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Research article

Economic Covid-19 effects analysed by macro econometric models—the case of Norway

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Abstract: Counterfactual analysis of the impact of Covid-19 can be based on a solution of a macroeconomic model for a scenario without the coronavirus interfering with the macroeconomic system. Two measures of impact are introduced with the aid of a simple theoretical model, and then used in the empirical analysis: (I) The difference between the counterfactual without Covid-19 and a baseline model solution. (II) The difference between the counterfactual and the actual development of the economy. In order to analyze the impact on GDP we use two model categories. First, empirical final form model equations, which were purpose-built with the aid of a machine learning algorithm. Second, an operational multiple-equation model of the Norwegian macroeconomic system. Empirically, we find a significant impact of Covid-19 on the GDP Mainland Norway in 2020. For some of the estimator/model combinations, the impacts are also significant in the two first quarters of 2021. Using the multiple-equation model, the assessment is extended to the impact of Covid-19 on value added in four Mainland Norway industries, on imports and exports, and on final consumption expenditure and gross capital formation.

Keywords: Covid-19; simulation of impact; counterfactual; unit-root; cointegration; structural breaks; parameter invariance; propagation; machine learning; impulse indicator saturation

JEL Codes: C32, C53, C54, E17, E27, E32, E37, E65

1. Introduction

The list of topics covered by the Covid-19 literature has become broad and varied, cf. Susskind and Vines (2020) and Padhan and Prabheesh (2021). Considerable diversity in methodology and in data types is another trait, also in the sub-branch of the literature that has a macroeconomic focus. However, analyses based on empirical macroeconometric models have been far between hitherto. In this paper, empirical models, both univariate and multiple-equations, are used to obtain quantitative results for the impact of Covid-19 on the Norwegian economy. Because a multiple-equation model implies (univariate) final form equations for its endogenous variables, there is no logical inconsistency between the two approaches. The single equation approach is simple and transparent, but may offer little in terms of interpretation. A multiple-equation model is by definition more complex but it can have other appealing features. For example, since it may include behavioral relationships from different sectors of the economy, it can be used for efficient analysis of the broader impact of Covid-19 on the national economy.

In general, across methodologies, the analysis of the impact of Covid-19 builds on the idea about a hypothetical counterfactual development without a pandemic that interfered with the macroeconomic system. When a macroeconometric model (small or large) is used, a feasible counterfactual will be a "no Covid-19" solution of the model, obtained in practice by dynamic simulation. The impact and dynamic effects of Covid-19 can then be estimated as the differences between the counterfactual and the baseline, which can be a solution of the model with the estimated impact of Covid-19 included or, more directly, the actual development of the economy over the pandemic period.

As with all comparative dynamics, a challenge to the interpretation and validation of the results is the defensibility of the assumption about the invariance of the model parameters with respect to the Covid-19 phenomenon. It is a strong assumption and seems unlikely to be fully met for all model equations. However, invariance is a relative model property, and it can hold partly if not completely. In practice therefore, the question about the validity of a counterfactual is not completely black and white. Moreover, as more data becomes available, the degree of invariance of coefficients of model equations can be investigated empirically, hopefully leading to more reliable estimates of the impacts and propagated effects of Covid-19.

Since the counterfactuals used in this paper are simulated model solutions, they are also conditioned by the many decisions that were made in the construction of the models. One important decision is about the econometric treatment of trends in the time series. As is well known, if a time series is modeled as trend stationary, ie., without a unit-root of +1, a short-run impact of Covid-19 will eventually die out and there is no long-run effect. Conversely, when the time series variable Y_t is modeled as integrated, usually of order one, denoted by $Y_t \sim \mathrm{I}(1)$, impacts of Covid-19 become transformed to permanent shifts in the level of the variable.

In the models used in this assessment, real GDP, value added by industry and other main variables in the national accounts have been modeled as I(1) series. Testing has shown that the null hypothesis of a zero-frequency unit-root cannot be rejected, using augmented Dickey-Fuller tests and the standard significance levels, while such a hypothesis can typically be rejected for the first-differences of the variables. In a final form equation, an implication of this is that any catch-up in the level of the variable after the first impact depends on counteracting impacts later in the pandemic period, which of course may happen. For a multiple-equation econometric model, the full dependency on counteracting shock for recovery may be more of an empirical question. It can in practice be studied by empirical identification of impulses (impacts), and by simulation of the propagation effects of the macroeconomic system.

The rest of the paper is organized into sections we here will summarize. In section 2, counterfactual measures of Covid-19 impact are defined, and illustrated by using a stylized theoretical model with a closed form solution. What makes the illustrative model relevant, is that is has an important feature in common with the models used in the empirical assessment of Covid-19, namely that macroeconomic time series are assumed to be generated by stochastic difference equations with the dominating unit-root of +1. The first measure we define is the difference between a counterfactual solution of the model, which is like a forecast, and a baseline solution of the model. The baseline can include the estimated impact of Covid-19 in the form of impulse indicator variables. The second measure is the difference between the counterfactual solution and the actual, hence it is like a forecast error from a dynamic macroeconometric model.

In section 3, the two measures are used in an assessment of the effects of Covid-19 on Norwegian GDP. Two categories of models are used. First, empirical final form equations which are empirical counterparts to the theoretical final form equations from section 2. They have been purpose-built through the use of a machine learning algorithm. Hence, the final form equations are transparent and are simple to use in practice. The second model is a dynamic multiple-equation explanatory model called Norwegian Aggregate Model, NAM (https:normetrics.no/nam/). NAM is efficient to use to study the wider impact of Covid-19 on the economy, as there is no need to specify separate final form model equations for each variable of interest. Since the model includes the main national account identities, internal logical consistency is also secured, for example between the impacts on aggregate demand and on aggregate supply. The evaluation period in section 3 is the nine quarters from 2020(1) to 2022(1). We find empirically the same qualitative impact of Covid-19 and the responses to it, as in the illustrative model. Negative Covid-19 shocks early in the pandemic resulted in relatively persistent differences between counterfactual and baseline solutions paths. This is true for both the final equation model and for NAM, and for both types of measures of Covid-19 impact.

Quantitatively, the impact was largest in the second quarter of 2020. Two years later, the simulated effect had become much smaller in magnitude. The reduction in the gap between counterfactual and baseline can partly be due to counteracting shocks later in the simulation period. The catch-up variables can be rationalized by fiscal policies that were introduced, and by a temporary return to "almost business as usual" in the periods when non-pharmaceutical measures were lifted.

In addition to the comparison of the results for the two models used in this offering, section 3 compares the findings with the assessment made in work commissioned by the government's Coronavirus commission.[1] In sum, we find that the loss in income generation due to the impact of Covid-19 has been substantial in Norway and that it may have been larger than reported in the studies made for the government's commission. The final analysis of section 3 addresses the wider impact of Covid-19 on the components of the general budget. In that analysis, a simulation of NAM is used.

In section 4 the conclusions are given. It also contains a discussion of areas for further research related to the use of model-based counterfactuals in the analysis of big shocks to the economy. One question that can be addressed empirically as more data becomes available, is the relative invariance of the parameters in the model that determines how the impact of Covid-19 becomes propagated into medium-term effects. Another area of research, which is complementary, is the development of models with non-linear propagation mechanisms.

2. Model-based counterfactual analysis

In order to define the counterfactual, notation for a dynamic macro model is given at the start of the section. Thereafter, difference-from-counterfactual measures of the impact of Covid-19 are defined

[1] https://www.regjeringen.no/en/dokumenter/nou-2022-5/id2910055/.

and they are illustrated with the aid of a model with a closed-form algebraic solution.

2.1. *Models of the macroeconomy*

A dynamic model can be expressed compactly as:

$$y_t = f_y(y_{t-1}, ..., y_{t-p}, x_t, ..., x_{t-p}, D_{yt}, \varepsilon_{yt}), \text{ where } f_y(\cdot) \text{ denotes a function.} \tag{2.1}$$

y_t denotes a vector with n endogenous variables in period t while x_t has the m exogenous variables as elements.

D_{yt} represents deterministic terms which may include constants, trends, seasonal dummies and indicator variables for interventions or shocks. ε_{yt} represents random error-terms that are unpredictable by conditioning on the other arguments in the function.[2]

If the model's solution is to mirror reality in a reasonable degree and generate many of the properties of the data, the model equations must capture the normal economic behavior of firms and households (and rule-based policy responses), cf. Granger (1992), Visco (2005), Spanos (2021).

An operational definition of a large shock is that it can be found as a significant impulse indicator variable, by the use of statistical tests and conventional significance levels. In practice this means that the shock can be "picked up" by a (zero-one) indicator variable which is an element in D_{yt}. A shock can be large in this meaning of the term without *necessarily* leading to further structural changes in the equations of the model which describes normal economic behavior, although that can clearly happen as well. Investigation of the degree of invariance of the deeper parameters of the model is of great importance for the continued relevance of any operational model after a large shock has hit the economy. Meanwhile, methods like impulse indicator saturation (IIS) estimation can give robust estimation with respect to shocks within a sample period, Johansen and Nielsen (2009).

A large shock can affect the data generation of all the economic variables of the model, not just the endogenous ones. However, the model can be completed by a module that endogenizes the variables in the x_t vector:

$$x_t = f_x(x_{t-1}, ..., x_{t-p}, D_{xt}, \varepsilon_{xt}). \tag{2.2}$$

We refer to Equations (2.1)–(2.2) as the extended model. Below, in section 3.2, a practical example is given when an extended version of NAM is used to analyse the macroeconomic impact of Covid-19.

The extended model can be compactly written by stacking y_t and x_t in the $m + n$ vector $\boldsymbol{y}_t$, the two error-terms in $\boldsymbol{\varepsilon}_t$ and the deterministic terms in $\boldsymbol{D}_t$:

$$\boldsymbol{y}_t = f(\boldsymbol{y}_{t-1}, ..., \boldsymbol{y}_{t-p}, \boldsymbol{D}_t, \boldsymbol{\varepsilon}_t). \tag{2.3}$$

2.2. *Difference-from-counterfactual*

The extended model is a system of difference equations, with general functional form. The solution of $\boldsymbol{y}_t$ that we use in the following is a function of initial conditions $\boldsymbol{y}_0$, $\boldsymbol{y}_{-1}$,....,$\boldsymbol{y}_{-p+1}$, errors $\boldsymbol{\varepsilon}_t$,...,$\boldsymbol{\varepsilon}_1$ and deterministic terms $\boldsymbol{D}_t$,...,$\boldsymbol{D}_1$. It is custom to refer to it as a causal solution, as apart from the "forward" solutions that does not condition on initial conditions, see eg., Nymoen (2019, Ch. 3).

We let $T_0 - 1$ denote the end of the pre-pandemic sample. The Covid-19 sample is therefore: $t = T_0, T_0 + 1, ..., T_0 + H$ where H is the evaluation horizon.

[2]Equal lag length (p) for the two variables is to save notation, it is without loss of generality.

The ordered sequence of random variables $\boldsymbol{y}_t; t = T_0, T_0 + 1, ..., T_0 + H$ generated by the model is in particular function of the sequence of deterministic terms $\boldsymbol{D}_{T_0+h}, h = 0, 1, ..., H$.

Let $\mathcal{I}_D$ denote the information set that the solution is based on. For ease of exposition, we distinguish between two main cases:

1. All the dummies in $\mathcal{I}$ are zero in all time periods, denoted by $\mathcal{I}_{D=0}$.
2. At least one dummy is set to 1 in at least one period, denoted by $\mathcal{I}_{D=1}$.

In the following, we refer to a solution based on $\mathcal{I}_{D=1}$ as a baseline solution and denote it by $\boldsymbol{y}_t^b$. A solution based on $\mathcal{I}_{D=0}$ is a counterfactual solution, denoted by $\boldsymbol{y}_t^c$.

The impact and dynamic effects of a large shock to the system can be estimated as the difference between the conditional expectations of the two solutions:

$$\text{Diff}_I \boldsymbol{y}_t = E(\boldsymbol{y}_t^c \mid \mathcal{I}_{D=0}) - E(\boldsymbol{y}_t^b \mid \mathcal{I}_{D=1}); t = T_0, T_0 + 1, ..., T_0 + H. \tag{2.4}$$

For ease of exposition, and with little loss of generality (cf. section 2.3), we can assume a closed, univariate, system ($m + n = 1$).

Under the assumption that the other parameters of the model are invariant to the impact of corona virus in period T_0, $\text{Diff}_I \boldsymbol{y}_{T_0}$ is simply:

$$\text{Diff}_I \boldsymbol{y}_{T_0} = -\gamma_{1,0}, \tag{2.5}$$

where $\gamma_{1,0}$ denotes the coefficient of the indicator variable D_{T_0}. Hence, testing the null hypothesis H_0: $\text{Diff}_I \boldsymbol{y}_{T_0} = 0$ is equivalent to testing H_0: $\gamma_{1,0} = 0$.

Since the model is dynamic, the impact will be propagated, as indicated by:

$$\text{Diff}_I \boldsymbol{y}_{T_0+h} = -\gamma_{1,h}, h = 1, 2, ..., H. \tag{2.6}$$

If the process is stationary, $\gamma_{1,H} \to 0$ as $H \to \infty$. However, in the typical case of a low-frequency unit-root, the impact becomes a lasting effect, ie., $\gamma_{1,H} \neq 0$ for all H.

Realistically, there may be impacts later in the pandemic period. For simplicity, consider a second impact at $T_0 + 1$, in which case Equation (2.6) is replaced by:

$$\text{Diff}_I \boldsymbol{y}_{T_0+h} = -\gamma_{1,h} - \gamma_{2,h}, h = 1, 2, ..., H, \tag{2.7}$$

where $\gamma_{2,0}$ is the coefficient of the indicator variable D_{T_0+1}. Again, it is only in the stationary case that it is implied that $\text{Diff}_I \boldsymbol{y}_{T_0+h} \to 0$ as $H \to \infty$. Stationarity put to one side, it appears that "anything can happen", even in this stylized example: The second impact can either strengthen or counteract the effect of the first one. Hence, already for this stylized case, it appears that the question could only be answered empirically, using a combined strategy of estimation of impacts, and simulation of propagated effects.

The second measure we consider is the difference between the counterfactual and the actual $\boldsymbol{y}_t$:

$$\text{Diff}_{II} \boldsymbol{y}_t = E(\boldsymbol{y}_t^c \mid \mathcal{I}_{D=0}) - \boldsymbol{y}_t; t = T_0, T_0 + 1, ..., T_0 + H, \tag{2.8}$$

which is similar to the estimator proposed by Pesaran and Smith (2016) to analyze the effects of economic policy changes.

Formally, Equation (2.8) is like a time series of forecast errors. The significance of the impact of Covid-19 can therefore be tested by using statistical tests of forecast failure. Hence, a t-value of the forecast error for period $T_0 + h$, which is statistically significant, can be interpreted as evidence of a significant impact.

However, care must be taken: If, during the evaluation period, the economy changes in other ways than those that are captured by a indicator variable set, a significant forecast error can be due to those other changes, at least in part. Note that this is somewhat different from the assumption made in connection with Diff_I, which was about invariance of the coefficients in the *model* of the economy. In order to have a clear cut interpretation of Diff_{II} the invariance assumption applies to the economy itself, rather than to "just" our model of the economy.

The relationship between the two impact measures can be expressed as:

$$\text{Diff}_{II}\boldsymbol{y}_t = \text{Diff}_I\boldsymbol{y}_t + \boldsymbol{e}_t^b; t = T_0, T_0 + 1, ..., T_0 + H, \tag{2.9}$$

where $\boldsymbol{e}_t^b$ denotes the forecast errors associated with the baseline solution:

$$\boldsymbol{e}_t^b = E(\boldsymbol{y}_t^b \mid \mathcal{I}_{D=0}) - \boldsymbol{y}_t; t = T_0, T_0 + 1, ..., T_0 + H. \tag{2.10}$$

In general, the two measures will not be equal, due to any non-zero forecast errors associated with the baseline simulation.

2.3. An algebraic example

As an illustration of the properties of the $\text{Diff}_I\boldsymbol{y}_t$ function we use an algebraically tractable model with two variables, X_t and Y_t, and with first order dynamics.

In the case where one of the variables is exogenous, say X_t, we speak of an open system. In that case we can think of $f(\cdot)$ in Equation (2.3) as a function that incorporates the exogeneity restrictions implied by Equations (2.1) and (2.2). In the closed system interpretation, $f(\cdot)$ in Equation (2.3) is a function that allows the mutual temporal dependencies between the two variables.

As noted, macroeconometric models have over time become adapted to be consistent with the idea that trend non-stationarity is a typical feature of many macroeconomic time series. Hence, we specify the example model with variables that are integrated of order one. In a common notation this assumption is written as $X_t \sim I(1)$, $\Delta X_t \sim I(0)$, and the same applies for Y_t. A special case of Equation (2.3) which is consistent with this is:

$$\Delta Y_t = \tilde{c}_{10} + \tilde{c}_{11}\Delta X_t + \tilde{c}_{1d}D_t + \tilde{\alpha}_{11}(Y_{t-1} + \beta_{12}X_{t-1}) + \tilde{\varepsilon}_{1t}, \tag{2.11}$$

$$\Delta X_t = c_{20} + c_{2d}D_t + \alpha_{21}(Y_{t-1} + \beta_{12}X_{t-1}) + \varepsilon_{2t}, \tag{2.12}$$

where β_{12} denotes the cointegration parameter. In one interpretation, Equation (2.11) is a conditional model equation and Equation (2.12) is a marginal model equation, and (by valid conditioning) the error-terms are uncorrelated. Another interpretation is that the model is a semi-reduced form with Equation (2.11) as a structural equation in a simultaneous equation model (SEM), while Equation (2.12) is the reduced form equation for X_t from that SEM (in which case the error terms are correlated).

The reduced form (or VAR) is obtained by eliminating ΔX_t from Equation (2.11). For completeness, we can write the reduced form as:

$$Y_t = Y_{t-1} + c_{10} + c_{1d}D_t + \alpha_{11}(Y_{t-1} + \beta_{12}X_{t-1}) + \varepsilon_{1t}, \tag{2.13}$$

$$X_t \quad = \quad X_{t-1} + c_{20} + c_{2d}D_t + \alpha_{21}(Y_{t-1} + \beta_{12}X_{t-1}) + \varepsilon_{2t}, \tag{2.14}$$

where it is understood that $c_{10} = \tilde{c}_{10} + \tilde{c}_{11}c_{20}$, and similarly for c_{1d}, α_{11} and ε_{1t}, as result of the substitution.

2.3.1. The closed system final equations

Assume that both equations of the model are of the equilibrium correction type, hence $\alpha_{11} < 0$ and $\alpha_{21} > 0$. The properties of the solutions for X_t and Y_t can be studied through the final form equations, Wallis (1977). As a consequence of cointegration, the two final form equations for ΔY_t and ΔX_t become:

$$\Delta Y_t \quad = \quad \gamma_{10} + \lambda_2 \Delta Y_{t-1} + c_{1d}D_t + [\alpha_{11}\beta_{12}c_{2d} - (\alpha_{21}\beta_{12} + 1)c_{1d})]D_{t-1} + \epsilon_{1t}, \tag{2.15}$$

$$\Delta X_t \quad = \quad \gamma_{20} + \lambda_2 \Delta X_{t-1} + c_{2d}D_t + [\alpha_{21}c_{2d} - (1 + \alpha_{11})c_{2d}]D_{t-1} + \epsilon_{2t}, \tag{2.16}$$

where λ_2 is the second of two characteristic roots. The first root is $\lambda_1 = 1$, while λ_2 is given by:

$$\lambda_2 = 1 + \alpha_{11} + \alpha_{21}\beta_{12}, \tag{2.17}$$

where $\alpha_{11} + \alpha_{22}\beta_{12} < 0$ for consistency with the assumed stationarity of ΔX_t and ΔY_t.

The error terms of Equation (2.15) and (2.16) are linear combinations of the two reduced form error-terms ε_{1t} and ε_{2t}, and the first lag of those two variables.[3]

Let D_t denote a single impulse indicator. Hence, D_t takes the value 1 in a period when a shock hits the economic system, and zero in all other time periods. As the equations show, D_t and D_{t-1} shift the two constant terms in the final form equations. Therefore, a shock will have permanent effects on the solutions of the two level variables given by the identities $Y_t = \Delta Y_t + Y_{t-1}$ and $X_t = \Delta X_t + X_{t-1}$.

In line with the definitions above, and if the impact period is set to $t = T_0$, the counterfactual solution is the conditional expectation: $E(Y^c_{T_0+h} \mid \mathcal{I}_{D=0})$, and the baseline solution is $E(Y^b_{T_0+h} \mid \mathcal{I}_{D=1})$, hence $\text{Diff}_I Y_{T_0+h} = E(Y^c_{T_0+h} \mid \mathcal{I}_{D=0}) - E(Y^b_{T_0+h} \mid \mathcal{I}_{D=1})$.

We let $D_t = 0$ in all time periods of the counterfactual solution, while in the baseline solution:

$$D_t = \begin{cases} 1 & \text{, if } T_0 \\ 0 & \text{, for all other } t. \end{cases} \tag{2.18}$$

The difference between the counterfactual and basis for the change ΔY_{T_0+h} is denoted $\text{Diff}_I \Delta Y_{T_0+h}$. It is a stationary first order process, hence:

$$\text{Diff}_I \Delta Y_{T_0} = -c_{1d}, \tag{2.19}$$

$$\text{Diff}_I \Delta Y_{T_0+1} = -\alpha_{11}c_{1d} - \alpha_{11}\beta_{12}c_{2d}, \tag{2.20}$$

$$\text{Diff}_I \Delta Y_{T_0+1+h} = \lambda_2^h \text{Diff}_I \Delta Y_{T_0+1}, h = 1, 2, ...H, \mid \lambda_2 \mid < 1 \tag{2.21}$$

while $\text{Diff}_I Y_{T_0+h}$ follows the updating formula:

$$\text{Diff}_I Y_{T_0+h} = \text{Diff}_I \Delta Y_{T_0+h} + \text{Diff}_I Y_{T_0-1+h}, \; h = 0, 1, 2, ..., H, \tag{2.22}$$

[3] $\epsilon_{1t} = \varepsilon_{1t} - \varepsilon_{1t-1}(1 + \alpha_{11}\beta_{12}) + \varepsilon_{2t-1}\alpha_{11}\beta_{21}$ and $\epsilon_{2t} = \varepsilon_{2t} - \varepsilon_{2t-1}(1 + \alpha_{11}) + \varepsilon_{1t-1}\alpha_{21}$.

with the remark that $\text{Diff}_I Y_{T_0-1+h} = 0$ initially ($h = 0$).

The values of function $\text{Diff}_I Y_{t_0+h}$ are non-zero for all h, even if the impact lasted for only one period. This is an implication of the unit-root of +1. As noted above, it is a typical feature of integrated series.

However, although both Y and X are permanently affected by the impact of a single one-period shock, the long-run relationship between the variables is not disrupted. Let Z_t denote the disequilibrium variable $Z_t = Y_t + \beta_{12} X_t$ which is I(0). $\text{Diff}_I Z_{T_0}$ and $\text{Diff}_I Z_{T_0+1}$ will be directly influenced by D_{T_0} and D_{T_0+1}, but $\text{Diff}_I Z_{T_0+2}$ and later differences are given by:

$$\text{Diff}_I Z_{T_0+1+h} = \lambda_2^h \text{Diff}_I \Delta Z_{T_0+1},\ h = 1, 2, ...H, \mid \lambda_2 \mid < 1,$$

which is another consequence of the correspondence between the counterfactual and a forecast: Cointegration between variables is preserved in forecasts, Engle and Yoo (1987).

Under the assumption that there are no parameter changes other than the shift in the intercept captured by D_t, the second measure $\text{Diff}_{II} Y_{T_0+h}$ can be expressed as:

$$\text{Diff}_{II} Y_{T_0+h} = \text{Diff}_I Y_{T_0+h} - \sum_{j=0}^{h} \xi_j \epsilon_{1T_0+j},\ h = 0, 1, 2, .. \tag{2.23}$$

which is a special case of (2.9), with $\boldsymbol{e}^b_{T_0+h} = -\sum_{j=0}^{h} \xi_j \epsilon_{1T_0+j}$.

In Equation (2.23), ξ_j (h=0,1,...) does not represent a well-behaved linear filter, Nymoen (2019, p 167). Therefore, the stationarity of the error-term process ϵ_{1T_0+j}, $j = 0, 1, ..., h$ is not preserved in the second term of (2.23). Hence, the conditional variance $Var(\text{Diff}_{II} Y_{T_0+h} \mid Y_{T_0})$ is strictly increasing in h.

2.3.2. The open system

In the case of $\alpha_{21} = 0$, X_t is an exogenous variable. Under this restriction on Equations (2.11)–(2.12), the model equation for Y_t, ie., (2.11) is an example of a macro-econometric model of the open type, and the X_t-equation takes the role of the completing equation in the extended model.

The open-system version of (2.13)-(2.14) is:

$$Y_t = c_{10} + Y_{t-1} + c_{1d} D_t + \tilde{\alpha}_{11}(Y_{t-1} + \beta_{12} X_{t-1}) + \varepsilon_{1t}, \tag{2.24}$$

$$X_t = c_{20} + X_{t-1} + c_{2d} D_t + \varepsilon_{2t}, \tag{2.25}$$

with the remark that the equilibrium correction coefficient in the final form equation for Y_t is the same as in the conditional model equation. In this case, the solution for X_t can be found first and can be taken as given in the solution for Y_t.

The qualitative effects of single period shocks are the same as for the closed system. However, if we more generally include separate impulse indicators in Equation (2.24) and (2.25), say D_{Yt} and D_{Xt}, it is only D_{Xt} that affects the level of both X_t and Y_t. If the shock is "limited to" Y_t so that it is captured by D_{Yt}, the solution for the level of Y_t will not be permanently affected, because the level of Y_t is linked to the level of X_t in this case.

2.3.3. Generalization

If the macroeconomic model has several endogenous variables and higher order dynamics, the implied final forms equations are completely general ARMA model equations. However, in practice, such ARMA processes are approximated by high order AR processes, $Y_t \sim AR(p)$.

Hence, more generally we can think of the final equation of Y_t as an $AR(p)$ model equation. Under the assumption that the *largest* root of the associated characteristic equation is +1, it follows that $Y_t \sim I(1)$ and ΔY_t is $AR(p-1)$ with $p-1$ characteristic roots that are less than one in magnitude. Hence $\Delta Y_t \sim I(0)$.

In the illustrative model there was one stable root and one unit root. The generality stems from the famous result known as the *typical spectral shape* of economic time series noted above, Granger (1966), Granger and Newbold (1986, Ch. 2.7). The theorem states that if there is a large number of stable roots, a single root equal to +1 at the zero-frequency implies that the time series properties become dominated by a random-walk component, Nymoen (2019, Ch. 9.3). Hence, because the random-walk component is a common feature of both simple and complex models, the results obtained for Diff_I and Diff_{II} may be more general than first thought. It can be useful to have them in mind when interpreting the simulation results from empirical models consisting of multiple-equations with higher order dynamics.

3. Impact of Covid-19 using models of the Norwegian economy

We now turn to the results of the impact of Covid-19 on the macroeconomic system of Norway. We focus on GDP Mainland-Norway, which is the income variable followed most closely in the discussion of fiscal policy and the interest rate path decided by Norges Bank [Central Bank of Norway].

In section 3.1, a single equation approach is used to estimate and simulate an empirical final form equation for (log of) GDP Mainland-Norway.

In section 3.2 the results from using NAM are presented. NAM is a complete model of the economy. Each of the behavioral equations are investigated for significant impulse indicator variables in the period 2020(1) – 2022(1).

Using the complete model is efficient as it gives relevant information about where in the system the impact first came, and how the effects were propagated by the dynamics of the multivariate system. The multiple-equation model is also suited to study the wider impact of Covid-19 on the economy, as demonstrated in section 3.4.

On the other hand, the results of a larger explanatory model are conditional on the decisions that have been made in the construction of the model. The single equation approach has the advantage of being transparent and easy to replicate and update. In addition, since the counterfactual is a forecast, using a parsimonious single equation model of for example GDP may produce more reliable results than a larger model which may be misspecified in ways that bias the forecast, Pesaran and Smith (2016).

3.1. Results from empirical final form equations

As noted above, a system of I(1)-variables implies final form equations for each variable in differenced form. Equations (2.15) and (2.16) are particular special cases. All final form equations obtained from a closed system have identical autoregressive structures (the homogeneous parts of the difference equations are identical). But they also contain moving average errors, which are not the same for all variables, and impulse indicators will in general also enter differently in the equations (ie., they belong to the inhomogenous parts of the difference equations).

In general, each final form equation is therefore an ARMA(p,q) model, augmented by impulse indicators. However, as noted, an ARMA model can be approximated by an AR with $p' > p$. The machine learning algorithm Autometrics was used to decide the lag-specification and which indicator variables to include, Doornik (2009), Hendry and Doornik (2014), Doornik and Hendry (2018a).

Autometrics is a computer algorithm that automates variable selection by starting from a general unrestricted model (GUM) with the aim of discovering the local data generating process (LDGP). The methodological foundation of Autometrics is found in general-to-specific (gets) modeling and the theory of reduction, see eg., Hendry et al. (1984), Hendry (1995, Ch. 9). The GUM defines the whole space of models which we can picture as a search tree with the GUM at the root. At every node in the tree is a unique model. In order to be operationally efficient, Autometrics implements several strategies to speed up the tree search, see Doornik (2009) for details. Simulation studies indicate that Autometrics in general performs well compared to other algorithms for variable selection, see Hendry and Doornik (2014, Ch. 17) (comparisons with LASSO and RETINA) and Epprect et al. (2013) and Muhammadullah et al. (2022) (comparison with LASSO).

Autometrics with IIS extends the general unrestricted model (GUM) by one indicator variable for each observation. We then create a complete set of impulse indicator variables, $\left\{1_{\{j=t\}}\right\} = 1$ when $j = t$ and 0 otherwise for $j = 1, \ldots, T$, and add T indicators and estimate a "saturated" GUM. The deadlock created by creating more parameters than observations, is elegantly resolved by the feasible split-sample IIS algorithm. In the simplest case it is to add the indicators in blocks of $T/2$, noting that all the indicators are mutually uncorrelated. The algorithm then adds half of the indicators to the GUM (*eg* the null model in the simplest case) and selects as usual, records the outcome and drops that first indicator set. Next, add the second set of $T/2$ indicators and select again. Then the retained indicators from the first two selections are combined and added to the GUM, and the selection algorithm is run again as is if we commenced with a number of indicators well below T, see Castle et al. (2012) and Hendry and Doornik (2014, Ch. 15) for details. The estimators of retained economic variables have been shown to have an interpretation as robust estimators statistically speaking, Hendry et al. (2008), Johansen and Nielsen (2009).

We analyze the impact on GDP Mainland-Norway, the term used by Statistics Norway to refer to GDP without valued added in oil and gas extraction, pipeline transportation and ocean transport.[4] As noted above, GDP Mainland-Norway is the preferred variable for analysis of economic activity and of income generation in Norway. The time series is quarterly and from the national accounts.[5]

Letting Y_t denote GDP Mainland-Norway, the relative changes in GDP are given by the differenced series $\Delta log(Y_t)$. A general unrestricted model (GUM) with twelve autoregressive terms, constant and three seasonal dummies was found to be not misspecified when evaluated by a standard test-battery.

Variable selection algorithms involve repeated testing, which can lead to inflated Type-I error probability levels. In Autometrics, the overall significance level depends on the user-set *Target size*. Conceptually, Target size is one-to-one with the rate k^{irr}/k^{GUM}, where k^{irr} denotes how many irrelevant variables are found to be acceptable on average in the final model, and k^{GUM} denotes the number of regressors in the GUM. Hence, with 15 regressors in the GUM, and accepting 0.15 irrelevant variable on average, the target level can be set to 1 %. Increasing it to 5 % implies on average 0.75 irrelevant variables in the model equation delivered by the algorithm.

[4]https://www.ssb.no/en/nasjonalregnskap-og-konjunkturer/nasjonalregnskap/statistikk/nasjonalregnskap.

[5]https://www.ssb.no/en/statbank/ltable/09190/. The unit is million NOK, constant 2019 prices. The valuation is market values, and the series is not seasonally adjusted.

The model equation needed to simulate the forecasts for the $\mathrm{Diff}_{II}Y_t$ measure was selected by using a sample that ends in 2019(4). Using Target size = 0.01, Autometrics retained $\Delta log(Y)_{t-1}$, $\Delta log(Y)_{t-2}$, $\Delta log(Y)_{t-12}$, the constant term, three seasonals and nine impulse indicators, denoted by $D_{\text{year(quarter)}}$.The model is shown in equation (3.1).

$$\begin{aligned}
\Delta log(Y)_t \;=\; & -\underset{(0.0652)}{0.608}\;\Delta log(Y)_{t-1} - \underset{(0.0651)}{0.2764}\;\Delta log(Y)_{t-2} + \underset{(0.0545)}{0.1468}\;\Delta log(Y)_{t-12} \\
& + \underset{(0.00148)}{0.00857} - \underset{(0.00701)}{0.04032}\;\mathrm{CS}_t - \underset{(0.00631)}{0.06038}\;\mathrm{CS}_{t-1} \\
& - \underset{(0.00543)}{0.06526}\;\mathrm{CS}_{t-2} + \underset{(0.0149)}{0.04329}\;D_{1984(1)} + \underset{(0.015)}{0.06846}\;D_{1985(1)} \\
& + \underset{(0.0149)}{0.05462}\;D_{1986(2)} - \underset{(0.015)}{0.0348}\;D_{1988(3)} + \underset{(0.015)}{0.04121}\;D_{1996(1)} \\
& + \underset{(0.015)}{0.05826}\;D_{1997(2)} + \underset{(0.0149)}{0.03583}\;D_{2001(1)} + \underset{(0.015)}{0.0421}\;D_{2005(2)} \\
& - \underset{(0.015)}{0.04317}\;D_{2009(1)}
\end{aligned} \tag{3.1}$$

OLS	Sample: 1981(2) - 2019(4)	Number of obs.: = 155
$\hat{\sigma}100 = 1.46$	$R^2 = 0.89$	
AR_{1-5} :	F(5,124) = 1.208[0.31]	
ARCH_{1-4} :	F(4,147) = 0.40[0.81]	
Normality :	$\chi^2(2) = 0.68[0.71]$	

On the right hand side of the equation, CS_t denotes the centered seasonal dummy variable for the first quarter of the year.[6] Estimated standard errors are in round brackets below the coefficients. Four of the indicators are from the 1980s, a decade marked by high but volatile economic growth, and which ended with a crash in the housing market, a reduction of GDP Mainland Norway in 1988, and the start of a period with high unemployment, Nymoen (2017). The two indicators from the second half of the 1990s come from the period after recovery from that crisis. The first decade of the new millennium was marked by growth, and unusually high growth in some individual quarters. However, the international financial crisis had a significant negative impact, picked by the indicator variable $D_{2009(1)}$.

R^2 and the residual percentage standard deviation ($\hat{\sigma}100$) indicate quite good fit. The residual misspecification tests AR_{1-5}, ARCH_{1-4} and Normality are reported with their respective p-values. The tests of no (autoregressive) residual autocorrelation and of ARCH are both insignificant, see Harvey (1981), Engle (1982). The same is the case for the test of departures from the assumption about normally distributed error-terms, Jarque and Bera (1980).

The $\mathrm{Diff}_I\Delta$Y measure requires that a baseline solution can be simulated for the pandemic period. When the sample was extended 2022(1), Autometrics selected two quarters, 2020(2) and 2021(4). The estimated coefficient of $D_{2020(2)}$ was −0.08 (t-value −5.7). For $D_{2021(4)}$ the estimate was +0.03 (t-value of 2.4). Equation (3.1), with the two Covid-19 impact indicators included was estimated on the full sample and used to simulate the baseline solution used in Diff_IY. As already noted above, Equation (3.1) was used to generate the forecast used to obtain Diff_{II}Y. In both models, the identity $log(Y_t) = \Delta log(Y_t) + log(Y_{t-1}$ was included to give simulated levels of (log) GDP Mainland-Norway.

[6]Each centered seasonal sum to zero over the year.

Table 1. GDP Mainland-Norway. Results using empirical final form equation.

(1) Horizon	(2) Actual	(3) C-factual	(4) Baseline	(5) Diff_IY (3)–(4)	(6) Diff_IY Percent	(7) Diff_{II}Y (3)–(2)	(8) t-value	(9) Diff_{II}Y Percent
2020(1)	762903	774610	774610	0	0	11707	1.0	1.5
2020(2)	701430	765681	706114	59567	7.8	64251	5.7	8.4
2020(3)	732876	766497	741599	24898	3.2	33621	2.6	4.4
2020(4)	794793	816347	786286	30061	3.7	21554	1.4	2.6
2021(1)	754553	788096	750397	37700	4.8	33543	2.2	4.3
2021(2)	754712	784506	753082	31424	4.0	29794	1.8	3.8
2021(3)	769877	780345	748021	32324	4.1	10468	0.6	1.3
2021(4)	835909	833809	824849	89605	1.1	−2100	−0.1	−0.3
2022(1)	798592	806328	783249	23079	2.7	7736	0.4	1.0

Notes: (2)–(5) and (7): numbers are in million 2019 kroner. (6) and (9) are percents of (3). (8) is t-value of (7).

The results of the simulations are shown in Table 1. Note that, since Autometrics-IIS did not keep the indicator for 2020(1) in the model equation, Diff_IY gives zero impact of Covid-19 in that quarter. Diff_{II}Y (column (7)) shows a negative impact, as the actual value in 2022(1) was lower than the forecasted. In percent, the estimated impact was a 1.5 reduction compared to the counterfactual without the coronavirus pandemic.

't-values" for Diff_{II}Y can be used to test the null hypothesis of zero difference between counterfactual and actual in a given quarter. In column (8), simulated forecast error standard deviations were used for comparability with the result in the next section, where simulation was the only feasible method. For the single equation dynamic forecasts errors in Table 1 exact variance formulas exist, see Clements and Hendry (1998), and for practical purposes they could be treated as equal to the simulated.[7]

Table 1 column (8) therefore indicates that for 2020(1), Diff_{II}Y is not significantly different from zero. However, for 2020(2) the t-value in column (8) is 5.7, which is evidence of a significant impact statistically speaking. The numerical impact, corresponding to 8.4 percent, is also the largest seen for Diff_{II}Y over the period as a whole. The same can be said of Diff_IY, as the huge estimated impact of 7.8 percent is due to Autometrics-IIS keeping the indicator variable for 2020(2).

For 2020(3) and 2021(1) the t-values are also larger than 2, indicating significant impact of Covid-19 and the responses to it. As the forecast error variances are increasing with the horizon, a given numerical difference between the counterfactual and the actual will be more significant early in the period than it will be in a later quarter of the pandemic.

When we compare the numbers in column (6) and (9) we see that the Diff_{II}Y estimates are largest early in the pandemic. From 2020(4) and onward, it is Diff_IY that gives the higher estimates of Covid-19 impact. The reduction in the simulated impact in 2021(4) is due to the positive indicator variable for that quarter.

[7]The simulated standard errors were obtained by using Eviews 12 and were checked against the analytical versions by using Oxmetrics 8.0-PcGive 15, Doornik and Hendry (2018b).

3.2. Impact of Covid-19 using the multiple-equation model NAM

NAM is a multiple-equation dynamic model, but it has in common with the simple models above that the solutions of the endogenous variables are in principle given by final form equations and their associated characteristic roots. Another common feature is that cointegration has been an important modelling concept, hence shocks and short-lived impulses integrate into shifts in level of variables, without necessarily disrupting relationships between those variables.

NAM originated from the econometric assessment of wage-and price formation in Nymoen(1989a, 1989b, 1991), further developed in Bårdsen et al. (1998), Bårdsen and Fisher (1999), Bårdsen and Nymoen (2003), and the monetary transmission model of Bårdsen and Klovland (2000). An early version of the model was presented in Bårdsen et al. (2003), while a more complete version was documented in Bårdsen and Nymoen (2009). The methodological orientation of the model was also represented by the book on macroeconometric modeling by Bårdsen et al. (2005).

Theoretically, and true to its origin, NAM is an incomplete competition model. The assumption of monopolistically competitive firms is combined with a model of wage formation that brings in the collective aspect of the national system of wage formation in Norway. Taken together, these modules imply a determination of the nominal path of the economy which is distinctly different from the implications of natural rate models that make use of price and wage Phillips curves, Kolsrud and Nymoen (2014), Nymoen (2021).

NAM is an operational empirical econometric model. Regular updates of the model is synchronized with the releases of the Quarterly National Accounts.[8] The documentation is on the internet, Bårdsen and Nymoen (2022).[9]

Examples of variables that were exogenous in the version of the model that we took as our starting point were: An export market indicator, a foreign short-term interest rate, and other variables from financial and product markets abroad (ie., the foreign sector of the model). As the pandemic was global, several of these non-modelled variables were likely to be affected by Covid-19 impacts. Hence, in line with the argument above, the standard model was extended with empirical equations for this category of variables.

In more detail, the extended model included equations for thirteen variables. *Foreign sector*: Export market indicator, oil price, foreign producer price index, foreign money market interest rate, euro area consumer price index, European bank sector CDS index, US stock market volatility index. *Domestic sector*: Policy interest rate, exports of oil and natural gas, public consumption expenditure, gross fixed capital formation in the general government, number of persons on active labor market programs, NPISH disposable income.

We used the extended version of the model in the simulations that we report the results of below. The second category of model change was to add to each econometric equation in the standard version, the set of indicator variables for the nine "Covid-19 quarters" from 2020(1) to 2022(1) Dummies were retained in the model if they achieved t-values that were significant at the 5 % level.

[8] NAM users include Financial Supervisory Authority of Norway, NHO (Confederation of Norwegian Enterprise) and LO (Norwegian Confederation of Trade Unions).

[9] https://www.normetrics.no/nam.

Table 2. Number of NAM equations where Covid-19 impulse indicators are included

Quarter	Impulse Indicator	Model version	
		Standard (120 eqs)	Extended (133 eqs)
2020(1)	$D_{\text{Covid},t}$	12	23
2020(2)	$D_{\text{Covid},t-1}$	26	38
2020(3)	$D_{\text{Covid},t-2}$	15	26
2020(4)	$D_{\text{Covid},t-3}$	13	23
2021(1)	$D_{\text{Covid},t-4}$	11	21
2021(2)	$D_{\text{Covid},t-5}$	12	20
2021(3)	$D_{\text{Covid},t-6}$	11	28
2021(4)	$D_{\text{Covid},t-7}$	7	9
2022(1)	$D_{\text{Covid},t-8}$	9	12

Table 2 shows that the indicator variable for 2020(2) was included most frequently, in 38 of the equations. The third quarter of 2020 had the second-highest number of inclusions. We note also that the indicator variable for 2020q1 was included in as many as 23 equations. The automatic variable selection used above did not include the first Covid-19 quarter, hence it is a factor that will contribute to difference between the results of the two analyses.

Table 2 also shows that the indicators from 2020(4) to 2021(3) have 20 or more entries. There is a marked drop in the inclusion numbers for 2021(4) and 2022(1). There may be caveats about the choice of including 2022(1) in the analysis. In Norway, there were some non-pharmaceutical measures still in place at the start of the year. However, the society opened up in February 2022 when the Norwegian government concluded that the damaging effects of the measures had become more serious than the effects of the spread of the virus in the population. Later in February, Russia invaded Ukraine and several global markets, energy markets, in particular, were immediately affected by the war. Hence, in 2022(1) the economy was influenced by two shocks, and care must therefore be taken when interpreting the results for that quarter in particular.

It can also be noted that several of the affected equations in the Extended-column represent the foreign sector, indicating that a substantial part of the total effects may come from international trade and financial markets.

Table 3 shows the results in the same format as for the final equation model above. One notable difference from Table 1 is that, when NAM is used, the Diff_IY measure indicates that Covid-19 had an impact on GDP already in 2020(1). This is the joint impact of the 23 indicators mentioned above.

Closer inspection shows that one of the relationships with an early impact was the equation for valued added in service activities, which includes retail trade, accommodation and food service activities. Although the first full lockdown occurred quite late in the first quarter of 2020, on 12 March, it is not unreasonable that the value added in service activities became reduced compared with a scenario without Covid-19. For example, because of the voluntary social distancing phenomenon.

Other equations with a 2020(1) dummy include the consumption function and the equation for the growth in Norwegian export markets. In NAM, both of these impacts have indirect effects on value added, not only in retail and private service production, but also in other industries, like manufacturing.

Both Table 1 and Table 3 indicate that the largest impact of Covid-19 came in 2020(2). The estimates are a little higher in Table 3 and that remark also applies for the later quarters in the period. In

Table 3. GDP Mainland-Norway. Results using NAM.

(1) Horizon	(2) Actual	(3) C-factual	(4) Baseline	(5) Diff_IY (3)-(4)	(6) Diff_IY Percent	(7) Diff_{II}Y (3)-(2)	(8) t-value	(9) Diff_{II}Y Percent
2020(1)	762903	788467	768423	20043	2.5	25564	3.2	3.2
2020(2)	701430	765312	698992	66320	8.7	63882	7.0	8.3
2020(3)	732876	774473	736370	38103	4.9	41597	4.2	5.4
2020(4)	794793	825513	793295	32217	3.9	30720	2.7	3.7
2021(1)	754553	807255	747218	60038	7.4	52702	4.0	6.5
2021(2)	754712	799268	739736	59532	7.4	44556	3.2	5.6
2021(3)	769877	800020	752769	47251	5.9	30143	2.0	3.8
2021(4)	835909	852804	809529	43278	5.0	16895	1.0	2.0
2022(1)	798592	818998	770505	48493	5.9	20406	1.1	2.5

Notes: (2)–(5) and (7): numbers are in million 2019 kroner. (6) and (9) are percents of (3). (8) is t-value of (7).

particular, the Diff_IY measure in the NAM table indicates the largest and most persistent effects of the impact and responses to Covid-19. That said, as the standard deviation of the differences are increasing with the horizon, the t-values of Diff_IY become insignificant towards the end of the evaluation period. It can be taken as a reminder of the large uncertainties involved in this type of assessment.

3.3. Comparison with other studies

There are existing studies of national and regional Covid-19 effects in Norway, written as reports for the Norwegian corona commission, cf. Bjertnæs et al. (2021) and von Brasch et al. (2022).

The two reports for the commission used a forecast prepared by Statistics Norway late in 2019 as the counterfactual, while the baselines were constructed by a combination of available national account data with forecasts for the remaining quarters of the period (ie., until 2023(4)). Hence, the numerical assessment of the impact of Covid-19 in the two corona commission reports appear to have combined $\mathrm{Diff}_I \boldsymbol{y}_t$ and $\mathrm{Diff}_{II} \boldsymbol{y}_t$ type measures in order to be able to give results for the whole period from 2020 to 2023.

For comparison with the corona commission, annual percentage numbers are given in Table 4. For 2020, the results are quite similar. The average of the four estimates in our study is 4.5 percent, against 4.6 percent in the 2022 version of the commissioned estimate. Bjertnæs et al. (2021) and von Brasch et al. (2022). Table 4 shows that after 2020, the simulated losses in income generation using NAM are systematically larger than in the other assessments. The percentage numbers in von Brasch et al. (2022) for 2021,2022 and 2023 give the smallest loss in income generation. For the 2020-2023 period as a whole, von Brasch et al. (2022) sets the discounted reduction in GDP Mainland-Norway to 270 billion kroner in fixed 2019-prices. When the same discounting factor (of 4%) is applied to the NAM results, the reductions amount to 578 billion. Although the calculations must be interpreted with care because of the many uncertainties, they indicate large income losses over the period of the pandemic, 9 percent (von Brasch et al. (2022)) and 18 percent of the 2019 Mainland-Norway GDP (NAM). Large

Table 4. GDP Mainland-Norway. Difference between counterfactual (no Covid-19) and baseline/actual in percent of counterfactual.

	2020	2021	2022	2023
Diff_IY				
Final equation	3.7	3.5	2.6	2.8
NAM	5.0	6.4	4.8	3.1
Diff_{II}Y				
Final equation	4.2	2.5		
NAM	5.1	4.4		
Corona commision:				
Bjertnæs et al. (2021)	4.7	3.8	2.2	0.5
von Brasch et al. (2022)	4.6	2.4	2.1	−0.9

as they are in a Norwegian context, such a number shrink in comparison with the results for the US economy reported in Rio-Chanona et al. (2020). Based on inter alia detailed input-output tables, they estimated that compared to 2019 the shocks in the first year of the pandemic alone would threaten around 20 percent of the US economy's GDP.

3.4. The broader impact of Covid-19

A multiple-equation model like NAM is efficient to use in the analysis of the wider impact of Covid-19 and the responses to it, as the endogenous variables of the model cover several sectors and markets in the economy.

As one example, Table 5 shows effects in percent of the no-covid solutions for GDP (total), GDP Mainland-Norway and four sectors of Mainland Norway. In order to get a broader picture of how the effects evolve with the horizon, the simulation was extended through 2023, and we give the annual numbers in this table.

For GDP total, and GDP Mainland-Norway, the reductions are monotonous after 2021. The smaller estimates for GDP total follow directly from finding empirically that the petroleum industry was practically unaffected by the pandemic. The estimate reported in Rungcharoenkitkul (2021), of a 3 percent reduction in Norwegian GDP in 2020, is close to the estimate in Table 5.

The rows with the numbers for value added in main industries, show that Covid-19 and the responses to it had a wide impact. The numbers for value added in service activities (which includes eg., travel and accommodation as mentioned above) are the largest. However, the impacts on manufacturing and other production (which includes construction) were also estimated to be of numerical significance. The large reduction in value added in service activities, compared to 2019 (so not a counterfactual) is also found for Denmark and Sweden, Blytt et al. (2022). According to the results in Table 5, the government sector was the least affected by Covid-19.

The line for imports in table 5 shows a huge impact. In the model, imports are driven by components of aggregate demand and by the real exchange rate. Hence it is not surprising to find several huge estimated impacts of Covid-19 for the "demand components" in the bottom half of the table. Public consumption is an exception, and more interestingly, also capital formation in residential housing. Housing prices grew in real terms during the pandemic, which in the model is one of the main

Table 5. Difference between counterfactual (no Covid-19) and baseline/actual in percent of counterfactual. $\mathrm{Diff}_I y_t$ results using NAM.

	2020	2021	2022	2023
GDP	3.4	4.3	2.6	1.1
GDP Mainland-Norway	5.0	6.4	4.8	3.1
Value added, manufacturing	6.1	4.9	2.7	1.5
Value added, other products	3.4	5.1	3.9	2.5
Value added, service activities	8.1	9.7	7.5	5.0
Value added, general government	1.1	0.6	0.1	−.3
Imports	13.4	14.8	8.7	4.8
Exports	4.2	2.6	−.5	−0.8
- Mainland-Norway	10.7	9.3	6.6	4.5
Private consumption	9.4	8.6	2.8	1.7
Public consumption	0.6	−.3	−.4	−.4
Gross capital formation	6.1	10.5	7.4	3.4
- Mainland-Norway private business	12.1	19	12	3.2
- Housing	0.3	2.1	0.5	−1.2

explanatory variables of real investments in housing.

4. Conclusions and discussion

Counterfactual analysis is required to estimate the economic impact of Covid-19. In this offering, macroeconometric models are used to simulate counterfactual developments of GDP Mainland Norway, and of a selection of other variables in the Norwegian national accounts. Two operational definitions of the difference between the counterfactual (without Covid-19) and the baseline (or actual) were defined. They were illustrated with the aid of a model which, despite its simplicity, incorporates the double feature of low-frequency unit-root and cointegration, which has also become common features of empirical models.

There is no logical inconsistency between using both "small" and "large" models to elucidate Covid-19 impacts on the economy. Dynamic econometric modeling can be used to specify a model equation which can be interpreted as an approximation to the unknown final equation of for example GDP. For an existing operational model, dynamic simulation gives the solution path of the final equation implied by the model structure.

Empirically we found, by simulating NAM ("large model") and empirical final equations ("small model"), that the differences between the counterfactual and the baseline/actual for GDP Mainland Norway were large and statistically significant in 2020 and in the first half of 2021. Towards the end of the evaluation period the differences were no longer significant. This was due to smaller numerical differences, and increased standard deviations associated with the counterfactual.

The simulated impact of Covid-19 on Mainland-GDP in our study can be said to confirm the results in studies made for the government's coronavirus commission, specifically for 2020. For 2021, 2022 and 2023 the tendency was that our simulations indicated larger impacts. As both data and models used for the counterfactual simulations were different, exact correspondence was not to be expected, and as noted, the uncertainties are large and they increase with the length of the horizon.

In addition to GDP Mainland-Norway, we reported the wider impact of Covid-19 and the response to it for a set of national accounts variables that are endogenous in the model used (NAM). This demonstrates the efficiency of using a multiple-equation empirical model of the macroeconomy, if it is available and operational.

As the counterfactual is like a dynamic forecast, many of the challenges to interpretation and validation are the same for the two model usages. Ideally, any model which is used to measure the impact of Covid-19 should characterize the normal behavior of the economy as accurately in the pandemic (forecast) period as it did over the estimation sample. Hence, although model intercepts must necessarily change significantly to capture the impact of Covid-19, the other model parameters should be invariant with respect to this shock. This is a strong requirement and seems unlikely to be met for all behavioral equations of an empirical macroeconomic model.

However, invariance is a relative concept and a property that can hold partly if not completely. The more practical requirement may be that although the other parameters than the intercepts may not remain completely constant after the Covid-19 shock, they do not change so much that the model-based counterfactual becomes uncorrelated with the true counterfactual. It was beyond the scope of this study to do formal testing of parameter invariance, but as more data becomes available research will no doubt shed light on this important issue.

More generally, the experience from modeling the economy during the pandemic can be used in a progressive way to improve on existing models. One area for research is dynamic models with non-linear cointegration, Johansen (2004). In such models, equilibrium coefficients may change while cointegration parameters are still assumed to be invariant to the shock.

The values of the function $\mathrm{Diff}_I(\boldsymbol{y}_t)$ will be different with linear and non-linear equilibrium correction. As illustrated in the example above, the reason is that the medium-term difference between the counterfactual and the baseline depends on the adjustment coefficients, and not only on the cointegration parameters. A deeper form of structural breaks would be changes in the equilibrium relationships in the model of the economy that existed before the shock, ie., in the cointegration parameters.

However, cointegration is a rare phenomenon and can be said to represent deep structural relationships that do not break down easily. However, the possibility of breaks in cointegration coefficients deserves to be given particular attention in model maintenance. It would represent an impact of Covid-19 with a consequence for model building, maybe in the direction suggested by Vines and Wills (2020) for rebuilding macroeconomic theory with the use of models with multiple equilibria.

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Conflict of interest

The author declares no conflicts of interest in this paper.

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