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THE INFRA EFFECT OF INVESTMENTS

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1. The capacity effect and the infra effect of an investment

An investment may have two effects: *First,* it may increase the *capacity* in a given sector. *Second* it may change the current account input coefficients in a given sector. Sometimes the sole or at least the main purpose of an investment is, indeed, to save certain kinds of inputs. This aspect of the problem will be considered here.

The effect produced by changing the input coefficients in the receiving sector K – i.e. in the row K in an input–output table – we will call the *infra effect* in K.

Terminologically we may look upon this effect as something which is "below" or "before" the effects that are *visible* in an ordinary interflow table built on constant coefficients. Compare infrared as the designation of that part of the spectrum which is "below" or "before" the red end of the visible part of the spectrum. Sometimes we will speak of the coefficients in the row K as *modulated*[1], but most of the time we will stick to the term infra effect. One advantage of the word infra is that it is *short* and very easy to handle phonetically.

2. Inversion with one changed row

We will first consider the following formal problem: Given the input coefficient matrix X'_{kh} (k, h = 1, 2 . . . n) where k is a delivering sector and h a receiving sector. Suppose we change the column K by adding here the *infracoefficients* X^{*}_{kK}. That is, the column K now to the considered is $(X'_{kK} + X^{*}_{kK})$ (k = 1, 2 . . . n).

[1] The term modulated was suggested by Dr. Abdel Rahman, Chief of the National Planning Committee, Cairo.

If we now require that the final deliveries shall be X_{k*}, what will the total productions X_h have to be?

To answer this question we start from the definitional equation

$$\sum_h (X'_{kh} + X^*_{kh}) X_h + X_{k*} = X_k \qquad (k = 1, 2 \ldots n) \tag{2. 1}$$

Here we now assume

$$X^*_{kh} = 0 \text{ except for } h = K \tag{2. 2}$$

In the usual way these equations can be written

$$\sum_h (e - X' - X^*)_{kh} X_h = X_{k*} \qquad (k = 1, 2 \ldots n) \tag{2. 3}$$

where

$$e_{kh} = \begin{cases} 1 \text{ if } h = k \\ 0 \text{ otherwise} \end{cases} \tag{2. 4}$$

Moving X_K over into the right member of (2. 3) we get

$$\sum_{h \neq K} (e - X')_{kh} X_h = X_{k_*} - (e - X' - X^*)_{kK} X_K \qquad (k = 1, 2 \ldots n) \tag{2. 5}$$

Writing out explicitly the equations for $k \neq K$ and for $k = K$ we get

$$\sum_{h \neq K} (e - X')_{kh} X_h = X_{k_*} + (X' + X^*)_{kK} X_K \qquad (k \neq K) \tag{2. 6}$$

$$(1 - X' - X^*)_{KK} X_K = X_{K*} + \sum_{h \neq K} X'_{Kh} X_h \tag{2. 7}$$

We use the notation

(2. 8) $\overset{-1}{(e - X')}_{hk \neq K}$ = the inverse of the $(n - 1)$ rowed submatrix obtained from $(e - X')_{kh}$ by omitting the row K and the column K.

This submatrix can be computed from the complete inverse $\overset{-1}{(e - X')}_{hk}$ by the single step building down procedure indicated in Section 18 of the Memorandum of 12 September 1958 from the University Institute of Economics, Oslo.

From (2. 6) we get

$$X_h = \Big[\sum_{k \neq K} \overset{-1}{(e - X')}_{hk \neq K} X_{k*} \Big] + X_K \Big[\sum_{k \neq K} \overset{-1}{(e - X')}_{hk \neq K} (X' + X^*)_{kK} \Big] \quad (h \neq K) \tag{2. 9}$$

The first bracket in (2. 9) represents an unmodulated effect.

Inserting the expression (2. 9) in (2. 7), we get

$$(2.10)\ X_K = \frac{X_{K*} + \sum_{k \neq K} \left[\sum_{h \neq K} X'_{Kh} (e - X')^{-1}_{hk \neq K} \right] X_{k*}}{(1 - X' - X^*)_{KK} - \sum_{h \neq K} \sum_{k \neq K} X'_{Kh} (e - X')^{-1}_{hk \neq K} (X' + X^*)_{kK}}$$

The last equation permits to determine $\underset{K}{X_K}$ explicitly as a linear form in the final deliveries X_{k*} ($k = 1, 2 \ldots n$). And inserting these linear forms into (2. 9) we get the remaining X_h expressed as linear forms in the deliveries X_{k*} ($k = 1, 2 \ldots n$).

The *coefficients* of these linear forms constitute an n rowed matrix, which is the *infra corrected inverse*. It is corrected with respect to a modulation of the input coefficients for sector K. If the complete inverse in the uncorrected coefficients is known, it does not take too much work to determine the infra corrected inverse.

If we had a simple model where the basis equations are expressed in terms of investment startings and with all sectors as lee sectors (i.e. as sectors with the sector product as a variable which is not necessarily at its upper bound – the capacity bound –), the above procedure permits to determine the *coefficients* as functions of the startings. If we have a model like the channel model used in the National Planning Committee, Cairo, with only a limited number of lee sectors, the reasoning would be the same, except that the inversion would be of a lower order.

If we not only ask for the *coefficients* in the new situation but want to express sector products and, perhaps, other things in terms of the startings, we run into a *quadratic* problem. For the moment we will not approach the problem in this perfectly general setting but instead investigate if we can find a simpler way of measuring approximately the infra effect of an investment.

3. The National Product and its maximization

Let us consider the definition of the national product and certain broad formal questions connected with its maximization.

Let X_{k*} be final delivery from sector k and let X_{*h} be the primary input into sector h. Simplifying the concepts to their lowest terms we can say that we have

(3. 1) $\quad \Sigma_k X_{k*} =$ gross national product

(3. 2) $\quad \Sigma_h X_{*h} =$ value added (input of primary factors)

Since the sum of the row sums in any table is equal to the sum of the column sums, the two expressions (3. 1) — (3. 2) must be equal. It has therefore no meaning to say that we want to maximize (3. 1) under a given (3. 2), or minimize (3. 2) under a given (3. 1).

The maximization of (3. 1) gets a meaning if we split each X_{*h} in two terms

$$X_{*h} = X_{Wh} + X_{\varepsilon h} \qquad (h = 1, 2 \ldots n) \tag{3. 3}$$

The first of these two terms indicate primary factors input (labour) and the second term indicates net gain (residual input). We then have

$$\Sigma_k X_{k*} = \Sigma_h X_{Wh} + \Sigma_h X_{\varepsilon h} \tag{3. 4}$$

The maximization of national product may now be looked upon as the maximization of the second term in (3. 4) under a given value of the first term, i.e.

$$\Sigma_h X_{\varepsilon h} = \max \tag{3. 5}$$

$$\Sigma_h X_{Wh} = \text{given} \tag{3. 6}$$

Let us assume fixed and given input coefficients for all inter-deliveries as well as for labour, i.e.

$$X'_{kh} \text{ and } X'_{Wh} \text{ fixed and given} \tag{3. 7}$$

In this case the residual input coefficients are also fixed and given, because

$$X'_{\varepsilon h} = 1 - \Sigma_k X'_{kh} - X'_{Wh} \qquad (h = 1, 2 \ldots n) \tag{3. 8}$$

The problem can now be formulated as the following problem in the n variables X_h $(h = 1, 2 \ldots n)$:

$$\Sigma_h X'_{\varepsilon h} X_h = \max \tag{3. 9}$$

$$\Sigma_h X'_{Wh} X_h = \text{given} \tag{3. 10}$$

Suppose for a moment that all the coefficients $X'_{\varepsilon h}$ and X'_{Wh} are strictly positive, not zero, and let

$$\varphi_k = \frac{X'_{\varepsilon k}}{X'_{Wk}} \qquad (k = 1, 2 \ldots n) \tag{3. 11}$$

be the profitability of sector k.

If in any situation we raise a particular X_k and lower a particular X_h correspondingly according to (3. 10) with all other sector products constant, there will be an increase or a decrease in the preference function

(3. 9) accordingly as $\varphi_k > \varphi_h$ or $\varphi_k < \varphi_h$. The case $\varphi_k = \varphi_h$ can be considered later as a limiting case.

Suppose that all the X_k are bounded, i.e.

$$\underline{X}_k \leqq X_k \leqq \overline{X}_k \qquad (k = 1, 2 \ldots n) \tag{3. 12}$$

where the bounds $\underline{X}_k$ and $\overline{X}_k$ are given.

The argument after (3. 11) shows that we have the following rules:

(3. 13) If in the optimum point sector k is at its upper bound, no sector with a *larger* φ can be at its lower bound.

(3. 14) If in the optimum point sector k is at its lower bound, no sector with a *smaller* φ can be at its upper bound.

From this we deduce the following simple rule for determining the solution of (3. 9) — (3. 10):

(3. 15) (a) Start by putting all sectors at their *lower* bounds. If this gives a too large value of (3. 10), the problem has no solution. If the value (3. 10) is not yet filled, the problem may have a solution, and we proceed as follows.

(b) Take the sector with the *largest* φ and move it towards its upper bound, keeping all the other sectors at their lower bounds. If we fill up (3. 10) before reaching the upper bound of the moving sector, a solution has been found. The moving sector is now marginal. If we reach the upper bound of the moving sector before having filled up (3. 10), the problem may have a solution. We leave the moved sector at its upper bound and proceed as follows.

(c) Take the sector with the *next to largest* φ and move it from its lower towards its upper bound, keeping all the other sectors at the bounds were they are now. If we fill up (3. 10) before reaching the upper bound of the sector that is now moving, the solution is found. The moving sector is now marginal. If we reach the upper bound of the now moving sector before (3. 10) is filled up, the problem may have a solution. We leave the now moved sector at its upper bound and proceed as follows.

(d) Take the sector with the *third largest* φ and move it from its lower towards its upper bound. The same cases will occur as under (c).

In this way we continue. Then one of two things will happen. Either we find a solution, or we reach the upper bound of all sectors without having as yet filled up (3. 10), in which case the problem has no solution.

It may be remarked in passing that if a solution is found, we will have (n — 1) sectors at one of their bounds and one marginal sector, except in the case where accidentally all n sectors are at one of their bounds at the same time as (3. 10) is exactly filled up.

In the above analysis we have assumed that there is at all times exactly full utilization of the labour force, compare (3. 10). It would have been more effective and more realistic only to impose an upper bound on (3. 10) instead of the exact equational condition. The technique of solution would have been the same. In this case a higher (or at least a not smaller) value of the preference function would have been found[2]. This means that we could have *reached a larger gross national product by allowing some unemployment than by insisting on full employment.*

We could have increased the realism by making more breakdowns and entering more conditions. This would gradually have led us to a complete decision model. This is not the object of the present analysis. We only need a few broad indications as a basis for the subsequent reasonings on the infra effect.

4. Comparing two structures of coefficients

When we consider changes in structural coefficients-whether in the simple case of an interflow table with one or two rows of primary input and one column of final deliveries, or a more complicated model – a basic problem will be to compare the whole interflow constellation in one of these structures with the whole interflow constellation in another of the structures.

In so doing we obviously need some sort of *correspondence principle* which permits us to associate a specific constellation in the first structure with a specific constellation in the second structure. Otherwise the whole comparison is meaningless, because there is in each structure a great number of degress of freedom in the constellation.

For simplicity take the simple case discussed in section 3 when the labour force – i.e. the value of (3. 10) – is given. We could then imagine that we made a maximization of the gross national product *separately in each structure* assuming *the same labour force* in the two structures and the same bounds on sector products. This given value of the labour force together with the bounds on sector products would then express the correspondence principle between the two structures.

[2] Or more precisely: We would have avoided the above infeasibility case where all sectors are at their upper bounds and yet (3. 10) is not filled up.

In each structure the gross national product would be determined, and we could consider the *difference* between these two gross national products as an expression for how much is gained by passing from one structure to another. This gain would be the *infra effect.*

A similar consideration could be made if we did not fix the labour force that shall be in actual operation, but only fix its upper bound. This upper bound would then enter as one of the features defining the correspondence principle.

In more elaborate models – such as the channel model – we could conceive of a *whole optimal solution* performed in the two structures with everything equal except whatever differences there may be in the coefficients in the two structures (current account input coefficients, consumption coefficients, Government expenditure coefficients, investment carry-on coefficients etc. . .). The difference between the gross national product in the two solutions would measure the infra effect by going from one structure to another.

In practice it will, of course, not be possible to go to such extensive computations in order to measure the intra effect of a given project that aims at changing certain coefficients. Nor would it be practical to do so in order to measure the infra effect of investments in a channel which represents a more or less conventional aggregation of a group of projects.

When it comes to a description of the infra effects of investments, it becomes even more clear how unsatisfactory it is to aggregate projects into channels *before* the optimal programming. Once again we have forcefully brought out the conclusion that the only really satisfactory model is *a model on project basis.* One should by all means work towards such a model. I believe that the project model is not only more satisfactory in principle, but it will be *more practical,* because the *actual decisions* which the authorities have to make are on a project basis. The language of a project model is a language they will *understand more easily.*

Since the above definition of the infra effect can only be applied in principle, we must look for some drastic simplifications in order to arrive at a measure that can be applied to any given project description.

5. Actual measurement of the infra effect of a project

To arrive at a practically applicable measure of the infra effect of a project we take as a starting point the preference function (3. 9).

Let us as a *correspondence principle* simply take the equality of the sector products X_h in the two structures. This is admittedly a very rough type of approximation and in a closer study of the infra effect it should be replaced by something better. But this principle does at least take account of a great part of what could in a more refined way be taken as a measure of the infra effect.

The above simple definition of the correspondence principle being adopted, we see that the infra effect in years of a given project – assumed to be started with no delay i.e. d = o – can simply be defined as

$$(5.1) \qquad X^{*s}_{\varepsilon K} X^{s}_{K} \qquad (s = 1, 2 \ldots)$$

where

(5. 2) $X^{*s}_{\varepsilon K}$ = the *addition* to the current account input coefficient $X'_{\varepsilon K}$ which the project will create in year s, assuming that the starting will take place with no delay, i.e. starting in year 1.

Further X^{s}_{K} is an *apriori estimate* of what the total production in K will be in the year s. Perhaps it may be put equal to the assumed *capacity* (which may or may not be estimated as changing with s).

The sector K is the receiving sector whose current account input coefficients will be affected if the project is accepted. On a project basis this "sector" is simply the current account operation of the factory or other special activity considered.

If K is an aggregated sector, the magnitude X^{s}_{K} in (5. 1) will also have to be estimated apriori. From the decisional view point this is, of course, unsatisfactory. It is a consequence of the simplifying procedure we have adopted in order to avoid quadratic programming.

The distribution of the *modulational term* $X^{*s}_{\varepsilon K}$ over s = 1, 2 . . . will have to be determined from the project description in the same way as the capacity effects.

As a rule the modulational term $X^{*s}_{\varepsilon K}$ itself will not be given but will have to be computed by the formula

$$(5.3) \qquad X^{*s}_{\varepsilon K} = - \sum_{k} X^{*s}_{kK} - \sum_{\alpha = B, W} X^{*s}_{\alpha K}$$

where B indicates non-competitive imports (and W is interpreted as including not only wages in the proper sense but also distributed ownership income).

The individual modulational terms in the right member of (5. 3) can be read off from the project description.

When the modulational term (5. 3) has been computed, the *total infra effect* in year s of the project – if started with no delay – is computed by (5. 1).

If we only want to describe the total effect which the project will have if accepted, the above calculations are all that is needed. But if we want to let the infra effect enter as a part of the optimal programming, we must express (5. 1) in relation to the total starting $H_{\Gamma o}$ (i.e. the total estimated investment cost), where Γ is the channel (the project) that produces coefficient modulations in the current account sector K. The subscript o indicates no delay, i.e. starting in year 1.

This ratio we denote

$$(5.4) \qquad R^{*s}_{\Gamma} = \frac{X^{*s}_{\varepsilon K} X^{s}_{K}}{H_{\Gamma o}}$$

The ratio as thus considered gives the coefficients of *an infra term to be added to the gross national product taken as a preference function* in a programming analysis where the current account input coefficients are originally assumed constant and the total startings are basis variables.

In the channel model a term

$$(5.5) \qquad R^{*s}_{g}$$

might be estimated for each channel g by a conventional aggregation of the projects in each channel, considering the infra effect of the individual projects in each channel. In some channels the infra effect may be taken as zero.

The complete term

$$(5.6) \quad R^{*t} = \sum_{s=1} \sum_{g[\mathrm{all}]} R^{*s}_{g} H_{g,t-s} \qquad (t = 1, 2 \ldots T)$$

$$T = \text{repercussion horizon}$$

can be taken as additional equations in the reduced form of the standard equations of the programming [3].

In (5. 6) we have assumed that the infra effect is *stationary* in the sense that the total effect in year t is

$$(5.7) \qquad R^{*t-d}_{\Gamma} H_{\Gamma d} \qquad (d = 0, 1 \ldots)$$

where R^{*s}_{Γ} is a coefficient with a single superscript s as defined in (5. 4). This, of course, is only an approximation if X^{s}_{K} in (5. 4) has to be estimated apriori on the assumption that the starting is in year 1. But again this

[3] The equation (5. 6) will, of course, have to be processed by moving the extradecisional startings into a constant term, if any of the extradecisional startings have an infra effect.

is a consequence of the rough sort of approximation we have adopted here to avoid quadratic programming.

When R^{*t} has been defined in this way, and its expression (5.6) has been included in the pre-programming equations, we can take

$$F^t = R^{gr.t} + R^{*t} \tag{5.8}$$

as the preference function. Here $R^{gr.t}$ is the function expressing gross national product in terms of the old, unmodulated coefficients.