96

Frisch on Wicksell

Knut Wicksell (1851–1926) stands out among the Scandinavian school of economists, which includes such men as Gustav Cassel (1866–1945), Eli F. Heckscher (1879–), Bertil Ohlin (1899–), and Gunnar Myrdal (1898–) in Sweden; Ragnar Frisch (1895–) in Norway; and Frederik Zeuthen (1888–) in Denmark. Inspired by the work of the Austrians and of the Lausanne school, Wicksell developed the marginal productivity theory of distribution, integrating it with the theory of capital and interest. His principal claim to fame rests on his contributions to monetary theory, based on the notion of monetary equilibrium and on the distinction between the actual rate of interest and the "natural" one which would equate the amount of loan capital demanded and that of savings supplied. Wicksell's influence on modern economic thought has been profound and far-reaching. It is noticeable in Hayek's overinvestment theory of the business cycle, with its emphasis on the notions of capital shortage and forced saving, in Schumpeter's theory of economic development, in Frisch's dynamic theory of the business cycle, and in the recent discussion of saving and investment. Moreover, Wicksell's use of mathematics, although often hidden by the literary form of presentation, has set an influential precedent.

Wicksell's principal works are available in English translations: *Interest and Prices* (1936; first published 1898), and *Lectures on Political Economy* (2 vols., 1934; first published 1901–1906).

Ragnar Frisch, associated with the University of Oslo as a student and teacher, is himself a distinguished member of the Scandinavian school and a leading mathematical economist. His appraisal of Wicksell is a model of sustained technical reasoning. Readers unfamiliar with his technique may supplement the study of his essay by one of the articles cited on pp. 655 f., below.

HOMAGE

IN THE *Archiv für Sozialwissenschaft und Sozialpolitik*, 1927, Joseph Schumpeter wrote an introductory article to a paper by Knut Wicksell. Schumpeter's article was written in his incomparable, brilliant style and with the warmth of his fine heart. Nobody who wants to understand Wicksell's greatness must miss this article. The first part of it is in all its brevity so much to the point that it is entirely useless to try to produce anything better by way of introduction to a study of Knut Wicksell. So here it is in translation:

"When the *Archiv* introduces to its readers the last work of Knut Wicksell, it brings, as an exception to its usual policy, a work which has already appeared elsewhere, namely in the *Ekonomisk Tidskrift*, 1925. There are two reasons for this.

"First of all we wanted to honour the Swedish Marshall, and once again see the greatest name in Nordic national economy in our pages. And it is fitting in this case to derogate still more from our usual practice and bestow on the man and his works some words, as his significance in wider circles of colleagues is not yet sufficiently valued, his message not yet exhausted. This is owing to the fact that his character excluded every kind of advertisement, that his amiable modesty left no room for any emphasis on his own contributions, and that he never stressed his powerful originality and never neglected to give the researchers to whom he was attached what was their due. However, it is not only our sense of duty which leads us to render him a justice which he never claimed himself, but also the recognition that scarcely any other of the architects who have laid the foundations of modern analysis, have so much to give us to-day—to give everyone of us who are growing, developing and struggling for new ways and views—as he. This is not only due to his wealth of thought, but also to the traits of his character. As he always thought of the subject only and never of himself and what could serve his own best, he had a style which, indeed, is neither smooth nor simple, but which for that very reason gives us a look into his workshop. We trace the vivid flash of constructive imagination, we see the original formulations, the difficulties and doubts such as they presented themselves

[Not published before.]

to the author. Therefore, he gives us more than the actual result; he teaches research itself and points in every line beyond himself. This is very rare. We perceive most clearly how rare it is when a master shows in detail the machinery of an inquiring mind, as did Ernst Mach concerning mechanics and the theory of heat. For a number of reasons, the most important one being the lack of competent professional criticism, that way of presenting and teaching is nowhere as rare as with us. All the more we must admire and be thankful to the man who, despising any personal success, has been teaching this way. And while considering his lifework, we remember some words that Mach said of Huyghens: 'The remotest generation will know that he was only a human being, but they will also know what kind of a human being he was.'"

Personally, I never met Knut Wicksell. I saw him once when he delivered a lecture in Oslo, but being an unassuming student at the time, I did not have the courage to talk to him. I only remember the appearance of a friendly, obliging, intelligent-looking, elderly gentleman. So my knowledge of his theory came only through his writings. That, however, was a very intense and absorbing form of making his acquaintance. Already from my early student days, I read his writings (in German and Swedish) avidly. And I continued to do so later. There is probably no other economist who has had so much influence on my thinking, at least not in monetary theory. When looking through old notes from lectures I delivered in the Oslo University 1934–35 on modern monetary theories—including besides Wicksell also Lindahl, Myrdal, Marco Fanno, Robertson, Pigou, Keynes, and others—I found that what I did all the time was to classify and treat these theories more or less as so many different ways of formulating Wicksell's fundamental ideas—or of *misunderstanding* them.

When I started my study of Wicksell, I found that his works were not easy reading. Often it was only at the third or fourth reading that I grasped his ideas. Invariably, each new reading made me more and more enthusiastic. Sometimes it happened that I thought I had finally caught him in an inconsistency or in unclear thinking. Every time this happened, it turned out, however, that the error was mine. After a number of such experiences, I reached the conclusion that whenever a person thinks that he has found an inconsistency or a piece of unclear thinking in Wicksell's works, and wants to "correct it," that is only a sure criterion that the person in question has not yet penetrated to the bottom of Wicksell's ideas. The discovery of the fact that Wicksell is, after all, right, will always be a matter only of patience and intelligence on the part of the reader. That conclusion I reached rather early in my study of economics, and later have never had any reason to change it. The impression has been reinforced these days as I have gone through his works anew when writing this paper.

On the economic thinking in the Scandinavian countries the works of Wicksell have had an enormous influence. I think it is correct to say that all living, outstanding economists in these countries have a good knowledge, in many cases a thorough knowledge, of Wicksell's ideas, and have to a large extent applied types of reasoning similar to those of Wicksell.

I don't propose to discuss here the innumerable ways in which Wicksell's ideas and teaching have influenced economic policy. Only one significant fact might be mentioned: Wicksell was usually far ahead of his time in constructive, practical suggestions. One example is the uses that were made of the gold exchange standard and similar systems which he advocated a long, long time ago. Another example in point is the use of the proceeds from export duties for subsidies in order to keep internal prices down. This plan was worked out by Wicksell during World War I.[1] It was not put into effect then, but measures of this sort have in recent years played an important role in the economic policy of Scandinavian countries.

In the following I shall not only give a brief account of the basic points that are to be found explicitly in Wicksell, but also give certain mathematical developments aiming at a condensed synthesis of the main structure of his reasoning in matters of economic theory. I shall confine myself to those parts of the structure, which, as I see it, are the vital ones, leaving all details aside. The essential points, on the other hand, I shall try to cover fairly thoroughly.

1. WICKSELL'S LIFE

Facts about Knut Wicksell's life and works are reported in a number of special papers.[1–14] Suffice it here to give a brief outline.

[1] See Eli Heckscher, p. 82, in *Penningväsenet och penningpolitik*, Stockholm, 1926, with references to Wicksell's work in the question.

[1a] *Lunds Universitets årsberättelse* 1916–17, p. 8.

[2] Oskar Jaeger, "Johan Gustaf Knut Wicksell." *Statsøkonomisk Tidsskrift*, Oslo, 1926.

[3] Bertil Ohlin, "Knut Wicksell (1851–1926)." *Economic Journal*, 1926.

(*Footnotes 4–14 continued on next page*)

THE PERIOD UNTIL 1885

Johan Gustaf Knut Wicksell was born 20 December 1851 in Stockholm. After having passed his first examination (candidatus philosophiæ) in the University of Uppsala in May 1872, he embarked upon extensive postgraduate works with strong emphasis on mathematics. His postgraduate university studies were not very regular—he worked for instance occasionally as a teacher—and thirteen years would elapse before he passed his final university examination (licentiatus philosophiæ) in mathematics in 1885.

Wicksell had come from a religious home, but passed through a religious crisis and became an opponent of the orthodox form of the Christian religion. His basic attitude in matters of religion is, however, not clear because he never talked about this even with his nearest friends.[15] One thing, however, is clear. He had a high moral standing, a great, warm heart, and a deep sympathy for those not

[4] E. Sommarin, "Minnesord över professor Knut Wicksell." *Kungl. Hum. Vetenskapssamfundet*, Lund, 1927.

[5] E. Sommarin, "Förord" to the Swedish 1927 edition of Wicksell's *Föreläsningar*, Vol. I.

[6] Joseph Schumpeter, "Zur Einführung der folgenden Arbeit Knut Wicksells." *Archiv für Sozialwissenschaft und Sozialpolitik*, 1927.

[7] E. Sommarin, "Das Lebenswerk von Knut Wicksell." *Zeitschrift für Nationalökonomie*, 1931.

[8] Valfrid Spångberg, "Knut Wicksell och Verdandi." *Verdandi genom femtio år.* Stockholm, 1932.

[9] Johan Åkerman, "Knut Wicksell, a pioneer of econometrics." *Econometrica*, 1933.

[10] Lionel Robbins, "Introduction" to the English edition, 1935, of Wicksell's *Lectures on Political Economy.*

[11] Bertil Ohlin, "Introduction" to the English edition, 1936, of Wicksell's *Interest and Prices.*

[12] Gustav Cassel, "Konkurrensen om Lundaprofessuren" (In *I förnuftets tjänst*), Stockholm, 1940.

[13] Eli Heckscher, "David Davidson." *Minnesteckning foredragen på Vetenskapsakademiens höglidsdag den 31 mars 1951.* (Certain parts also concern Knut Wicksell.)

[13a] [C. G. Uhr, "Knut Wicksell—A Centennial Evaluation." *American Economic Review*, 1951—Ed.]

[14] For the collection of printed source material on Knut Wicksell's life and publications, I am obliged to Mr. Arne Amundsen, research associate at the University Institute of Economics, Oslo. Mr. Amundsen has also worked out a list of Knut Wicksell's published works, books as well as articles. The list may be obtained in mimeographed form from the Institute.

[15] Sommarin Ref. 4, p. 22.

living on the sunny side. He was willing to fight for them even though it might mean sacrifices to himself.

As a student he came in contact with a group of radicals, including amongst others the (somewhat older) great poet August Strindberg, the physiologist Hjalmar Öhrvall, and the politician Hjalmar Branting. In the spring term of 1880 he delivered two addresses (on 19 February in a temperance society and on 25 February before an academic public) on poverty, drunkenness, prostitution, and neo-malthusianism. He had witnessed how the unrestricted production of children kept the lower classes in misery, and spoke openly of the remedy: neo-malthusianism. His views and conclusions in this matter also appeared in print. As could be expected a storm of protests arose from conservative quarters. To this Wicksell replied in a booklet (1880).

The Period 1885–1900

Having passed his final university examination in mathematics in 1885 he was awarded a stipend of the Lorén Foundation. Now his scientific studies in economics developed in full. He went to England, France, Switzerland, Austria, and Germany. The progress of his economic studies and the type of problems and the authors to which he devoted his energy during this period (John Stuart Mill, Böhm-Bawerk, Karl Menger, and others) can be followed from his public speeches and addresses, which in many cases appeared in print afterwards.[16]

At the same time the discussion on neo-malthusianism and the questions connected with it continued. In 1886 and 1887 he gave several talks on this, not only in Sweden, but also in Denmark and Norway. On 16 March 1887 he talked in the radical students' association "Verdandi" that counted many members who have later become prominent in the scientific and political life of Sweden. The association was founded (1882) more or less in protest against the attempts at curtailing the freedom of speech that had followed Wicksell's talks and publications in 1880.[17]

In 1889, at the age of 38, he married the Norwegian Anna Margrethe Kristine Bugge. Wicksell's wife had passed a university examination in Kristiania (now Oslo) in 1886, and she graduated in Law in Lund 1911. She took part in public life, being amongst others a Swedish representative in League of Nations activities.

[16] Sommarin Ref. 4, p. 30. [17] Spångberg Ref. 8, p. 217.

In 1895 Wicksell passed the final university examination in economics and became a doctor of philosophy the same year. It is interesting to note that as late as January 1898 at the age of 46, he wrote:[18] "I hold no teaching post, so that my scientific work is made possible only by special grants. I have in the first place to express my profound gratitude to the administrators of the Lorén Foundation, who for the third time have made me a generous grant. It further gives me particular pleasure to express my respectful appreciation to the Government of Sweden for making me a grant towards this work.".

In 1899 he graduated in Law, and the same year became a docent (assistant professor) in Economics and Public Law.

The Period 1900–1926

In 1901 Wicksell competed with Gustav Cassel for the professorship in Economics and Public Law which had become vacant in Lund after the retirement of Professor G. K. Hamilton.[19] The affair developed in a rather dramatic way. Wicksell's position as an economic scientist was at that time well established and it was quite probable that he would be appointed to the vacant post if the decision was made on the basis of scientific competence alone. That, however, did not seem to be what was going to happen. Conservative quarters did not want to see Wicksell appointed. A chief argument was that a person who had taken such a position regarding neo-malthusianism as Wicksell had, could not be considered fitted for the task of guiding and enlightening others. These quarters therefore worked for the appointment of Cassel. Cassel was scientifically in strong opposition to Wicksell and wanted to have a competent, scientific scrutiny of the points at issue. He wished, of course, very much to see a decision in his own favour, but could not take advantage of any support offered him for other reasons than strictly scientific ones. Therefore, in protest against the kind of arguments that had been used against Wicksell, he withdrew his application, and Wicksell was appointed to fill the vacancy. Even if Wicksell would on the strength of his outstanding scientific contributions have been appointed in any case, Cassel's gesture commands great respect for his scientific integrity and sense of fair play.

In November 1908 Wicksell gave a talk in Stockholm on "The throne, the altar, the sword and the purse." For his forceful, pointed

[18] In the preface to "Geldzins und Güterpreise."
[19] Cassel. Ref. 12, p. 33.

and open way of presenting his views on these things he was sentenced to two months' imprisonment. The terms of his imprisonment cannot have been very severe because some of his well-known writings were produced during this imprisonment.

With the fall term of 1916 he retired from his chair in Lund and moved back to Stockholm for which he had been longing during the professorship years in Lund.[20] His wife had built a home for them on the seaside near Stockholm, about twenty minutes' rail trip from the centre of the town. Wicksell did not like the idea of becoming countryfied and of stagnating. From 1917 he was nearly always present in the meetings of the Stockholm economic society and frequently took part in the discussions, always injecting valuable viewpoints and penetrating theoretical remarks, many of which are preserved in the printed proceedings of the society. He was also busy writing promemoria and articles on monetary questions, but most of all he liked to be in his quiet study at home working on some theoretical problem that appealed to his keen, still unfailingly sharp intelligence. In this way his years passed by, full of activity, until an accidental cold, which unexpectedly developed into pneumonia, ended his life on 3 May 1926.

2. THE THEORY OF CAPITAL AND THE PRODUCTIVITY RATE OF INTEREST

It is unfortunate that Wicksell did not put down his theory of capital in a complete mathematical form. If he had done so, he would have saved his commentators a lot of trouble and helped tremendously to popularize his ideas in our generation. The word popularize here is not a printer's. error. We have now—I don't hesitate to say fortunately—reached a stage where the younger generation is very reluctant to use its time and energy on discussions of really complicated points of economic theory unless these points are expressed in rigorous mathematical terms. At the time of Wicksell the situation was entirely different. He simply had to write in a semi-mathematical and literary style if he wanted his writings to be read outside a small group of specialists. It therefore seems worth while to attempt a brief mathematical summary of Wicksell's theory of capital. In presenting such a summary I shall try to reduce the theory to its lowest terms and use a notation which fits in with modern macro-economics.

[20] Sommarin Ref. 4, p. 1.

Wicksell's theory of capital is a theory of the *stationary state* in a society where there are two primary factors of production, land and labour. Suppose that each year there is performed a number of units of services of land equal to x. Out of this total amount a part x_0 is used in such a way that its fruits become immediately available, another part x_1 in such a way that its fruits only become available 1 year hence, still another part x_2 used in such a way that its fruits become available 2 years hence, etc. Similarly for the services of labour (for instance for labour hours) $y_0, y_1, y_2 \cdots$, etc. We then have by definition

$$(2.1) \qquad x = x_0 + x_1 + \cdots + x_n \qquad y = y_0 + y_1 + \cdots + y_n$$

where n is the longest period of delay between an input element and the output element which is attributable to it. The total amount of services of land rendered each year x and of labour done each year y—both constant in a stationary society—are in Wicksell's theory taken as *data* not to be explained.

In this stationary society there will each year emerge a certain amount of finished goods. Wicksell assumes that these goods can be measured in a technical unit. This is equivalent to assuming that only one single kind of good is produced. Let z be the quantity of it that emerges each year. Thus x, y, z all have the denomination "per year."

We assume that z is a technically given production function of the $2n + 2$ input elements $x_0 \cdots x_n, y_0 \cdots y_n$, i.e.,

$$(2.2) \qquad z = f(x_0 \cdots x_n, y_0 \cdots y_n)$$

This means that, if we compare different stationary states and z, $x_0 \cdots x_n, y_0 \cdots y_n$ are the magnitudes belonging to any such state, these magnitudes will always be connected by the relation (2.2) where f is a function whose form is independent of the state considered.

The marginal productivities are denoted

$$(2.3) \qquad f_\tau(x_0 \cdots x_n, y_0 \cdots y_n) = \delta f/\delta x_\tau$$

$$f_{(\tau)}(x_0 \cdots x_n, y_0 \cdots y_n) = \delta f/\delta y_\tau$$

$$(\tau = 0, 1, \cdots, n)$$

The meaning of the phrase "the technical superiority of the roundabout way of production" (Böhm-Bawerk's third ground) can be expressed by certain assumptions about the forms of the functions f_τ and $f_{(\tau)}$. See below.

All the above is purely technical. Now for prices. Let p, q, and P be the prices of land, labour, and the product, respectively, all measured in an arbitrary unit. Wicksell assumes that $P = 1$, that is, all prices are expressed in terms of the good produced. I think, however, that the formulae are more efficiently handled by leaving P as an arbitrary parameter. Wicksell assumes that there exists a possibility of *trade* in the concretizations of land and labour which was performed Θ years ago and then applied in such a form that their fruits would be given off τ years after the services had been rendered ($\Theta = 0, 1 \cdots \tau$; $\tau = 0, 1 \cdots n$). Such concretizations will to-day still have $(\tau - \Theta)$ years to go before the finished goods emerge as part of z. If we wanted to, we could handle the problem by considering all these concretizations as separate goods with prices $p_{\tau\Theta}$ and $q_{\tau\Theta}$, respectively, and determine all these prices through equilibrium equations. It is, however, quicker to introduce Wicksell's next assumption immediately. It is to the effect that borrowing operations are possible at an interest rate ρ which is the same for all forms of borrowing. To begin with, nothing is assumed about this rate; it may be positive, negative, or zero. The only assumption at this point is that some rate exists. If this is so and no gain is possible through mere lending and borrowing operations, the prices $p_{\tau\Theta}$ and $q_{\tau\Theta}$ must be correlated in the sense that

$$(2.4) \quad p_{\tau\Theta} = (1 + \rho)^{\Theta} p \qquad q_{\tau\Theta} = (1 + \rho)^{\Theta} q$$

$$(\text{for all } \Theta = 0, 1 \cdots n, \tau = 0, 1 \cdots n)$$

In particular the exchange price of concretizations of land and labour that have no more time to go but are just on the point of giving off their fruits, will be

$$(2.5) \quad p_{\tau\tau} = (1 + \rho)^{\tau} p \qquad q_{\tau\tau} = (1 + \rho)^{\tau} q \qquad (\tau = 0, 1 \cdots n)$$

Now suppose that equilibrium is produced *as if* an entrepreneur each year tries to maximize the entrepreneurial profit

$$(2.6) \quad \pi = Pf(x_0 \cdots x_n, y_0 \cdots y_n) - \sum_{\tau=0}^{n} (p_{\tau\tau} x_\tau + q_{\tau\tau} y_\tau)$$

under constant prices P, $p_{\tau\tau}$, $q_{\tau\tau}$ ($\tau = 0, 1 \cdots n$) and freely variable $x_0 \cdots x_n$, $y_0 \cdots y_n$. There is no contradiction between the assumption that *actually* (2.1) is fulfilled with given x and y, and the assumption that the equilibrium is reached *as if* profit maximization takes place under free variation of $x_0 \cdots x_n$, $y_0 \cdots y_n$. The latter assumption pertains indeed only to the conjectural action of entrepreneurs, not to the final situation produced. Under this conjectural action we must reach a point which is substitutional in the sense that

$$Pf_\tau(x_0 \cdots x_n, y_0 \cdots y_n) = (1 + \rho)^\tau p \tag{2.7}$$

and

$$Pf_{(\tau)}(x_0 \cdots x_n, y_0 \cdots y_n) = (1 + \rho)^\tau q$$

$$(\tau = 0, 1 \cdots n)$$

The number of variables in the above argument is $(2n + 7)$, namely $(2n + 2)$ for x_τ, y_τ, and 5 for p, q, P, ρ, π. The number of equations are $(2n + 5)$, namely $(2n + 2)$ for (2.7), 2 for (2.1) when x and y are given, and 1 for (2.6). Since one degree of freedom is disposed of by the arbitrary selection of P, there remains one degree of freedom. We may represent it in various ways, for instance by saying that for each given magnitude of ρ all the other variables are determined.

There is, however, another and more fruitful approach, chosen by Wicksell. We may compute *the exchange value of the existent capital stock* and consider this value as representative of the remaining one degree of freedom. That is, for each given magnitude of this value all the other variables, including the interest rate ρ, will be determined. This will give an analysis of the demand side for capital. When this demand is finally compared with the supply of capital as it emerges through the saving in society, the equilibrium position—now including also the value of the capital stock—will be determined.

The exchange value of capital (L. I. 204)[21] is computed as follows. Take the capital stock as it exists at the beginning of any year, and let us—in conformity with the way of reasoning in (5.1)—assume that all the productive services are rendered at the beginning of the year while all the finished product emerges at the end of the year. At the beginning of any year there will then be present τ layers of that kind of land concretizations which have the property that the fruits emerge τ years after the service is performed. Each such layer consists of x_τ units of land service. One of these layers consists of land service just rendered; its exchange value will consequently be px_τ. Another layer consists of land service that was rendered one year ago; its exchange value will consequently be $(1 + \rho)px_\tau$, and so on up to the layer which consists of land service that was rendered $(\tau - 1)$ years ago, and whose fruits will therefore emerge at the end of the year we are now considering. This applies for all $\tau = 1, 2 \cdots n$.

[21] References are abbreviated thus: (L. I. 204) means "Lectures," Vol. I, p. 204; (I. P. 101) means "Interest and Prices," p. 101.

Similarly for the work performed. In other words, the exchange value of the capital stock that is present at the beginning of any year is

$$(2.8) \qquad K = \sum_{\tau=1}^{n} \sum_{\Theta=0}^{\tau-1} (1 + \rho)^{\Theta}(px_{\tau} + qy_{\tau})$$

$$= 1/\rho \sum_{\tau=0 \text{(or 1)}}^{n} [(1 + \rho)^{\tau} - 1](px_{\tau} + qy_{\tau})$$

Wicksell's argument in (L. I. 204) is equivalent to putting $(\Theta + 1)$ instead of Θ in (2.8). The difference is only a conventional one depending on whether the output is assumed to emerge at the beginning or at the end of the year. The definition (2.8) gives the simplest structure of the formulae, and is in full harmony with the reasoning in (5.1).

K in (2.8) is measured in absolute units—say dollars—so that it will depend on the conventional choice of P. Since all the equations are of the well-known form encountered in static equilibrium theory, all equilibrium prices will simply be proportional to P, and so will K. Our assumption about K may therefore be formulated either by saying that $k = K/P$ is given or by saying that K and P are given, or for brevity by saying that K is given, remembering that P is also given.

The above gives a formal determination of the equilibrium values of the variables for any chosen value of K. To study the structure of the solution we must look into certain relations that can be deduced from the above.

From (2.1), (2.6), and (2.8) follows immediately

$$(2.9) \qquad px + qy + \rho K + \pi = Pz$$

All the terms of (2.9) represent values per unit of time.

From the axiomatic viewpoint (2.9) is a fundamental relation which shows that when the exchange value of capital is computed by (2.8), the difference between the total value of the annual product on the one hand and on the other the sum of the entrepreneurial profit and what is paid annually to the primary factors, land and labour, at the moment when the services of these factors are rendered, is an amount per year equal to one year's interest on the existing capital. This indicates that although capital is not in the technical sense just another factor juxtaposited with the primary factors, but rather a new dimension on each of the primary factors (L. I. 148–150), yet capital has in one particular sense the same position in the problem as the primary factors: it receives a remuneration which forms part of the total value of the product. Incidentally, Wicksell assumes most of the time that the entrepreneurial profit in the equilibrium point is zero. This will always be the case if production follows a pari-passu law, i.e., if the function (2.2) is homogeneous of the first degree. Otherwise the assumption $\pi = 0$ introduces an additional assumption which in general will not be compatible with the other assumptions.[22]

Also from another angle will capital retain a position similar to that of the other factors: We may speak of the marginal productivity with respect to real capital k, dz/dk, calculated under constant x and y (and constant prices and interest rate), and this marginal productivity turns out to be equal to ρ in the equilibrium point. Indeed consider any differential variation $dx_0 \cdots dx_n, dy_0 \cdots dy_n$ compatible with (2.1) under constant x and y, i.e., $\Sigma_\tau dx_\tau = 0$ and $\Sigma_\tau dy_\tau = 0$. For any such variation we have by (2.8) $\rho dK = \Sigma_\tau(1 + \rho)^\tau(pdx_\tau + qdy_\tau)$. On the other hand we have $dz = \Sigma_\tau f_\tau dx_\tau + f_{(\tau)}dy_\tau$; hence in the equilibrium point $Pdz = \Sigma_\tau(1 + \rho)^\tau(pdx_\tau + qdy_\tau) = \rho dK$, so that

$$(2.10) \quad d(Pz)/dK = \rho \qquad \text{i.e., } dz/dk = \rho \qquad (\text{when } P = \text{const.})$$

This conclusion holds no matter how large π may be in the equilibrium point. We may also prove (2.10) under another set of assumptions, which do not use the equilibrium conditions and are also weaker in other respects. Dividing (2.9) through by P we see indeed immediately that under any variation which leaves ρ and $(px + qy + \pi)/P$ unchanged while (2.9) holds, the result must be the last formula in (2.10).

Thus, if we "apply more capital," i.e., arrange for dK in any of the senses above, we will—*regardless of how this marginal dose of capital is composed*, provided only that it satisfied the conditions specified—always find that the increment in the product reckoned per unit of increment in real capital is equal to the existing rate of interest ρ.

There is still another sense in which capital behaves as a factor of production: The interest rate can be looked upon as the marginal productivity (or more precisely the relative marginal productivity) with respect to waiting. To see this we must define the concept of the *average period of production* $\bar{\tau}$. If it would have been sufficiently accurate to reckon with simple interest, we could have defined $\bar{\tau}$ simply as the weighted arithmetic average of all the individual periods $\tau = 0, 1 \cdots n$. In this case the definition of $\bar{\tau}$ would have been independent of ρ (L. I. 184). Since in fact we have to reckon with compound interest, we must define $\bar{\tau}$ more precisely by saying that

[22] See my paper "Overdeterminateness and optimum equilibrium," *Nordisk Tidsskrift for Teknisk Økonomi*. Copenhagen, 1948.

if the amount $(px + qy + \pi)$ which is paid annually to the primary factors and to the entrepreneurs (if they receive a remuneration at all), increases in value at interest ρ, compounded continuously for this period $\bar{\tau}$, we get the value of the product that emerges annually. In other words the average period of production $\bar{\tau}$ is defined by $(px + qy + \pi)e^{\rho\bar{\tau}} = Pz$, that is

$$(2.11) \qquad \rho\bar{\tau} = \log \text{nat } (Pz) - \log \text{nat } (px + qy + \pi)$$

Applying to (2.11) a differential variation that leaves ρ and $(px + qy + \pi)$ unchanged we have

$$(2.12) \qquad \rho = d \log \text{nat } (Pz)/d\bar{\tau} = 1/Pz \cdot d(Pz)/d\bar{\tau} = dz/d\bar{\tau} \cdot 1/z$$

This is the precise meaning in which the interest rate which exists in the equilibrium point can be looked upon as the (relative) marginal productivity of waiting (L. I. 177 and 184).

Incidentally, if we had defined $\bar{\tau}$ as $K/px + qy + \pi$, we would have obtained $\rho/1 + \rho\tau$ instead of ρ in the left member of (2.12), and if we had defined $\bar{\tau}$ as K/Pz, we would have obtained $\rho/1 - \rho\bar{\tau}$. Both expressions are close to ρ if either ρ or $\bar{\tau}$ is small.

In the equilibrium point we must by (2.7) have

$$(2.13) \qquad f_\tau(x_0 \cdots x_n, y_0 \cdots y_n) = (1 + \rho)^{\tau-\theta} f_\theta(x_0 \cdots x_n, y_0 \cdots y_n) \qquad \text{(for all } \tau \text{ and } \theta\text{)}$$

And similarly for the services of labour. These relations are important for a discussion of the much debated question of whether the equilibrium rate ρ is positive.

Wicksell thinks it will be positive. He is definitely aware of the fact that a proof of this proposition cannot be given simply by referring to the fact that in the equilibrium point we usually have

$$(2.14) \qquad f_\tau(x_0 \cdots x_n, y_0 \cdots y_n) > f_\theta(x_0 \cdots x_n, y_0 \cdots y_n) \qquad \text{when } \tau > \theta$$

And similarly for the services of labour. In order to deduce that the equilibrium point will show a positive ρ, we must build on the existence of the inequality (2.14) taken in the *schedule sense*, that is in the sense that it holds everywhere within a certain domain of the variables involved. Such a formulation of (2.14) is the essence of Böhm-Bawerk's third ground. Wicksell seems to think that the third ground—taken in the schedule sense—is *sufficient* to prove the positivity of the equilibrium interest rate, and that we consequently do not need to evoke Böhm-Bawerk's first ground (less adequate supply of goods in the present than in the future) or his second ground (the undervaluation of future needs). "Thus there remains only the third of Böhm-Bawerk's main reasons" (L. I. 155; see also 150). Let us examine this a little closer.

If we assume technical superiority in a sufficiently strong schedule sense, we can always prove that this superiority is a sufficient condition for the equilibrium ρ to be positive. For instance, if we assume that (2.14) holds identically for all conceivable magnitudes of the variables $x_0 \cdots x_n$, $y_0 \cdots y_n$, we see immediately from (2.13) that ρ must be positive, *wherever* the equilibrium point might fall. But to assume technical superiority in so strong a sense would certainly be going too far. By transferring enough land services from Θ-use (and possibly from other uses) to τ-use ($\tau > \Theta$) it must be possible to press down f_τ and to increase f_0 to a point where the inequality (2.14) is reversed.

We could weaken the assumption on the technical superiority by considering only those points ($x_0 \cdots x_n$, $y_0 \cdots y_n$) where the individualized marginal productivities satisfy the proportionality conditions

$$(2.15)\quad f_\tau = \alpha(1+\rho)^\tau \qquad f_{(\tau)} = \beta(1+\rho)^\tau \qquad (\tau = 0, 1 \cdots n)$$

α, β, ρ being any three numbers, positive, negative, or zero. This would leave us with three degrees of freedom only. If we assumed that (2.14) holds identically over the field defined by (2.15), the positivity of the equilibrium ρ would follow. However, to assume technical superiority in the sense just considered would only weaken the assumption in a very formal way. The fundamental problem would be left: If we throw the elements of the given sum x sufficiently far off into the future, it must be possible to reverse the inequality (2.14) even if we limit ourselves to considering points ($x_0 \cdots x_n$, $y_0 \cdots y_n$) compatible with (2.15) (where no assumption is made about ρ being positive). Numerical examples of this can undoubtedly be constructed.

So, in order to prove that the equilibrium ρ will be positive, it seems that we must fall back on *something that limits the size of* K. We can, for instance, formulate the technical superiority by assuming that (2.14) holds identically in ($x_0 \cdots x_n$, $y_0 \cdots y_n$) within *that* region where (2.1) and (2.7) are fulfilled and where K/P defined by (2.8) has a "reasonable" size. I think that this is, after all, Wicksell's meaning of technical superiority. When this formulation is accepted, a positive equilibrium ρ follows. It seems, however, that it is stretching the terminology a bit to call this set of assumptions a "technical" superiority. We have here assumed so many factors on the *supply side* of "waiting" that we have really used something equivalent at least to Böhm-Bawerk's first and second ground.

This whole question should be analysed further. I believe that when the analysis is carried through in real terms only and the assumptions—for instance Böhm-Bawerk's three grounds—are formulated in so weak a sense that we can adopt them unhesitatingly, then they will not form a set of sufficient conditions for a positive equilibrium rate. The concrete facts which make the occurrence of a zero or a negative interest so unlikely must, in my opinion, rather be sought on the *monetary* side. Under present monetary institutions liquidity may take on such forms that it would require very drastic measures to produce a zero or a negative interest. One would, for instance, have to use monetary notes (and coin) automatically losing in denomination with time.

Assuming that the equilibrium rate ρ is positive, how will it change when we shift our attention from a stationary state with one value of K to another stationary state with a larger value of K (and constant P)? Wicksell says (L. I. 157 and 162) that ρ then will *go down*, which by (2.13) is only another way of saying that the marginal productivity of the long-period uses of land and labour will go down *in relation to* the marginal productivity of the short-period uses. He also claims that the marginal productivities f_0 and $f_{(0)}$ will actually increase and that the uses of land and labour will be shifted in the direction of the longer periods.

In the case where $n = 1$ and the production function expresses a pari-passu law (i.e., is homogeneous of the first degree) these propositions can easily be proved as follows. To simplify we consider only land; the inclusion also of labour would not materially alter the argument. Putting again for brevity $K = kP$, we now have the three equations

(2.16) $\quad x_1 f_0(x_0, x_1) = k \qquad x_1 f_1(x_0, x_1) = (1 + \rho)k \qquad x_0 + x_1 = x$

between the four variables x_0, x_1, ρ, k. From these equations we immediately deduce

(2.17) $$1 + \rho = f_1(x - x_1, x_1)/f_0(x - x_1, x_1)$$

The variation from x_1 to $x_1 + dx_1$ (under constant x) produces the variations

(2.18) $$df_0(x - x_1, x_1) = (f_{01} - f_{00})dx_1$$

and

$$df_1(x - x_1, x_1) = (f_{11} - f_{10})dx_1$$

where $f_{ij} = \delta f_i/\delta x_j$. So long as we are within the region of substitution (i.e., where f_0 and f_1 are positive) and the production law is a pari-passu law, the second order derivatives always have the following signs: f_{00} and f_{11} are negative, f_{01} and f_{10} positive. Thus, if dx_1 is positive, df_0 must necessarily be positive and df_1 negative, from which all the above conclusions follow. In the more general case the proof is not so simple and I have not gone through it rigorously, but it seems probable that Wicksell's conclusions hold in general on reasonable assumptions.

In other words, if we measure K along the horizontal axis and ρ along the vertical axis of a diagram, the curve representing the (ρ, K) relation in the sense just specified will—under constant P and within the domain considered by Wicksell—*be a downward-sloping curve.* This entails, among other things, the fact that the ordinate is a single-valued function of the abscissa and at the same time that the abscissa is a single-valued function of the ordinate.

If we take K as the independent variable, the ordinate read off from the curve indicates the interest rate which would emerge in the production process under the given technical conditions and the given kind of adaptation (profit maximization) and the given constant P when the value of the capital employed is K. This interest rate we may call the *productivity* rate of interest and denote ρ^*. Wicksell does not use any special term or symbol for what I have called the productivity rate. Sometimes he speaks about a natural or a real rate (for instance L. II. 207) as if he should have the ordinate of the above curve in mind, but most of the time his natural, real, or normal rate is handled as an equilibrium concept (compare [6.6] below), so I have found it more convenient to retain a special term for the ordinate of the curve.

It should be noted that Wicksell focussed his attention primarily on the relation between the interest rate and *the existing capital:* ". . . why a given amount of existing social capital gives rise to a certain rate of interest" (L. I. 171). The annual net *addition* to capital, i.e., that concept which we would term net investment and denote I, is a flow concept and must not be confounded with the capital in use K.

When we read Wicksell and consider the context with a willingness to understand, we will never have any real difficulty in understanding whether he thinks of the flow concepts or the stock concepts, but he might not always satisfy our most pedantic claims regarding terminological rigor. My friend and colleague Professor Trygve Haavelmo has insisted strongly on the distinction between flow and stock concepts in connection with Wicksell. I should certainly not have been so careful in my formulation if I had not profited by his remarks.

It is also necessary to distinguish between the demand and supply aspect of these concepts. Saving S is the supply concept corresponding to the demand concept I. The stock concept corresponding to S may be termed capital held and denoted H.

Before proceeding to an application of these concepts in Wicksell's monetary theory, a word must be said about the sense in which investment can be different from saving.

3. HOW CAN INVESTMENT BE DIFFERENT FROM SAVING?

There is no logical difficulty in conceiving of the desired (planned, ex-ante) magnitude of investment as being different from the desired (planned, ex-ante) magnitude of saving, or from actual saving. Nor is there any difficulty in conceiving of a difference between actual investment and actual saving if one of these magnitudes refers to one period of time and the other to another period. The problem concerns the case where investment and saving are both actual (ex-post) figures and refer to the same period.

There is something peculiar about these concepts of actual investment and actual saving, both referring to the same period. Sometimes we find outselves involved in an argument where there seems to be good common sense in assuming that these two magnitudes are different. At other times, equally good common sense would indicate that the two must be equal.

When one has become aware that the problem must be handled from the "ecocirc" (tableau économique) viewpoint, the path of least mental resistance undoubtedly leads to saying that actual investment is by definition always equal to actual saving. I am convinced, however, that this is not the solution we need. To adopt it would mean that we take something important out of the problem. I vividly remember the deception I felt one evening in King's College many years ago when Keynes told me that he had finally decided to make actual investment by definition equal to actual saving. I am sure that this was a step backwards in the "General Theory" as compared with his "Treatise on Money." Keynes' remark: "No one can save without acquiring an asset whether it be cash or a debt or capital-goods" ("General Theory," 1936, p. 81) is in my opinion not to the point. The nature of the asset is essential. It is particularly important in a theory where liquidity is a central theme.

We need a system of concepts which allow both kinds of differences to exist, both an investment-saving difference between two desired or one desired and one actual magnitude, or between magnitudes referring to different periods of time, *and* a difference between two actual magnitudes referring to the same period of time. The first kind of differences will in a sense explain the ultimate "cause" of the movement, the second will explain how that cause in concreto *operates*.

A mechanical analogy will bring out more precisely what I have in mind. Take a U-shaped tube with constant internal diameter, both branches being open in the upper end. When there is liquid in the tube, there will be two columns of fluid connected at the bottom. Let x be the level of the fluid in the left branch, and y that in the right, both measured from the same conventional base. In static equilibrium we must, of course, have $x = y$. Now suppose that a person *desires* to add a certain amount of fluid in the left branch. This desired amount will illustrate the concept of a planned or ex-ante magnitude, and it will be a useful tool when we want to explain the motivations back of what happened on such and such an occasion, but it will not explain the mechanism by which fluid flows from the left to the right branch when the person carries out his intention. To explain *this* it is necessary to use a logical model where the observed actual x at a given point of time may be different from the observed actual y at that same point of time. Without such a model we would not be able to understand what really happens when fluid flows from the left to the right branch and thus re-establishes static equilibrium. On the other hand, if we do use an appropriate model, we can give a very complete and understandable account of what happens. We may for instance explain that a constant inflow per unit of time in the left branch will entail a difference $(x - y)$ which is constant over time, at least approximately if there is no friction in the tubes themselves, but only friction at one point, say in a valve at the bottom, where the two branches communicate. The magnitude of the constant difference $(x - y)$ will depend on how strong the friction in the valve is, and on how heavy the fluid is. An increasing inflow in the left branch will entail a faster increase in x than in y. And so on.

This mechanical analogy may be applied to a great number of economic phenomena. We may for instance let x and y be capital in use and capital held, respectively. Or we may let x and y represent investment and saving per unit of time at any given point of time.

Whether we interpret the analogy one way or the other, there are undoubtedly certain equilibrium situations which it would be natural to characterize by $x = y$, but we would not have perceived the problem sufficiently broadly if we constructed a model where by definition $x = y$ and tried to use this model to explain *what happens* when persons or groups in society try to change the existing investment-saving situation.[23]

To reach a concrete workable definition of the money value of the actual rate of investment per unit of time and the money value of the actual rate of saving per unit of time in such a way that the two may be different, I think we should start by saying that however we finally define these two variables, the difference between them should in one way or the other be connected with the frequently and loosely used concept "credit expansion" or more specially with the concept of an "inflationary credit expansion" produced through the intermediary of a special sector, "the banks."

Let K_t be total loans (which in Wicksell's analysis is more or less the same thing as capital in use)[24] and let H_t be total deposits (capital held) at the point of time t. Further let the divided differences (rates of change) be denoted

$$\dot{K}_t = K_t - K_{t-\kappa}/\kappa \qquad \dot{H}_t = H_t - H_{t-\kappa}/\kappa \tag{3.1}$$

When $\kappa \rightarrow 0$, we get derivatives.

Wicksell focusses attention on the *appreciation part* of the change in K_t, that is, the increase in value produced by the mere fact that prices are changing. In modern works on national accounting this aspect of the problem is not considered as explicitly as one could wish. In this respect there is still much to be learned from Wicksell. The following is a suggestion for a system of concepts which may satisfy at the same time the requirements of the theory of the Wicksellian cumulative process and that of national accounting.

Let P_t be an index of prices such that it can be used for deflating the amount K_t, so that, as before, the ratio $k_t = K_t/P_t$ can be looked upon as the volume of real capital. If K_t is defined simply as loans, k_t would be the deflated value of loans. We have

$$\dot{K}_t = \dot{k}_t P_t + k_{t-\kappa}\dot{P}_t = \dot{k}_t P_{t-\kappa} + k_t \dot{P}_t \tag{3.2}$$

[23] My friend and colleague Edgard B. Schieldrop, professor of mathematical mechanics at the Oslo University, has at my suggestion worked out a number of equations which can undoubtedly be translated into economic terms. I hope to be able to revert to this on another occasion.

[24] More precisely: the value of circulating real capital.

where

(3.3) $\dot{k}_t = k_t - k_{t-\kappa}/\kappa \qquad \dot{P}_t = P_t - P_{t-\kappa}/\kappa$

When $\kappa \to 0$, we get derivatives and (3.2) becomes the usual formula for the derivative of a product. The following concepts—all reckoned per unit of time—must be distinguished:

(3.4) $\dot{K}_t$ = increase in the value of capital (investment reckoned inclusive of appreciation on capital)

(3.5) $\dot{k}_t$ = the volume of real investment

(3.6) $\dot{k}_t P_t$ = the value of real investment, or shorter investment $= I_t$

(3.7) $\dot{K}_t/P_t$ = deflated increase in the value of capital

(3.8) $k_t \dot{P}_t$ = appreciation on capital

When the price index P_t is constant, (3.8) is zero and all the concepts (3.4–7) are practically synonomous. Otherwise they must be kept distinct. I believe that (3.6) comes nearest to expressing "investment per unit of time" in the minds of the majority of those who work on national accounts.

Similar distinctions must be made for the concept capital held = deposits H_t. Let its deflated value be $h_t = H_t/P_t$. Various rates of change can be derived from H_t similar to (3.1–8). In particular saving may be defined $S_t = \dot{h}_t P_t$.

In order that a change in the difference between loans and deposits shall become conceivable, the model must contain some means of storing purchasing power outside of the banks. The most natural way to introduce this possibility is to assume a circulating medium—notes and coin—held by the public, i.e., by "non-banks." Let M be the amount of this circulating medium and $m = M/P$ its deflated value. Assuming that M, K, and H are all measured from conventional origins, we may put

(3.9) $$M_t = K_t - H_t$$

hence $\dot{M}_t = \dot{K}_t - \dot{H}_t$ and $m_k = k_t - h_t$

In a concrete case one would have to specify carefully all the items that would come under the headings "loans," "deposits," and "circulating medium," respectively. However complicated the banking system and its operations are, it will always be possible to make the classification in such a way that the definitional equations (3.9) hold. From the above definitions follows

$$(3.10) \qquad I_t - S_t = P_t \dot{m}_t = P_t(d/dt)(M_t/P_t)$$

The formula (3.10) indicates, in my opinion, the way in which we should fundamentally introduce a difference between I_t and S_t.

The above concept $S_t = \dot{h}_t P_t$ is saving in the restricted sense of including only such values which (through the banking system) *are made available for someone else.* And it is defined exclusive of appreciation, i.e., it expresses the increase in a volume figure (in a deflated figure), this increase being however expressed in monetary units of the current year. An increase in cash holdings is not included in this savings concept. In a broader sense we may consider $(S_t + P_t \dot{m}_t)$ as "saving." This is the value of the increase in the deflated value of *all* reserves whether in the form of deposits or cash holdings. *This* "saving" would by definition be equal to I_t, but this savings concept is not well adapted for a study of how cash holdings are absorbed by the public. For this purpose the appropriate concepts would seem to be I_t and S_t. This is illustrated by the following special cases.

In a model where no cash holdings M_t exist, we will by definition have $I_t = S_t$. This equality will also apply if prices always move *immediately in strict proportion to the amount of cash holdings.* Indeed, if $\dot{m}_t = 0$ for all t, the ratio M_t/P_t would be constant. The existence of a difference between I_t and S_t—that is, the non-proportionality of M_t and P_t—will express a *buffer effect* produced by the cash holdings. In terms of the usual equation of exchange, this effect might be translated as a change in the velocity of circulation of money. The buffer effect is expressed by the difference (3.10). It characterizes the way in which new loans are absorbed by the public. A period of expansion—not necessarily accompanied by rising prices—would be characterized by a positive value of the difference (3.10)—in the mechanical analogy fluid would be driven from the left to the right branch—and a period of contraction would be characterized by a negative value of (3.10). In many cases the average value of (3.10) taken over a year may be small, just as the average difference of level of the fluid in the two branches in the mechanical analogy may be small, but the existence of this small difference may account for the flow of a considerable amount in the course of a year, i.e., a considerable value of $\dot{K}_t$ and of $\dot{H}_t$.

When the above definitions are adopted, the cash holdings should be looked upon as *a draft on the social product,* not—as one would from a purely formal viewpoint—as a draft on the institution that has issued the notes or coin. So far as economic effects in a modern society are concerned, the concept of the note as a draft on the social product is undoubtedly the more relevant. From this viewpoint the notes and coin appear as something half way between real objects and credit instruments. The latter have both a debtor and a creditor —and are indeed nothing but an expression of the relation between these two parties—the former have neither a debtor nor a creditor, but are simply owned by someone. From the social-product viewpoint each note has a creditor—the person who holds it—but no individualized debtor. Having adopted this view on money, we may—to use a term by D. H. Robertson—call the expression (3.10) a "levy" on the public. This levy constitutes the difference between investment and saving.

Two things are essential in order to arrive at a distinction between I and S along the lines here developed: first, that we have segregated one sector, "the banks," to be treated in a special way in the definition of investment and saving; and second, that we have segregated a special kind of objects, namely notes and coin, to be treated in a special way.

If some investment takes place within the enterprises without passing through the banks, one would count this both as investment and saving, and with equal amounts. The same would apply if some of the savings of individuals were invested in real form directly by these individuals. This would follow logically because a given enterprise has not created any "circulating medium" that gets a meaning because of its circulation within this enterprise. And the same would apply to the household of an individual. Equation (3.10) will therefore hold even if I and S are defined as *totals for society.*

If we want to use a model where all transactions are performed by notes and coin alone, we will have to distinguish between two parts of this circulating medium, a "hoarding" part that can, so to speak, be kept out of sight when this is wanted, while the other, "the active part" can be inserted for M in the right member of (3.10). When a part of the circulating medium is transferred from one to the other of these two compartments, there would sometimes emerge a positive and sometimes a negative value of the right member of (3.10). If we introduce "the banks" as a special sector to be treated in a special way from the viewpoint of investment and saving, a distinction between two different parts of the cash holding does not become necessary for the investment-saving definition, although it might of course still be desirable from other viewpoints. It is unessential which one of the above two ways of thinking we accept, provided we arrive at an equation of the form (3.10).

Equation (3.10) must be tied in with the definition of sectorial income (national income if the sector is a nation). Let C be the money value of the sector's consumption per unit of time and, as before, I the money value of the sector's net investment per unit of time, and let A and B denote total exports and imports, respectively, taken in the broadest sense. Finally let G be unilateral transfers (taxes, gifts, etc.) from this sector to the rest of the world, reckoned net, so that G may be positive, negative, or zero. As before, M is the amount of sectorial monetary circulation. Then we have the following hierarchy of income concepts:

(3.11) Internal income $R^{\text{in}} = C + I$

(3.12) Disposable income $R^{\text{dis}} = C + I + (A - B)$ ("the internally and externally disposable sector income").

(3.13) Accruing (released) income $R^{\text{ac}} = R = C + I + (A - B) + G$

(3.14) Produced income $R^{\text{prod}} = C + S + (A - B) + G$

Since the accruing (released) income will as a rule be the most important income concept to consider, we have for brevity denoted it R without any superscript. Accruing (released) income is the money value of income as it emerges when investment I is reckoned at actual prices at the end of the year, i.e., at prices compatible with the credit expansion that has taken place, if any, while produced income is the money value of income after deducting from the accruing (released) income the amount $(I - S)$, i.e., the "levy" on the public. The difference between accruing (released) income and produced income is the same thing as the difference between the money value of actual investment and that of actual saving, i.e.,

(3.15) $$R^{\text{ac}} - R^{\text{prod}} = I - S \doteq P\dot{m} = P(d/dt)(M/P)$$

Adopting (3.6) as the definition of investment and, similarly, of saving means that the income concepts (3.11–3.14) are defined exclusive of value appreciation on capital. If we had taken $\dot{K}$ as the definition of investment and $\dot{H}$ as that of saving, the above income concepts would have included value appreciation on capital.

For any individual or any subsector, say No. α, the accruing income R^{α} plus the increase in the loans $\dot{K}^{\alpha}$ to this individual or subsector is the total purchasing power at the individual's or subsector's disposal. It can use this purchasing power for the following six purposes: consume C^{α}, invest at home I^{α}, invest abroad $(A^{\alpha} - B^{\alpha})$, pay unilateral transfers to the rest of the world G^{α}, increase its cash holdings $\dot{M}^{\alpha}$, and increase its deposits $\dot{H}^{\alpha}$. If this equation is summed over all α, and we use (3.9), we get (3.13) where the magnitudes refer to the sector as a whole.

In the sequel the above investment-saving concepts will be applied in expounding Wicksell's theory.

4. CURRENCY THEORY VERSUS BANKING PRINCIPLE

To understand the genesis of Wicksell's monetary theory, one should begin by considering certain other theories which he partly accepts and partly discards.

In one specific sense, Wicksell accepts—as practically every sensible economist would—the quantity theory: "that a large issue of paper currency progressively depreciates in value and thereby raises the prices of all other commodities, calculated in paper money, has been proved too often in history to be open to doubt. Similarly, there are some, though by no means many, examples of a successive withdrawal of paper money rehabilitating its value and causing a fall in commodity prices, in terms of paper money" (L. II. 170).

All this is simple. The essence of Wicksell's problem is something different, namely how *bank credit to the public*, either in the form of notes or of fictitious deposits, will affect prices. On this point we should consider in particular the currency theory, whose principle exponent was Ricardo, and the banking principle defended by Tooke.

In a nutshell, Ricardo's views can be expressed by saying that "the banks possess, by the granting of credit, and especially by the issue of notes, an unlimited power to increase the circulating medium, and therefore to raise commodity prices" (L. II. 171). In this process, the interest rate plays, according to Ricardo, an important role. The height of the interest rate will be causally connected with the increase or decrease in circulating medium, not with the existing amount of it. A liberal issuance of bank credits in the form of notes or fictitious deposits would tend to produce an easy money market, and this easiness would lower the interest rates. Viewed from the other side, this low interest rate would, under ordinary circumstances, be the means by which the banks could make the public absorb the enlarged amount of circulating medium. This easiness of the money market would be maintained as long as, and no longer than, the outflow of new circulating medium took place. As soon as the outflow had ceased and the increased circulating medium had produced its effects on prices, the easy money market would be gone, and the money rate would move up again to its former height. In other words, the low money rate represented so to speak the position of a valve through which the new circulating medium flowed into the system. A changed position of the valve and a changed strength of the flow would, more or less, be the same thing.

Wicksell accepted the essence of this conclusion: ". . . Ricardo rightly insists that a fall in money interest can only take place so long as the surfeit of money has not led to a corresponding increase in prices. As soon as this occurs, there no longer exists any surfeit of money, relative to the requirements of turnover" (L. II. 179). But Wicksell is dissatisfied with the *analysis* which led Ricardo to his conclusions. In Wicksell's opinion, Ricardo was too narrowly concerned with the "high price of bullion" aspect of the problem (L. II. 176) and did not distinguish sufficiently sharply between internal commodity prices and the external premium on gold. He thinks Ricardo's proof on this point is all too slender (L. II. 177). On the whole, he holds that Ricardo does not go sufficiently deeply into the *mechanism* which connects rising prices and the issuance of bank credit. It is precisely on this point that Wicksell thinks his own contribution will be illuminating.

Tooke—in opposition to Ricardo—holds the view that "the volume of exchange media is never the cause, but on the contrary, always the effect, of fluctuations in prices and of the requirements of turnover for the medium of exchange" (L. II. 173). Tooke's view on this point could perhaps be characterized as the "small-coin view." Everybody will, of course, agree that the level of commodity prices is not (to any sensible degree) affected by the amount of pennies and other small coins in circulation. Under ordinary circumstances, the need for these types of circulating media will be *determined by* the existing level of commodity prices (and by trading habits). Any attempt to force a larger amount of pennies and small coins on the public, would simply result in a flow of these denominations back to the banks. Tooke's view is that the issuance of bank credit has a similar effect on prices. The commodity prices are determined by speculation, taking account of production costs and specific factors in the market. In general, there is no speculation in the commodity markets based on easy bank credit and a low interest rate.

To this Wicksell remarked that "Tooke has . . . confused two essentially different phenomena" (L. II. 184), namely, speculation in goods owing to political events, failure of harvest, etc., and the regular element of speculation that enters into current business transactions under capitalistic production. In the first case, the interest rate is an element of minor importance—on this point Tooke is correct—but in the second case, the interest rate is of paramount importance, and it acts precisely in the way which Wicksell himself wanted to explain.

Thus, to sum up, Wicksell admits the correctness of important elements both in the currency theory and in the banking principle, but he considers neither as giving a satisfactory *systematic analysis* of the mechanism which connects bank credit, interest rate and prices. This analysis is furnished by his own "positive solution."

5. PRELIMINARY EXPOSITION OF WICKSELL'S MONETARY THEORY

The main line of Wicksell's argument in his monetary theory is concerned with what would happen if the primary factors of production—land and labour—remained constant in real-terms equilibrium, while certain constellations within the *monetary* system changed. The evolution to which this would give rise is the famous Wicksellian cumulative process which manifests itself as a mere *price movement.* The underlying real-terms equilibrium, which in the monetary theory is assumed constant, is explained fully in that part of his theory which in his lectures was treated in volume I (compare Section 2 above). Wicksell was, of course, well aware of the fact that the evolution of the monetary factors actually influences also the underlying real factors, and in his monetary theory he makes frequent remarks on this. But they remain as side remarks. To quote but one example: ". . . It is, of course, not impossible for the rise in prices to be counteracted to a certain extent by an increase in production, for example if previously there had been unemployment, or if higher wages had induced longer working hours, or even by the increasing roundaboutness which is undoubtedly invoked by a fall in interest rates, even if it occurs artificially. But all these are secondary considerations" (L. II. 195).

The essential fact which, according to Wicksell, distinguishes the monetary market mechanism from the market mechanism of a real good, he explains as follows: "The movement and equilibrium of actual money prices represent a fundamentally different phenomenon, above all in a fully developed credit system, from those of *relative* prices. The latter might perhaps be compared with the mechanical system which satisfies the conditions of *stable* equilibrium, for instance a pendulum. Every movement away from the position of equilibrium sets forces into operation—on a scale that increases with the extent of the movement—which tend to restore the system to its original position, and actually succeed in doing so, though some oscillations may intervene. The analogous picture for *money* prices should rather be some easily movable object, such as a cylinder, which rests on a horizontal plane in so-called *neutral* equilibrium" (I. P. 100–101).

The mechanism by which this effect is produced, is explained in "Interest and Prices" (pp. 136–141) and later in his "Lectures." The reasoning in "Interest and Prices" can most effectively be translated into an accounting system. See table (5.1). The assumption here is that no addition is made to the stock of fixed capital goods. For simplicity, the production of consumption goods (by means of a stock of capital goods and an inventory of consumption goods) is assumed to take place in cycles of one year's duration. Wicksell considers the four groups or parties indicated in table (5.1).

TABLE (5.1). TRANSACTIONS IN A SOCIETY CONSISTING OF THE FOUR WICKSELLIAN GROUPS

		Cash Transactions								*Loans*	*Deposits*
		Entrepreneurs		Landlords and Workers		Capitalist-Commodity Dealers		Banks			
Transactions at the beginning of the year	(1)	K							K	K	
	(2)		K'	K'							
	(3)				K'	K'					
	(4)		K''			K''					
	(5)						K	K			K
Transactions at the end of the year	(6)					ρK			ρK		
	(7)					K			K		0
	(8)	$(1+\rho)K$					$(1+\rho)K$				
	(9)		$(1+\rho)K$					$(1+\rho)K$		0	
Grand total		$(2+\rho)K$	$(2+\rho)K$	K'	K'	$(2+\rho)K$	$(2+\rho)K$	$(2+\rho)K$	$(2+\rho)K$		

The *entrepreneurs* own fixed capital goods, but have otherwise no capital of their own. At the beginning of the year, they borrow from the banks an amount K (line 1), which they immediately, that is to say, at the beginning of the year, pay out in two sums, K' and K'' where $K' + K'' = K$, as follows: K' (line 2) is paid in advance to the *landlords and workers* in remuneration for the services they will render in the year's production. These services are used by the entrepreneurs to repair and renew (but not more than renew) the fixed capital goods which the entrepreneurs own, and also used to produce the year's consumption for society as a whole. The consumption goods emerge as finished products at the end of the year. The sum K' received by the landlords and workers from the entrepreneurs at the beginning of the year is immediately used by them (line 3) for buying from the capitalist-commodity dealers all the consumption goods which the landlords and workers need for the whole year. The sum K'' (line 4) is paid out by the entrepreneurs at the beginning of the year to buy from the capitalist-commodity dealers the consumption goods which the entrepreneurs need for the whole year.

The *capitalist-commodity dealers* are pure rentiers. At the beginning of the year they hold for one moment their whole fortune—which is $(1 + \rho)K$—in real form, namely as an inventory of all the consumption goods which are used in society as a whole for one year. At the beginning of the year, the rentiers immediately sell for an amount K' to the landlords and workers (line 3) and for an amount K'' to the entrepreneurs (line 4) and keep for themselves (not shown) an amount ρK to be consumed in the course of the year. Having done that, the capitalist-commodity dealers immediately deposit the proceeds of the sales, that is, the amount $K = K' + K''$, in the banks. It is unessential how we imagine that the various payments around new year are performed. It may be done by an instantaneous use of cash (notes), or by drafts on the banks, or by clearing in the banks, the account of one party being credited at the same time as the account of another party is debited with the same amount.

The *banks* receive at the beginning of the year a deposit K from the capitalist-commodity dealers, and lend this sum immediately to the entrepreneurs. The interest to be paid on the loan granted to the entrepreneurs is to be the same as that paid on the deposits made by the capitalist-commodity dealers.

At the end of the year, the following transactions take place. First, the entrepreneurs sell to the capitalist-commodity dealers as much of the year's production of consumption goods as is necessary in order to pay off the entrepreneurs' debt to the banks as of the end of the year. This debt is equal to K plus the interest ρK. In other words, the entrepreneurs are assumed to sell commodity goods for a total amount of $(1 + \rho)K$.

Let C be the value of total net output at current prices, that is, the value of the consumption goods that emerge annually in the productive process directed by the entrepreneurs. This value need not be exactly equal to the value $(1 + \rho)K$ of the consumption goods which the entrepreneurs must sell in order to cover exactly their debts to the banks as of the end of the year. In order to describe the difference we may define a coefficient $\bar{\rho}$ by the equation

$$(1 + \bar{\rho})K = C \tag{5.2}$$

All prices are assumed given and constant during the year. The coefficient $\bar{\rho}$ may provisionally be taken as the definition of the natural (or normal) rate of interest.

Wicksell considers the difference $(1 + \bar{\rho})K - (1 + \rho)K = (\bar{\rho} - \rho)K$. When this difference is positive he visualizes it as consisting of a stock of consumption goods which the entrepreneurs *need not sell*, but may put aside for themselves: ". . . and laying them on one side for the consumption of the coming year" (I. P. 142).

Thus, the total value of consumption goods which the entrepreneurs receive annually (after the first year) as remuneration for their taking part in the economic activity can be looked upon as consisting of two parts: K'' and $(\bar{\rho} - \rho)K$. The former is a "normal" part, and the latter is a "surplus" which is positive or negative accordingly as the natural rate of interest is larger than or less than the market rate.

The existence of such a surplus will exert a profound influence on the course of affairs. Take for instance the case where it is positive: "If entrepreneurs continue, year after year, perhaps, to realise some surplus profit of this kind, the result can only be to set up a tendency for an expansion of their activities. I emphasize once again that so far it is purely a question of a *tendency*. An *actual* expansion of production is quite impossible, for it would necessitate an increase in the supply of real factors of production . . . such changes . . . we need not consider . . . at this point" (I. P. 143). That is to say, *if* prices remained constant, the entrepreneurs *would*, at least after some time, discover that they realized an unusually large profit, and so would, amongst them, start a scramble for expansion. Since in real terms there could not be any expansion, the *bidding of the entrepreneurs* would push prices up. This bidding and driving up of prices would necessitate larger bank credit (on this point Wicksell comes very close to a Tooke-ian reasoning) and these credits would be available: "In our ideal state every payment, and consequently every loan, is accomplished by means of cheques or giro facilities. . . . No matter what amount of money may be demanded from the banks, that is the amount which they are in a position to lend" (I. P. 110). The price movement would spread—as a first approximation roughly proportionally—over all sectors of the economy. In other words, all the real factors would be maintained exactly as they are assumed in table (5.1). *But there would be a proportional increase of K, K', K'',* while $\bar{\rho}$ and ρ, and consequently also the difference $(\bar{\rho} - \rho)$ remained constant. So long as the increase in K, K', and K'' is proportional, nothing would be disturbed in the balancing of the accounts of the table (5.1). In other words, *the same argument* regarding the scramble for expansion under constant real factors, leading to a price inflation, could be *constantly repeated.* This is in essence the famous Wicksellian cumulative process. Any possible side effects on the underlying real situation could only take place within rather narrow limits, and would not be cumulative because the natural resources are not unlimited as is the potential bank credit. The above argument would work both ways, that is, both under an inflation and under a deflation.

So, the whole development depends on the difference between $\bar{\rho}$ and ρ. The market rate ρ can be changed more or less at will through a decision of the banks. For the natural rate $\bar{\rho}$, the situation is entirely different. As is seen from (5.2), this rate will depend only on the ratio C/K, which, on the assumptions accepted, is a *technical* datum. In other words, the natural rate $\bar{\rho}$ cannot be changed except as the result of a change in the underlying real factors. When we ask for the "cause" of the price inflation, it is, therefore, plausible to express it by saying that the market rate of interest "is kept too low."

This whole argument depends obviously on the possibility of segregating out a part of the entrepreneurs' annual consumption, which can be considered a "surplus" part, and therefore will create the incentive to expand entrepreneurial activity. Since this is a crucial part of the argument, it should be considered a little closer. What precisely is the criterion on which a part of the remuneration to the entrepreneurs can be segregated out as a "surplus" distinct from the "normal" part of the remuneration? Are not the entrepreneurs at liberty to draw the line of demarcation arbitrarily? If the answer is yes, the whole theory would really amount to saying that prices move up whenever the entrepreneurs happen to be in a mood to make them move up, and vice versa.

This apparent indeterminacy of the price tendency can be expressed in terms of the elements of table (5.1), by saying that any example of the form (5.1) which leads to a difference between the natural rate and the market rate, can, within the framework of Wicksell's ideas, be replaced by another where no such difference exists. To see that such a transformation is possible, we note that the set-up has four degrees of freedom which we may represent, say, by the parameters

K, K', C and ρ. Suppose that these four numbers are given. Using these data, let us transfer the activity which consists of simply *possessing* an amount $(\bar{\rho} - \rho)K$ of consumption goods, from the group "entrepreneurs" to the group "capitalist-commodity dealers," and maintain all the rest of the example. This, certainly, would not be contrary to Wicksell's way of reasoning in this matter. It would, indeed, seem to be the only reasonable thing to do when the capitalists are defined the way they are in Wicksell's reasoning here. This transfer would mean that we retain the original magnitudes K', C and ρ, while replacing K by $\bar{K}$ and determining this $\bar{K}$ by the condition that all the consumption goods produced are sold by the entrepreneurs in order to pay off their debts with the banks at the end of the year; i.e., we would have $C = (1 + \bar{\rho})K = (1 + \rho)\bar{K}$. This gives $\bar{K} = \dfrac{1 + \bar{\rho}}{1 + \rho} K = \left(1 + \dfrac{\bar{\rho} - \rho}{1 + \rho}\right) K$. The new example thus constructed would be of the same form as (5.1), with K replaced by $\bar{K}$, and K'' by $K'' + (\bar{K} - K)$, while K' and C would be unchanged. The new natural rate would by (5.2) be equal to ρ; i.e., the entrepreneurial "surplus" profit would now have disappeared.

The solution of the puzzle is that Wicksell has an additional consideration which gives an independent determination not only of K', but also of K'', so that a transformation of the above kind is excluded (and only proportional changes of K, K' and K''—for instance those that occur during an inflation or a deflation—are permitted). Indeed, on the "normal" part of the entrepreneurial profit Wicksell says: ". . . he (the entrepreneur) . . . obtains the same return for the trouble of conducting his business as he would have obtained for conducting similar business on behalf of others, for instance of a company" (I. P. 140). In other words, the "surplus" profit that starts the scramble for expansion emerges when the entrepreneur, who carries on business on his own account, realizes a *higher* remuneration for his own services and for that of the fixed capital goods which he possesses than he *could have* obtained by taking a salaried job and letting out on hire his fixed capital goods.

In the subsequent section I shall attempt to put the concept of the natural rate into a broader perspective and connect it with the other parts of Wicksell's theory.

6. SYNTHESIS BETWEEN CAPITAL THEORY AND MONETARY THEORY

In his capital theory Wicksell discusses the effects produced by a *change* in real capital, a "change" being interpreted as the shifting of our attention from one stationary situation to another. In his monetary theory, on the contrary, he assumes in essence a *constant* real capital. It is by no means easy to see how these two lines of thought gear into each other. And yet, there is an intimate connection between the two parts of the theory.

To bring out this connection we must have recourse to a "roundabout" way of reasoning. We must consider more closely what I called the productivity rate of interest (end of Section 2). This should not be taken as synonomous with what Wicksell calls the natural or normal or real rate nor with what he calls the market rate. It is only a parameter by which we so to speak *temporarily add one dimension to the problem.* This device elucidates, in my opinion, the whole problem. The reasoning about the Wicksellian cumulative process and the meaning of the concepts natural, normal, or real rate will then follow consistently—and indeed in a rather obvious way—by a consideration of a special case obtained by an additional assumption which again takes out one degree of freedom. When the theory is formulated in this way, it becomes, as I see it, immune to the special kind of criticism that has been directed against it by Lindahl, Myrdal, and Ohlin.[25]

In fig. (6.1) let the productivity rate ρ^* be measured on the vertical axis and the volume of real capital in use $k = K/P$ on the horizontal axis. This real value k is the same thing as the value of capital K when we put $P = 1$, i.e., when we measure value in terms of the product. In other words k is exactly the concept which Wicksell uses in his theory of capital. This volume k at any given point of time is a technical datum which is defined independently of the market rate. Indeed, the magnitudes $x_0 \cdots x_n, y_0 \cdots y_n$ of Section 2 are technical variables defined without any reference to a rate of interest. Similarly for the production function (2.2). Suppose that we are in a point $(x_0 \cdots x_n, y_0 \cdots y_n)$ belonging to the field (2.15) where α, β, ρ are any numbers. This is a necessary condition for the point to be in real term equilibrium. The value of the parameter ρ in this point as determined by (2.13) is the productivity rate of interest. Thus, in any point $(x_0 \cdots x_n, y_0 \cdots y_n)$ belonging to the field (2.15) the productivity rate is a technical datum. And so are p and q. Inserting these values in (2.8), we get K, and hence $k = K/P$, which is independent of P. In other words, in any point $(x_0 \cdots x_n, y_0 \cdots y_n)$ belonging to the field (2.15) the volume k of real capital is a technical datum, independent of the market rate. I am convinced that this is a true rendering of the essence of Wicksell's thought. It is not

[25] Summarized by Ohlin in his Introduction to *Interest and Prices*, 1936, p. xvii.

necessary to build the definition of k on an existing market rate. And even if we wanted to let the market rate influence the definition of k in some way, we might still arrive at two different concepts ρ^* and ρ.

Draw—as defined at the end of Section 2—the down-sloping curve that connects ρ^* and k. Let $k = F_k(\rho^*)$ be the function that expresses

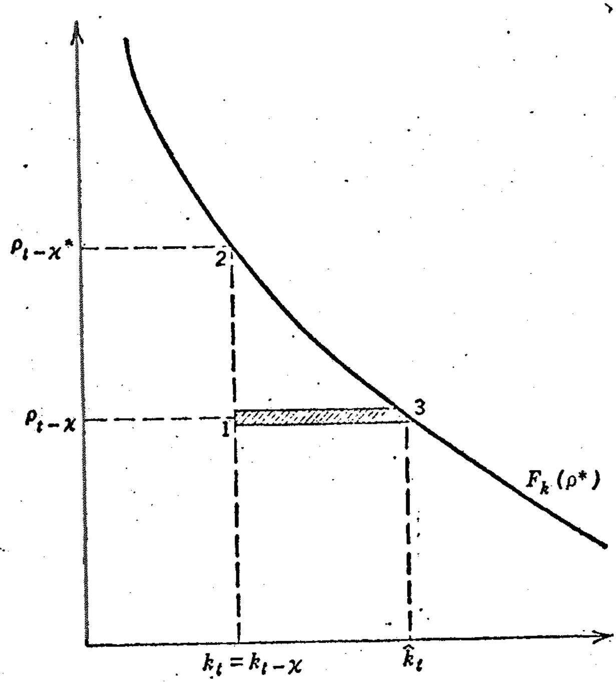

FIG. 6.1.

how k depends on ρ^* along this curve, and let $F_k^{-1}(k)$ be its inverse. The relative prices p and q of the input elements are determined by k and will therefore in general change with k along the curve. On the vertical axis of fig. (6.1) we also measure the market rate of interest ρ.

The curve of fig. (6.1), which was originally—in Section 2—defined as a static curve expressing a comparison between stationary alternatives, may also help to explain certain *changes over time*, provided we add some convention on the *rapidity* with which things might move.

Consider two points of time t and $t - \kappa$ separated by an interval of time κ, which we may call the entrepreneurial *reaction period*. This means that the situation at $t - \kappa$—characterized by the market rate $\rho_{t-\kappa}$ and the volume of real capital $k_{t-\kappa}$—will determine the plans of the entrepreneurs for t. If we retain the assumption of the abstract example (5.1), we may imagine that the production is split up into separate periods of length κ, so that the productive services of land and labour are performed at the beginning of each such period while the product falls due at the end of the period. In this case the reaction period will be exactly equal to the period of production. In reality the whole process is, of course, of a more continuous sort: Certain entrepreneurial reactions take place rapidly, others more slowly, due to institutional circumstances and the psychology of the entrepreneurs. In our model κ will therefore be some sort of average which need not be equal to the average period of production, but is simply a datum characteristic for the rapidity of entrepreneurial reaction. Wicksell was, of course, well aware of this concrete background of the problem, but his fast-working, abstract mind did not stop to go into a detailed discussion of the circumstances which in fact determine the length of the reaction period. So we simply take κ as a technically and institutionally given number.

If $\rho_{t-\kappa}$ and $k_{t-\kappa}$ are given, we may use the function $F_k(\rho^*)$—i.e., the curve of fig. (6.1)—for two purposes. In the first place we note that when real capital is $k_{t-\kappa}$, the productivity rate $\rho_{t-\kappa}{}^*$ is the ordinate of the curve for the value $k_{t-\kappa}$ of the abscissa, i.e., it is $F_k{}^{-1}(k_{t-\kappa})$. This is indicated by the point 2 of the diagram. Hence the difference between the productivity rate and the market rate at $t - \kappa$ is given by the vertical distance between the points 2 and 1. In the second place we note that the volume of real capital $\hat{k}_t$ which the entrepreneur *would like* to see realized at t is equal to $F_k(\rho_{t-\kappa})$, that is, the abscissa read off from the point 3 of the diagram. Hence the difference between the planned (ex-ante) volume of capital $\hat{k}_t$ and the existing volume of capital $k_{t-\kappa}$ is given by the horizontal distance between the points 3 and 1.

This being so, what will happen if the situation at $t - \kappa$ is given by the two numbers $\rho_{t-\kappa}$ and $k_{t-\kappa}$? According to the basic assumptions indicated in Section 5 the entrepreneurs will through the banking system be in a position to command the nominal (money) capital K_t which they need for carrying out their plans, but they might *not* be in a position to control the actual magnitude k_t of the volume of real capital at t. This is crucial for the whole reasoning. Wicksell assumes $k_t = k_{t-\kappa}$, but that is not an essential part of the argument. What is essential at this point is only to take the actual volume of capital k_t which will be realized at t as some given magnitude. The development of prices—that is, of the *general price level* of the goods which form the product of the production process, i.e., the price P of the volume z defined in Section 4—can now be determined. Indeed, let $P_{t-\kappa}$ and P_t be these price levels at $t - \kappa$ and t, respectively, and let us for the time being assume that the entrepreneurs do *not* take account of anticipated price changes. They will then want to employ at t a nominal (money) capital equal to $P_{t-\kappa}F_k(\rho_{t-\kappa})$, and will by assumption actually employ it. That is,

$$(6.3) \qquad K_t = P_{t-\kappa}F_k(\rho_{t-\kappa}) \qquad \text{i.e., } k_tP_t = P_{t-\kappa}F_k(\rho_{t-\kappa})$$

One should note the fundamental logical difference between $\hat{k}_t = F_k(\rho_{t-\kappa})$ and $K_t = P_{t-\kappa}F_k(\rho_{t-\kappa})$. The difference is more than just a transformation from a volume figure into a value figure. The former is an ex-ante figure and the latter an ex-post figure. From (6.3) follows immediately

$$(6.4) \qquad P_t - P_{t-\kappa}/\kappa P_{t-\kappa} = F_k(\rho_{t-\kappa}) - k_t/\kappa k_t$$

The divided difference to the left in (6.4) is the relative rate of change of the general price level over the interval κ (for $\kappa \to 0$ we get the logarithmic derivative d lognat P_t/dt). The expression to the right in (6.4) is obtained graphically from fig. (6.1). Consider the point 1 with ordinate $\rho_{t-\kappa}$ and abscissa k_t (which now need not be equal to $k_{t-\kappa}$), and also consider point 3. The numerator in the right member of (6.4) is the horizontal distance between the points 3 and 1 (positive in the example), and the denominator is equal to κ times the abscissa of the point 1. In other words the ratio which the numerator of (6.4) bears to the denominator *is directly and easily read off from the graph*, apart from the factor κ. The smaller the factor κ, under a given shape of the curve in (6.1), the larger will be the relative rate of change of the price level. This is only an expression for the obvious fact that the price level will move all the faster the quicker the entrepreneurs react. A finite rate of change of the price level is due to a non-zero reaction period of the entrepreneurs.

The above gives already a first part of the more elaborate theory of the cumulative process. Assuming that the actual volume of real capital at t (the abscissa of the point 1 in fig. (6.1) is *given*, we can say that the general price level will *increase* when the market rate is below the productivity rate that corresponds to the given volume of capital, and that it will *decrease* in the opposite case. And we can further say that the price level will move all the faster the greater the difference between the market rate and the productivity rate (the greater the vertical distance between the points 2 and 1). These conclusions follow immediately from a mere inspection of fig. (6.1) provided the curve is sloping down (which was one of the main results of the analysis of Section 2).

The above analysis uses *two* data: a given market rate and a given volume of capital. This is the sense in which we may call the analysis two-dimensional. So long as only the shape of the curve in fig. (6.1) is given, the point $(\rho_{t-\kappa}, k_t)$ may fall anywhere in the diagram. To reach the final formulation of the theory of the cumulative process we must add a new datum, namely a schedule expressing the supply of capital, i.e., the willingness of the public to wait. First assume that there are no cash holdings and that $F_h(\rho_t)$ is a function that indicates the real (deflated) value of capital held h_t, i.e., of deposits, which the public wants to maintain when the market rate is ρ_t (the interest on deposits and that on loans are assumed equal). In other words $F_h(\rho_t)$ is a *reserve-preference* schedule for the public (when there are no cash-holdings). In this connection we do not discuss the other factors (income, etc.) on which F_h might depend.

It is assumed that the public at any time is in a position actually to make the deposits it wants to make, so that actual deposits H_t measured in current monetary units are at any time equal to $H_t = P_tF_h(\rho_t)$. Let the upward-sloping dotted curve of fig. (6.2) represent the shape of the function $F_h(\rho_t)$—the supply curve for the volume of (the deflated value of) deposits, the ordinate of the curve being ρ_t and the abscissa the value of the function $F_h(\rho_t)$. Wicksell assumes that $F_h(\rho_t)$ is an *increasing* function of the market rate ρ: "A high rate of interest encourages saving" (L. II. 113). In fig. (6.2) we have indicated the curve rather steep because h might not be very strongly influenced by ρ. The whole argument leading up to the Wicksellian cumulative process can be applied even though $F_h(\rho_t)$ is independent of ρ_t (the curve a vertical line). If there are no cash holdings, we must by (3.9) have $k_t = h_t$, so that $k_t = F_h(\rho_t)$. Any actually realized (ρ_t, k_t) point, such as 1, must therefore lie on the supply curve. In other words, having introduced this curve we are now confronted with a one-dimensional analysis of points along this curve. By (6.4) this gives

$$(6.5)\quad P_t - P_{t-\kappa}/\kappa P$$

$$= F_k(\rho_{t-\kappa}) - F_h(\rho_t)/\kappa F_h(\rho_t) \qquad (\text{when } M_t = 0)$$

The right member of this formula only depends on the number κ, on the market rates at t and $t - \kappa$ and on the shapes of the two curves in fig. (6.2). If $\rho_{t-\kappa} = \rho_t$, the ratio expressed by the right member of (6.5) is obtained by drawing a horizontal line indicating the level of the market rate and expressing the horizontal distance between the points 1 and 3 as a fraction of the abscissa of point 1. This ratio divided by κ will give the actual relative rate of change of the price level over the interval between $t - \kappa$ and t.

The analysis of how the price change is determined can be further simplified. Indeed, when the time interval κ and the shapes of the two curves of fig. (6.2) are given, we may compute the ratio in question for each level ρ of the market rate (assumed the same in $t - \kappa$

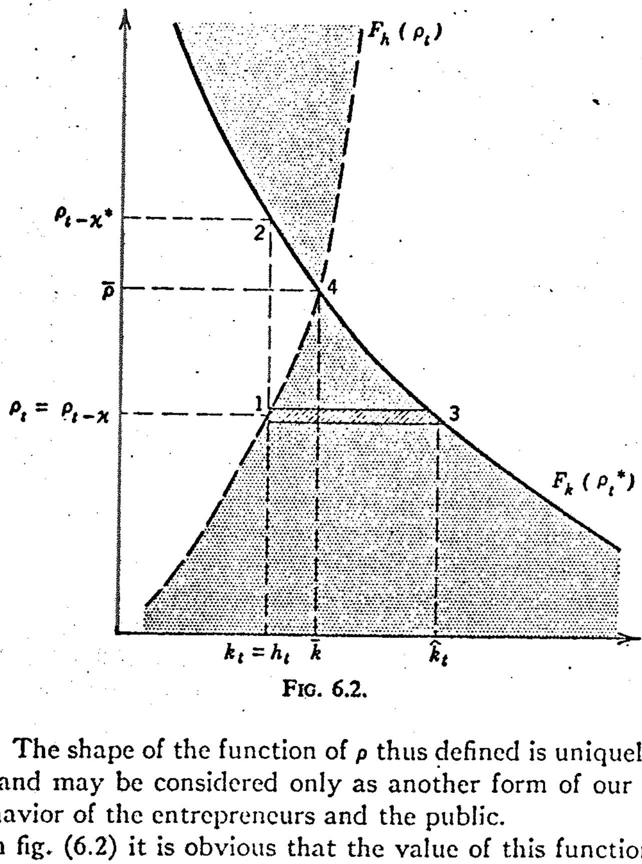

FIG. 6.2.

and t). The shape of the function of ρ thus defined is uniquely determined and may be considered only as another form of our data on the behavior of the entrepreneurs and the public.

From fig. (6.2) it is obvious that the value of this function *passes from positive to negative* when ρ passes a specific value $\bar{\rho}$ which is uniquely determined by the shapes of the two curves. This value $\bar{\rho}$ is the root of the equation

$$F_k(\bar{\rho}) = F_h(\bar{\rho}) \tag{6.6}$$

The root of this equation—in fig. (6.2) the ordinate of the point 4—is the *natural*, or if we like, the *normal* rate of interest. It is ". . . the rate of interest which would be determined by supply and demand if no use were made of money and all lending were effected in the form of real capital goods" (I. P. 102). Or shorter: "The rate of interest at which *the demand for loan capital and the supply of savings* exactly agree" (L. II. 193). Thus, while the market rate may be observed directly in the market place, the natural or normal rate is only the root of an equation. It is, however, in principle just as realistic as the market rate and may be computed when the shapes of the two curves are known.

In the above-mentioned function giving the right member of (6.5) as a function of ρ, let us introduce $\delta = (\rho - \bar{\rho})$ as argument instead of ρ. This gives

$$(6.7) \qquad P_t - P_{t-\kappa}/\kappa P_{t-\kappa} = \phi(\delta_t) \qquad (\text{where } \delta_t = \rho_t - \bar{\rho})$$

The form of the function ϕ is defined by

$$(6.8) \quad \phi(\delta) = F_k(\delta + \bar{\rho}) - F_h(\delta + \bar{\rho})/\kappa F_h(\delta + \bar{\rho})$$

(where $\bar{\rho}$ is determined by [6.6])

It is obvious from fig. (6.2) that the function $\phi(\delta)$ is positive when δ is negative (the situation exemplified in fig. [6.2]), and vice versa. It is further clear that $\phi(\delta)$ is a monotonically decreasing function of δ. This function $\phi(\delta)$ gives the essence of the theory of the cumulative process: *So long as the market rate of interest is maintained at a level below the normal rate, the general price level will be increasing all the time, and the increase will be all the faster the greater the difference between the market rate and the normal rate.* Vice versa if the market rate is maintained at a level above the normal rate. Thus, the difference between the market rate and the normal rate is responsible for the *change* in prices, not for the absolute level of prices. If an inflation has been going on for some time because the market rate has been lower than the normal rate, the price level will be brought to a *standstill*, but will not be brought back to its previous state if the market rate is made equal to the normal rate (L. II. 196).

In my opinion this conclusion points out something of great importance in a free competitive economy. This is not the place to discuss the modifications necessary in an economy which is more or less directly controlled. Nor do I propose to discuss *how low* the market rate would have to be if it should under exceptional circumstances in a free economy (for instance in the depression of 1930 in U. S.) be able to turn a tide of deflation. It might then possibly have to be negative.

The equilibrium point where $\rho = \bar{\rho}$ has several interesting properties. When we speak of the "interest rate" at this point, it does not matter whether we think of the market rate ρ, the productivity rate

ρ^* or the normal rate $\bar{\rho}$ because all these concepts are here equal. And when we speak of "capital" at this point, it does not matter whether we think of the volume of capital which the entrepreneurs would like to employ $\hat{k}_t$ or that which they actually employ k_t. When the market rate deviates from $\bar{\rho}$, we may from fig. (6.2) and the preceding formula derive a great number of concrete practical interpretations on which I need not dwell here.

The argument can easily be generalized to the case where ρ_t and $\rho_{t-\kappa}$ may be different. If they are, the points 1 and 3 would be at different levels, 1 at the level $\rho_{t-\kappa}$ and 3 at the level ρ_t. For instance if $\rho_{t-\kappa}$ is below the normal rate the ensuing increase in the price level may be *somewhat* reduced by raising ρ_t—perhaps even above $\bar{\rho}$—but this compensating effect would not be very great if the F_h curve is steep. If this curve is vertical, ρ_t is without influence, only $\rho_{t-\kappa}$ will then effect the ratio (6.5).

If cash holdings exist, we let as before $F_h(\rho_t)$ denote the preference schedule for (the deflated value of) deposits and introduce a new function $F_m(\rho_t)$ expressing the preference schedule for (the deflated value of) cash holdings, so that $F_{hm}(\rho_t) = F_h(\rho_t) + F_m(\rho_t)$ will be a total reserve preference function of the public. Assuming that nothing prevents the public from maintaining at any time the cash holdings they like, we see by (3.9) that the real (deflated) value of actual loans will be $k_t = F_{hm}(\rho_t)$. Inserting this for k_t in (6.4) we get

$$(6.9)\quad P_t - P_{t-\kappa}/\kappa P_{t-\kappa} = F_k(\rho_{t-\kappa}) - F_{hm}(\rho_t)/\kappa F_{hm}(\rho_t)$$

(in the general case)

Essentially this leads to the same sort of theory as in fig. (6.2) only with a different meaning of the supply curve. In any case the essence of Wicksell's explanation of the relative price change is the *difference* between a demand and a supply magnitude.

Both these magnitudes in Wicksell's theory represent a *stock* concept. We shall now look into a circumstance which at first sight seems rather surprising, namely that the relative price change can also be expressed in several ways as a ratio whose numerator is a difference between certain *flow concepts*, which in essence are rates of change with respect to time of the nominal values of the magnitudes in the numerators of (6.5) and (6.9).

From the first expression in (3.2) we get $\dot{K}_t - \dot{k}_t P_t = k_{t-\kappa}\dot{P}_t$, hence by (3.6) $k_{t-\kappa}\dot{P}_t = \dot{K}_t - I_t$, or by (3.10) $k_{t-\kappa}\dot{P}_t = \dot{K}_t - (S_t + \dot{m}_t P_t)$. That is,

$$(6.10)\quad P_t - P_{t-\kappa}/\kappa P_{t-\kappa} = \dot{K}_t - (S_t + \dot{m}_t P_t)/K_{t-\kappa}$$

$$= \dot{K}_t - I_t/K_{t-\kappa} \qquad \text{(in the general case)}$$

If the cash holdings are not identically zero, we may make a further transformation. In (6.10) we insert for I_t the expression $\dot{k}_t P_t = (\dot{h}_t + \dot{m}_t)P_t = \dot{H}_t - h_{t-\kappa}\dot{P}_t + \dot{m}_t P_t$, and carry the term with $\dot{P}_t$ over to the left. When $M_{t-\kappa} \neq 0$, this gives

$$(6.11) \quad \dot{P}_t - P_{t-\kappa}/\kappa P_{t-\kappa} = \dot{K}_t - (\dot{H}_t + \dot{m}_t P_t)/K_{t-\kappa} - H_{t-\kappa}$$
$$= \dot{M}_t - \dot{m}_{t-\kappa}/M_{t-\kappa} \qquad (\text{if } M_{t-\kappa} \neq 0).$$

If there is no buffer effect, that is, if $\dot{m}_t = 0$ for all t, an obvious simplification takes place in the formulae (6.10–11). The first numerator to the right in (6.10) then becomes $\dot{K}_t - S_t$, and that in (6.11) $\dot{K}_t - \dot{H}_t$.

In (6.10–11) the relative price increase is expressed in various ways as a fraction whose numerator is the difference not between two stock concepts as in (6.9), but between two flow concepts. At the same time the factor κ in the denominator has disappeared. The numerator in the first expression to the right in (6.10) is the difference between *investment*, reckoned inclusive of appreciation on capital in use, and *saving*, reckoned exclusive of appreciation on the reserves but embracing all reserves, both deposits and cash holdings. The numerator in the last expression in (6.10) is the difference between investment, reckoned inclusive of appreciation on capital in use, and investment, reckoned exclusive of such appreciation. Similarly the numerator of the second expression in (6.11) is the difference between investment, reckoned inclusive of appreciation on capital in use, and saving, reckoned inclusive of appreciation for the deposits part and exclusive of appreciation for the cash holdings part.

In the argument leading up to (6.5)—or more generally up to (6.9)—and further to (6.10–11) we have, with Wicksell, assumed that a complete renewal of the capital stock takes place within the time interval considered, and that therefore the "general" price level P_t is also the price level of the goods of which the capital stock is made up. Compare the example (5.1). This, of course, is a strong simplification. In a further development of the analysis along these lines one would want to consider separately the value of gross annual investment J (defined as $J = I + D$, where D is annual depreciation on capital). This would lead amongst others to an explicit distinction between the price of capital goods and that of consumption goods, but I shall not follow up this line of thought here.

Any of the numerators to the right in (6.10) may be looked upon as a definition of an "inflationary credit expansion." For instance, in a case where all three terms in the first equation in (3.2) are positive, we may look upon $\dot{K}_t$ as total credit expansion or, if we like, as investment in a broad sense, including appreciation on capital. This investment we may say is financed *in part* by actual saving k_tP_t (including in this expression both S_t and $\dot{m}_tP_t$) and *in part* by the "inflationary credit expansion" ($\dot{K}_t - k_tP_t$). Since the latter expression is the same as $k_{t-\kappa}\dot{P}_t$ (which leads to [6.10]), we see how the inflationary credit expansion causes the price level to rise. When Wicksell speaks about "credit as between man and man" (L. II. 193), I think he simply means such credit operation that does *not* give any chance for producing an "inflationary credit expansion" because it does not pass through the banks. On the subsequent page (L. II. 194) he explains the elastic credit supply through the banks.

Since all the ratios (6.10–11) are by definition identically equal to (6.5), or more generally to (6.9), we may, if we so choose, consider the motivations expressed by the right members of (6.5) or (6.9) *equally as motivations concerning* the ratios (6.10–11), and we could, if we wanted to, develop the whole theory by starting from a consideration of the difference in the numerators in any of these ratios. Therefore, there is no contradiction between the facts expressed by (6.10–11) and Wicksell's reasoning on the stock concepts in (6.6) and (6.9). In the above analysis both aspects of the problem—that relating to the stock concepts and that relating to the flow concepts—are brought out in one coherent system of concepts and notations.

7. A DYNAMIC THEORY OF THE BUSINESS CYCLE

A cumulative process may start any time and continue for a considerable period because the normal rate may be changed for a great number of causes, technical and otherwise while ". . . the banks never alter their interest rates unless they are induced to do so by the force of outside circumstances" (L. II. 204). Such circumstances will eventually emerge for institutional and other reasons. Thus there exists a tendency for the market rate to gravitate towards the normal rate, but this tendency only comes about through the price movement. In a period of expansion the price movement will (à la banking principle) increase the need for cash holdings by the public and thereby put a strain on the banks. This conception of the price movement as the vehiculum for forcing the market rate in line with the normal rate is a major point in Wicksell's reasoning: ". . . there exists . . . no other connection between the two than the *variations in commodity prices* caused by the difference between them. And this link is elastic, just like the spiral springs often fitted between the body of a coach and the axles" (L. II. 206).

To bring out the manner in which these various factors act and react on each other during an upswing, a turning phase and the subsequent downswing, according to Wicksell's ideas, it will be well to use a simple dynamic model which admittedly disregards many concrete details, also many details discussed by Wicksell, but in return brings out clearly the main structure of the argument.

If we should incorporate (6.7) as it stands—with a finite κ—into a dynamic theory, we would be led to mixed difference and differential equations of an extremely complicated nature. Such an analysis will, however, not be necessary for the present purpose as we only aim at a rough description of the course of affairs. We simply replace the *average* rate of change of P over the interval κ, which is written in the left member of (6.7) by the *instantaneous* rate at t. Introducing for brevity Q_t = lognat P_t, the equation (6.7) will then take the form

$$(7.1) \qquad \dot{Q}_t = \phi(\delta_t) \qquad \text{where } \dot{Q}_t = dQ_t/dt$$

The effect of the finite number κ will still be present because it affects the form of the function ϕ in the right member of (7.1). The larger κ, the smaller ϕ. Compare (6.8). Since $\phi(\delta)$ is a monotonically decreasing function of δ and positive for $\delta < 0$, negative for $\delta > 0$, we may as a rough linear approximation put $\phi(\delta) = -\beta\delta$, where β is a positive constant, all the smaller the larger κ. This gives

$$(7.2) \qquad \dot{Q}_t = -\beta\delta_t \qquad (\beta \text{ a positive constant})$$

In the course of a cycle the attitude of entrepreneurs as well as that of the public will change. In particular it will change through *anticipations*. Wicksell considers them very explicitly. Here it will be sufficient to take account of how the *price changes* themselves affect the anticipations and thereby the behaviour and preferences of the individuals: "The upward movement of prices will in some measure 'create its own draught.' When prices have been rising steadily for some time, entrepreneurs will begin to reckon on the basis not merely of the prices already attained, but of a further rise in prices . . ." (I. P. 96). "Indeed, if the rise in prices itself gives birth to exaggerated hopes of future gains, as often happens, the demand for bank credit may far exceed the normal . . ." (L. II. 207).

We can express this by saying that the functions F_k and F_h (or more generally F_{hm}) depend explicitly on the rate of change of prices and that consequently also $\bar{p}$ depends on this rate. This does not necessarily mean that we must assume the form of the function ϕ to the right in (7.1)—and β in (7.2)—to depend explicitly on the price change. Indeed an important part of the anticipation effects produced by price change is already taken care of when we say that the normal rate $\bar{\rho}$ depends on the price change. Let us suppose that this is sufficiently accurate, so that we can still assume in the right member of (7.1) a given function ϕ of constant form and depending only on the single variable δ_t. As an example of such a situation consider the case where the entrepreneurs as well as the public are motivated by interest in real terms instead of in nominal terms. They will then consider a nominal market rate ρ as equivalent to a real-terms interest rate of $(\rho - \dot{Q})$. In other words, at the market rate ρ there would now be a demand for real capital equal to $F_k(\rho - \dot{Q})$ and a supply of real deposits equal to $F_h(\rho - \dot{Q})$, where F_k and F_h are the functions previously considered. In other words, the supply and demand curves in fig. (6.2) would simply get *an equal vertical shift* which means that the root of the new equation (6.6)—or more generally of the equation with F_{hm} instead of F_h—would simply be $\bar{\rho}_t = \bar{\rho}_0 + \dot{Q}$ where $\bar{\rho}_0$ is the root of the original equation. This is an example of a case where we would still have (7.1) with a function ϕ depending only on the one variable $\delta_t = \rho_t - \bar{\rho}_t$. More generally let us put

$$(7.3) \quad \bar{\rho}_t = \bar{\rho}_0 + \gamma \dot{Q}_t + \lambda Q_t \qquad \text{(where } \bar{\rho}_0, \gamma, \lambda \text{ are constants)}$$

and let us assume that this transformation is sufficient to take care of the anticipations so that we still have an equation of the form (7.1). The above example shows that a value of γ in the neighbourhood of 1 might not be too unrealistic. The inclusion of a term with Q_t besides $\dot{Q}_t$ in (7.3) can be looked upon more as a means of producing a shift in phase of the time variable part of ρ_t (as one would by aggregating, with weights, a sine function and its derivative) than as a means of introducing the effect of a *partial* variation of Q_t under constant $\dot{Q}_t$.

Now for the other factors that change cyclically. In the strictly monetary part of the theory Wicksell assumes constant output and constant real capital, but in his special comments on the business cycle (L. II. 209) he very much stresses the real factors, recognizing however also the importance of money (L. II. 210). All in all I think it is correct to say that his attitude is that the essence of the problem resides more in the interplay of the factors than in any one of them taken separately.

To incorporate total output in this interplay, it will for the present purpose suffice simply to use the well-established fact that total output and prices, roughly speaking, *move together* in the cycle. We might express this by assuming

$$(7.4)\quad \dot{z}_t/z_t = \mu(\dot{P}_t/P_t),$$

$$\text{i.e., } \dot{z}_t/z_t = \mu\dot{Q}_t \qquad (\mu \text{ a positive constant})$$

In Wicksell's theory the velocity of circulation of money V is conceived of in such a manner that $Pz = MV$ becomes the equation of exchange. For the velocity V as thus defined it would seem reasonable in our simplified model to assume a relation similar to that of (7.4).

$$(7.5)\quad \dot{V}_t/V_t = \nu(\dot{P}_t/P_t),$$

$$\text{i.e., } \dot{V}_t/V_t = \nu\dot{Q}_t \qquad (\nu \text{ a positive constant})$$

Finally we may condense Wicksell's argument about the behaviour of the banking authorities into a relation between the *acceleration* of δ_t, i.e., $\ddot{\delta}_t$, and the increase in the cash holdings of the public. This increase is indeed in Wicksell's theory the main element which puts a strain on the liquidity situation of the banks and thus finally produces a motivation for an adjustment of the market rate. In the first approximation we may express it by putting

$$(7.6)\qquad \ddot{\delta}_t = \Theta(\dot{M}_t/M_t) \qquad (\Theta \text{ a positive constant})$$

The coefficient Θ in (7.6) must be positive because it is a positive increase in the cash holdings of the public that produces the incentive for the banks to raise the market rate.

From the above equations we get by simple substitutions

$$(7.7)\qquad \ddot{\delta}_t = -\alpha^2\delta_t \qquad \text{where } \alpha^2 = \beta\Theta(1 + \mu - \nu)$$

The meaning of the coefficients in (7.7) is such that it is realistic to assume $1 + \mu - \nu > 0$, hence the time shape of δ_t is

$$(7.8)\qquad \delta_t = -A\cos(a + \alpha t)$$

the amplitude A and the phase a being determined by initial conditions which may be given conventionally.

From (7.7) is seen that the cycle is all the *shorter:*

1. The larger β, i.e., the stronger the price effect of the difference between market rate and normal rate (in particular: all the shorter the shorter the reaction period κ).
2. The larger Θ, i.e., the stronger the banking authorities react on a tightening of the liquidity situation.
3. The larger μ, i.e., the more elastic the reaction in total output is.

4. The smaller ν, i.e., the more constant the velocity of circulation of money is.

Inserting (7.8) into (7.2) we get

(7.9) $\dot{Q}_t = \beta A \cos (a + \alpha t) \qquad Q_t = \beta A/\alpha \sin (a + \alpha t) + \text{const}$

Carrying this into (7.3) and again using (7.8) we get after some reduction

(7.10) $\bar{\rho}_t = A\sqrt{\alpha^2\gamma^2 + \lambda^2} \sin (a' + \alpha t) + \text{const}$

(7.11) $\rho_t = A\sqrt{\alpha^2(\gamma - 1/\beta)^2 + \lambda^2} \sin (a'' + \alpha t) + \text{const}$

The phase relations of a' and a'' to a are given by

(7.12) $tg(a' - a) = \alpha\gamma/\lambda \qquad tg(a'' - a) = \alpha(\gamma - 1/\beta)/\lambda$

If by convention the square root in (7.10–11) is taken positive, the angle $(a' - a)$ is to be taken in the first quadrant and $(a'' - a)$ in the first or fourth quadrant.

These phase relations are interesting. Indeed the three time series $\bar{\rho}_t$, Q_t, ρ_t may be taken as characterizing the same three groups of phenomenon as the famous Harvard A, B, C-curves: A = speculation, i.e., industrial stock prices, etc., leading the movement; B = cost of living index and production, moving in the middle of the cycle; and C = interest rate, lagging behind. From the Wicksellian viewpoint $\bar{\rho}_t$ is undoubtedly the nearest we can come to an expression for speculation: The entrepreneurs look to $\bar{\rho}_t$ to find motivation for what to do next, and this determines the course of prices. On the other hand the market rate is pulled towards $\bar{\rho}_t$. Q_t represents the B-curves and ρ_t the C-curves. The nature of the A, B, C lags and a rough estimate of how they depend on the structural coefficients of the system are brought out by the above simplified model: from (7.12) follows that $\bar{\rho}_t$ must always lead over Q_t. And Q_t will lead over ρ_t when $\gamma < 1/\beta$. This inequality will certainly prevail when the movement of prices does not exert an exceptionally strong influence on the anticipations of the entrepreneurs and of the public, and the entrepreneurs are not extremely quick in demanding the new loans which a high level of the productivity rate may warrant (κ not too small). Since it is an empirical fact that the market rate will usually lag behind the price level (about one-eighth of a period), it seems reasonable to conclude that the two conditions just mentioned are fulfilled in reality.

The above extremely simplified account of the cyclical behaviour of prices, interest rate, and output proceeds on the assumption that there is no friction or similar phenomena which will eventually dampen the oscillations. In reality such dampening factors are, of course, present, and a fundamental problem arises of what is the source of energy that *maintains* the oscillations.

On this point we find what, in my opinion, is one of the most profound contributions of Wicksell: The source of energy that maintains the oscillations is to be found in the fact that the human population is *increasing* and that the technical progress does not proceed in perfect *simultaneity* with the population increase. The population increase proceeds fairly smoothly, but technical progress is distributed irregularly in time, and this very irregularity, paradoxical as it may seem, is just the explanation of why the observed, approximately regular economic fluctuations are maintained.

In a meeting of the "Nationalekonomiske Föreningen" on 27 March 1924 (where Professor Harald Westergård had delivered an address on Economic Barometers) Wicksell took part in the discussion, and the following precious words from him are conserved in print (Nationalekonomiske Föreningens Forhandlinger, 1924, p. 86. Published in *Ekonomisk Tidskrift*): "I beg to be excused for bringing in an old thought I have had, but for which I have nowhere found any response. I can't drop the idea, however, because I cannot find that it has been disproved. In my opinion there is one particular fact in the human economy which by necessity must produce a disturbance in it. It cannot proceed evenly from one year to another so long as there is an increase in population. The increase in population, which goes on all the time, does not only require that the new men get employed like the old, nor is it enough that capital accumulation goes on at the same rate as the increase in population, but it requires in addition—because the large factor nature is unchanged—that there are all the time introduced new methods of production, that is, technical progress goes on. The question then is if this technical progress can proceed according to a curve that increases as smoothly as the curve of the increasing population. It is difficult to escape the conclusion that here there must be a certain lack of harmony. The technical progress will either come a little before or a little after the increase in population. In the former case there ought to be an increase and in the latter case temporarily a decrease in the standard of living. This cause in itself is, of course, all too irregular to produce a true periodicity, even if we do not go very far in our requirements to regularity. It may be, however, that there is something else which is responsible for the periodicity, namely the structure of the human society itself. The difference between technical progress and human wants causes a jerk in the organism, and this jerk is transformed into a wave proceeding in a certain rhythm because of the structure of the human society itself. It takes for instance a certain time before one summons courage after having passed through economic disasters, etc. I have many times used the analogy that if one hits a rocking horse with a hammer, the blows may fall quite irregularly and still the movement of the rocking horse be more or less regular because of its own form."

These words, in all their brevity, give a fairly complete statement, I think, of the basic principles of the theory of erratic shocks which have come to mean so much in modern economic theory. They form so to speak the final link in the long chain of Wicksell's thoughts that lead all the way from the ultra-simple, abstract assumptions concerning the fundamentals of capital theory, through the somewhat less abstract theory of the cumulative process to a conception of the full-fledged modern society in its progressing and swinging form.