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PRICE INDEX COMPARISONS BETWEEN STRUCTURALLY DIFFERENT MARKETS

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An important problem in economics is to compare the *price of living* (the price level of consumption goods) in two different places or in two different points of time. The most usual formula expressing the price of living in the situation 1 relative to that of the situation 0 is

$$P_{01} = \frac{q^1 p_1^1 + q^2 p_1^2 + \cdots + q^N p_1^N}{q^1 p_0^1 + q^2 p_0^2 + \cdots + q^N p_0^N} = \frac{\Sigma q p_1}{\Sigma q p_0} \tag{1}$$

where q stand for physical quantities and p for prices, and the various commodities and services — N in number — are denoted by superscripts.

The quantities q in (1) are assumed the *same* in both situations. The difficulty is that in fact the quantities consumed by a typical individual or household are *not* the same in the two situations. They will even as a rule change just because of the price change that it is wanted to study. But (1) would loose its meaning as a *price* concept if the q's were made to reflect the actual change in quantities.

To get around this difficulty one may proceed as follows: Assume an *indicator,* i. e., a function $I(q^1 \cdots q^N)$ or shorter $I(q)$ such that if the typical individual has the choice between any two quantity combinations q_0 and q_1, then $I(q_0)$ will be $\overline{\gtrless} I(q_1)$ accordingly as q_0 is equivalent to, preferred to or given up for q_1. Obviously a function $\bar{I}$, obtained by an arbitrary monotonically increasing transformation of I, will also be an indicator. If the indicator has certain convexity properties, the individual in the situation 0 will move along a certain *expansion-path* in the $(q^1 \cdots q^N)$ space as his income increases. Similarly a path, corresponding to the situation 1. Along the 0-path we consider ϱ_0 as a function $\varrho_0(I)$. Similarly $\varrho_1(I)$. We define *equivalence* of ϱ_0 and ϱ_1 by requiring that they correspond to the same value of I. Accordingly

$$P_{01} = \frac{\varrho_1(I)}{\varrho_0(I)} \tag{2}$$

will be the price index sought. In general it will depend on I, if not, we have *expenditure proportionality.* In this case the poor and the rich need to have their incomes multiplied by the *same* number in order to be just as well off in situation 1 as in 0. Satisfactory approximation methods of computing (2) exist when data regarding the two paths are given. (One such method is my "double expenditure" method.)

If not only the quantities consumed, but even the *description* of the commodities are different in the two situations, we have *structurally different* markets. A method of comparison, applicable to this general case, ought of course also to furnish the correct solution (2), if applied to the case where (2) has a meaning. The line of attack will therefore be to consider this simple case, but try here to determine equivalence by a principle which in its final form does not involve neither quantities nor the values of I. For economic reasons it seems appropriate to consider the function

$$\theta_t = 1 - \frac{dE}{dI} \tag{3}$$

where $E = q^1 \frac{\partial I}{\partial q^1} + \cdots + q^N \frac{\partial I}{\partial q^N}$, the differential in (3) being taken along the path t. θ is invariant for a linear transformation of I, the only transformation admissible if we assume *independence,* i. e., assume that $I = J + K$, J depending on some of the q's and K of the others. This assumption is economically plausible. On this assumption I have developped statistical methods of computing θ, using data regarding the paths. Actual computations have been made by authors in different countries. Therefore θ may be looked upon as observable. θ is a pure number, independent of the nature of the commodities.

If ϱ_0 and ϱ_1 are equivalent in the sense defined, they satisfy the differential equation.

$$(1-\theta_0) = (1-\theta_t)\frac{d \log \varrho_t}{d \log \varrho_0} + \frac{d \log \dfrac{d \log \varrho_t}{d \log \varrho_0}}{d \log \varrho_0}. \tag{4}$$

(4) can be solved explicitly, but for the present purpose is taken as it stands. If we have expenditure proportionality, (4) reduces to $\theta_0(\varrho_0) = \theta_1(\varrho_1)$. Economically this seems plausible a first approximation. Starting with this, a correction may be computed using (4). And the process may be iterated. So far I cannot give general criteria for whether the process converges towards a definite functional relationship between ϱ_0 and ϱ_t, and if so which one of the solutions of (4) that is obtained. Numerical examples tend to show, however, that if the indicator fulfills certain conditions (convexity etc.) that are economically plausible, at least a semi-convergence towards the correct solution — i. e., ϱ_0 *equivalent* to ϱ_t — is obtained.