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define the marginal cost of a complex output of branches and tree-trunks obtained in certain proportions.

If the village should be unable to procure from "its own" forest the construction timber it needed, it would have to be purchased from an outside source in exchange, for instance, for a certain quantity of firewood; this would define the value of timber in terms of firewood. With the aid of this rate of exchange, *determined by the market*, it will now be possible to calculate the quantity of firewood which is equivalent to the complex product "timber-firewood", and derive a marginal cost in terms of firewood, which could then be compared with the marginal cost of mining coal.

(*d*) *"Social" Conditions*

What should the community do if it be found that miners contract silicosis, while young lumbermen acquire desirable qualities, both physical and moral? One could not seriously maintain that it is only a matter of adding the cost of treating the disease to the coal bill, and deducting from the cost of wood and timber the expected savings of the wage of a local policeman. The community will, in fact, have to make an essentially political decision, whose terms cannot be measured by a monetary yardstick. However, marginal cost will re-emerge within the framework of the conditions that the community chooses to adopt.

(*e*) *Marginal Costs and Technical Development*

If one day it should occur to a villager that wood could conveniently be moved by cable railway, one of the factors making for rising marginal costs as exploitation is extended to higher slopes, would diminish in importance, and the relative level of lumbering and mining operations will have to be modified to the detriment of the latter. However, such modifications will not be introduced immediately, for the investments made in connection with the mine, which would become redundant by reason of the extension of lumbering operations, still exist. It may pay not to embark upon the construction of the cable railway before these installations are worn out: this is a case where the "historical" nature of economic decisions plays its part.

Moreover, the scheme itself must be given precision: it will be necessary, for example, to know such details as the number of cables, the position of the winding-drums, etc. There may, from a strictly economic standpoint, exist an optimum scheme for the transportation of wood, but various considerations may prevent its introduction. For instance, an influential landowner may be reluctant to see his property spoilt, or it will be found that the cable railway could also with advantage be utilised for transporting hay, and in this case the optimum project may well assume a completely different form.

It follows that the marginal cost of lumbering operations will only be defined within the framework of a particular scheme of operations: it is fallacious *a posteriori* reasoning to maintain that the project has been determined by marginal cost considered as an abstract pre-existing datum.

MONOPOLY—POLYPOLY—THE CONCEPT OF FORCE IN THE ECONOMY

By RAGNAR FRISCH

Translated from French* by W. Beckerman

In this paper I propose to discuss the play of market forces and their interrelations, and the tendency towards equilibrium or disequilibrium in the market. I also intend to formulate a number of definitions, and in particular to discuss the concept of force in the economic field; but my main subject will be the manner in which an important part of the economic mechanism functions.

My examples are drawn from the sphere which may be called the theory of polypoly. It will be convenient, at the outset, to say a few words on the nature of that theory and on its place in general economic theory.

In the classical studies of political economy the concept of free competition played a fundamental part. It is not an exaggeration to say, I think, that this concept is the foundation of almost all classical analytical theories.

Economists studied how prices are formed in the market, how production finds its equilibrium, how the national income is shared out amongst the factors of production etc., and almost all their studies were based on the fundamental assumption that the economic units entering into the analysis, that is to say entrepreneurs, owners of factors of production etc., were subject to the working of free competition.

It is not necessary, here, to formulate exactly all the implications of this assumption; it is sufficient to point out that it implies that none of the relevant economic individuals, namely the entrepreneurs, the owners of factors of production, etc., is, alone, important enough to affect the total situation to any significant degree. In other words, the dispositions of all the individuals could be considered as virtual displacements and consequently it was possible to develop a theory where each individual acted as if the total environment were given for him. For example, buyers and sellers in a market acted as if prices were fixed, producers adjusted the factors of production as if factor prices were given, etc.

This was a very great simplification in theoretical analysis, and it cannot be denied that this extremely simple system has rendered very great services in a large number of cases, particularly in the explanation of the economic situation of the last half of the 19th century.

Cases which could not be resolved with the simple assumption of free competition, were disposed of by the development of a special theory, namely, the theory of monopoly. This case is the complete opposite to free competition. The individual or single unit of enterprise here under consideration is important enough to influence at will certain parameters which characterise the total situation, for example, the price of a monopolised good.

* "Monopole—Polypole—La notion de force dans l'économie", "*Festschrift til Harald Westergaard*", supplement to *Nationaløkonomisk Tidsskrift*, April, 1933.

The concept of pure monopoly was in a way the result of the same preoccupation which had led to the theoretical framework of free competition, namely, the preoccupation with simplicity. Obviously the idea of a single monopolist was an enormous simplification, and it must be admitted that this simplification, too, has, in the past, rendered considerable services in the analysis of certain economic phenomena.

But with the uninterrupted evolution of social and economic institutions, the simplifying hypotheses of free competition and pure monopoly have come to conform less and less with reality. We live in an economic era which is characterised more than ever by a tendency towards trustification in all its forms, by the concentration of financial interests, and by the organisation of technical processes of production in larger and larger units.

At the same time these tendencies have not eliminated the element of competition. On the contrary, in some cases they have made this element more acute than it was under the system which was considered in classical theory. In fact there are but very few cases in which the tendency towards concentration is pushed to the extreme limit of absolute monopoly. It is more usual to find situations where some very large units compete against each other by more or less belligerent methods. There can also be competition between large enterprises on the one hand, and a group of small enterprises on the other, these small concerns acting in the same way as the typical enterprises of classical theory.

In these circumstances it has become essential for an economic theory which is to keep abreast of reality, to supplement the analysis based on the hypotheses of pure competition and pure monopoly, by another theoretical approach which takes account of the possible existence of a certain number of individuals or firms powerful enough to influence the total situation appreciably, without, however, exerting a dominating influence. The dispositions of these units, that is to say these individuals or firms, would no longer be *virtual* displacements but *real* displacements. In mechanical terms one could say that these units would no longer be atomistic but would be finite objects. It is the interplay of forces between these finite units which must be studied. This is the content of a theory of polypoly.

It is customary to speak of "duopoly" when there are two units, that is to say, two individuals or firms which enter into competition; in the same way we speak of "triopoly" when there are three units, and of polypoly when the number of units tends to *n*. Each of these units can be called a "polypolist", or more briefly, a "polist".

Any study of polypolistic situations must first take account of the considerable diversity in the modes of strategy which can be exploited. A preliminary, though very important, section of the theory of polypoly must be devoted to the study of these types. It is necessary to classify them and to take account of the nature of the influences which they can bring to bear on the economic mechanism.

It would seem that Professor Bowley was the first to draw attention to the need for distinguishing between the different modes of strategy. I shall indicate briefly the different modes considered by Professor Bowley, and extend the list to include certain other modes, for the purposes of the subsequent arguments.

MODES OF STRATEGY

- I Elementary Adaption
  - A. Quantity adaptor
  - B. Stochastic price adaptor
  - C. Receiver of an option
  - D. Proposer of an option
- II Parametric action
  - A. Autonomous action
  - B. Conjectural action
  - C. Superior action
- III General negotiation.

First, let us consider certain types which we will call elementary modes of strategy. The simplest of these is the quantity adaptor. This is an individual in a position to buy or to sell a certain good, the price at which the transaction must take place being given, whilst the volume of the transaction can be fixed by the individual himself. A typical example of a quantity adaptor is the individual buyer in a free market, where the quantities involved are very large relative to the quantities which the particular individual could conceive of buying. For such an individual the price is given while the quantity is a variable which he can control at will.

The ordinary concept of demand and supply curves is essentially linked to this mode of strategy. In the case of a quantity adaptor it is indeed possible to consider different possible alternative prices and to register what quantity he will buy at each price. The curve defined in this way, which expresses the relation between price and quantity, is the ordinary supply or demand curve. The quantity adaptor is the simplest conceivable type, and the ordinary supply curves apply to this simple case. This demonstrates the excessive degree of simplification which is implicit in classical analyses based on the concept of ordinary demand and supply curves.

We often encounter situations which in some ways are the inverse of those which we have just considered. The quantity may be given and the price be a variable element which the individual can fix. A typical example is a buyer asking a seller at what price he would supply a certain quantity of goods of a specified quality. In short, it is a situation of "tender". The seller must adapt his price to the datum, which in this case is the quantity.

The most important difference between this situation and that previously considered, is that in the present case the individual is not certain of being able to conclude the transaction after having fixed the parameter which is at his disposal. Whether or not the transaction takes place in the case of "tender" depends on the price fixed by the price adaptor—in our example the seller.

The lower the price he quotes, the greater will be the chances that the transaction will be effected. It can be assumed that for each given quantity the individual will fix his price in such a way that his mathematical expectation of profit is maximised. That is to say, he will fix his price in such wise that the potential increase in profit to be derived from a small increase in price will be exactly balanced by the reduction in the probability of being able to effect the transaction. For this reason we will call the individual who fixes the price in such conditions a stochastic price adaptor.

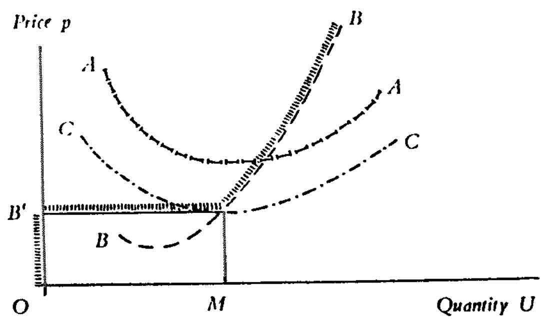

Fig. 1

In this case, too, it is obviously possible to consider a series of alternatives of different quantities and, at each given quantity, to register the price which the individual quotes. We thus obtain a curve which we will call a stochastic supply curve. In the same way we can construct stochastic demand curves. These curves are completely different from the ordinary demand and supply curves. The most important difference is that on the curve now under examination, each point is determined by considerations of probability.

For a producer the stochastic supply curve will always be situated above the average cost curve, abstraction being made from any motive which the producer may have for selling temporarily at a loss.

If the curve *CC* of figure 1 is the average cost curve, the stochastic supply curve may, for example, be the curve *AA*.

If the producer does not make very complicated appraisals of chances in adapting his price to a given quantity, the shape of the stochastic supply curve will resemble the shape of the average cost curve. As a first approximation it can even be said that the stochastic supply curve is obtained by simply shifting the average cost curve upwards by a certain distance. In a more general way, it can perhaps be assumed that the price on the stochastic supply curve is a linear function of the price on the average cost curve.

To emphasise clearly the difference between the stochastic supply curves and the ordinary supply curves I shall indicate briefly how the latter will be situated in figure 1. For this purpose let us consider the marginal cost curve, that is to say, the curve of the derivative of total cost with respect to the quantity produced. This curve will clearly cut the average cost curve at the point where the latter is at a minimum. For example let *BB* be the marginal cost curve: if the producer is a quantity adaptor, he will not enter into the market at a price below the minimum average cost, always abstracting from motives which might induce him to sell temporarily at a loss. At higher prices, he will increase his supply in such a way that the price always covers his marginal cost. The ordinary supply curve will thus be plotted by the intersection of a vertical line *OB′* and a horizontal line at a level equal to minimum average costs, and then by the—always rising—portion of the marginal cost curve which is situated above the average cost curve. A comparison between the curves *BB′O* and *AA* in figure 1 shows quite clearly the essential difference between the ordinary supply curve and the stochastic supply curve.

Let us now consider a third strategic type. Suppose that a buyer or a seller can fix neither quantity nor price but that someone offers him a transaction in which the price and the quantity are both given, the individual having no other alternative for his reply than to say yes or no. We shall call an individual who finds himself in such a situation a receiver of an option. Obviously such an individual is in a much weaker position than a quantity adaptor or a stochastic price adaptor. We can also consider a supply or demand curve for a receiver of an option. This will simply be the line of the boundary which separates the price-quantity combinations where the individual replies "yes" from those combinations where he replies "no". In other words it is simply an indifference line for the individual. This curve we will call the curve of "forced supply".

For a producer the curve of "forced supply" will simply be the curve of average cost, because obviously the producer will accept an option or not according to whether his profits will be positive or negative, assuming always that he has no motive to sell temporarily at a loss. The supply curve for the producer will thus be the curve *CC* in figure 1. This illustrates clearly the weakness of the position of a receiver of an option confronted with someone who knows his average cost curve and who can draw his conclusions. Such a person could, so to speak, pursue the receiver of the option all the way along his average cost curve and, at every point, reduce the latter's profit to zero.

In the economic literature these three types of supply and demand curves, the ordinary, the stochastic and the forced curve, are often confused. For example, the statement is often made that along the falling part of the average cost curve, that is to say to the left of the point *M* of figure 1, the supply curve will be the curve of average cost, whilst beyond point *M*, where the average cost increases, the supply curve will be the curve of marginal cost. In my opinion this is wrong. It is true that the supply curve can sometimes be the curve of average cost, and at other times be the curve of marginal cost, but this does not depend on the position on the quantity axis, but on other assumptions. On the hypothesis that the producer is a quantity adaptor, the supply curve will never be falling, but will be determined by a vertical line, a horizontal line, and a rising portion of the marginal cost curve. Alternately, on the hypothesis that the producer is a receiver of an option, the supply curve will always be the curve of average costs throughout its whole length, and not only along the first part.

The opposite party to the receiver of an option is the proposer of the option, who, for various reasons, is in a position where he can force other individuals to act as receivers of options. The proposer of an option is obviously in a very strong position.

The theoretical cases which we have considered here will be called the cases of elementary adaption. In this elementary group there is one particularly simple type, namely the quantity adaptor, and it is fundamental to the theory of completely free competition.

Let us now proceed to more complicated types. For this purpose we must first define what we mean by a parameter of action. Let us suppose that we have a situation in which there are some polists, and assume that the economic relation between these polists is such that each of them has the power to fix at his will a certain number of economic parameters which characterise the total situation. These parameters will then be called the parameters of action for the different polists.

An example will make this concept more precise. Let us consider a monopolistic producer who employs a certain factor of production which is also sold monopolistically. Let p be the price of the finished article and q be the price of the monopolised factor, u the quantity of the goods and v the quantity of the factor which is employed in the production of the quantity u of the finished goods. It will then be fairly reasonable to suppose that price p as well as quantity v are fixed by the monopolistic producer, whilst price q is fixed by the owner of the factor of production. Such a situation is not the only one which one could imagine, but in any case it is a situation which often occurs. In this case we say that p and v are the action parameters of the monopolistic producer and q an action parameter for the owner of the factor of production.

The action parameters, defined in this way, must be considered as independent parameters. Before finally closing the list of action parameters for any given problem, it is necessary to make certain that there is nothing in the definition of the situation which could prevent the independent variation of these parameters.

If we return for example to the case of a monopolistic producer who employs a monopolised factor, let us suppose that the final demand for the product is atomistic and that we can therefore draw an ordinary demand

curve. Thus u cannot be considered an action parameter for the monopolistic producer at the same time as the parameter p, because if the monopolistic producer fixes the price p, he is obliged to sell the quantity u which corresponds to the demand function. This is a very simple illustration of the need for enquiring whether all the parameters are independent. In more complex situations this investigation into the independence of action parameters can be a fairly difficult question.

There is another point which is relevant to the definition of action parameters. In principle, each polist has the power to fix his parameters at will. But that does not mean that he will act without taking into account the actions of other polists. On the contrary, his final decision concerning the fixing of his parameters will be influenced by a whole series of often very complex considerations, which include the known actions and even the potential actions of other polists. Thus the action of all the polists can have an *indirect* influence on any given parameter. Nonetheless, the only direct influence that can affect a given parameter is that of a polist who possesses, and exercises, the actual power to fix that parameter. This is an assumption we have made in introducing the idea of action parameters. We have, so to speak, classified the independent variables which figure in the problem and we have designated each parameter as belonging to one of the polists; this is a fairly realistic procedure which will make systematic analysis possible.

The action parameters of polists No. 1, 2 .... m, shall be designated by

$$
\begin{array}{c}
z_1^1 \dots z_\alpha^1 \\
z_1^2 \dots z_\beta^2 \\
\dots\dots \\
z_1^m \dots z_\gamma^m
\end{array}
$$

The total number of action parameters will thus be:

$$\alpha + \beta + \dots\dots + \gamma = N$$

A theoretical system based on this concept can be called a system of parametric adaption or parametric action, or, yet more briefly, simply a parametric system. This is a much more general system than that defined by the elementary strategic types. A parametric system is a mechanism which possesses N degrees of freedom, that is to say, as many degrees of freedom as there are action parameters; we have to study the way in which this mechanism functions, for example its equilibrium or its lack of equilibrium, etc.

For this purpose it is now necessary to introduce and study the profits of the m polists. These profits can be of very different types. Some polists may for example be producers and their profits will be determined as functions of the price of their goods and the prices of the factors of production, etc. Other polists may be intermediaries between producers and final consumers.

Yet other polists may be final consumers, their profits being defined in terms of utility functions or indices of utility. From a general methodological point of view it does not matter much how we define the different profits. We shall simply assume that the profits of each polist are unequivocally defined as being a function of the N independent parameters which constitute the problem, that is to say the N action parameters. Let these profit functions be represented by

$$r^h = r^h\,(z_1^1 \ldots z_\alpha^1, z_1^2 \ldots z_\beta^2, \ldots z_1^m \ldots z_\gamma^m) \qquad (h = 1,2 \ldots m)$$

Obviously each polist tries to maximise his profits, and in order to do so, he can vary his action parameters. However, his profit depends not only on his own action parameters, but also on some of the action parameters of other polists, perhaps even on all the other parameters. It is therefore important for each polist to know in what direction and to what degree a change in his own parameters is likely to provoke a change in the parameters of other polists.

Let us return once more to our example of the monopolistic producer and the owner of a monopolised factor. If the owner of the monopolised factor sees that the monopolistic producer increases the price of the final product, he will probably be inclined to increase the price of the factor. And on the other hand, if the owner of the factor increases the price of the factor, this will probably lead the monopolistic producer to employ less of that factor. If there is any possibility of substitution, he will probably try to replace the monopolised factor to a certain extent by other factors.

The manner in which each polist forms an opinion about the repercussions on the action of other polists which might be provoked by a change in his own parameters, is absolutely fundamental to the functioning of the mechanism which we are examining. It seems quite plausible to take this aspect of the problem as a starting point and to classify the different cases according to the opinions of the different polists on this question. This is a line of approach which seems very natural, but which nevertheless does not so far seem to have been used in the literature in any systematic manner.

I propose to consider here three different types, according to the opinion of the polists on the above question. First, I shall consider *autonomous action*. This is the case in which each polist is aware of the importance of the different action parameters which actually exist on the market, but acts as if a small change in his own parameters would not induce a change in the parameters of others. In other words, each polist considers his own parameters as variables and the parameters of other polists as constants given by the actual situation. When a polist tries to maximise his profits on this assumption I say that he acts according to a system of autonomous adaptation.

The classical example of such a situation is the case studied by Cournot, in which he considered two or more producers of the same good, each producer adapting his quantity on the assumption that a change in his own quantity would not lead to a change in the quantities produced by the others, but would probably produce a change in the market price by virtue of the fact that the increase or decrease in the quantity produced by him will constitute an addition to, or a subtraction from, the total quantity on the market.

Let us now consider the situation where polists take account of the possibility that a change in their parameters will induce a change in the parameters of the others. We shall first take the simplest case imaginable; namely the case where each polist acts as if the possible change in the other parameters was going to be a continuous function of the change in his own parameters, or more precisely, a function which possesses a derivative. To specify the nature of this function we introduce elasticities:

(1) . . . . $$z_{ij}^{hk} = \frac{\text{đ} z_i^h}{\text{đ} z_j^k} \cdot \frac{z_j^k}{z_i^h}$$

The coefficient defined by equation (1) expresses the change in the parameter z_i^h which polist k believes he will induce by a change in his parameter z_j^k.

I wish to stress the fact that these coefficients do not necessarily express what will *in fact* happen if polist k slightly changes the parameter z_j^k; but what polist k *thinks* will happen. For this reason I will call these coefficients *conjectural* coefficients or *conjectural* elasticities, to distinguish them from *real* elasticities which express what actually happens.

To indicate the nature of the increments in definition (1) I have employed a special symbol đ which can be called the symbol of conjectural partial differentiation. It is convenient to write

(2) . . . . $$z_{ii}^{hh} = 1$$

because if polist h changes his parameter i in a certain proportion, he knows that this parameter will be changed in that proportion. The adaptation which takes place in a system of conjectural coefficients I shall call conjectural adaptation.

It will readily be appreciated that the case of autonomous adaptation is the special case of conjectural adaptation, where the matrix of the conjectural coefficients is quite simply the unit matrix, that is to say the matrix in which all the elements are zero except those which are defined by equation (2).

Finally let us consider the case of *superior adaptation*. Here the assumption is that amongst the polists there is a group of individuals who act in an autonomous manner; that is to say, their conjectural coefficients are all equal to zero with the exception of the direct coefficients defined by equation (2). Let us further assume that there is another group of polists who know that the polists in the first group act in an autonomous manner and who are also aware of the nature of the profits which the polists of the first group try to maximise. In such a situation the polists of the second group can play, so to speak, on the entire mechanism within the sphere of action of the polists of the first group. In the relation between the polists of the latter group, *vis-a-vis* those of the former group, the conjectural attribute no longer applies: the coefficients in question are now all real coefficients. The conjectural element which enters into the considerations of the polists of the latter group consists only of the conjectural coefficients amongst the individuals of the second group themselves. In this case we say that the individuals of the second group act under a system of superior adaptation.

A special case is that in which the second group is composed only of a single individual; here the manner in which this individual acts will be similar to that of a monopolist who finds himself faced with a series of markets between which he can discriminate.

It is possible to conceive of a whole hierarchy of superior adaptors. There may perhaps be a third group which knows the reactions of the polists of the second group. Thus the polists of the third group can play upon and exploit indirectly the whole mechanism which is defined by the polists of the first and second group. Further there may also be a fourth group composed of polists who are acquainted with, and who can thus exploit, the manner in which the third group acts and so on.

And if in addition there is a last group which is composed only of a single polist, of ultimate power, he will be able to exploit the whole mechanism in order to maximise his profit.

In considering types of adaptors so far, we have assumed throughout that each parameter of the problem can be classified as belonging to one or the other of the polists as an action parameter. There exist, however, much more complex situations in which such a classification cannot be made. These are situations in which the different parameters of the problem, or at least a certain number of these parameters, are no longer fixed in a decisive manner by any one polist, but are subject to negotiation, the particular technique of which exercises a profound influence on the whole functioning of the economic mechanism under consideration.

A concrete example of such a situation would be a case of negotiation between a group of organised employers and a group of trade unionists. Here it can no longer be said that one or the other of the two parties fixes the wage on his own initiative. To reach agreement on new wage scales the parties nominate delegates who enter into negotiation. These negotiations involve quite particular techniques in which both parties use the strategy which they think best for achieving their aim. The concept of an action parameter is, in this case, too simple a concept. The problem of negotiation has been studied particularly by Professor Zeuthen.

I do not wish to dwell any further on these more complicated cases, and will return to conjectural adaptation to see how the economic mechanism finds its equilibrium in this case.

To analyse this situation I have introduced the concept of "force of attraction" for the different parameters. These are coefficients by which I try to express the intensity of the motive which induces any polist to increase or

decrease the parameter under consideration. This motive obviously depends on two things: First on the nature of the function which expresses how his profit depends on all the parameters of the problem, in other words on facts which one must assume the polist knows in an objective manner. Secondly, his motive for increasing or decreasing the parameter under consideration, depends also on the change that he thinks will occur in the parameters of the other polists as a result of a change in his own parameters. These are the conjectural consequences which are characterised by the conjectural elasticities which we have defined above.

By combining these two elements a given polist can make some estimate of the total change which might occur in his profit if he slightly changes one of his action parameters while keeping his other parameters constant, and assuming that the parameters of the other polists vary with the parameter in question in the manner defined by the conjectural coefficients. On these assumptions of the variability of the parameters we can define the elasticity of profits r^h with respect to the parameter z_i^h, that is to say the coefficient

$$(3) \quad \omega_i^h = \frac{\bar{d} r^h}{\bar{d} z_i^h} \cdot \frac{z_i^h}{r^h}$$

This coefficient may be called the coefficient of attraction of the parameter z_i^h, because it somehow expresses the intensity of the motive which induces polist h to increase or decrease this parameter.

It would be easy to make the variables in formula (3) more explicit and to express them with the aid of the actual partial derivatives of the profit functions and the conjectural coefficients, but I do not wish to stress this point.

Let us now represent the N parameters z_i^h as the coordinates of a point in an N-dimensional space. We can consider z_i^h as the symbol representing a point in this space. If we assume that the conjectural coefficients are functions of the point z_i^h, which appears to be fairly plausible, we can consider the N quantities ω_i^h as the components of a vector attached to the point z_i^h. On account of the significance of these components, this vector will express the intensity of the attraction which exists on the market as a whole, and which tends to change the situation.

We are thus led to study the movement of an actual point in a field of forces defined by the vector ω_i^h. From a formal point of view this analysis is completely analogous to the analysis of the mechanical case. Nevertheless we shall not pursue this analogy. It will be more interesting to consider a particular economic case and to examine it in some detail.

Suppose that we have two polists producing the same product, the demand being differentiated in some way so that the two polists can fix two different prices, say p^1 and p^2, without all the demand being absorbed by the one who fixes the lower price. In other words, there exists a certain friction in the market so that the law of price indifference does not apply.

But apart from that we assume the usual atomistic state of demand. Thus p^1 can be taken as an action parameter for the first polist and p^2 as an action parameter for the second polist.

The state of demand being given as well as the technical conditions of production for the two producers, it is easy to formulate the profit functions r^1 and r^2, that is to say, to formulate how r^1 and r^2 depend on p^1 and p^2. It is not necessary to write these functions here explicitly.

If the conjectural coefficients are given, then the two attractions ω^1 and ω^2 are given. The vector ω, the components of which are ω^1 and ω^2, will thus be defined at each point of the space (p^1, p^2). This space we will call the adaptation space.

The structure of the vector field, so defined, now characterises the tendency towards equilibrium or disequilibrium. In figure 2, we have represented a case of stable equilibrium. The direction of the vector is so distributed as to make the lines of forces of the field tend towards a central point T.

The most important feature which characterises the nature of the field are the two lines in figure 2, which we can call the frontiers of attraction for polists No. 1 and No. 2 respectively. Let us examine, for example, the frontier of attraction (I), this being a line which separates the points in the adaptation space in which polist No. 1 is inclined to increase his price from the points where he is inclined to reduce it. If we find ourselves at any point to the left of this line, polist No. 1 will increase his price. The vector of the field here

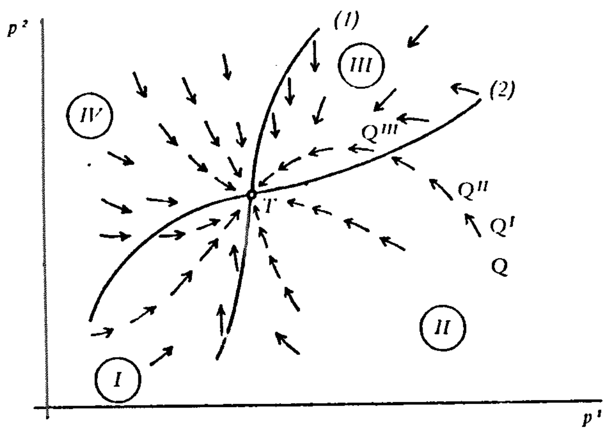

Fig. 2

lies in the first or the fourth sector, and at any point to the right of this position we have the opposite situation. Finally, at any point actually on the line, polist No. 1 is interested neither in a reduction nor in an increase in his price.

In other words the vector of the field is vertical along this line. In an analogous fashion the frontier of attraction for polist No. 2 separates the points where he wishes to increase the price from the positions where he wishes to reduce it.

The field is thus divided into four sectors indicated I, II, III, IV, in figure 2. In sector I both polists are interested in raising the price. In sector II, polist No. 1 is interested in reducing his price, whilst polist No. 2 wishes to increase his. In sector III both polists wish to reduce their prices and in sector IV the first polist wishes to increase his price whilst the second wishes to reduce his. With such a construction it is obvious how equilibrium will be brought about. Let us take an actual point representing the situation in the market. If, for example, we take an actual point in position Q it will follow the path Q^I, Q^{II}, Q^{III}, towards T. In more concrete terms we can say that in the position Q the first polist believes that, taking all the factors into account, he can increase his profit by reducing his price, whilst polist No. 2 believes that he would be wise to increase his price. The result will thus be a change in the market situation towards the positions Q^I, Q^{II}. After a very marked change in this direction, more precisely beyond the position Q^{III}, polist No. 2 discovers that he has increased his price too much, and the competition of polist No. 1 is beginning to trouble him. This is expressed geometrically by the fact that the vector in the line of force which passes through Q^I, Q^{II}, Q^{III}, enters into the third quarter past the point Q^{III}. The action of polist No. 1 on the contrary remains the same throughout the whole movement. He reduces his price up to the point T which represents the position of market equilibrium.

The above study of the movement towards equilibrium in which we have used the concept of the field of attraction, is obviously more general than the simple study which merely stipulates that the two attractions together must equal zero. This last condition also leads to the determination of the central point T, but it gives us no information at all about the manner in which the market reaches its point of equilibrium, and it does not even enable us to say whether this point of equilibrium is a stable point or not. The concept of the field permits us to state in fairly realistic terms how the equilibrium is reached. We can express the elastic links, so to speak, between the two polists in the course of the actions which finally lead to the point of equilibrium.

The theoretical tool which we have employed can also be used to study the effects of changes in the conjectural coefficients. It is, for example, interesting to compare the equilibrium position which will be reached in the case of autonomous adaptation with that which will result in the conjectural case. Suppose that the two solid lines of figure 3 are the boundaries of attraction in the conjectural case and that the two dotted lines are the boundaries of attraction in the autonomous case. In assigning values to the derivatives in the definitions of ω^1 and ω^2 it can be demonstrated, in the general case, that when polist No. 1 changes his adaptation from the conjectural system to the autonomous system, his boundary of attraction moves to the left. And if polist No. 2 changes his adaptation from the conjectural system to the autonomous system, his boundary of attraction moves downwards.

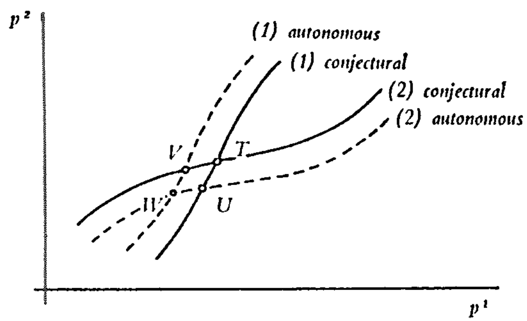

Fig. 3

Let us now consider the four curves of the diagram: T is the point of equilibrium when the two polists act in a conjectural manner; U is the point of equilibrium when polist No. 1 acts in a conjectural manner and polist No. 2 in an autonomous manner; V is the point of equilibrium when polist No. 2 acts in a conjectural manner and polist No. 1 in an autonomous manner; and finally W is the point of equilibrium when both polists act in an autonomous manner.

An analysis such as we have now made contains elements which are almost dynamic. In effect, the introduction of the vector has permitted us to pose the problem in terms of force, and we have considered an equilibrium determined by this force. However, one essential dynamic element is still lacking, namely the analysis of the speed of movement and the connection between the concept of speed and that of force. That is a subject with which I propose to deal elsewhere. This concept, which is essentially dynamic, will lead to the notion of cyclical oscillation. We shall there again meet the concept of friction, and we will have to discuss this fundamental dynamic problem: what is the source of energy which maintains these oscillations and which keeps economic life in a state of perpetual flux where static equilibria are never realised?