglitz (2001).

[2] For some reviews of the literature see Chand and Jaeger (1996), Orszag and Stiglitz (2001), and Lindbeck and Persson (2003).

Nonetheless, several questions can be raised about the recommended mix. Why should there be three pillars and not some other number? The principal rationale advanced for making a fully funded, individual account, pillar mandatory is alleged myopia of individuals. While this may be of practical importance, an analytical treatment based on standard optimising behaviour under appropriate informational conditions would dispense with this feature and focus instead on a voluntary fully funded pillar. The analysis then reduces to two pillars, the mandatory PAYGO and a voluntary fully funded one.[3] Having both can be readily justified on standard portfolio diversification grounds (see for example, Merton (1983)). However, what should their respective sizes be? The traditional theoretical literature has tended to focus on one or the other pillars as mutually exclusive alternatives, but a more recent literature has begun to address the issue of the optimal composition of a mixed system.[4] This paper focuses on the latter issue. It asks whether the de-emphasis on PAYGO systems is justified, and concludes that while a mixed system is preferable to either extreme, there should be a heavier weighting on the PAYGO component, at any rate for most countries. In developing this argument, the paper abstracts from distributional considerations that are frequently presented in favour of the PAYGO.

The approach in this paper is two-fold. It looks first at the prospective pensioner's decision-making problem regarding how best to prepare for retirement, and then the implications at the aggregate level of the choices made. A lifetime portfolio choice perspective is adopted in the context of a two-period overlapping generations framework. Agents in this model are subject to at least two sources of retirement risk involving respectively the return on their savings and the length of their lives. Experiencing an unexpected decline in the return to savings would be discomforting to the retiree, but so would living longer or shorter than the period for which provision has been made. Since a retiree's hedging capacity through income generation from work is limited, risk aversion is likely to be high, or at least higher

[3] The PAYGO pillar would presumably have to be mandatory so as to prevent future generations from reneging on their acquired obligations to earlier generations, and also possible free-rider problems.
[4] For a review of this literature see Matsen and Thøgersen (2003). The present paper draws in part on Dutta, Kapur, and Orsag (1999), who present the portfolio choice perspective and some empirical evidence in its favour, and on the paper by Matsen and Thøgersen, who develop a partial equilibrium analysis. The latter's approach is closest in spirit to the development of the static model of section 2 below.

than when young. Obtaining some form of insurance against such risks is obviously desirable and retirees will seek to do so.

Risk averse prospective pensioners are therefore likely to trade some return on saving for greater security. As a significant part of the literature has emphasised, the PAYGO is an innovative social, intergenerational, institution for mitigating such risks. But others, including prominently Feldstein (1996), have criticised the PAYGO system as being economically inefficient. Not only does the individual receive lower rates of return on their savings than with a fully funded equity based alternative, but the economy as a whole will exhibit a reduced capital intensity resulting in a lower inter-temporal consumption profile. These potentially adverse effects are aggravated by possible deadweight losses on the supply of labour occasioned by social security contributions. The plausibility of Feldstein's criticisms of the PAYGO system depend in part on the mechanisms provided for containing or hedging risk. To what extent does PAYGO provide a relatively risk-free asset compared to a fully funded, individual accounts system? How much are individuals willing to pay for this advantage? What are some of the aggregate economy implications in an inter-temporal context?

Developing transparent closed-form solutions that integrate the portfolio choice aspect with the savings and capital accumulation aspects raises some analytical obstacles. While it would be convenient to undertake the portfolio analysis on traditional mean-variance lines, there are well known difficulties involving the unattractiveness of having to assume either a quadratic utility function or normally distributed returns.[5] A better approach for identifying risk with variance is to assume that asset returns follow a log-linear distribution. This allows for limited liability (Merton, 1992). Based on this assumption, a simple model is presented in Section 2 that provides a solution to the inter-temporal choice problem under uncertainty involving a mix of two assets, one of which is the lower risk PAYGO asset and the other is a higher risk fully funded one. As is to be expected, the optimal portfolio depends on the degree of risk aversion, the spread between asset rates of returns, and

[5] The former confers the undesirable characteristic of increasing risk aversion with higher levels of wealth. Assuming normally distributed returns is more defensible but it carries the unacceptable implication that people can lose more than they have invested.

their variance-covariance structure. Applying conventionally used measures of the degree of risk aversion shows that the pensioner prefers a mix that is heavily weighted in favour of the PAYGO asset.

The selected mix both affects the portfolio rate of return and the flow of saving into capital accumulation. In particular, the return to capital is affected, which feeds back into the portfolio choice decision. It is only when the economy has settled down at its long-run equilibrium rate of return to capital that the sustainable PAYGO contribution rate can be determined. These issues are examined in Section 3, together with the scope for a PAYGO induced decline in the long-run equilibrium capital intensity. It is argued there that provided pensioners are sufficiently risk averse, this need not be unduly adverse and could even be beneficial.

However, a pension system that includes a PAYGO pillar will be more vulnerable to systemic demographic aging shocks than one that relies solely on funded individual accounts. Solutions widely proposed and implemented for addressing the problem involve various types of parametric adjustments to contribution and benefit rates and periodic increases in retirement ages. Such solutions tend to increase the degree of uncertainty surrounding the PAYGO asset, robbing it of its superior risk characteristics relative to the equity asset. If the welfare benefits of the mixed system are to be retained, a solution is needed that avoids adding to the riskiness of the PAYGO asset. Section 4 outlines an approach for immunising the PAYGO asset against demographic shocks.

2. Portfolio choice and pension savings in a one-period context

The model involves a representative agent who saves for retirement, but is concerned about the variability of returns, which the individual hedges through a diversified portfolio. The choice provided in the model is between a relatively risk free asset, which is subsequently identified with the PAYGO system, and a risky asset, say, equity in the capital stock, which is associated with the fully funded approach. The basic set-up is that of an overlapping generations model (OLG) with risk. Individuals live two periods. In their first period they work the capital that the retired generation owns, while they live off that capital during the second period. There are no intended bequests.

The expected utility maximization problem can be specified as

$$Max_{S_t,\alpha_t} : U_t(C_{1,t}) + \delta E_t U_{t+1}(C_{2,t+1}) \tag{1}$$

s.t

$$\begin{aligned}
&C_{1,t} = W_t - S_t; \quad C_{2,t+1} = S_t(1+R_{p,t+1}) \\
&where \\
&sub\ 1 = young;\ \ sub\ 2 = retired \\
&R_{p,t+1} = portfolio\ rate\ of\ return, \\
&\delta = \frac{1}{1+\rho}; \quad \rho > -1,
\end{aligned}$$

Here *W*= wages, *S*= savings, E_t is the expectations operator based on information available at time t, ρ is the time preference rate taking a positive value when current consumption is preferred to future consumption, and α is the share of the risky asset in portfolios, which features in the maximization subsequently.

A constant relative risk aversion (CRRA) utility function is assumed:

$$\begin{aligned}
U &= \frac{C^{1-\lambda}}{1-\lambda} \quad \lambda \neq 1 \\
&= \log C \quad \lambda = 1
\end{aligned} \tag{2}$$

Even under this simple assumption it is difficult to develop transparent closed-form solutions. The approach here is to develop a solution in stages. First, we focus on the portfolio solution for a given amount of saving and then generate a compatible solution for first-period saving.

$$\begin{aligned}
&Max_{\alpha_t} : \quad E_t \delta \frac{C_{2,t+1}{}^{1-\lambda}}{1-\lambda} \\
&s.t \\
&C_{2,t+1} = S_t(1+R_{p,t+1}); \quad S_t = \overline{S}_t
\end{aligned} \tag{3}$$

The choice of α_t, the portfolio share of the risky asset, determines the portfolio return $(1+R_{p,t+1})$, which is assumed to be lognormal and i.i.d. As a consequence, second-period consumption is also lognormal from the multiplicative form of the second-period budget constraint. The utility function is thus a CRRA function of a lognormal variable.

A solution to the above optimisation problem can be developed using a procedure outlined in Campbell and Viceira (p. 26, 2002)[6]. Optimising the preceding function is the same as optimising a logarithmic transformation of the function (from which the scale factor $\delta/1-\lambda$ has been dropped since it does not affect the solution):

$$Max_{\alpha_t} = LnE_t C_{2,t+1}{}^{1-\lambda} \tag{4}$$

From the definition of a log-normal variable X

$$Ln\,X \sim N(\mu_{\ln X}, \frac{1}{2}\sigma^2_{\ln X}),\;\; E(X) = e^{\mu_{\ln X} + \frac{1}{2}\sigma^2_{\ln X}}$$

Hence,

$$\ln E(X) = E\ln X + \frac{1}{2}Var.\ln X$$

Using this property in (4) yields an equivalent transformation

$$Max_{\alpha_t}(\ln E_t C^{1-\lambda}_{2,t+1} \equiv (1-\lambda)E_t \ln C_{2,t+1} + \frac{1}{2}(1-\lambda)^2\sigma_c^2) \tag{5}$$

where σ_c^2 is the assumed constant variance of consumption.

Taking logarithms of second-period consumption in (3),

$$\ln C_{2,t+1} = \ln S_t + r_{p,t+1},\; where\; r_{p,t+1} \equiv \ln(1+R_{p,t+1}) \tag{6}$$

[6] See also Matsen and Thøgersen.

Substituting the result back in (5) and dividing through (5) by $1-\lambda$ converts the optimisation problem to

$$Max_{\alpha_t} E_t r_{p,t+1} + \frac{1}{2}(1-\lambda)\sigma^2_{p,t+1} \quad (7)$$

where $\sigma^2_p = \sigma^2_c$ from (6) and the assumptions made.

The portfolio comprises two assets: a relatively risk-free asset whose return is denoted $r_{b,t+1} = \ln(1+R_{b,t+1})$ and variance σ^2_b, and a risky asset with return $r_{k,t+1} = \ln(1+R_{k,t+1})$ and variance σ^2_k. Given α_t, the proportion of saving devoted to the risky asset, the portfolio return is

$$r_p, t+1 = \alpha_t r_{k,t+1} + (1-\alpha_t) r_{b,t+1} \quad (8)$$

This can be viewed as an excess return over the pure risk free rate r_f, which we normalise to *ln1=0*. Excluding short sales as well as borrowing to finance investment in the risky asset imposes the restriction $0 \le \alpha \le 1$.

Using Taylor's approximation to log-linearise the portfolio return around the benchmark rate and taking expectations yields

$$E_t(r_{p,t+1}) = \alpha_t(\mu_{k,t+1} - \mu_{b,t+1}) + \mu_{b,t+1} + \frac{1}{2}\alpha_t(1-\alpha_t)H \quad (9)$$

$$H \equiv \sigma^2_k + \sigma^2_b - 2\rho_{k,b}$$

Here $\rho_{k,b}$ is the covariance between the risky and benchmark assets and μ_k, μ_b are the expected mean returns of equity and the benchmark asset, respectively.

When α_t=1 in (9), only the risky asset is held and the portfolio return is then the mean return on capital. In addition to the expected portfolio return, to complete the maximization problem of equation (7) an expression is needed for the variance of the portfolio return in terms of the component assets. Applying the rule regarding the sum of variances to (8).

$$\begin{aligned}\sigma_p^2 &= \alpha^2\sigma_k^2 + (1-\alpha)^2\sigma_b^2 + 2\alpha(1-\alpha)\rho_{k,b} \\ &= \alpha^2 H + \sigma_b^2 - 2\alpha(\sigma_b^2 - \rho_{k,b})\end{aligned} \tag{10}$$

The single-period maximisation problem in (7) can now be expressed as

$$Max_{\alpha_t}:$$

$$\alpha(\mu_{k,t+1} - \mu_{b,t+1}) + \frac{1}{2}(\alpha - \alpha^2)H + \frac{1}{2}(1-\lambda)\left[\alpha^2 H + \sigma_b^2 - 2\alpha(\sigma_b^2 - \rho_{k,b})\right] \tag{11}$$

The solution for α is derived from the first order condition for (11)

$$\alpha = \frac{1}{2\lambda} + \frac{(\mu_{k,t+1} - \mu_{b,t+1}) - (1-\lambda)(\sigma_b^2 - \rho_{k,b})}{\lambda H}$$

$$0 \le \alpha \le 1 \tag{12}$$

Examining the case where the risk aversion coefficient for the CRRA utility function is $\lambda = 1$ yields

$$\alpha_t = \frac{1}{2} + \frac{(\mu_k - \mu_b)}{H} \tag{13}$$

The proportion of the portfolio invested in the risky asset is one-half plus a term depending on the excess of that asset's mean return over the benchmark asset. If the assets have identical means, the second term disappears and the optimum portfolio allocation is simply one-half to each asset.

Some properties of α of use in subsequent analysis are given below.

Lemma 1: *Assume* $\mu_k > \mu_k$ *and* $\sigma_k^2 > \sigma_b^2$

Case (i): $\lambda = 1$

$$\frac{\partial\alpha}{\partial\mu_k}>0,\ \frac{\partial\alpha}{\partial\mu_b}<0,\ \frac{\partial\alpha}{\partial\sigma_k^2}<0,\ \frac{\partial\alpha}{\partial\sigma_b^2}<0,\ \frac{\partial\alpha}{\partial\rho_{k,b}}>0,$$

These are derived from differentiating equation (13).

Case (ii): $\lambda \neq 1$

$$\frac{\partial\alpha}{\partial\lambda}<0, \frac{\partial\alpha}{\partial\mu_k}>0,\ \frac{\partial\alpha}{\partial\mu_b}<0,\ \frac{\partial\alpha}{\partial\sigma_k^2}<0,\ \ \frac{\partial\alpha}{\partial\rho_{k,b}}>0,\ \ \frac{\partial\alpha}{\partial\sigma_b^2}<0 \text{ if } \lambda<1, \text{ and } ><0 \text{ if } \lambda>1$$

These are derived from differentiating (12).

From the lemma, the higher the risk aversion coefficient λ the smaller α, which ranges from a maximum of 1 for very low levels of λ to a minimum of 0 for high values. This relationship is illustrated in Figure 1 as movements along the given curve. Shifts in the curve are attributable to changes in mean returns or their variance-covariance matrix. Greater variability in the return for capital shifts the curve down, reducing α, while an increase in the risky asset's rate of return displaces the curve upwards. The point A in the diagram is drawn for λ=1. Of particular note is the implication that if the covariance between the two assets increases then more of the higher yielding asset will be held. The effect of an increase in the variance of the less risky asset is ambiguous for the case of $\lambda>1$. This is because with greater risk aversion, an increase in the riskiness of the less risky asset increases the attractiveness of the more risky asset, but reduces the attractiveness of the whole portfolio. Introducing a pure risk free asset such as treasury bills would have led to an unambiguous reduction in desired holding of the risky asset.

To obtain a perspective on α, it is instructive to look at the data presented in Table 1. This is drawn from the study by Dutta, Kapur, and Orszag (1999) and covers the period 1900-1989 for five major industrial countries. The growth rate in GDP is used here as a proxy for the growth rate of labour income. According to this table, its mean growth is a good deal lower than that of the total return on equity, in some cases amounting to less than one-third of the latter.[7] This is of interest since labour income provides the base for the PAYGO system, and its growth rate is a measure of the "return" to PAYGO contributions. The low PAYGO rate of return is associated with variances that are much smaller than those on the return to equity, for example for the USA amounting to less than two percent. Substantial welfare gains, depending on the degree of risk aversion, are therefore likely from portfolio diversification. The scope for such gains rises from limited covariance between the alternative assets.

Using equation (12) and the data in Table 1, the value of α is computed for five countries for different degrees of risk aversion. The results presented in Table 2 indicate considerable variation in desired portfolio allocations. The first column shows that when risk aversion is set at $\lambda = \frac{2}{3}$, the desired share of portfolio to be invested in equity amounts to over 80 percent for the sample. However, as risk aversion increases, the desired share for the risky asset falls rapidly. For $\lambda=3$, it falls to less than 20 percent. Since experimental studies suggest relatively high degrees of risk aversion, risk-averse pensioners are likely to want to invest heavily in the less risky asset that is here identified with PAYGO rather than fully funded accounts.

In principal, the riskiness of fully funded accounts may be reduced through acquisition of safe bonds, etc. However, this raises the issue of how risk is intermediated in the economy, which complicates the analysis. Furthermore, since some of the rationale for fully funded accounts is provided on the grounds of its superior return, this will be at its highest when only equities are held.

3. The optimal pension mix in a multi-period setting

The portfolio analysis in the preceding section assumed a given level of savings and capital. To pave the way for an inter-temporal, general equilibrium analysis, it is essential to obtain a solution for first- period savings. This is undertaken

[7] It would be interesting to speculate on the reasons for this phenomenon. One explanation would be on the lines that since labour income tends to governed by contracts, greater volatility is forced onto profits.

here first for the benchmark CRRA logarithmic utility function for which the degree of risk aversion is 1. Subsequently, this assumption is relaxed.

$$
\begin{aligned}
&Max_s: \ \ln C_{1,t} + \delta E_t \ln C_{2,t+1} \\
&s.t \quad C_{1,t} = W_t - S_t \\
&\qquad C_{2,t} = S_t(1+R_{p,t+1})
\end{aligned}
\tag{14}
$$

Maximising the two period expected utility function with respect to savings yields the Euler condition, which indicates that saving is a constant proportion of wages.

$$
\frac{1}{W_t - S_t} = \delta E_t \frac{1}{S_t} \quad \Rightarrow S_t = \frac{1}{2+\rho} W_t \tag{15}
$$

To derive longer-term implications for the evolution of the capital stock of the postulated savings and portfolio behaviour in Section 2, it is convenient to employ the standard assumptions of the overlapping generations (OLG) model.[8] The labour market is assumed to adjust throughout so as to ensure full employment. Let the population size at *t* be N_t, growing at an exogenous rate *n* so that $N_{t+1} = (1+n)N_t$. Competitive firms supply output in accordance with a well-behaved constant returns to scale, net (of depreciation) production function, $Y = F(K,N)$, which is given explicit form as a Cobb-Douglas technology in much of the subsequent analysis. Factors of production are paid in accordance with their marginal products. In particular, the rate of interest is equated to the marginal product of capital. Using small case letters to denote per-capita magnitudes, the production side of the model is

$$
\begin{aligned}
y_t &= f(k_t) = k_t^{\gamma} \\
R_{k,t} &= f'(k_t) = \gamma k_t^{\gamma-1} \\
w_t &= f(k_t) - k_t f'(k_t) = (1-\gamma)k_t^{\gamma}
\end{aligned}
\tag{16}
$$

Here $R_{k,t}$ equals the marginal product of capital, and w_t is per capita wage.

[8] For example, Romer (2001)

Goods market equilibrium prevails at each point in time and is reflected by the condition that investment is equal to saving: $K_{t+1} = N_t S_t$. When stated in per-capita terms, on taking account of the population rate of growth and using the savings rate stated in (15), the law of motion governing the capital stock is

$$k_{t+1} = \frac{1}{(1+n)(2+\rho)}\left(f(k_{t)} - f'(k_t)k_t\right) \qquad (17)$$

With Cobb-Douglas technology this takes the form

$$k_{t+1} = \frac{1}{(1+n)(2+\rho)}(1-\gamma)k_t^{\gamma} \qquad (17a)$$

In the logarithmic utility case, next period's portfolio allocation between risky and non- risky assets is of no consequence to the current worker's saving, which is a fixed proportion of that period's income. This assumption is subsequently relaxed.

Next, introduce a social security system that is based on intergenerational transfers. Even though from the individual's perspective the contributions made are a form of saving, the aggregate implication is quite different since a transfer is consumed by the elderly rather than added to the capital stock. The individual may regard entering into and continuing with this intergenerational transfer system as desirable insofar as the asset acquired confers greater stability than would withdrawal from the system and fully funding one's own retirement. Let 1-α of saving represent the desired acquisition of this transfer based "asset". Aggregate investment would be reduced by this amount and the resulting law of motion of capital in the Cobb-Douglas case becomes

$$k_{t+1} = \frac{\alpha_t}{(1+n)(2+\rho)}(1-\gamma)k_t^{\gamma} \qquad (18)$$

With the exception of the term α, this is a standard difference equation in growth theory, which is readily shown to have a globally stable solution $k=k^*$.

With variable k, α also varies. The lower the capital intensity the higher the marginal product of capital. From (12) and (13), the higher the rate of return on equity capital to which it is equated the greater the proportion of the portfolio that will be held in the form of equity. Thus for some $k \leq k_l$, the entire portfolio would be specialized in equity. Likewise, the higher k the lower the equity rate of return. For some $k \geq k^u$ the risky asset would be completely dominated and the portfolio would specialize in the less risky asset. These limits on α are shown in Figure 2. The behaviour of the dynamical system in (18) can now be stated.

Proposition 1: When $\lambda = 1$ and the production technology is Cobb-Douglas,

(i) The dynamical system has a unique stable equilibrium at k_m^*, where the subscript refers to a mixed system,

(ii) If $k_m^* > k_l$, $\alpha < 1$, *then* $k_m^* < k_f^*$, where the subscript *f* refers to a fully funded system

(iii) If $k_m^* < k_l$, $\alpha = 1$, *then* $k_m^* = k_f^*$

Proof:

From (13), α is a decreasing function of expected next period's capital intensity. Assuming rational expectations, and using lemma 1, define $\alpha_t = \alpha(k_{t+1})$.

Now $\alpha'(k_{t+1}) < 0, \; when \; k > k^l, and \; 0 \; otherwise$. Using this in (18)

$\frac{k_{t+1}}{\alpha_t} = \frac{1}{(1+n)(2+\rho)}(1-\gamma)k_t^{\gamma}$. Taking total derivatives, $\frac{dk_{t+1}}{dk_t} = \frac{\alpha_t}{1-\varepsilon_{\alpha,k}}\gamma\frac{(1-\gamma)k_t^{\gamma-1}}{(1+n)(2+\rho)}$

where the elasticity of the portfolio share of the risky asset with respect to capital is

$\varepsilon_{\alpha,k} = \frac{\alpha'(k_{t+1})k_{t+1}}{\alpha} < 0$. Hence

$\frac{dk_{t+1}}{dk_t} > 0$ and $\frac{\mathrm{d}}{dk_t}\left(\frac{dk_{t+1}}{dk_t}\right) = \frac{\alpha_t}{1-\varepsilon_{\alpha,k}}\gamma\frac{(1-\gamma)(\gamma-1)k_t^{\gamma-2}}{(1+n)(2+\rho)} < 0$. This demonstrates part (i).

Parts (ii) and (iii) are self-evident. See also Figure 3.

The above proposition states that under the assumed conditions, the resulting steady state capital intensity will be at or below that of the system without the

PAYGO component. The latter case would yield some potential for welfare increasing net tradeoffs between portfolio diversification and greater output, assuming that the solutions are in the dynamically efficient ranges for the equilibrium capital stock.

Relaxing the benchmark assumption of unitary coefficient of risk aversion for the more realistic case of greater risk aversion gives rise to a richer analysis. Savings are now no longer unaffected by the portfolio allocation decision. This is because the latter influences the expected return on the portfolio, which in turn influences the amount of saving desired. For the CRRA utility function, if the degree of risk aversion is greater than 1, then an increase in the expected rate of interest will reduce savings (Romer (2001), p.78). Note that in the present portfolio model, the appropriate rate of interest to consider is the portfolio rate of return rather than the standard growth model's real rate of return, which is equated to the marginal product of capital. The savings ratio now takes the form

$$s(r_{p,t+1}) = \frac{(1+R_{p,t+1})^{\frac{1-\lambda}{\lambda}}}{(1+\rho)^{\frac{1}{\lambda}} + (1+R_{p,t+1})^{\frac{1-\lambda}{\lambda}}} \qquad \begin{array}{l} s'(r_{p,t+1}) \geq 0 \text{ , } \lambda < 1 \\ s'(r_{p,t+1}) \leq 0, \quad \lambda > 1 \end{array} \qquad (19)$$

The evolution of the capital intensity is now described by

$$k_{t+1} = \frac{\alpha\left(k_{t+1}\right)_t s\left(r_{p,t+1}\right)}{(1+n)}(1-\gamma)k_t^{\gamma} \qquad (20)$$

A particular implication of note is that in the case of risk-averse agents, allocating part of their pension investments to PAYGO will have the effect of reducing the portfolio rate of return, which from (19) will induce them to increase their overall saving. Hence an offset, whose strength will depend on the curvature of the savings and utility functions, is present for the PAYGO induced reduction in capital accumulation. Insofar as the system settles at a level of $\alpha < 1$ it is, therefore not necessarily the case that the equilibrium capital intensity will be less than that implied by a fully funded system, which is indicated in the following proposition.

Proposition 2: When $\lambda > 1$, the possibility arises that $k_m^* > k_f^*$.

Proof: From (20), $\frac{k_{t+1}}{\alpha(k_{t+1})_t\, s(r_{p,t+1})} = \frac{1}{(1+n)}(1-\gamma)k_t^{\gamma}$. Hence,

$\frac{dk_{t+1}}{dk_t} = \frac{\alpha(k_{t+1})_t\, s(r_{p,t+1})_t}{1-(\varepsilon_{\alpha,k}+\varepsilon_{s,k})}\gamma\frac{(1-\gamma)k_t^{\gamma-1}}{(1+n)}$, where from

(19), $\varepsilon_{s,k_{t+1}} = \frac{s'(r_{p,t+1})}{s(r_{p,t+1})}\frac{\partial r_{p,t+1}}{\partial k_{t+1}}k_{t+1} \geq 0$ is the elasticity of the savings ratio with respect to the capital intensity. Since the two elasticity terms move in opposite directions, a richer variety of dynamical behaviour is now possible. Assuming that the sum of the elasticities is negative will reproduce the standard growth behaviour for the benchmark model (λ=1). However, the non-linear product term $\alpha(k_{t+1})_t\, s(r_{p,t+1})$ could exceed $\frac{1}{(2+\rho)}$, or the savings ratio for the benchmark case, proving the proposition. See also Figure 4.

Assuming that the system has settled on its long-run equilibrium path, the optimal transfer rate τ^* from wages, i.e., the social security tax, can then be determined. This is simply the proportion 1-α^* of the long-run equilibrium portfolio that is desired to be held as a PAYGO asset. Depending on the underlying model, a specific parametric form can be computed.

4. Addressing the demographic vulnerability of the PAYGO component

The discussion so far suggests that the optimal pension system would include a sizable PAYGO component. Unlike the fully funded system, which automatically takes care of the aging problem, defined as systemic increases in average life expectancies (as distinct from idiosyncratic movements that wash out in aggregate insurance systems), the PAYGO system is vulnerable. If this issue is not addressed, the superiority of a mixed system with a large PAYGO component over one with a smaller component or none will be difficult to sustain. Systemic risk, depending on the welfare calculus, could outweigh individual risk-return considerations. However, in principle it is quite easy to address this source of vulnerability.

The progressive increase in life expectancies over the past century for many countries, roughly at the rate of one year for each decade, has placed social security systems that operate a PAYGO scheme under considerable pressure. This arises when defined benefit rates are kept unchanged at levels that may have been appropriate when life expectancies were lower, effectively increasing the rate of return of the PAYGO system above the growth rate of the labour force.

A simple way of obtaining a handle on the aging process is to begin with the requirement that in a mature, actuarially sound PAYGO system, incomings must equal outgoings. Assuming that the contribution and defined benefit rate have been determined optimally as in Section 3, if the population and labour force growth rate for now is exogenously given as *n*, the following relation holds in steady state

$$b_{t+1}N_t = \tau^* w^*_{t+1}\ N_{t+1} \qquad (21)$$

The LHS of the above equation indicates at time t+1the total benefits of pensioners, which are financed by total contributions from wage earners in that period. Since the labour force is growing at the rate *n*, the return obtained by the individual pensioner on contributions that he or she made in the preceding period follows on dividing through by the number of pensioners

$$b_{t+1} = \tau^* w^*_{t+1}(1+n) \qquad (22)$$

The above computation assumes an unchanging life expectancy and retirement period. Suppose these are normalised at 1, respectively. Representing a period increase in life expectancy by *p(s)*, the increase in life expectancy at any time *t* would be

$$P_t = \sum_{s=0}^{t} p(s)\,; \qquad \text{where} \qquad \lim_{t\to\infty} P_t = \widetilde{P} < 1 \qquad (23)$$

For financial viability of the system in a context of increasing life expectancy a downward adjustment to a given benefit level defined in *t+1* is needed at time *t+i*

$$b_{t+i} = b_{t+1}(1-\sum_{s=t}^{t+i} P(s)) = \tau^* w^* (1+n) \qquad (24)$$

In the above equation the benefits rate paid out at time t+1when life expectancy was 1 is reduced by the amount of the increase in life expectancy, assuming that the full amount is carried over into the retirement period. An alternative would be to increase the minimum retirement age, say from an initial normalized value of 1 to $1+\sum_{s=t+1}^{t+i} h(s)$.

$$b_{t+i} = b_{t+1} \frac{1}{1+\sum_{s=t}^{t+i} h(s)} = \tau^* w^* (1+n) \qquad (25)$$

The next proposition combines both possibilities.

Proposition 3: For an unchanged benefit rate to be financially sustainable when life expectancies increase, the early retirement rate should be adjusted commensurately with increases in life expectancies.

Proof: Combining the preceding two equations yields

$$b_{t+i} \equiv b_{t+1} \frac{1}{1+\sum_{s=t+1}^{t+i} h(s)} (1-\sum_{s=t}^{t+i} P(s)) = \tau w^* (1+n)$$

It immediately follows that keeping the benefits rate unchanged requires

$$b_{t+1} = \frac{1+\sum_{s=t}^{t+i} h(s)}{(1-\sum_{s=t}^{t+i} P(s))} \tau^* w^* (1+n)$$

The adjustments noted in the proposition are not routinely undertaken in countries. As a consequence the effective benefits rate tends to increase over time. Instead of contributions paid in providing a return of *n*, they would now be higher since they would include payments for the duration of the increased life expectancy, which may be politically popular. The financial viability of the system would be compromised creating a financial gap that would have to be financed by future generations and the current working generation. Furthermore, to the extent the PAYGO rate of return has now gone up, savers will want to reduce their holdings of the risky asset, which could imply a reduction in the capital intensity.

There is often recourse to the practice of periodically raising contribution rates or reducing benefits, but this increases the risk associated with the PAYGO system, while reducing its rate of return, eroding the very qualities that makes it desirable. Neither the frequently resorted to upward adjustments in contribution rates nor downward adjustments in benefit rates are advisable, unless they are completely out of line and need to be restructured. The concern here is with frequent adjustments rather than one-off changes, since the former add to the riskiness of the PAYGO asset.

The most acceptable solution that would preserve the quality of the asset is to increase the minimum retirement age. Although there have been upward adjustments in retirement rates these have not been popular. It would seem that most people prefer to retire earlier, an option they are better able to exploit with a PAYGO scheme, but at the expense of the latter insofar as benefit rates have not been commensurately reduced. This is in marked contrast to fully funded schemes, where even though individuals might like to retire earlier they are subject to the hard budget discipline of inadequate portfolios. Consequently, they postpone retirement or engage in various part-time work/ semi-retirement schemes.

A way of reintroducing budget discipline into the PAYGO scheme that might be more acceptable than episodic adjustments in minimum retirement ages, would be to gradually, but continuously, raise the minimum retirement age in step with rising longevity. For example, if longevity is rising at the rate of one year per decade, a cohort facing retirement in 30 years time would eventually, and on a cumulative basis, have 3 years added to their minimum retirement age, whereas a cohort only a year away from retirement would have one-tenth of a year added. Minimum retirement ages adjusted in this manner would be compatible with keeping unchanged defined benefits. If people choose to retire earlier, defined benefits should be adjusted downwards in an actuarially sound manner. The benefits received would still remain defined on correcting for actuarial increases in the length of the retirement period thereby dispelling uncertainty and contributing to more effective lifetime planning.

5. Conclusion

This paper set out to examine the robustness of the conventional wisdom that a pension system with a small or negligible PAYGO component is superior to one with a large PAYGO component. It was not found to be a robust assessment when allowance is made for risk aversion and portfolio diversification needs. While it is true that investing in a PAYGO type asset could divert savings from capital accumulation into consumption transfers, the analysis of an admittedly highly simplified model showed that even this could be limited. Under certain circumstances the result might even be the opposite. Recommendations based on the allegedly superior capital accumulation outcome of a fully funded, individual accounts scheme, that would outweigh the benefits from intergenerational risk sharing of the PAYGO type, may therefore need to be qualified.

The case for a more heavily weighted PAYGO system is reinforced when fully funded self-insurance schemes collapse, as occurred in the United States before the creation of the present social security system in the mid-1930s. The simplest and quickest way of meeting the needs of the destitute aged, a problem that is endemic in much of the developing and transition world, is to institute or rehabilitate a PAYGO weighted system. However, for it to retain its considerable merit as a lower-risk alternative to more risky pension system designs it will need to be adequately structured to tackle demographic shocks. Much of the disenchantment with PAYGO systems is attributable to the fiscal strains they have been associated with. These are often attributable to benefit structures becoming progressively, and perhaps surreptitiously, more generous as a consequence of keeping rate structures unchanged in the face of increasing life spans. The paper has outlined one option for addressing the latter problem.

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Table 1. Basic Data: Selected Industrial Countries, 1900-1989

Country	GDP Growth (%) Mean μ_b	Variance σ_b^2	Return on Equity Mean μ_k	Variance σ_k^2	Cov. $\rho_{k,b}$
USA	3.16	0.67	6.33	47.6	0.67
UK	1.87	0.97	4.42	60.9	4.16
France	2.42	5.37	8.4	80.7	6.57
Germany	2.88	9.72	8.93	89.4	16.2
Japan	4.42	15.71	7.76	287.9	47.52

Source: Dutta, Kapur, and Orszag (1999)

Table 2. Desired portfolio share of equity capital (α) for different degrees of risk aversion (λ)

	λ = 0,67	λ = 1	λ = 2	λ =3
USA	0,80	0,57	0,28	0,19
UK	0,82	0,55	0,24	0,14
FRANCE	0,85	0,58	0,28	0,18
Germany	0,95	0,59	0,25	0,13
Japan	0,85	0,52	0,18	0,07

Source: Table 1 and equation (12).

Figure 1. The portfolio share of equity capital α and degree of risk aversion

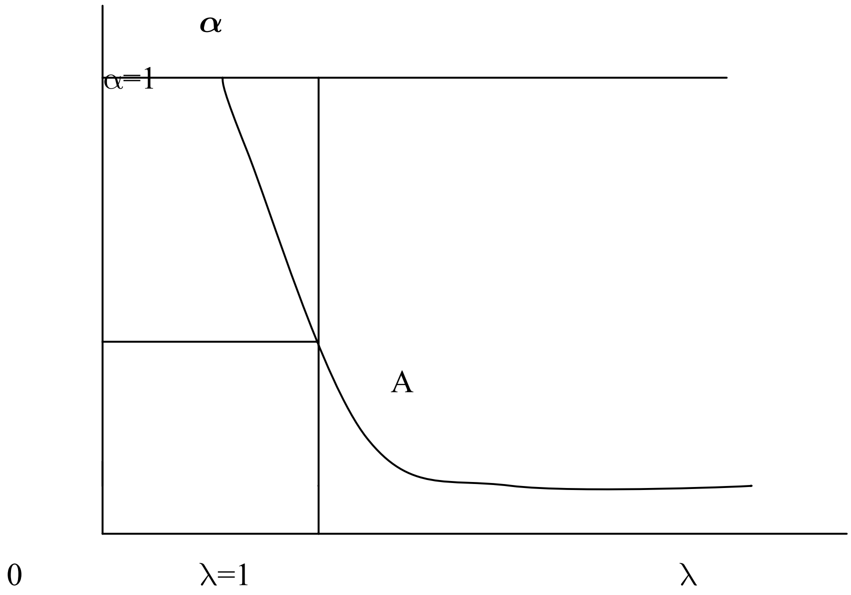

Figure 2. PAYGO, the portfolio share α and capital intensity k

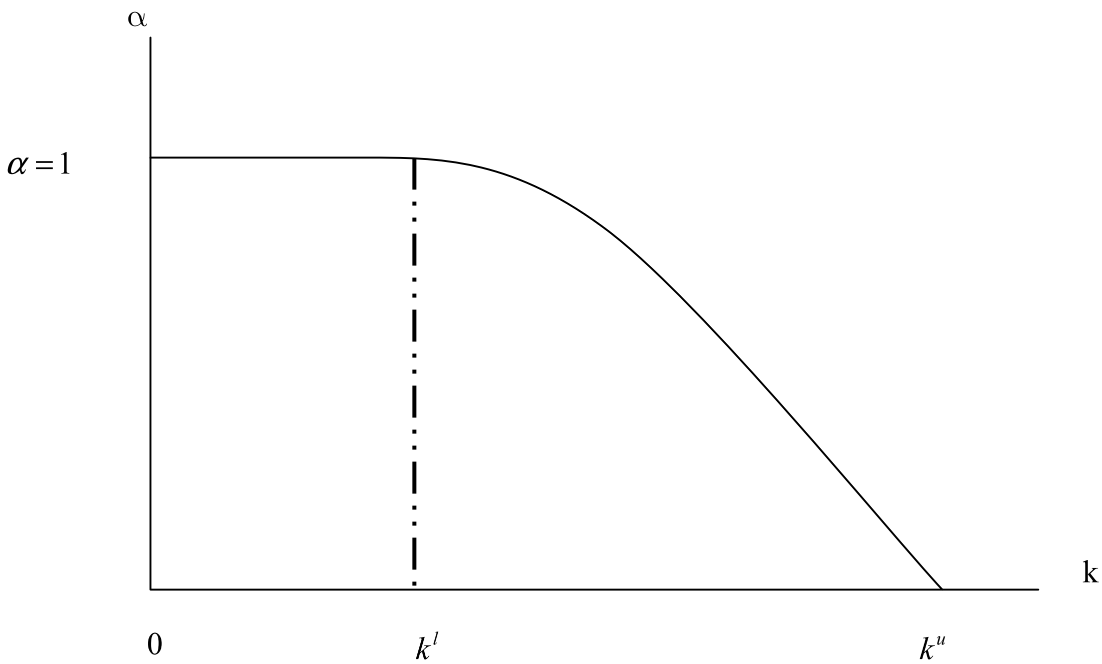

Figure 3. PAYGO and the dynamics of k when λ=1

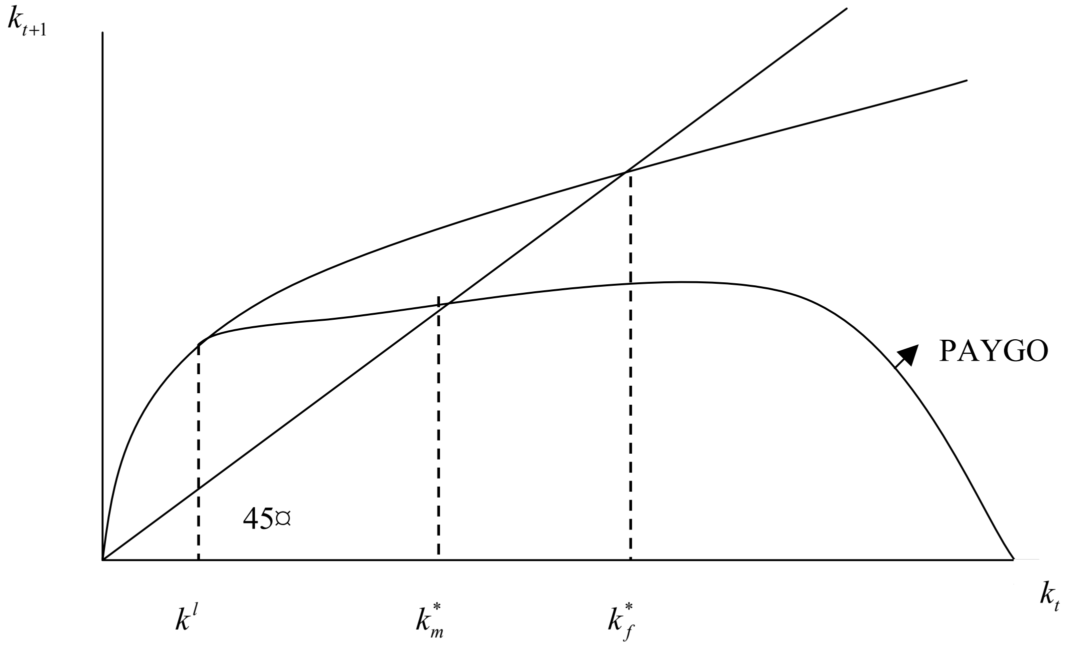

Figure 4. PAYGO and the dynamics of k when λ>1, a scenario