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KOPI
Originalen sendt UB's
håndskriftsamling

It would be most useful I think to concentrate on the non-linear equations first and take the integral equations later.

15th October 1955.

I think you should also look at the determination of the parameters, which is a very important part of the book. Please drop me a word and let me know what you think.

Dear Dr. de Wolff,

Best regards,

I have just received a letter from Allen explaining the contents of his book and I expect in the near future to sit down and work out, somewhat in detail, the plans of our book. Part of it can probably be translated from a Norwegian book, which it has just been decided that my colleague Professor Guldberg and I shall write for the use of our students. Other parts of the book by you and me, however, would have to be worked out afresh, and on more advanced lines.

Even without having yet settled the question of the detailed programme of our book, you may of course start work, because there are certain parts of it which we know ought to be included and for which you would have to take the responsibility. I am thinking in particular of a chapter on integral equations and another chapter on Non-linear Difference and Differential Equations. Particularly the latter topic is exceedingly important from the economic point of view. The type of non-linear equations that occurs most frequently is the one where product terms of pairs of variables (but not second power terms) occur, the product terms expressing the value: price times quantity of a certain commodity. Examples of this type of equations can easily be found. Tinbergen will be able to tell you about some of his problems leading to equations of the type considered. Enclosed you will find another example of the same sort, i.e. equations ensuing from problems discussed in Namur between Koopmans, Tinbergen and me. The line of attack of these equations would be to try and find out what can be said in general about the character of the solutions. The more generalities that can be formulated the better. If the theory is to be really interesting, it would have not only to be a simple numerical process of finding an approximation to the solution in a case where the coefficients have given numerical

Of course the whole question of non-linear difference and differential equations is a vast topic, and an attempt would have to be made to single out those fundamentals that appear the most interesting from our point of view. You would, of course, have to get somewhat acquainted with the literature. It would perhaps be a good plan if you were to look through bibliographical periodicals, such as the "Fort-schritteder Matematik".

It would be most useful I think to concentrate on the non-linear equations first and take the integral equations later.

In your analysis I think you should ^{assume} include all you need about the determinants, matrices, linear transformations, quadratic forms (definite forms, Hermitian forms, etc.).

Please drop me a word and let me know whether this suggestion meets with your approval.

Best regards,

Dear Dr. ...

Cordially Yours,

I have just received your letter from Allen explaining the contents of his book and I am glad to hear that you are working on it. I have just finished a book on the theory of linear transformations which is now being published. I shall write you about it as soon as I have had a chance to do so. I am sure you will find it very interesting. I am sure you will find it very interesting. I am sure you will find it very interesting.

Even without having yet settled the question of the detailed programme of our book, you may of course start work, because there are certain parts of it which we know ought to be included and for which you would have to take the responsibility. I am thinking in particular of a chapter on integral equations and another chapter on non-linear difference and differential equations. Particularly the latter topic is exceedingly important from the economic point of view. The type of non-linear equations that occur most frequently is the one where product terms of pairs of variables (but not second power terms) occur, the product terms expressing the value of the variables, the quantity of a certain commodity. Examples of this type of equation can easily be found. Tinbergen will be sure to tell you about some of his problems leading to equations of the type considered. Enclosed you will find another example of the same sort, i.e. equations arising from problems discussed in Hamar between Koopmans, Tinbergen and me. The line of attack to these equations would be to try and find out what can be said in general about the character of the solutions. The more generalities that can be formulated the better. It is important to be really interested in the problem of finding numerical solutions in case there are any. I am sure you will find it very interesting.

KOPI
Originalen sendt U
håndskriftsamling

Amsterdam den 16 oktober 1935.

Høfraktede herr Professor,

Jeg takker Dem meget for Deres venlighed å forære
mig et exemplar av Deres bok om Confluence Analysis.
Tillike vilde jeg minde Dem om hvad vi har avtalt
i Haag, nemlig at De, hvis det er muligt, vilde
sende mig *Econometrica*, skjønt jeg enda ikke er
mottatt som medlem av foreningen.
For det tredje vilde jeg gjerne abonnere mig på
det "Nordisk Tidsskrift for Teknisk Økonomi".
Jeg har fått kopien av Deres brev til Mr Allen
(min adresse er forresten: 2^{de} Leliedwarstraat 17^e
Amsterdam (C), brevet var avsendt til herr S. de Wolff
Jan Huykenstraat 62 Amsterdam (C)) Jeg har nettop hentet
Nörlunds bok om "Differenzrechnung", jeg tror nok at
denne boken inneholder meget, som kan være oss
til stor hjelp. Videre håper jeg snart å ha leilighet til å
tale litt nærmere om saken med Prof. Finbergen. Jeg synes
ikke at det er muligt å gjøre noe annet, for jeg har fått
Deres notiser.

Med høfraktelse

S. de Wolff

KOPI
Originalen sendt UB's
håndskriftsamling

23^{de} oktober 1935.

Østjakkede professor Frisch,

Utan deres brev har jeg forstått at de selv ikke har skrevet nogle notiser om ikke-lineare differensialligninger. Jeg synes derfor også at det er bedst at jeg skal begynde med å samle på litteraturen over dette emnet. Jeg har endnu ikke kommet til å gjøre dette, fordi jeg ikke visste, hvis de allerede hadde utført en del av arbeidet. Forresten tror jeg nok at jeg nu har fått en temmelig viktig forståelse om hvad slags ligninger vil være av verdi til økonomene. (Jeg har nemlig også sett nogle av Prof. Tinbergens problemer).

Så snart jeg har en oversikt vil jeg sende Dem en liste av hvad efter min mening må tas med. Til sist vil jeg endnu en gang feste deres oppmerksomhet på at min adresse er:

og ikke Dr. S. de Wolff
Jan Luykenstraat 62
Amsterdam (Z)

(Den herre er medlem av "The Econometric Society")

Kanskje er det anledningen til feiltagelsen)

Jeg har også spurt dem å notere mig som abonnent
på „Nordisk Tidsskrift for Teknisk Økonomi“.

Kanskje vil de kunne se til at det blir sendt til den
riktige adressen.

Med høflighet

J. de Wolff

KOPI
Originalen sendt UB's
håndskriftsamling

26th October 1935.

Dr. R. de Wolff,
2de Leliedwarstraat 17¹¹,
Amsterdam (C).

My dear de Wolff,

Thank you for yours of October 16th.

I am writing to Colorado regarding your request to have
Econometrica as soon as possible.

I have also forwarded your subscription for Nordisk Tids-
krift for Teknisk Ökonomi to the office of that periodical.

I have noticed your change of address.

I agree with you that Nörlund's book on the calculus of
differences is an excellent and useful book. I have myself used
it quite a lot. I think you ought to look up my doctoral disser-
tation where I go into certain aspects of this topic. Its title
is "Sur les semi-invariants et moments employes dans l'etude
des distribution statistique." Skrifter utgitt av Den Norske
Videnskaps akademi i Oslo. II Hist.Filos. Klasse 1926. No.3.

With vest regards,

Sincerely Yours,

KOPI

Originalen sendt UB's
håndskriftsamling

Amsterdam den 13 desember 1935.

Høfraktede Herr Professor,

I anledning av et spørsmål av Prof. Tinbergen har jeg utarbeidet en metode, hvorved det er mulig å nedskrive en spesiell løsning av en linear differens likning (med konstante koeffisienter) med annet led, ifall det andre led er en Fouriers serie (^{med}alminnelig: hvis det er en summe av potenser: a^x) det er ikke nødvendig å kjenne løsningen av den karakteristiske likning.

Metoden har jeg gjennomført aldeles symbolisk og Prof. T. tenkte at det kanskje vilde være noe for *Econometrica*. Hvis det er også Deres mening, skulde jeg gjerne få høre det. I så fall skal jeg oversette artikkelen til engelsk og sende Dem den. Hvis det ikke vilde være verd til å optas i *Econometrica*, kunde det kanskje være noe for "boken".

Følgende punkter hadde jeg tenkt mig å skrive om:

$$P (\Delta \phi(x) = \phi(x+1)!))$$

Proof of the following theorem:

If we have a system of linear difference equations with constant coefficients:

$$\sum_{k=1}^{i-1} a_{ik}(\lambda) \phi_k = b_i(x) \quad (i=1 \dots n)$$

(a_{ik} are integer rational functions of λ , b_i are known functions of x)

all ϕ_i are satisfying the equations:

$$D \phi_k = \sum_{i=1}^n b_{ik} \phi_i \quad (k=1 \dots n)$$

$D = (a_{ik})$ Δ_{ik} is the minor of a_{ik} in D .

- 2° Relations between the general solutions of the ϕ 's.
- 3° If the right hand member of the equation:
 $(\sum_{i=0}^n a_i \Lambda^i) \phi = b$ is of the following form $\sum \alpha_i \rho_i^x$
 there exists always a particular solution of the same form
 with other constants. (If one of the numbers ρ is a root of the
 characteristic equation $\sum_{i=0}^n a_i x^i = 0$ the theorem holds still with
 a little alteration)
- 4° If the right hand member is a Fourier series, ~~this~~ we have
 a special case of 3°. But is still possible to write down the
 particular solution in a very simple way
 (This case would probably be the most important in
 economics)
- [5° The same problem if there is a damped right hand member
 of the form $e^{-\alpha x} (a \cos \beta x + b \sin \beta x)$, this is of course an
 other special case].

Jeg håber at de snart vil ha leilighet til å skrive mig
 Deres mening.

Med vennlige hilsener

Deres

Falck-Wolf.

2^{de} Lieliedvarstruak 17^{te}

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Originalen sendt UB's
håndskriftsamling

Slendalsveien 98,
OSLO.
14th December 1935.

Dear Mr. de Wolff,

I talked to a man the other day who suggested an idea about non-linear differential equations. In the theory of atomic functions, for instance, developed by Hartree and others, use is made of an iterative process which consists in first finding the solution of the equation obtained by leaving out non-linear terms. May be this technique can be applied also to the type of non-linear equations we meet in economics.

With best wishes for Christmas and the New Year.
Cordially Yours

Ragnar Frisch.

Ragnar Frisch

Slendalsveien 98, Oslo.

Professor R. de Wolff,

gde, Leliedwarstraat 17^{II}

AMSTERDAM (C),

KOPI
Originalen sendt UB's
håndskriftsamling

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4th January 1936.

Dr. R. de Jolff,
2de Belliedwarstraat 17,
amsterdam (G)

Dear Dr. de Jolff,

Thank you for yours of December 13th. I was much interested to hear about your solution of the difference equation. Of course from the hints you make it is difficult to say whether your results will be suitable for Econometrica or not.

In your proposition (1) I take it that the symbol Δ is an operator meaning a displacement of one unit in the argument of the function written after the operator. The a_{ik} considered as functions of Δ are also operators I take it, hence also the determinant D . The expression $D\phi_k = D\phi_k(x)$ is consequently not the function ϕ_k (for some value of its argument x), but a linear combination of ϕ_k for different values of the argument. In other words, it is only the left member of one single difference equation now involving the single function $\phi_k(x)$. It is, therefore, not yet the "solution" in $\phi_k(x)$. But of course it may still be of considerable interest because the new equation may be easier to solve. In essence I think the theorem is equivalent to the well known fact that a system of difference equations may be replaced by one single difference equation of a much higher order. I think you should bring this connection out so as not to present this theorem as fundamentally novel. The form you have given to the theorem is interesting. You should look up the literature to see whether this same form has been presented before. In view of the obvious analogy to the solution of an ordinary system of linear equations (not symbolic equations), it would be surprising if this theorem has not already

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Once the above theorem is established, I can see that conclusions of the forms which you list under 2, 3, 4 & 5 may be drawn.

I suggest that you try to formulate the whole of this in a very condensed, but nevertheless entirely exact form, using a telegram style, but all the same a precise one. (Do you know Landau's small book on the Theory of Numbers? It is I think the limit of telegram style presentation, but yet absolutely exact.) Having written such a paper, look up the literature and make sure that the results are not already to be found. Then send me the paper and I will see whether it is suitable for *Econometrica*. Please write the paper in English.

Good luck to you, and please give my best regards to Professor Tinbergen when you see him.

Best regards,

Yours sincerely,