

English Translation of Parts of R.F.'s Paper: "Quantitative Formulation of the Laws of Economic Theory"

"Kvantitativ formulering av den teoretiske økonomikkens lover", Statsoekonomisk Tidsskrift, Oslo, 1926. ~~2-125~~

From the end of the 80's and until the end of the century was the time of the great discussion about the ^{cause} ~~cause~~ of value. Böhm-Bawerk characterized its extension when he said in 1892 "Dieselbe Polemik steht im Augenblicke sonzusagen auf der internationalen Tagesordnung".* On one side we find the representatives of the Austrian school: Wieser, Böhm-Bawerk, and their followers, Patten, Smart and a number of other American and European economists. On the other hand, we find those theoreticians who more or less decidedly stuck to the classical theory of value: MacVane, Dietsel, Scharling.** This discussion has left its traces in the theoretical economic thought in the following years, perhaps not so much directly by the results which ^{were} ~~was~~ obtained concerning the very subject of discussion: What is the ^{cause} ~~cause~~ of value, because this in itself is really a futile question, but more by the fact that the discussion came to be a starting point for a profound ^{and} ~~critical~~ revisional work concerning the logical system of theoretical economics. The discussion, namely, showed that the matter of dispute for a large part was due to a lack of accuracy in the statement of the assumption, and the basis for the theoretical arguments. A typical example in this respect was the objection that

*Jahrbucher für Nat. Ök. u. Stat. 1892, p. 321.

**Böhm-Bawerk: Jahr. f. Nat. Ök. 1890, 1892. Quart. J. of Ec. 1890. Z. f. Volksw. 1894. Ec. Journ. 1894. Positive Theorie des Kapitals (3. og 4. utg. Exkurs VIII).

Wieser: Annals of the Am. Ac. 1892.

Smart: Introd. to the Theory of Value on the Lines of Menger, Wieser and Böhm-Bawerk. London, 1891.

Patten: J. f. N. 1891. Theory of Dynamic Economics. Phil. 1892.

McVane: Quart. J. of Ec. 1890, 1895. Ann. Am. Ac. 1895.

Dietsel: J. f. N. 1890 (litt. heuv. p. 561). 1891.

Scharling: J. f. N. 1888, 1904.

marginal utility was not a quantitative notion ^{that is} ~~just~~ did not represent a measurable quantity.

More clearly than at any earlier time it became evident during the discussion that the relatively young economic theory was logically and systematically in a very rudimentary stage, as compared with the older sciences.

This was the situation when a number of scientists went in for systematic work to revise the logical foundation of economic theory. Fisher wrote in 1892: "The truth is, most persons, not excepting professed economists, are satisfied with very hazy notions. How few scholars of the literary and historical type retain from their study of mechanics an adequate notion of force! Muscular experience supplies a concrete and practical conception but gives no inkling of the complicated dependence on space, time, and mass. Only patient mathematical analysis can do that. This natural aversion to elaborate and intricate analysis exists in Economics and especially in the theory of value. The very foundations of the subject require new analysis and definition."²

One would give an incomplete characterization of the movement by saying that it was only a reaction against the foggishness of the value discussion. It had deeper roots and would sooner or later have developed by historical necessity. In any science whose object is the things in the exterior world there always comes a moment when the logical want is awakened, when the immediate, more or less emotional, concepts of the fundamental notions must give way to an objective and exact definition.

But when the critical work with the fundamental notions of economics that started in the 90's at once encountered understanding within a wider group and developed to be more than scattered manifestations of the interest of a few individual men, ^{the} reason was doubtless that the soil was prepared through the value discussion.

²"Mathematical Investigations in the Theory of Value and Prices", Yale University Press, New Haven, 1926 (reprinted) ^{1892.}

The work is far from having been carried to completion. It is going on today and it is one of the most interesting features in the situation in theoretical economics at the moment.

In the following I shall try to picture how this work has progressed. In particular, I shall concentrate on the significance which this development has had regarding the quantitative formulation of the laws of economic theory. I start by mentioning a common characteristic feature, namely the way in which the movement has influenced the relation between economic theory and concrete economic life. Then I shall proceed to some of the special ~~relations~~^{results} that have been obtained so far, and finish by mentioning some of the problems that are still left.

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(The paper then proceeds to analyze the possibilities of quantitative definition of various notions of pure economics, in particular the notion of utility. It states, among other things, the results obtained by Fisher in his doctoral thesis. Furthermore, it states ~~the~~^{some} of the results obtained by R.F. ~~himself~~ in his paper, "Sur un Problème d'économie pure". Then it goes on to analyze the relation between the marginal utility of money and the supply curve of labor.)

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The quantity supplied x of a certain kind of labor measured in hours (of a certain average effectivity) ~~hours~~ per day can as a first approximation be considered as a function $x = f(p)$ of the price of labor, that is, of the wage per hour, p . I shall here not enter upon the controversial question as to what extent it can be considered as possible for the wage earner under the wage system of ~~hour~~^{one}-days based upon collective bargaining to vary the amount of labor. Quite regardless of this possibility, one of the factors that comes in by the fixation of the collective wage agreement is doubtless the question as to how the relation

between hours of work and wage per hour is on the average valued in the group of workers included in the collective wage agreement. The function $f(p)$ must be looked upon as an expression for this average valuation. There is reason to believe that the course of the function $f(p)$ will be as indicated in Fig. 8.

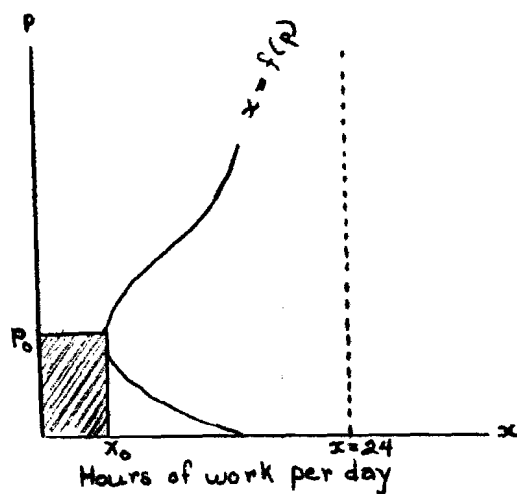


Fig. 8

If the wage per hour p first is very high and then decreases, the amount of labor supplied x will decrease until a certain point x_0 from which point the amount of labor supplied begins to increase if the wage per hour decreases further.

The kinematic theory of prices would limit itself to accepting this form on the supply curve as a fact, a fact that could enter as an assumption in a further argument but which should itself not be the object for a further analysis.*

Marginal utility ^{theory} goes one step further. It wants to investigate why the supply curve for labor has this characteristic shape. If possible it wants

*Cassel's remark: (Theoretische Sozialök., p. 514) is in this respect characteristic "...können wir als normalen Zug der Entwicklung feststellen dass mit steigendem Arbeitslohn die Ansprüche auf Verkürzung der Arbeitszeit immer stärker hervortreten..." It is only a question of "feststellen", not a question of a theoretical explanation of the phenomenon. In this same connection Cassel speaks about the disutility theory in a way which could be interpreted to the effect that a monotonously increasing supply curve follows from the disutility theory. As shown above, the reverse is true. The disutility theory has as its objective first to explain why one part of the supply curve has the shape indicated by Cassel, and second to investigate in which points the change takes place.

to locate the point where the change takes place. This can be done by assuming the following data.

First the marginal disutility of labor being given as a function $y = \varphi(x)$ that after a certain point x_1 is positive and increasing with increasing hours of work per day. (Fig 9),

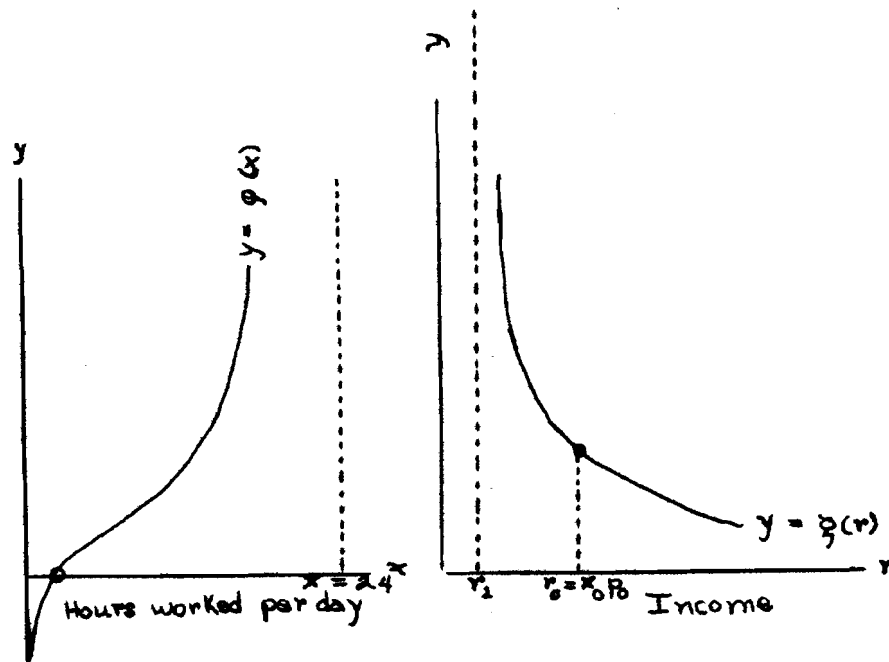


Fig 9

Fig. 10

Second, the marginal utility of money (under a constant price level) as a function $y = g(r)$ which from a certain point r_1 (the minimum of existence) is positive and decreasing with increasing income per day r . (See Fig. 10)

In any point (i.e., for any magnitude of income) the rate of change of the marginal utility of money (Fig. 10) is characterized by the flexibility which is the ratio between the percentage ~~decrease~~ ^{(negative) increase} in marginal utility and the corresponding percentage increase in income. Let us denote the flexibility of the marginal utility of money by \check{g} ; \check{g} is just as g itself a function of the income r .

In the same way can we consider the flexibility of the marginal disutility of labor for that part of the curve in Fig. 9 where the disutility is positive. Let us denote the flexibility of marginal disutility of the labor by $\check{\phi}$. $\check{\phi}$ is here only defined for that part of the curve in Fig. 9 where the marginal disutility of labor is positive (namely for $x > x_1$). In concrete life it is only this part of the curve we need to take account of. It is easy to prove and for the rest nearly evident without ~~proving~~ ^{proof} that the magnitudes of x (hours of work per day) that occur on the supply curve in Fig. 8 must all be larger than x_1 , that is larger than that number of hours of work per day from which the marginal disutility of labor becomes ~~inactive~~ ^{negative} by a further decrease in hours of work per day. For shorter hours per day than x_1 an increase in the number of hours worked will be wanted on account of the enjoyment which is connected with the work.

In the same way, we can consider the flexibility of hours worked $x = f(p)$ as a function of the wage per hour (Fig. 8). Let us denote its flexibility \check{f} . The curve in Fig. 8 is increasing in a given point if \check{f} in this point is positive and vice versa.

This being so, one can prove (proof not given here) that \check{f} (the flexibility of hours of labor as a function of the wage per hour) can be expressed in terms of $\check{\phi}$ (the flexibility of the marginal disutility of labor as a function of ^{hours} _{worked})

and η^v (the flexibility of the marginal utility of money as a function of the income) in the following way.

$$\bar{r} = \frac{1 - (-\eta^v)}{\eta^v + (-\eta^v)}$$

Here are both η^v and $(-\eta^v)$ positive. \bar{r} will therefore be positive or negative accordingly as $(-\eta^v)$ is less than or larger than unity. The supply curve for labor will consequently be decreasing or increasing accordingly as the flexibility of marginal utility of money in the income point in question is less than or larger than unity in absolute value.

The point x_0 where the change takes place is characterized by the fact that the area of the rectangle $x_0 p_0$ is equal to that income r_0 where the absolute value of the flexibility of marginal utility of money (Fig. 10) passes unity.

The result of the previously cited investigation of the marginal utility of money for the group of customers in the Union des coopérateurs, Paris, seems to indicate that the absolute value flexibility of marginal utility of money is larger than unity for the entire income interval of the ^{ordinary} wage earners. This means that the wage equilibrium will be realized in some point on the lower branch of the curve in Fig. 8. The situation will probably be similar for most of the European markets. It is a question whether this will also be the situation for the United States. As yet one has no means of getting a definite answer to this question. It would be extremely interesting if there could be made a comparative statistical study of the flexibility of the marginal utility of money for various European and American markets. This would contribute to throw some light on the collective bargaining fight. So long as the equilibrium occurs in a point on the lower branch of the curve in Fig. 8 this fight must be particularly violent from the laborers' point of view. Here the laborers must either keep status quo or they must simultaneously obtain both wage increase and shorter hours. On the upper branch this does not hold good.

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It is not improbable that one of the reasons why the American labor market is more peaceful than the European is to be found just in this fact that the income interval where the absolute value of the flexibility of the marginal utility of money is less than 1, that is to say, where the collective bargaining fight takes place in some point on the upper branch of the curve, Fig. 8. If this is true, we should here have an example of the opposite extreme of the situation that was realized under the development of the factory system about the end of the Eighteenth Century when a decreasing wage per hour forced the laborers to work fourteen to sixteen hours a day, that is to say, when the wage equilibrium was realized in some point very low on the lower branch of the curve in Fig. 8.