Economic COVID-19 effects analysed by macro econometric models—the case of Norway*

Ragnar Nymoen[a]

[a] University of Oslo, Norway, ragnar.nymoen@econ.uio.no

Abstract

Counterfactual analysis of the impact of COVID-19 can be based on a solution of a macroeconomic model for the scenario without the corona virus interfering with the macroeconomic system. Two measures of impact are defined and put to use: (I) The difference between the counterfactual and a baseline model solution. (II) The difference between the counterfactual and the actual development of the economy. In order to analyse the impact on GDP we use two model categories. First, empirical final form model equations, which were purpose-built with the aid of a machine learning algorithm. Second, an existing multiple equation model of the Norwegian macroeconomic system. Empirically we find significant impact of COVID-19 on GDP Mainland Norway in 2020. For some of the estimator/model combinations, the impacts are also significant in the two first quarters of 2021. Using the multiple-equation model, the assessment is extended to the impact of COVID-19 on value added in four Mainland Norway industries, on imports and exports, and on final consumption expenditure and gross capital formation.

1 Introduction

There is an emerging literature on the impacts and effects of COVID-19. A survey by Padhan and Prabheesh (2021) showed that the list of topics analysed has already become broad and varied. However, it appears that the impacts on GDP and other variables in the national accounts system have not been a main focus of the academic literature. Hence, there is a gap between the awareness shown by governments, businesses and the general public during the pandemic about its consequences for income generation, and the few assessments that hitherto have emerged in the literature.

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In particular, analyses using larger empirical macroeconometric models are rare. The hitherto low profile of explanatory models of variables in the national accounts in the COVID-19 literature can be a reflection of the long lasting scepticism towards this class of models among academic economists, Bårdsen et al. (2005, Ch. 1). However, modern empirical macroeconometric models are dynamic equation systems and have common features with models that are standard in research and in teaching of dynamic econometrics, for example VARs. Such models are also of interest because they include a number of behavioural relationships from different sectors of the economy. Hence the multiple equation models can be used for efficient analysis of the wider impact of COVID-19 on the national economy.

The analysis of the impact of COVID-19 builds on the idea about a hypothetical counterfactual development without a pandemic that interfered with the macroeconomic system. A feasible counterfactual is represented by a solution of a model of the macroeconomic system obtained in practice by dynamic simulation. The impact and dynamic effects of COVID-19 can then be estimated by the differences between the counterfactual and the "baseline", which can be a solution of the model with the estimated impact of COVID-19 included or, more directly, the actual development of the economy over the pandemic period.

The concepts are made precise in section 2 by the use of a small model which has a closed form algebraic solution. What makes the illustrative model relevant is that is has an an important feature in common with the models that we will use in the empirical assessment of COVID-19. The feature we have in mind is that the trends in macroeconomic variables are typically modelled as stochastic trends. Hence, a unit-root of +1 is implied, and that root is known to be a dominant feature of the solutions of models (small and large).

As a consequence of the low-frequency unit-root, impacts of COVID-19 that are captured by impulse indicator variables become transformed to permanent shifts in the levels of endogenous variables. In the illustrative model, the implication is that any catch-up in the levels of the variables after the initial impact depend on counteracting impacts later in the pandemic period, which of course may happen. For more multiple equation econometric models, the dependency on counteracting shock for recovery after the impact is more of an empirical question, and it can in practice be studied by simulation.

The rest of the paper is organized as follows. In section 2, the measures of COVID-19 impact are defined and illustrated by the use of the theoretical model mentioned above. Two measures are defined. One is formally like a forecast error from a dynamic macroeconometric model. The other is the difference between a counterfactual solution of the model, which is like a forecast, and a baseline solution of model. The baseline include the estimated impact of COVID-19 by the use of impulse indicator variables.

In section 3, the impact on COVID-19 on Norwegian GDP is analysed by the use of the two measures defined in section 2. Two categories of models are used. First, empirical final form equations which are empirical counterparts to the final form equations derived for the illustrative model in section 2. They have been purpose-built with the use of a machine learning algorithm. Hence, the final form equations are transparent and are simple to use in practice. The second model is a dynamic multiple-equation empirical model of the Norwegian economy called Norwegian Aggregate Model, NAM.[1] The model is an all-purpose model and it is therefore subject to the critique of being a "black box". On the other hand, NAM is

[1] https://normetrics.no/nam/

efficient to use to study the wider impact of COVID-19 on the economy, as there is no need to specify separate final model equations for each variable of interest. Since the model includes the main national account identities, internal logical consistency is also secured, for example between the impacts on aggregate demand and on aggregate supply.

The evaluation period in section 3 are the nine quarters from 2020(1) to 2022(1). We find empirically the same qualitative impact of COVID-19 and the responses to it, as in the illustrative model. Negative COVID-19 shocks early in the pandemic resulted in relatively persistent differences between counterfactual and baseline solutions paths. The is true for both the final equation model and for NAM, and for both types of measures of COVID-19 impact.

Quantitatively, the impact was largest in the second quarter of 2020. Two years later, the simulated effect had become much smaller in magnitude. The reduction in the gap between counterfactual and baseline can partly be due to counteracting shocks later in the simulation period. The catch-up variables can be rationalized by fiscal policies that were introduced, and by temporary return to "almost business as ususal" in the periods when non-pharmaceutical measures were lifted.

In addition to the comparison of the results for the two models used in this offering, section 3 compares the findings with the assessment made in work commissioned by the government's Coronavirus commission.[2] In sum, we find that the loss in income generation due to the impact of COVID-19 has been substantial in Norway and that it may have been larger than reported in the studies made for the government's commission.

Section 4 is a summary of result of the analysis, and a brief discussion of areas for further research related to model based counterfactuals in the analysis of big shocks to the economy. One question that can be addressed empirically as more data becomes available, is the relative invariance of the parameters in the model that determines how the impact of COVID-19 becomes propagated into medium term effects. Another, related area of research, is the development of models with non-linear propagation mechanisms.

2 Model based counterfactual analysis

In order to define the counterfactual, the general notation for a dynamic macro model is given first in this section. Thereafter the difference-from-counterfactual measures of the impact of COVID-19 are defined and illustrated with the aid of a model that has a closed form algebraic solution.

2.1 Models of the macroeconomy

A dynamic model can be expressed compactly as:

$$y_t = f_y(y_{t-1}, ..., y_{t-p}, x_t, ..., x_{t-p}, D_{yt}, \varepsilon_{yt}) \text{ where } \; f_y(\cdot) \; \text{ denotes a function..} \tag{1}$$

where y_t denotes a vector with n endogenous variables in period t while x_t has the m exogenous variables as elements.

[2]https://www.regjeringen.no/en/dokumenter/nou-2022-5/id2910055/

D_{yt} represents deterministic terms which are constants, trends, seasonals and dummy variables for interventions or shocks. ε_{yt} represents random error-terms that are unpredictable by conditioning on the other arguments in the function.[3]

The model equations must capture the normal economic behaviour of firms and households (and rule based policy responses) if the model's solution is to mirror reality in a reasonable degree, and generate many of the properties of the actual data, see Granger (1992), Visco (2005), Spanos (2021), among others.

An operational definition of a large shock is that it can be found as a significant impulse indicator variable, by the use of statistical tests and conventional significance levels. In practice this means that the shock can be "picked up" by a (zero-one) indicator variable which is an element in D_{yt}. A shock can be large in this meaning of the term without *necessarily* leading to further structural changes in the equations of the model which describes normal economic behaviour, although that can clearly happen as well. Investigation of the degree of invariance of the deeper parameters of the model is of great importance for the continued relevance of any operational model after a large shock has hit the economy. Meanwhile, methods like impulse indicator saturation (IIS) estimation which use below can give robust estimation with respect to shocks within the sample period, Johansen and Nielsen (2009).

A large shock can affect the data generation of all the economic variables of the model, not just the endogenous ones. Hence, in order to quantify the effects of shocks on the economy it is unsatisfactory to use (1) alone. The analysis becomes more relevant if the model is completed by module that endogenizes the variables in the x_t vector:

$$x_t = f_x(x_{t-1}, ..., x_{t-p}, D_{xt}, \varepsilon_{xt}). \tag{2}$$

We refer to (1) and (2) as the extended model, and will show an specific example below where we use an extended version of the NAM model to analyse the impact of COVID-19.

The extended model can be written compactly by stacking y_t and x_t in the $m+n$ vector $\boldsymbol{y}_t$, the two error-terms in $\boldsymbol{\varepsilon}_t$ and the deterministic terms in $\boldsymbol{D}_t$:

$$\boldsymbol{y}_t = f(\boldsymbol{y}_{t-1}, ..., \boldsymbol{y}_{t-p}, \boldsymbol{D}_t, \boldsymbol{\varepsilon}_t). \tag{3}$$

2.2 Difference-from-counterfactual

Mathematically, the extended model is a system of difference equations, with general functional form. The solution of $\boldsymbol{y}_t$ that we use in the following is function of initial conditions ($\boldsymbol{y}_0$, $\boldsymbol{y}_{-1}$,...,$\boldsymbol{y}_{-p+1}$, errors $\boldsymbol{\varepsilon}_t$,...,$\boldsymbol{\varepsilon}_1$ and deterministic terms $\boldsymbol{D}_t$,...,$\boldsymbol{D}_1$. It is custom to refer to it as a causal solution, as apart from the "forward" solutions that does not condition on initial conditions, see eg., Nymoen (2019, Ch. 3).

We let $T_0 - 1$ denote the end of the pre-pandemic sample. The COVID-19 sample is therefore: $t = T_0, T_0 + 1, ..., T_0 + H$ where H is the evaluation horizon.

The ordered sequence of random variables $\boldsymbol{y}_t; t = T_0, T_0 + 1, ..., T_0 + H$ generated by the model is in particular function of the sequence of deterministic terms $\boldsymbol{D}_{T_0+h}$, $h = 0, 1, ..., H$.

Let $\mathcal{I}_D$ denote the information set that the solution is based on. For ease of exposition, we distinguish between two main cases:

[3] Equal lag length (p) for the two variables is to save notation, it is without loss of generality.

1. All the dummies in $\mathcal{I}$ are zero in all time periods, denoted by $\mathcal{I}_{D=0}$.

2. At least one dummy is set to 1 in at least one period, denoted by $\mathcal{I}_{D=1}$.

In the following, we refer to a solution based on $\mathcal{I}_{D=1}$ as a baseline solution and denote it by $\boldsymbol{y}_t^b$. A solution based on $\mathcal{I}_{D=0}$ is a counterfactual solution, denoted by $\boldsymbol{y}_t^c$.

The impact and dynamic effects of a large shock to the system can be estimated as the difference between the conditional expectations of the respective solutions:

$$\text{Diff}_I \boldsymbol{y}_t = E(\boldsymbol{y}_t^c \mid \mathcal{I}_{D=0}) - E(\boldsymbol{y}_t^b \mid \mathcal{I}_{D=1}); t = T_0, T_0 + 1, ..., T_0 + H. \tag{4}$$

$\text{Diff}_I \boldsymbol{y}_t$ is the difference between two deterministic functions of time. Hence, the null hypotheses that the difference $\text{Diff}_I \boldsymbol{y}_t$ is significantly different from zero cannot be tested directly. However, inference can build on the statistical testing of the significance of the intervention dummies in $\boldsymbol{D}_t$.

Assume for example that we observe $\text{Diff}_I \boldsymbol{y}_{T_0} < 0$. Under the assumption that the other parameters of model are invariant to the (pandemic) intervention, the difference is:

$$\text{Diff}_I \boldsymbol{y}_{T_0} = -\gamma_{D_{T_0}}, \tag{5}$$

where $\gamma_{D_{T_0}}$ denotes the coefficient of the D_{T_0} indicator variable. Hence the null hypothesis of $\text{Diff}_I \boldsymbol{y}_{T_0} = 0$ is equivalent to testing H_0: $\gamma_{D_{T_0}}$.

More generally we have:

$$\text{Diff}_I \boldsymbol{y}_{T_0+h} = \sum_{j=0}^{h} -\gamma^w_{D_{T_0+j}}, \; h = 0, 1, 2, ..., H. \tag{6}$$

where each $\gamma^w_{D_{T_0+j}}$ denotes a weighted indicator variable coefficient (as implied by the solution). Testing the joint significance of the whole indicator set seems to be relevant. Another possibility is to test the significance of two groups of indicator variable coefficients that have opposite signs.

However, another measure to consider is the difference between the counterfactual and the actual $\boldsymbol{y}_t$:

$$\text{Diff}_{II} \boldsymbol{y}_t = E(\boldsymbol{y}_t^c \mid \mathcal{I}_{D=0}) - \boldsymbol{y}_t; t = T_0, T_0 + 1, ..., T_0 + H, \tag{7}$$

which is similar to the estimator proposed by Pesaran and Smith (2016) to analysis of effects of economic policy changes.

Formally, (7) is like a time series of forecast errors. The significance of the impact of COVID-19 can therefore be tested by using tests of forecast failure. Hence, a t-value of the forecast error for period $T_0 + h$ which is statistically significant can be interpreted as evidence of a significant impact.

However care must be taken: If, during the evaluation period, the economy changes in other ways than those that are captured by a indicator variable set, a significant forecast error can be due to those other changes, at least in part. Note that this is somewhat different from the assumption made in connection with Diff_I, which was about invariance of the coefficients in the *model* of the economy. In order to have a clear cut interpretation of Diff_{II} the invariance assumption applies to the economy itself, rather than to "just" our model of the economy.

The relationship between the two impact measures can be expressed as:

$$\text{Diff}_{II}\boldsymbol{y}_t = \text{Diff}_{I}\boldsymbol{y}_t - \boldsymbol{e}_t^b; t = T_0, T_0 + 1, ..., T_0 + H. \tag{8}$$

where $\boldsymbol{e}_t^b$ denotes the forecast errors associated with the baseline solution:

$$\boldsymbol{e}_t^b = E(\boldsymbol{y}_t^b \mid \mathcal{I}_{D=0}) - \boldsymbol{y}_t; t = T_0, T_0 + 1, ..., T_0 + H. \tag{9}$$

In general therefore, the two measures will not be equal since there will be non-zero forecast errors associated with the baseline simulation.

2.3 An algebraic example

As an illustration of the properties of the $\text{Diff}_{I}\boldsymbol{y}_t$ function we can use an algebraically tractable example with two time series variables, X_t and Y_t, and first order dynamics.

In the case where one of the variables is exogenous, say X_t, we speak of an open system. In that case we can think of $f(\cdot)$ in (3) as a function that incorporates the exogeneity restrictions implied by (1) and (2). In the closed system interpretation, $f(\cdot)$ in (3) is a function that allows the mutual temporal dependencies between the two variables.

As noted, macro econometric models have over time become adapted to be consistent with the idea that trend non-stationarity is a typical feature of many macroeconomic time series. Hence we specify the example model so that the variables become integrated of order one. In a common notation this assumption is written as $X_t \sim I(1)$, $\Delta X_t \sim I(0)$, and the same applies for Y_t. A special case of (3) which is consistent with this is:

$$\Delta Y_t = \tilde{c}_{10} + \tilde{c}_{11}\Delta X_t + \tilde{c}_{1d}D_t + \tilde{\alpha}_{11}(Y_{t-1} + \beta_{12}X_{t-1}) + \tilde{\varepsilon}_{1t} \tag{10}$$

$$\Delta X_t = c_{20} + c_{2d}D_t + \alpha_{21}(Y_{t-1} + \beta_{12}X_{t-1}) + \varepsilon_{2t} \tag{11}$$

where β_{12} denotes the cointegration parameter. In one interpretation, (10) is a conditional model equation and (11) is a marginal model equation, and then the error-terms are uncorrelated. Another interpretation is that the model is a semi-reduced form with (10) as a structural equation in a simultaneous equation model (SEM), while (10) is the reduced form equation for X_t from that SEM (then the error terms are correlated).

The reduced form (or VAR) is obtained by eliminating ΔX_t from (10). For completeness we can write the reduced form as:

$$Y_t = Y_{t-1} + c_{10} + c_{1d}D_t + \alpha_{11}(Y_{t-1} + \beta_{12}X_{t-1}) + \varepsilon_{1t} \tag{12}$$

$$X_t = X_{t-1} + c_{20} + c_{2d}D_t + \alpha_{21}(Y_{t-1} + \beta_{12}X_{t-1}) + \varepsilon_{2t} \tag{13}$$

where it is understood that $c_{10} = \tilde{c}_{10} + \tilde{c}_{11}c_{20}$, and similarly for c_{1d}, α_{11} and ε_{1t}, as result of the substitution.

The closed system final equations

Assume that there is equilibrium correction in both equations of the model, hence $\alpha_{11} < 0$ and $\alpha_{21} > 0$. The properties of the solutions for X_t and Y_t can be studied through the final form equations, Wallis (1977). As a consequence of cointegration, the two final form equations for ΔY_t and ΔX_t become:

$$\Delta Y_t = \gamma_{10} + \lambda_2 \Delta Y_{t-1} + c_{1d} D_t + [\alpha_{11}\beta_{12}c_{2d} - (\alpha_{21}\beta_{12} + 1)c_{1d})]D_{t-1} + \epsilon_{1t}, \quad (14)$$

$$\Delta X_t = \gamma_{20} + \lambda_2 \Delta X_{t-1} + c_{2d} D_t + [\alpha_{21}c_{2d} - (1 + \alpha_{11})c_{2d}]D_{t-1} + \epsilon_{2t}, \quad (15)$$

where λ_2 is the second of two characteristic roots. The first root is $\lambda_1 = 1$, while λ_2 is given by:

$$\lambda_2 = 1 + \alpha_{11} + \alpha_{21}\beta_{12}, \quad (16)$$

where $\alpha_{11} + \alpha_{22}\beta_{12} < 0$ for consistency with the assumed stationarity of ΔX_t and ΔY_t.

The error terms of (14) and (15) are linear combinations of the two reduced form error-terms ε_{1t} and ε_{2t}, and the first lag of those two variables.[4]

Let D_t denote a single impulse indicator. Hence, D_t takes the value 1 in a period when a shock hits the economic system, and zero in all other time periods. As the equations show, D_t and D_{t-1} shift the two constant terms in the final form equations. Therefore, a shock will have permanent effects on the solutions of the two level variables given by the identities $Y_t = \Delta Y_t + Y_{t-1}$ and $X_t = \Delta X_t + X_{t-1}$.

In line with the definitions above, and if the impact period is set to $t = T_0$, the counterfactual solution is the conditional expectation: $E(Y^c_{T_0+h} \mid \mathcal{I}_{D=0})$, and the baseline solution is $E(Y^b_{T_0+h} \mid \mathcal{I}_{D=1})$, hence $\text{Diff}_I Y_{T_0+h} = E(Y^c_{T_0+h} \mid \mathcal{I}_{D=0}) - E(Y^b_{T_0+h} \mid \mathcal{I}_{D=1})$.

We let $D_t = 0$ in all time periods of the counterfactual solution, while in the baseline solution:

$$D_t = \begin{cases} 1 & \text{, if } T_0 \\ 0 & \text{, for all other } t. \end{cases} \quad (17)$$

The difference between the counterfactual and basis for the change ΔY_{T_0+h} is denoted $\text{Diff}_I \Delta Y_{T_0+h}$. It is a stationary first order process, so:

$$\text{Diff}_I \Delta Y_{T_0} = -c_{1d}, \quad (18)$$

$$\text{Diff}_I \Delta Y_{T_0+1} = -\alpha_{11}c_{1d} - \alpha_{11}\beta_{12}c_{2d}, \quad (19)$$

$$\text{Diff}_I \Delta Y_{T_0+1+h} = \lambda_2^h \text{Diff}_I \Delta Y_{T_0+1},\ h = 1, 2, ...H, \mid \lambda_2 \mid < 1 \quad (20)$$

while $\text{Diff}_I Y_{T_0+h}$ follows the updating formula:

$$\text{Diff}_I Y_{T_0+h} = \text{Diff}_I \Delta Y_{T_0+h} + \text{Diff}_I Y_{T_0-1+h}, \ \ h = 0, 1, 2, ..., H, \quad (21)$$

with the remark that $\text{Diff}_I Y_{T_0-1+h} = 0$ initially (for $h = 0$).

The values of function $\text{Diff}_I Y_{t_0+h}$ are non-zero for all h, even if the impact lasted for only one period. This is an implication of the unit-root of +1, and it is typical of integrated series.

[4] $\epsilon_{1t} = \varepsilon_{1t} - \varepsilon_{1t-1}(1 + \alpha_{11}\beta_{12}) + \varepsilon_{2t-1}\alpha_{11}\beta_{21}$ and $\epsilon_{2t} = \varepsilon_{2t} - \varepsilon_{2t-1}(1 + \alpha_{11}) + \varepsilon_{1t-1}\alpha_{21}$.

However, although both Y and X are permanently affected by a the impact of a single one-period shock, the long-run relationship between the variables is not disrupted. Let Z_t denote the disequilibrium variable $Z_t = Y_t + \beta_{12} X_t$ which is I(0). $\text{Diff}_I Z_{T_0}$ and $\text{Diff}_I Z_{T_0+1}$ will be directly influenced by D_{T_0} and D_{T_0+1}, but $\text{Diff}_I Z_{T_0+2}$ and later differences are given by:

$$\text{Diff}_I Z_{T_0+1+h} = \lambda_2^h \text{Diff}_I \Delta Z_{T_0+1},\ h = 1, 2, ...H, \mid \lambda_2 \mid < 1,$$

which is another consequence of the correspondence between the counterfactual and a forecast: Cointegration between variables is preserved in forecasts, Engle and Yoo (1987).

Under the assumption that there are no parameter changes other than the shift in the intercept captured by D_t, the second measure $\text{Diff}_{II} Y_{T_0+h}$ can be expressed as:

$$\text{Diff}_{II} Y_{T_0+h} = \text{Diff}_I Y_{T_0+h} - \sum_{j=0}^{h} \xi_j \epsilon_{1T_0+j},\ h = 0, 1, 2, .. \tag{22}$$

which is a special case of (8), with $\boldsymbol{e}^b_{T_0+h} = \sum_{j=0}^{h} \xi_j \epsilon_{1T_0+j}$.

In (22), ξ_j (h=0,1,...) does not represent a well-behaved linear filter, Nymoen (2019, p 167). Therefore, the stationarity of the error-term process ϵ_{1T_0+j}, $j = 0, 1, ..., h$ is not preserved in the second term of (22). Hence, the conditional variance $Var(\text{Diff}_{II} Y_{T_0+h} \mid Y_{T_0})$ is strictly increasing in h.

The open system

In the case of $\alpha_{21} = 0$, X_t is an exogenous variable. Under this restriction on (10)-(11), the model equation for Y_t, ie., (10) is an example of a macro-econometric model of the open type, and the X_t-equation takes the role of the completing equation in the extended model.

The open-system version of (12)-(13) is:

$$\begin{aligned} Y_t &= c_{10} + Y_{t-1} + c_{1d} D_t + \tilde{\alpha}_{11}(Y_{t-1} + \beta_{12} X_{t-1}) + \varepsilon_{1t}, & (23)\\ X_t &= c_{20} + X_{t-1} + c_{2d} D_t + \varepsilon_{2t} & (24) \end{aligned}$$

with the remark that the equilibrium correction coefficient in the final form equation for Y_t is the same as in the conditional model equation.

In this case, the solution for X_t can be found first and can be taken as given in the solution for Y_t.

The qualitative effects of single period shocks are the same as for the closed system. However, if we more generally include separate impulse indicators in (23) and (24), say D_{Yt} and D_{Xt}, it is only D_{Xt} that affects the level of both X_t and Y_t. If the shock is "limited to" Y_t so that it is captured by D_{Yt}, the solution for the level of Y_t will not be permanently affected, because the level of Y_t is linked to the level of X_t in this case.

Generalization

If the macroeconomic model has several endogenous variables and higher order dynamics, the implied final forms equations are completely general ARMA model equations. However in practice, such ARMA processes are approximated by high order AR processes, $Y_t \sim AR(p)$.

Hence, more generally we can think of the final equation of Y_t as an $AR(p)$ model equation. Under the assumption that the *largest* root of the associated characteristic equation is +1, it follows that $Y_t \sim I(1)$ and ΔY_t is $AR(p-1)$ with $p-1$ characteristic roots that are less than one in magnitude. Hence $\Delta Y_t \sim I(0)$.

In the illustrative model there were one stable root and one unit root. The generality stems from the famous result known as the *typical spectral shape* of economic time series, Granger (1966),Granger and Newbold (1986, Ch. 2.7). The theorem states that if there is a large number of stable roots, a single root equal to +1 at the zero-frequency implies that the time series properties become dominated by a random-walk component, Nymoen (2019, Ch. 9.3). Hence, because the random-walk component is a common feature of both simple and complex models, the results obtained for Diff_I and Diff_{II} may be more general than first thought. At least, they can be useful to keep in mind when interpreting the simulation results from models with multiple equations and higher order dynamics.

3 Impact of COVID-19 using models of the Norwegian economy

We now turn to the results of the impact of COVID-19 on the macroeconomic system of Norway. We focus on GDP Mainland-Norway, which is the income variable followed most closely in the discussion of fiscal policy and the interest rate path decided by Norges Bank [Central Bank of Norway].

In section 3.1, I use a single equation approach to estimate and simulate an empirical final form equation for (log of) GDP Mainland-Norway.

In section 3.2 I present results for the multiple-equation model Norwegian Aggregate Model NAM. It is a complete model of the economy. Each of the behavioural equations are investigated for significant impulse indicator variables in the period 2020(1) – 2022(1), but apart from that there are no changes made in the the model structure.

Dynamic simulation of the complete NAM model is a feasible way of obtaining the solutions that define counterfactual and the baseline solutions, Hence, the solutions of the two models give rise to different estimates of the same phenomenon, namely impact and dynamic effects of COVID-19.

Using the complete model is in many ways efficient as it gives relevant information about where in the system the impact comes first, and how the effects are propagated by the dynamics of the multivariate system. The multiple equation model is also suited to study the wider impact of COVID-19 on the economy, as we show in section 3.4.

On the other hand, the results are conditional on the many decisions that have been made in the construction of the larger model ("black box" critique). The single equation approach has the advantage of being transparent and easy to replicate and update. In addition, since the counterfactual is a forecast, using a parsimonious single equation model of for example GDP may produce more reliable results than a larger model which may be mis-specified in ways that bias the forecast, Pesaran and Smith (2016).

3.1 Results from empirical final form equations

As noted above, a system of I(1)-variables implies final form equations for each variable in differenced form. Equation (14) and (15) are particular special cases. All final form equations obtained from a closed system have identical autoregressive structures (the homogeneous parts of the difference equations are identical). But they also contain moving average errors, which are not the same for all variables, and impulse indicators will in general also enter differently in the equations (ie., they belong to the inhomogenous parts of the difference equations).

In general, each final form equation is therefore an ARMA(p,q) model, augmented by impulse indicators. However, as noted, an ARMA model can be approximated by an AR with $p' > p$. The machine learning algorithm Autometrics was used to decide the lag-specification and which indicator variables to include, Doornik (2009), Hendry and Doornik (2014), Doornik and Hendry (2018a). Autometrics-IIS extends the general unrestricted model (GUM) by one indicator variable for each observation, and retain only a small number of them in the model delivered by the search algorithm, Castle et al. (2012).

The estimators of retained economic variables have been shown to have an interpretation as robust estimators statistically speaking, Hendry et al. (2008), Johansen and Nielsen (2009).

We analyse the impact on GDP Mainland-Norway, the term used by Statistics Norway to refer to GDP without valued added in oil and gas extraction, pipeline transportation and ocean transport.[5] As noted above, GDP Mainland-Norway is the preferred variable for analysis of economic activity and of income generation in Norway. The time series is quarterly and from the National accounts.[6]

Letting Y_t denote GDP Mainland-Norway, the relative changes in GDP is given by the differenced series $\Delta log(Y_t)$. A general unrestricted model (GUM) with twelve autoregressive terms, constant and three seasonal dummies was found to be not mis-specified when evaluated by a standard test-battery.

Variable selection algorithms involve repeated testing, which can lead to inflated Type-I error probability levels. In Autometrics, the overall significance level depends on the user-set *Target size*. As a rule-of-thumb, Target size = k^{irr}/k^{GUM} where k^{irr} denotes how many irrelevant variables one can accept on average in the final model, and k^{GUM} denotes the number of regressors in the GUM. Hence, with 15 regressors in the GUM, and accepting the retention of 0.15 irrelevant variable on average, the target level can be set to 1 %. Increasing it to 5 % implies 0.75 irrelevant variable retained in the model equation delivered by the algorithm.

In order to obtain the forecast that gives us the $\text{Diff}_{II}Y_t$ measure, the only search is over the pre-pandemic period, so it is convenient to first present the model used to obtain that measure.

Using Target size = 0.01, Autometrics retained $\Delta log(Y)_{t-1}$, $\Delta log(Y)_{t-2}$, $\Delta log(Y)_{t-12}$, the constant term, three seasonals and nine impulse indicators, denoted by $D_{\text{year(quarter)}}$.The

[5]https://www.ssb.no/en/nasjonalregnskap-og-konjunkturer/nasjonalregnskap/statistikk/nasjonalregnskap

[6]https://www.ssb.no/en/statbank/ltable/09190/. The unit is million NOK, constant 2019 prices. The valuation is marked values, and the series is not seasonally adjusted.

model is shown in equation (25).

$$
\begin{aligned}
\Delta log(Y)_t \quad = \quad & - \underset{(0.0652)}{0.608}\ \Delta log(Y)_{t-1} - \underset{(0.0651)}{0.2764}\ \Delta log(Y)_{t-2} + \underset{(0.0545)}{0.1468}\ \Delta log(Y)_{t-12} \\
& + \underset{(0.00148)}{0.00857} - \underset{(0.00701)}{0.04032}\ \mathrm{CS}_t - \underset{(0.00631)}{0.06038}\ \mathrm{CS}_{t-1} \\
& - \underset{(0.00543)}{0.06526}\ \mathrm{CS}_{t-2} + \underset{(0.0149)}{0.04329}\ D_{1984(1)} + \underset{(0.015)}{0.06846}\ D_{1985(1)} \\
& + \underset{(0.0149)}{0.05462}\ D_{1986(2)} - \underset{(0.015)}{0.0348}\ D_{1988(3)} + \underset{(0.015)}{0.04121}\ D_{1996(1)} \\
& + \underset{(0.015)}{0.05826}\ D_{1997(2)} + \underset{(0.0149)}{0.03583}\ D_{2001(1)} + \underset{(0.015)}{0.0421}\ D_{2005(2)} \\
& - \underset{(0.015)}{0.04317}\ D_{2009(1)}
\end{aligned} \tag{25}
$$

OLS	Sample: 1981(2) - 2019(4)	Number of obs.: = 155
$\hat{\sigma}100 = 1.46$	$R^2 = 0.89$	
AR_{1-5} :	F(5,124) = 1.208[0.31]	
ARCH_{1-4} :	F(4,147) = 0.40[0.81]	
Normality :	$\chi^2(2) = 0.68[0.71]$	

On the right hand side of the equation, CS_t, denotes the centered seasonal dummy variable for the first quarter of the year.[7] Estimated standard errors are in round brackets below the coefficients. Four of the indicators are from the 1980s, a decade which started with high and volatile economic growth, and which ended with a crash in housing prices in 1988 and later a banking crisis and, by post WW-II standards, very high unemployment, Nymoen (2017). The two indicators from the second half of the 1990s can be connected with the recovery form that crises. The first decade of the new millennium was marked by growth, and very high growth in some individual quarters. However, the international financial crisis had a significant negative impact, which is picked by the indicator variable $D_{2002(1)}$.

R^2 and the residual percentage standard deviation ($\hat{\sigma}100$) indicate quite good fit. The residual mis-specification tests AR_{1-5}, ARCH_{1-4} and Normality are reported with their respective p-values. The tests of no autoregressive autocorrelation and of ARCH are clearly insignificant, see Harvey (1981), Engle (1982), as is the test of departures from the assumption about normality, Jarque and Bera (1980).

In order to measure the difference between the counterfactual and a baseline simulation, Autometrics was used a second time to select an empirical final equation of $\Delta log(Y)$ with the pandemic period included in the sample. The model specification became the same as in equation (25), but with $D_{2020(2)}$ and $D_{2021(4)}$ included as two COVID-19 impulse indicators. The estimated coefficient of $D_{2020(2)}$ was -0.08 (t-value -5.7). For $D_{2021(4)}$ the estimate was +0.03 (t-value of 2.4). This equation was used to simulate the baseline solution needed to calculate Diff_IY. As noted, equation (25) was used to generate the forecast used to obtain Diff_{II}Y.

't-values" for Diff_{II}Y can be used to test the null hypothesis of zero difference between counterfactual and actual in a given quarter. Simulated forecast error standard deviations

[7] Each centered seasonal sum to zero over the year.

Table 1: GDP Mainland-Norway. Results using empirical final form equation.

(1) Horizon	(2) Actual	(3) C-factual	(4) Baseline	(5) Diff_IY (3)-(4)	(6) Diff_IY Percent	(7) Diff_{II}Y (3)-(2)	(8) t-value	(9) Diff_{II}Y Percent
2020(1)	762903	774610	774610	0	0	11707	1.0	1.5
2020(2)	701430	765681	706114	59567	7.8	64251	5.7	8.4
2020(3)	732876	766497	741599	24898	3.2	33621	2.6	4.4
2020(4)	794793	816347	786287	30060	3.7	21554	1.4	2.6
2021(1)	754553	788096	750397	37699	4.8	33543	2.2	4.3
2021(2)	754712	784506	753082	31424	4.0	29794	1.8	3.8
2021(3)	769877	780345	748024	32321	4.1	10468	0.6	1.3
2021(4)	835909	833809	824849	8960	1.1	-2100	-0.1	-.3
2022(1)	798592	806328	783249	23079	2.7	7736	0.5	1.0

Notes:
(2)-(5) and (7): numbers are in million 2019 kroner. (6) and (9) are percents of (3). (8) is t-value of (7)

were used for comparability with the result in the next section, where simulation was the only feasible method. For the single equation dynamic forecasts errors in Table 1 exact variance formulas exist, see Clements and Hendry (1998), and for all practical purposes they were identical.[8]

Because Autometrics-IIS did not keep the indicator for 2020(1) in the model equation, Diff_IY gives zero impact of COVID-19 in that quarter. As the actual value in 2022(1) was lower than the forecast. Diff_{II}Y (column (7)) on the other hand shows a negative impact, as the actual value in 2022(1) was lower than the forecasted. In percent, the estimated impact was a 1.5 reduction compared to the counterfactual without the coronavirus pandemic, which is however not significant when the t-value in column (8) is compared to a standard critical value.

Table 1 column (8) shows that Diff_{II}Y is significantly different from zero in 2020(2), the quarter with the largest numerical and percentage difference over the period as a whole. The Diff_{II}Y estimated impact of 7.8 percent is also significant as it is due to Autometrics-IIS keeping the indicator for 2020(2).

For 2020(3) and 2021(1) the t-values are also larger than 2, indicating significant impact of COVID-19 and the responses to it. As the forecast error variances are increasing with the horizon, a given numerical difference between the counterfactual and the actual will be more significant early in the period than it is in a later quarter.

When we compare the numbers in column (6) and (9) we see that the Diff_{II}Y estimates are largest early in the pandemic. From 2020(4) and onward it is Diff_IY that gives the higher estimates of COVID-19 impact. The reduction in the impact that occurs in 2021(4) is due to the positive indicator variable for that quarter.

[8]The simulated standard errors were obtained by using Eviews 12 and were checked against the analytical versions by using Oxmetrics 8.0-PcGive 15, Doornik and Hendry (2018b).

3.2 Impact of COVID-19 using a multiple-equation model

Norwegian aggregate model (NAM) is an multiple-equation econometric model that can be used to study the impact of COVID-19 on the Norwegian economy.

As a multivariate dynamic model, NAM has in common with the simple model above that the solution of the endogenous variables are in principle given by final form equations and their associated characteristic roots. Another common feature is that cointegration has been an important modelling concept, hence shocks and short-lived impulses integrates into shifts in level of variables, without necessarily disrupting relationships between those variables.

NAM originated from the econometric assessment of wage-and price formation in Nymoen(1989a,1989b,1991), further developed in Bårdsen et al. (1998), Bårdsen and Fisher (1999), Bårdsen and Nymoen (2003), and the monetary transmission model of Bårdsen and Klovland (2000). An early version of the model was presented in Bårdsen et al. (2003), while a more complete version was documented in Bårdsen and Nymoen (2009). The methodological orientation of the model is also represented by the book on macroeconometric modelling by Bårdsen et al. (2005).

NAM is an operational empirical econometric model. Regular updates of the model is synchronized with the releases of the Quarterly National Accounts.[9] The documentation of the latest version of the model is always on the internet, Bårdsen and Nymoen (2022).[10]

The standard version of NAM contains 120 estimated equations. There are several non-modelled (exogenous) time series in the model, meaning that model forecasts are conditional on projections for those variable made by the user. Examples of variables that are exogenous in the standard version of the model are: An indicator of growth in export markets, the foreign short term interest rate, and other variables from financial and product markets abroad (the foreign sector of the model).

Table 2: Number of NAM equations where COVID-19 impulse indicators are included

Quarter	Impulse Indicator	Model version	
		Standard (120 eqs)	Extended (133 eqs)
2020(1)	$D_{\text{Covid},t}$	12	23
2020(2)	$D_{\text{Covid},t-1}$	26	38
2020(3)	$D_{\text{Covid},t-2}$	15	26
2020(4)	$D_{\text{Covid},t-3}$	13	23
2021(1)	$D_{\text{Covid},t-4}$	11	21
2021(2)	$D_{\text{Covid},t-5}$	12	20
2021(3)	$D_{\text{Covid},t-6}$	11	28
2021(4)	$D_{\text{Covid},t-7}$	7	9
2022(1)	$D_{\text{Covid},t-8}$	9	12

As the pandemic is global, several of these variables are likely to be affected. Hence I have extended the operational model version (dubbed Standard in Table 2) by empirical equations

[9] However, updates are also made to accommodate model users' needs for model analysis, and after new and improved results from modelling results. NAM users include Financial Supervisory Authority of Norway, NHO (Confederation of Norwegian Enterprise) and LO (Norwegian Confederation of Trade Unions).

[10] https://normetrics.no/nam/

Table 3: GDP Mainland-Norway. Result using NAM.

(1) Horizon	(2) Actual	(3) C-factual	(4) Baseline	(5) Diff_IY (3)-(4)	(6) Diff_IY Percent	(7) Diff_{II}Y (3)-(2)	(8) t-value	(9) Diff_{II}Y Percent
2020(1)	762903	788681	768642	20039	2.5	25778	3.1	3.3
2020(2)	701430	765517	699166	66351	8.7	64087	6.9	8.4
2020(3)	732876	774523	736418	38105	4.9	41647	4.2	5.4
2020(4)	794793	825542	793334	32208	3.9	30749	2.6	3.7
2021(1)	754553	807422	747399	60023	7.4	52869	4.0	6.5
2021(2)	754712	799402	739884	59518	7.4	44690	3.2	5.6
2021(3)	769877	799843	752628	47215	5.9	29966	2.0	3.7
2021(4)	835909	852706	809452	43254	5.0	16797	1.0	2.0
2022(1)	798592	819091	770610	48481	5.9	20499	1.1	2.5

Notes:
(2)-(5) and (7): numbers are in million 2019 kroner. (6) and (9) are percents of (3). (8) is t-value of (7)

for this category of variables. As a result, the extended model version contains 13 additional equations. In addition to equations for variables for the foreign sector, they include model equations for public consumption expenditure (policy) and for capital formation in the public sector (general government).

It is the extended version of the model which is used in the results reported below. Indicator variables for the nine COVID-19 quarters from 2020(1) to 2022(1) were added to all the econometric equations of the model, and retained in the model if t-values were found to be significant at the 5 % level.

Table 2 shows the number of equations each COVID-19 indicator variable has been included in. The indicator variable for 2020(2) was included most frequently, in 38 of the equations. The third quarter of 2020 had the second highest number if inclusions. We note also that the indicator variable for 2020q1 was included in as many as 23 equations. The automatic variable selection used above did not include that first COVID-19 quarter, so this is one factor that will contribute to difference between the results of the two analyses.

Table 2 shows that the indicators from 2020(4) to 2021(3) have a little more than 20 entries each. There is a marked drop in the inclusion numbers for 2021(4) and 2022(1). There may even be caveats about the choice of including 2022(1) in the analysis. In Norway, there were some non-pharmaceutical measures still in place at the start of the year. However, the society opened up in February 2022 when the Norwegian government concluded that the damaging effects of the measures had become more serious than the effects of the spread of the virus in the population. Later in February, Russia invaded Ukraine and several global markets, energy markets in particular, were immediately affected by that war. Hence, in 2022(1) the economy was influenced by two shocks, and care must therefore be taken when interpreting the results for that quarter in particular.

A final thing to note is that several of the affected equations in the Extended-column represent the foreign sector, indicating that a substantial part of the total effects may come from international trade and financial markets.

Table 3 gives the result in the same format as for the final equation model above. One notable difference from Table 1 is that, when NAM is used, also the Diff_{II}Y measure indicates that COVID-19 had an impact on GDP already in 2020(1). This is due to the 23 indicators mentioned above.

Closer inspection shows that one of the relationships where there was an early impact was the equation for (private) service activities, which includes retail trade, accommodation and food service activities. Although the first full lockdown occurred quite late in the first quarter of 2020, on 12 March, it is not unreasonable that value added in service activities became reduced compared with a scenario without COVID-19.

Other equations that include the 2020(1) dummy include the consumption function and the equation for the growth in Norwegian export markets. In NAM, both of these impacts have indirect effects on value added, not only in retail and private service production but also in other industries, like manufacturing.

Both Table 1 and Table 3 show that the largest impact of COVID-19 came in 2020(2). The estimates are a little higher in Table 3 and that remark also applies for the later quarters in the period. In particular, the Diff_{I}Y measure in the NAM table indicates the largest and most persistent effects of the impact and responses to COVID-19. That said, as the standard deviation of the differences are increasing with the horizon, the t-values of Diff_{I}Y become insignificant towards the end of the evaluation period. It is a reminder of the large uncertainties involved in this type of assessments.

3.3 Comparison with other studies

There are existing studies of national and regional COVID-19 effects in Norway, commissioned by the government and done for the Norwegian corona commission, cf. Bjertnæs et al. (2021) and von Brasch et al. (2022).

The two reports for the commission used a forecast prepared by Statistics Norway late in 2019 as the counterfactual while the baselines were constructed by combination of available actual national account data with forecasts for the remaining quarters of the period (ie., until 2023(4)). Hence the numerical assessment of the impact of COVID-19 in the two corona commission reports appear to have combined $\text{Diff}_{I}\boldsymbol{y}_t$ and $\text{Diff}_{II}\boldsymbol{y}_t$ in order to be able to give results for the whole period from 2020 to 2023.

Table 4 shows the simulated differences in annual numbers. The simulated income reduction for 2020 amounts to 157 billion kroner, 5.2 % of the baseline GDP of Mainland-Norway. Interestingly, and as seen from the the table, this is somewhat larger than the percentage reduction reported in the two estimates made for the corona virus commission.

The table also shows that after 2020, the simulated losses in income generation using NAM are systematically larger than in the two other assessments. The percentage numbers in von Brasch et al. (2022) for 2021,2022 and 2023 give the smallest loss in income generation.

For the period 2020-2023 as a whole, von Brasch et al. (2022) sets the discounted reduction in GDP Mainland-Norway to 270 billion kroner in fixed 2019-prices. When the same discounting factor is used (4%), for the NAM results, the reductions amount to 578 billion. Clearly, both calculations must be interpreted with care because of the many uncertainties. That said, taken together the studies indicate huge income losses over the period of the pandemic, amounting to 9 percent (von Brasch et al. (2022)) and 18 percent of 2019 Mainland-Norway GDP (NAM).

Table 4: GDP Mainland-Norway. Difference between counterfactual (no COVID-19) and baseline/actual in percent of counterfactual.

	2020	2021	2022	2023
Diff_IYF				
Final equation	3.7	3.5	2.6	2.8
NAM	5.0	6.4	4.8	3.1
Diff_{II}YF				
Final equation	4.2	2.5		
NAM	5.1	4.4		
Corona commision:				
Bjertnæs et al. (2021)	4.7	3.8	2.2	0.5
von Brasch et al. (2022)	4.6	2.4	2.1	-0.9

In addition to methodological differences, one reason for the larger effects using NAM may be that they build on updated macroeconomic time series which covers more of the pandemic period. A similar remark applies to the result reported in Rungcharoenkitkul (2021), of a 3 percent reduction in Norwegian GDP in 2020.

3.4 The wider impact of COVID-19

A multiple equation model like NAM is efficient to use in the analysis of the wider impact of COVID-19 and the responses to it, as the endogenous variables of the model covers several sectors and markets in the economy.

As one example, Table 5 shows effects in percent of the no-covid solutions for GDP (total), GDP Mainland-Norway and four sectors of Mainland Norway. In order to get a clearer picture of how the effects decline with the horizon, the simulation was extended through 2023 and we give annual numbers.

For GDP total and GDP Mainland-Norway the reductions are monotonous after 2021. The smaller estimates for GDP total comes from the fact that the petroleum industry is quite large, and that it was estimated to be practically unaffected by the pandemic.

The rows that show estimated impact by industries show that COVID-19 and the responses to it had a wide impact. The numbers for value added in service activities (which includes eg., travel and accommodation as mentioned above) are largest. But the impacts on manufacturing and other production (which includes construction) are also estimated to be of numerical significance. The large reduction in value added in service activities, compared to 2019 (so not a counterfactual) is also found for Denmark and Sweden, Blytt et al. (2022).

According to the results in Table 5, the government sector was the least affected by COVID-19.

The line for imports in table 5 shows a huge impact. In the model, imports are driven by components of aggregate demand and by the real exchange rate. Hence it is not surprising to find several huge estimated impacts of COVID-19 for the "demand components" in the bottom half of the table. Public consumption is an exception, and more interestingly, also capital formation in residential housing. Housing prices grew in real terms during the

Table 5: Difference between counterfactual (no COVID-19) and baseline/actual in percent of counterfactual. $\text{Diff}_I\boldsymbol{y}_t$ results using NAM.

	2020	2021	2022	2023
GDP	3.4	4.3	2.6	1.1
GDP Mainland-Norway	5.0	6.4	4.8	3.1
Value added, manufacturing	6.1	4.9	2.7	1.48
Value added, other products	3.4	5.1	3.9	2.5
Value added, service activities	8.1	9.7	7.5	5.0
Value added, general government	1.1	.6	.1	-.3
Imports	13.4	14.8	8.7	4.8
Exports	4.2	2.6	-.5	-.8
- Mainland-Norway	10.7	9.3	6.6	4.5
Private consumption	9.4	8.6	2.8	1.7
Public consumption	.6	-.3	-.4	-.4
Gross capital formation	6.1	10.5	7.4	3.4
- Mainland-Norway private business	12.1	19	12	3.2
- Housing	.3	2.1	.5	-1.2

pandemic, which in the model is one of the main explanatory variables of real investments in housing.

4 Summary

Counterfactual analysis is required to estimate the economic impact of COVID-19. In this offering, macroeconometric models are used to simulate counterfactual developments of GDP Mainland Norway, and of a selection of other variables in the Norwegian national accounts. Two operational definitions of the difference between the counterfactual (without COVID-10) and the baseline (or "actual") were defined. They were illustrated with the aid of a model which, despite its simplicity, incorporates the double feature of low frequency unit- root and cointegration, which has also become common features of empirical models.

There is no logical inconsistency between using both "small" and "large" models to elucidate COVID-19 impacts on the economy. Dynamic econometric modelling can be used to specify a model equation which can be interpreted as an approximation to the unknown final equation of for example GDP. For an existing operational model, dynamic simulation gives the solution path of the final equation implied by the model structure.

Empirically we found, by simulating NAM ("large model") and empirical final equations ("small model"), that the differences between the counterfactual and the baseline/actual for GDP Mainland Norway were large and statistically significant in 2020 and in the first half of 2021. Towards the end of the evaluation period the differences were no longer significant. This was due to both smaller differences and to the increased standard deviation associated with the counterfactual, which is typical for dynamic forecasts of integrated time series.

The simulated impact of COVID-19 on Mainland-GDP in our study can be said to confirm the results in studies made for the government's coronavirus commission, specifically for 2020. For 2021, 2022 and 2023 the tendency was that our simulations indicated larger impacts. As both data and models used for the counterfactual simulations were different, exact correspondence was not to be expected, and as noted, the uncertainties are large and they increase with the length of the horizon.

In addition to GDP Mainland-Norway, we reported the wider impact of COVID-19 and the response to it for a range of main national accounts variables that are endogenous in NAM. This demonstrates the efficiency of using an multivariate empirical model of the macroeconomy, if it available and operational.

As the counterfactual is like a dynamic forecast, many of the challenges to interpretation and validation are the same for the two model usages. Ideally, any model which is used to measure the impact of COVID-19 should characterize the normal behaviour of the economy as accurately in the pandemic (forecast) period as it did over the estimation sample. Hence, although model intercepts must necessarily change significantly to capture the impact of COVID-19, the other model parameters should be invariant with respect to this shock. This is a strong requirement and seems unlikely to be met for all behavioural equations of an empirical macroeconomic model.

However, invariance is a relative concept and a property that can hold partly if not completely. The more practical requirement may be that although the other parameters than the intercepts may not remain completely constant after the COVID-19 shock, they do not change so much that the model-based counterfactual becomes uncorrelated with the true counterfactual. It was beyond the scope of this study to do formal testing of parameter invariance, but as more data becomes available research will no doubt shed light on this important issue.

More generally, the experience from modelling the economy during the pandemic can be used in a progressive way to improve on existing models. One area for research is dynamic models with non-linear cointegration, Johansen (2004). In such models, equilibrium coefficients may change change while cointegration parameters are still assumed to be invariant to the shock.

The values of the function $\text{Diff}_I(\boldsymbol{y}_t)$ will be different with linear and non-linear equilibrium correction. As illustrated in the example above, the reason is that the medium-term difference between the counterfactual and the baseline depends on the adjustment coefficients, and not only on the cointegration parameters. A deeper form of structural breaks would be changes in the equilibrium relationships in the model of the economy that existed before the shock, ie., in the cointegration parameters.

However, cointegration is a rare phenomenon and can be said to represent deep structural relationships that do not break down easily. However, the possibility of breaks in cointegration coefficients deserves to be given particular attention in model maintenance. It would represent an impact of COVID-19 with consequence for model building, maybe in the direction suggested by Vines and Wills (2020) for rebuilding macroeconomic theory with the use of models with multiple equilibria.

References

Bårdsen, G., Ø. Eitrheim, E. S. Jansen and R. Nymoen (2005). *The Econometrics of Macroeconomic Modelling*. Oxford University Press, Oxford.

Bårdsen, G. and P. G. Fisher (1999). Economic Theory and Econometric Dynamics in Modelling Wages and Prices in the United Kingdom. *Empirical Economics*, *24*(3), 483–507.

Bårdsen, G., P. G. Fisher and R. Nymoen (1998). Business Cycles: Real Facts or Fallacies? In Strøm, S. (ed.), *Econometrics and Economic Theory in the 20th Century: The Ragnar Frisch Centennial Symposium*, no. 32 in Econometric Society Monograph Series, chap. 16, 499–527. Cambridge University Press, Cambridge.

Bårdsen, G., E. S. Jansen and R. Nymoen (2003). Econometric Inflation Targeting. *Econometrics Journal*, *6*, 429—460.

Bårdsen, G. and J. T. Klovland (2000). Shaken or Stirred? Financial Deregulation and the Monetary Transmission Mechanism in Norway. *Scandinavian Journal of Economics*, *102*(4), 563–583.

Bårdsen, G. and R. Nymoen (2003). Testing Steady-State Implications for the NAIRU. *Review of Economics and Statistics*, *85*(3), 1070—1075.

Bårdsen, G. and R. Nymoen (2009). Macroeconometric Modelling for Policy. In Mills, T. and K. Patterson (eds.), *Palgrave Handbook of Econometrics Vol 2*, chap. 17, 851—916. Palgrave Mac-Millan.

Bårdsen, G. and R. Nymoen (2022). Documentation of Norwegian Aggregate Model, NAM. Https://normetrics.no/nam/.

Bjertnæs, G. H. M., T. von Brasch, Å. Cappelen, S. Holden, H. E, O. Slettebø and J. Zulavona (2021). COVID-19, tapt verdiskaping og finanspolitikkens rolle. Utredning for Koronakommisjonen. RAPPORTER/REPORTS 2021/13, Statistisk sentralbyrå, Statistics Norway, Oslo-Kongsvinger.

Blytt, J. P., A. Bougroug and P. Sletten (2022). Økonomisk utvikling gjennom Covid19. En oppdatert sammenligning av Norge, Sverige og Danmark. RAPPORTER/REPORTS 2022/14, Statistisk sentralbyrå, Statistics Norway, Oslo-Kongsvinger.

Castle, L., J, J. Doornik and D. Hendry (2012). Model Selection When There are Multiple Breaks. *Journal of Econometrics*, *169*, 239—247.

Clements, M. P. and D. F. Hendry (1998). *Forecasting Economic Time Series*. Cambridge University Press, Cambridge.

Doornik, J. A. (2009). Autometrics. In Castle, J. and N. Shephard (eds.), *The Methodology and Practice of Econometrics*, chap. 8, 88—121. Oxford University Press, Oxford.

Doornik, J. A. and D. F. Hendry (2018a). *Empirical Econometric Modelling PcGive 15. Volume 1*. Timberlake Consultants, London.

Doornik, J. A. and D. F. Hendry (2018b). *Modelling Dynamic Systems PcGive 15. Volume 2*. Timberlake Consultants, London.

Engle, R. F. (1982). Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation. *Econometrica*, *50*, 987–1007.

Engle, R. F. and B. S. Yoo (1987). Forecasting and Testing in Co-integrated Systems. *Journal of Econometrics*, *35*, 143–159.

Granger, C. W. J. (1966). The Typical Spectral Shape of an Economic Variable. *Econometrica*, *34*, 150–161.

Granger, C. W. J. (1992). Fellow's Opinion: Evaluating Economic Theory. *Journal of Econometrics*, *51*, 3–5.

Granger, C. W. J. and P. Newbold (1986). *Forecasting Economic Time Series*. Academic Press, San Diego.

Harvey, A. C. (1981). *The Econometric Analysis of Time Series*. Philip Allan, Oxford.

Hendry, D. F. and J. A. Doornik (2014). *Empirical Model Discovery and Theory Evaluation. Automatic Selection Methods in Econometrics*. Arne Ryde Memorial Lectures. MIT Press, Cambridge,Mass.

Hendry, D. F., S. Johansen and C. Santos (2008). Automatic Selection of Indicators in a Fully Saturated Regression. *Computational Statistics*, *23*, 317–335 and Erratum 337–339.

Jarque, C. M. and A. K. Bera (1980). Efficient Tests for Normality, Homoscedasticity and Serial Independence of Regression Residuals. *Economics Letters*, *6*, 255–259.

Johansen, S. (2004). Cointegration: An overview. Manuscript, Unversity of Copenhagen, Department of Applied Mathematics and Stastiics.

Johansen, S. and B. Nielsen (2009). Analysis of the Indicator Saturation Estimator as a Robust Regression Estimator. In Castle, J. L. and N. Shephard (eds.), *The Methodology and Practise of Econometrics*. Oxford University Press, Oxford.

Nymoen, R. (1989a). Modelling Wages in the Small Open Economy: An Error-Correction Model of Norwegian Manufacturing Wages. *Oxford Bulletin of Economics and Statistics*, *51*, 239–258.

Nymoen, R. (1989b). Wages and the Length of the Working Day. A Empirical Test Based on Norwegian Quarterly Manufacturing Data. *Scandinavian Journal of Economics*, *91*, 599–612.

Nymoen, R. (1991). A Small Linear Model of Wage- and Price-Inflation in the Norwegian Economy. *Journal of Applied Econometrics*, *6*, 255–269.

Nymoen, R. (2017). Between Institutions and Global Forces: Norwegian Wage Formation Since Industrialisation. *Econometrics*, *5*(1), 1–54. Doi:10.3390/econometrics5010006.

Nymoen, R. (2019). *Dynamic Econometrics for Empirical Macroeconomic Modelling*. World Scientific, Boston.

Padhan, R. and H. P. Prabheesh (2021). The Economics of COVID-19 pandemic: A Survey. *Economic Analysis & Policy*, *70*, 220–237.

Pesaran, M. H. and R. P. Smith (2016). Counterfactual analysis in macroeconometrics: An empirical investigation into the effects of quantitative easing. *Research in Economics*, *70*, 262–280.

Rungcharoenkitkul, P. (2021). Macroeconomic effects of Covid-19: a mid-term review. *Pacific Economic Review*, *20*(4), 439–458.

Spanos, A. (2021). Methodology of Macroeconometrics. *Oxford Research Encyclopedias*. Https://doi.org/10.1093/acrefore/9780190625979.013.175.

Vines, D. and S. Wills (2020). The Rebuilding Macroeconomic Theory Project Part II:Multiple Equilbria, Toy Models and Policy Models in a New Macroeconomic Paradigm. *Oxford Review of Economic Policy*, *36*(3), 427–497. Https://doi.org/10.1093/oxrep/graa066.

Visco, I. (2005). From Theory to Practice in Macroeconomic Models: Post Keynesian Eclecticism. *BNL Quarterly Review*, *LVIII*(233-234), 67–90.

von Brasch, T., Å. Cappelen, S. Holden, E. L. Lindstrøm and J. Skretting (2022). COVID-19, tapt verdiskaping og finanspolitikkens rolle. Utredning for Koronakommisjonen. RAPPORTER/REPORTS 2022/15, Statistisk sentralbyrå, Statistics Norway, Oslo-Kongsvinger.

Wallis, K. F. (1977). Multiple time series analysis and the final form of econometric models. *Econometrica*, *45*, 1481–97.