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Market Prices vs. Factor Costs and the Constancy of Production Coefficients

The question of whether it is most appropriate to use market prices or factor costs in national income statistics and more generally in economic model work, has been much discussed. I shall offer some comments on this question. Or rather, I shall look at this whole question from a somewhat more general viewpoint more adaptable for decision model work. Instead of factor input as distinguished from indirect taxes (minus subsidies), I shall consider the *proportional* input elements as distinguished from the residuum.

I will try to bring out that in so far as the *global national product* is concerned, the effects of including only the (proportional) factor costs instead of including the total market values, *disappear* to a large extent when the values are *deflated* so as to bring out the volume figures instead of the value figures.

In so far as the *production coefficients* are concerned, the difference between alternative kinds of figures is more complicated. We have to distinguish between at least four different types of magnitudes: *strictly physical* quantities, *volume* figures, *semi volume* figures and *current values*. For each of these types we must look into the question of the constancy of the coefficients.

For practical reasons the usual input output coefficients are as a rule computed as ratios between market values observed in a base year. This is also done in Norwegian work and in a general way I agree to this procedure for the reasons given in the sequel. There are, however, special purposes for which some modifications may be used. Compare the comments below on the method followed in the Oslo median model.

A. *The Strictly Physical Structure*

To bring out the essence of the problem as it appears in a decision model, let us first consider a table with all final deliveries aggregated, and with only strictly physical quantities involved so that no vertical summations are possible. The result of such a set up is given in Table 1. The strictly physical quantities are denoted by small letters.

TABLE 1

Input-Output of Strictly Physical Quantities

		Receiving sector No. $h = 1$	2	Final delivery	Total delivery
Delivering sector No.	$k = 1$	0	x_{12}	x_{1*}	x_1
	2	x_{21}	0	x_{2*}	x_2
Primary input	Labour	w_1	w_2	—	—
	Non competitive imports	b_1	b_2	—	—

If we do not impose any other relations than the definitions of the total products, i.e.,

$$
\begin{aligned}
x_{12} + x_{1*} &= x_1 \\
x_{21} + x_{2*} &= x_2
\end{aligned}
\tag{1}
$$

we have 10 — 2 = 8 degrees of freedom.

If we introduce the 6 production coefficients by

$$
\begin{aligned}
x_{12} &= x'_{12} x_2 & x_{21} &= x'_{21} x_1 \\
w_1 &= w'_1 x_1 & w_2 &= w'_2 x_2 \\
b_1 &= b'_1 x_1 & b_2 &= b'_2 x_2
\end{aligned}
\tag{2}
$$

and for the moment consider all these coefficients as variables, we have 16 variables and 8 equations, hence still 8 degrees of freedom. As basis variables we can choose for instance the 6 coefficients and x_1, x_2. Or we can choose the 6 coefficients and x_{1*}, x_{2*}. The 8 basis equations are in the first case (2) together with

$$
x_{1*} = x_1 - x'_{12} x_2 \qquad x_{2*} = -x'_{21} x_1 + x_2. \tag{3}
$$

In the second case they are

$$
x_1 = \frac{x_{1*} + x'_{12} x_{2*}}{1 - x'_{12} x'_{21}} \qquad x_2 = \frac{x'_{21} x_{1*} + x_{2*}}{1 - x'_{12} x'_{21}} \tag{4}
$$

together with the 6 expressions obtained by inserting (4) into (2).

This is the structure of the system expressed in strictly physical terms. The case of constant coefficients is covered simply by putting the coefficients in the basis equations equal to their constant values. This leaves us with 2 degrees of freedom, which may, for instance, be unfolded by x_1, x_2 or by x_{1*}, x_{2*}.

B. *The Structure Expressed in Volume Figures*

Now let us introduce a set of *product prices* π_1, π_2 and *factor prices* π_w, π_b. We call them *standard* prices. Let us see how the various constellations of the system which are physically possible with the degrees of freedom in (1), can be expressed in the value terms derived from the standard prices. We also introduce residual items ε_1, ε_2 in the two production sectors. The residual items may be the sum of taxes T_h and net profits (savings) δ_h. For practical purposes in a decision model these residual items are very important, but their introduction causes considerable complications in the definitional set up. These difficulties we must consider in a systematic way.

The new figures are listed in Table 2. We could, if we wanted to, introduce different wage rates in the two sectors and also different import prices, but that is unessential in the present connection.

TABLE 2

Interflow Table of Values Reckoned at Standard Prices and with Standard Residuum Elements

		Receiving sector No. $h = 1$	2	Final delivery	Total delivery
Delivering sector No.	$k = 1$	0	$\pi_1 x_{12}$	$\pi_1 x_{1*}$	$\pi_1 x_1$
	2	$\pi_2 x_{21}$	0	$\pi_2 x_{2*}$	$\pi_2 x_2$
Primary input	Labour	$\pi_w w_1$	$\pi_w w_2$	$-\pi_w(w_1+w_2)$	0
	Non competitive imports	$\pi_b b_1$	$\pi_b b_2$	$-\pi_b(b_1+b_2)$	0
Residual input		ε_1	ε_2	$-(\varepsilon_1+\varepsilon_2)$	0
Grand total		$\pi_1 x_1$	$\pi_2 x_2$	0	$\pi_1 x_1+\pi_2 x_2$

The prices π_1 and π_2 are actual prices per physical unit, say per kilogram or per kWh. The wage rate π_w is also reckoned per physical unit, say per hour of work. Similarly for π_b. The residual input is measured in money.

It should be understood that Tables 1 and 2 exist simultaneously, and that the actual physical quantities in the two tables are the same.

In Table 2 we have imposed the condition that the sum in column No. h shall be equal to $\pi_h x_h$. This is equivalent with defining the residual ε_h when the prices are given. Or we may inversely consider the condition as defining the prices π_h in terms of the residual inputs ε_1, ε_2. We will most of the time adopt the latter viewpoint.

The column sum conditions are expressed by

$$(5)\qquad \begin{aligned} \pi_2 x_{21} + \pi_w w_1 + \pi_b b_1 + \varepsilon_1 &= \pi_1 x_1 \\ \pi_1 x_{12} + \pi_w w_2 + \pi_b b_2 + \varepsilon_2 &= \pi_2 x_2. \end{aligned}$$

We define the residual *rates* $\bar{\varepsilon}_1$ and $\bar{\varepsilon}_2$ by

$$(6)\qquad \varepsilon_k = \bar{\varepsilon}_k x_k \quad (k = 1,\ 2).$$

If need be, these residual rates will be called *direct* residual rates to distinguish them from certain *aggregate* residual rates to be considered later.

This gives 4 equations in addition to the 8 we had before. The additional variables are

(7)

Number of variables	
4	$\pi_1,\ \pi_2,\ \pi_w,\ \pi_b$
2	$\varepsilon_1,\ \varepsilon_2$
2	$\bar{\varepsilon}_1,\ \bar{\varepsilon}_2$
8	Total

In other words we have $8 - 4 = 4$ more degrees of freedom than in the table of strictly physical quantities. As additional basis variables we choose the factor prices π_w, π_b and the residual rates $\bar{\varepsilon}_1$, $\bar{\varepsilon}_2$. Using x_1, x_2 rather than x_{1*} and x_{2*} as basis variables, the total set of basis variables will be

$$\begin{array}{ll} & x_1,\ x_2 \\ & \pi_w,\ \pi_b \\ (8) & \bar{\varepsilon}_1,\ \bar{\varepsilon}_2 \\ & x'_{12},\ x'_{21},\ w'_1,\ w'_2,\ b'_1,\ b'_2. \end{array}$$

If all the 6 coefficients listed on the last row in (8) are taken as given, there are 6 basis variable left, namely those on the first three rows of (8). Of these x_1, x_2 determine the physical constellation of the system — all the other physical features following from x_1, x_2 — while π_w, π_b, $\bar{\varepsilon}_1$, $\bar{\varepsilon}_2$ determine the price features — the other prices following from π_w, π_b, $\bar{\varepsilon}_1$, $\bar{\varepsilon}_2$.

The prices π_1 and π_2 as functions of the coefficients and the basis price elements, are determined by inserting into (5) from (2) and (6) which gives the system of two equations

$$(9)\qquad \begin{aligned} \pi_1 - \pi_2 x'_{21} &= \pi_w w'_1 + \pi_b b'_1 + \bar{\varepsilon}_1 \\ -\pi_1 x'_{12} + \pi_2 &= \pi_w w'_2 + \pi_b b'_2 + \bar{\varepsilon}_2. \end{aligned}$$

The solution of this is

$$(10)\qquad \begin{aligned} \pi_1 &= \frac{\pi_w(w'_1 + w'_2 x'_{21}) + \pi_b(b'_1 + b'_2 x'_{21}) + (\bar{\varepsilon}_1 + \bar{\varepsilon}_2 x'_{21})}{1 - x'_{12}x'_{21}} \\ \pi_2 &= \frac{\pi_w(w'_1 x'_{12} + w'_2) + \pi_b(b'_1 x'_{12} + b'_2) + (\bar{\varepsilon}_1 x'_{12} + \bar{\varepsilon}_2)}{1 - x'_{12}x'_{21}} \end{aligned}$$

In the case of n sectors (9) has the form

$$(11)\qquad \sum_{k=1}^{n} \pi_k (\delta - x')_{kh} = \pi_w w'_h + \pi_b b'_h + \bar{\varepsilon}_h \quad (h = 1,\ 2 \ldots n)$$

which is solved by

$$(12)\qquad \pi_k = \sum_{h=1}^{n} (\pi_w w'_h + \pi_b b'_h + \bar{\varepsilon}_h)(\delta - x')^{-1}_{hk} \quad (k = 1,\ 2, \ldots n).$$

The formulae (10) show that if not only the factor prices, but also the direct residual rates are constant, the product prices will also be constant. In other words the whole structure of standard prices will be fixed. We consider them as *base prices* and take the corresponding values as defining *volume figures*.

We put

(13) $X_k = \pi_k x_k \qquad X_{k*} = \pi_k x_{k*}$

(14) $X_{kh} = \pi_k x_{kh}$

(15) $W_h = \pi_w w_h \qquad B_h = \pi_b b_h.$

The volume figures defined by (13)—(15) are entered in Table 3. That is to say, if the numerical figures are entered, Table 2 and Table 3 will be exactly the same.

Keeping the price structure — as defined through π_w, π_b, $\bar{\varepsilon}_1$, $\bar{\varepsilon}_2$ — *constant* and varying x_1 and x_2, we get different constellations of the volume figures. Since it is simply a question of units of measurement to pass from the system of strictly physical quantities to the volume figures, we can just as well think of X_1 and X_2 as varying under constant π_w, π_b, $\bar{\varepsilon}_1$, $\bar{\varepsilon}_2$.

TABLE 3

Interflow Table of Volume Figures Reckoned under Base Year Prices and Base Year Residual Elements

		Receiving sector No. $h = 1$	2	Final deliveries	Total deliveries
Delivering sector No.	$k = 1$	0	X_{12}	X_{1*}	X_1
	2	X_{21}	0	X_{2*}	X_2
Primary input	Labour	W_1	W_2	$-(W_1+W_2)$	0
	Non competitive imports	B_1	B_2	$-(B_1+B_2)$	0
Residual input		ε_1	ε_2	$-(\varepsilon_1+\varepsilon_2)$	0
Grand total		X_1	X_2	0	X_1+X_2

The introduction of the volume figures, i.e. the magnitudes denoted by capital letters in (13)—(15), does not change the number of degrees of freedom because to each new magnitude corresponds one definitional equation. If we assume that in (8) not only x_1 and x_2, but also $\bar{\varepsilon}_1$ and $\bar{\varepsilon}_2$ are changing while the factor prices π_w and π_b as well as the 6 physical quantity coefficients are given, we have 4 degrees of freedom. (In n sectors $2n$ degrees of freedom). From the discussion in the sequel it will appear that we will reserve the terminology volume figures, as defined through (13)—(15), to the case of *constant* residual rates. These constant rates we can assume as *observed* by the actual situation in a base year. Now there remain only two degrees of freedom in the volume figures. They may be unfolded say by X_1 and X_2. (In n sectors n degrees of freedom).

Table 3 has at the same time the following two properties: (1) the magnitudes entering into it have the character of *volume* figures (because they represent values at base year prices), and (2) vertical summations in the table are possible.

I shall look a little closer into the particular aspect of the question that is represented by the constancy of the residual rates.

C. *Constant Factor Prices and Constant Residual Rates Entail Constant Input-Output Volume Coefficients.*

We define

(16) $$X'_{kh} = \frac{X_{kh}}{X_h}$$

(17) $$W'_h = \frac{W_h}{X_h} \qquad B'_h = \frac{B_h}{X_h}\,.$$

From the definitions (13)—(15) follows that the coefficients (16)—(17) are equal to

(18) $$X'_{kh} = \frac{\pi_k}{\pi_h} x'_{kh}$$

(19) $$W'_h = \frac{\pi_w}{\pi_h} w'_h \qquad B'_h = \frac{\pi_b}{\pi_h} b'_h$$

where the π_k are given by (12), and π_w, π_b are the given factor prices. If all the production coefficients x'_{12}, x'_{21}, w'_1, w'_2 etc. reckoned in strictly physical quantities are constant, we see from (10) and (16)—(17) that constant factor prices and constant residual rates $\bar{\varepsilon}_k$ entail constant X'_{kh}, W'_h and B'_h.

The conditions of constant residual rates $\bar{\varepsilon}_1$ and $\bar{\varepsilon}_2$ can be transformed into corresponding conditions about the residual rates expressed as fractions of X_1 and X_2. Indeed, from (6) and (13), we get

$$\varepsilon_k = \varepsilon'_k X_k \tag{20}$$

where

$$\varepsilon'_k = \frac{\bar{\varepsilon}_k}{\pi_k}. \tag{21}$$

The last formula taken in conjunction with (10) shows immediately that if we have constant factor prices and constant quantity coefficients, constant $\bar{\varepsilon}_k$ rates will entail constant ε'_k rates.

On the other hand, if the marginal rates ε'_k are given, instead of the $\bar{\varepsilon}_k$, we deduce from (9) by inserting from (21)

$$\begin{aligned} \pi_1(1-\varepsilon'_1) - \pi_2 x'_{21} &= \pi_w w'_1 + \pi_b b'_1 \\ -\pi_1 x'_{12} + \pi_2(1-\varepsilon'_2) &= \pi_w w'_2 + \pi_b b'_2. \end{aligned} \tag{22}$$

From the equations (22) the π_k are determined. That is to say: Constant coefficients in the strictly physical structure, constant factor prices and constant residual rates ε'_k entail constant prices π_k and hence by (21) constant $\bar{\varepsilon}_k$.

It is the same set of prices π_1, π_2 that is determined from (9) and (22) only the data are taken in a slightly different form. The generalization to n sectors is obvious.

This means that if the residual rates ε'_1 and ε'_2 as defined by (20) are constant, we can reason about the volume figures in Table 3 very much in the same way as we can about the strictly physical quantities.

To be more precise: In Table 3 there are 12 variables connected by the 12 equations

$$\begin{aligned} X_1 &= X_{21} + W_1 + B_1 + \varepsilon_1 = X_{12} + X_{1x} \\ X_2 &= X_{12} + W_2 + B_2 + \varepsilon_2 = X_{21} + X_{2x} \end{aligned} \tag{23}$$

and (16)—(17) and (20) where the coefficients X'_{kh}, W'_h, B'_h and ε'_h are constants (and hence may be determined by observing the content of Table 3 in a base year). The first two equations in (23) reduce to conditions on the coefficients. Hence two degrees of freedom in the variables. This checks with the remarks in connection with Table 3.

The two degrees of freedom that remain in Table 3 under the conditions specified — constant physical coefficients, constant factor prices and constant residual rates — may be generated by letting X_1 and X_2 vary. Or we may use X_{1*} and X_{2*} as basis variables and use other equations to express X_1 and X_2.

D. *Constant Coefficients in the Volume Sense Entail Constant Residual Rates*

If we assume a model of Table 3 where (16)—(17) hold with constant coefficients, we can conclude that the residual rates ε'_k defined by (20) are constants.

Indeed, introducing into the left hand equations in (23) — which follow from the balancing principles of Table 3 — the expressions for X_k, X_{kh}, W_h, B_h from (16)—(17), we get

$$\begin{aligned} \varepsilon'_1 &= 1 - (X'_{21} + W'_1 + B'_1) \\ \varepsilon'_2 &= 1 - (X'_{12} + W'_2 + B'_2). \end{aligned} \tag{24}$$

Hence: If X'_{kh}, W'_h, B'_h are constants, ε'_1 and ε'_2 must also be constants. We are thus back to the same type of analysis as was discussed under subsection *C*.

Having reduced in this way the whole formulation to the figures contained in Table 3, we may *drop* the assumption of an underlying strictly physical structure which we started from, and simply reason about the figures of Table 3 as *value figures reckoned at base year prices*. This formulation will apply even though there is a great variety of individual goods that enter into each aggregate X_k or X_{kh} etc. For all practical purposes these figures could be interpreted as volume indices. And it would seem plausible in many cases to make the assumption of constant input-output coefficients reckoned in such figures.

If we take the volume figures as the basis of the analysis, the product prices π_1, π_2 become indetermined, and the same is true of the factor prices π_w, π_b. Indeed, if in (22) we insert for x'_{kh} from (18), and similarly use (19) we simply get back to (24). The product prices and factor prices can now simply be looked upon as conventional multipliers by which we define "the strictly physical quantities" in (13)—(15). If the "strictly physical quantities" are well defined and observable, we can, of course, deduce the factor prices π_w, π_b and product prices π_1, π_2 that must prevail in order that we shall get the observed volume figures (in base prices year prices) X_1, X_2, W_1, W_2 etc.

E. *The Aggregate Residual Rates*

The residual rates ε'_h express the input of residual substance that is made *directly* into sector h, reckoned per unit of total output X_h from sector h. We can also consider the *aggregate* residual rates $\dot{\varepsilon}_h$ defined by

(25) $\dot{\varepsilon}_h =$ that part of X_h which is due to the input of residual substance in any sector, assuming that all residual substance is everywhere passed on to other sectors or to final output in the same proportion as the *volume* of cross deliveries or the final deliveries. In other words all units of output from a given sector contains the same amount of residual substance.

When the aggregate residual rates are defined in this way, they must satisfy the equations

$$\begin{aligned} \dot{\varepsilon}_1 - \dot{\varepsilon}_2 X'_{21} &= \varepsilon'_1 \\ -\dot{\varepsilon}_1 X'_{12} + \dot{\varepsilon}_2 &= \varepsilon'_2. \end{aligned} \tag{26}$$

The first equation in (26) is obtained by noticing that the total outflow of residual substance from sector 1 is $\dot{\varepsilon}_1 X_1$. This must be equal to the total inflow of residual substance into sector 1, namely the residual substance entered *directly* into sector 1 — this is $\varepsilon'_1 X_1$ — *plus* the residual substance that is entered into sector 1 through X_{21} — this is $\dot{\varepsilon}_2 X_{21}$ —. Dividing this equality by X_1, we get the first equation (26). Similarly for the second equation in (26).

The inputs of labour W_h and imports B_h are *not* to be entered in the above account as they are by definition *not* residual elements. But we could have singled out, say W_k, and considered the *direct* coefficient W'_h as distinct from the *aggregate* coefficient $\dot{W}_h$. The reasoning would be the same as in (26).

The generalization of (26) to n sectors is obvious, namely

$$\sum_{k=1}^{n} \dot{\varepsilon}_k (\delta - X')_{kh} = \varepsilon'_h \qquad (h = 1, 2, \ldots n). \tag{27}$$

The solution of this is

(28) $$\varepsilon_k^{\cdot} = \sum_{h=1}^{n} \varepsilon_h'(\delta - X')_{hk}^{-1} \quad (k = 1, 2, \ldots n).$$

If we take the volume figure coefficients X'_{kh} etc. as *data*, the meaning of the matrix in (27) is clear. If on the other hand we go back for a moment to the interpretation in terms of strictly physical quantities and with constant physical coefficients and constant factor prices, we must remember that the volume figure coefficients X'_{kh} in (27) *depend* on the ε_h'. The volume figure coefficients X'_{kh} will indeed in this case have to be looked upon as determined by (18) where the π_k are given by (22), and hence depend on the ε_h'. This means that if we fall back on the constancy of strictly physical coefficients, we cannot determine the $\varepsilon_k^{\cdot}$ for different ε_h' by retaining the left member matrix in (27) and just changing the right member vector ε_h'. Both the matrix and the vector will have to be changed. On the other hand, if volume figure coefficients are taken as given, we cannot change the ε_h' but must let these magnitudes be determined by (24). The equations (26) have therefore no use for determining the residual rates. The only purpose of the equations is to pass from the *direct* rates ε_h' to the *aggregate* rates $\varepsilon_h^{\cdot}$ or vice versa.

It is a fundamental proposition in input-output theory that equations of the form (26) will have non negative solutions $\varepsilon_h^{\cdot}$ if the ε_h' are non negative.

If we multiply the first equations in (26) by X_1 and the second by X_2 and add the equations, we get, using the equality between the left and right members of (23)

(29a) $$\varepsilon_1^{\cdot} X_{1*} + \varepsilon_2^{\cdot} X_{2*} = \varepsilon_1 + \varepsilon_2.$$

That is to say the total residual substance contained in the final delivery is equal to the total residual substance put into the system.

Since the $\varepsilon_h^{\cdot}$ are non negative (and at least one of them positive if at least one of the ε_h are positive), we see that the total residual substance contained in the sectors products must be larger than the total residual substance put into the system, i.e.

(29b) $$\varepsilon_1^{\cdot} X_1 + \varepsilon_1^{\cdot} X_2 > \varepsilon_1 + \varepsilon_2.$$

This double counting which prima facia appears a little puzzling, is easily explained: The global product $X_1 + X_2$ has itself emerged after some double counting. In $X_1 + X_2$ is indeed included not only the total primary and direct residual input, but also all *crossdeliveries*. This follows by taking the sum of the left hand equations in (23), which gives

$$(30) \quad X_1 + X_2 = (W_1 + W_2) + (B_1 + B_2) + (\varepsilon_1 + \varepsilon_2) + (X_{12} + X_{21})$$

That is to say the sum of all sector products will increase if we split the sectors further up.

The double counting is only in the total sector products, not in the aggregate residual rates $\dot{\varepsilon}_1$, $\dot{\varepsilon}_2$ as is seen from (29a).

For clear thinking in the variety of situations that arise according to the various systems of assumptions adopted it is essential to be very careful in the notation. It is indeed safe to be so explicit as nearly to appear pedantic.

We will from now on let the symbols used in subsections C and D, i.e. X_k, X_{kh}, X'_{kh} etc. and the corresponding coefficients be strictly interpreted as the *volume figures* and volume figure coefficients, that appear when the residual rates ε'_h are constants and *have a specific set of values.*

These volume figures themselves are recorded in Table 3 and the corresponding coefficients are defined in (16)—(17). With given and constant coefficients the degrees of freedom in this model is, as already stated, equal to n, the number of sections, i.e., in Table 3 it is equal to 2.

F. *The Structure in Semi-Volume Figures*

An essentially new situation arises if we drop the assumption of constant residual rates. We can discuss this situation by going back to the structure expressed in strictly physical terms. We assume constant technical coefficients in this physical structure and also constant factor prices, but the direct residual rates may now be changing. And as they change, they will produce changing product prices and hence changing value figures. These value figures we will term the *semi-volume* figures. In this way of thinking there are $2n$ degrees of freedom, represented, say, by the n physical quantities and the n residual rates.

Instead of discussing the semi-volume structure by the help of the strictly physical quantities, we can also do it through the residual rates and the *volume* figures as they were defined under subheadings B—E, see in particular the comments in the last part of subsection E.

All the semi-volume magnitudes will be denoted by the superscript sem (standing for semi-volume).

The new situation will be described by a table similar to Table 2 namely by Table 4 and it is through the balancing equations of this new table that the product price concept gets a meaning.

Using an interpretation in terms of the strictly physical structure, we are particularly interested in the connection between the π_k^{sem} and ε_1^{sem} and ε_2^{sem}.

We put up the following definitions, which are similar to (13)—(15).

$$X_k^{sem} = \pi_k^{sem} x_k \qquad X_{k*}^{sem} = \pi_k^{sem} x_{k*} \tag{31}$$

$$X_{kh}^{sem} = \pi_k^{sem} x_{kh} \tag{32}$$

$$W_h^{sem} = \pi_w^{sem} w_h \qquad B_{h*}^{sem} = \pi_{b*}^{sem} b_h. \tag{33}$$

The semi-volume figures X_k^{sem}, X_{kh}^{sem} etc. measure the production levels, the cross deliveries etc. when the residual rates are chosen as $\varepsilon_k'^{sem}$ instead of the ε_k' that are associated with the measurement of the volume figures X_k, X_{kh} etc.

In general we will assume

$$\pi_w^{sem} = \pi_w \qquad \pi_b^{sem} = \pi_b \tag{34}$$

but for the symmetry of the formulae we may retain the notation π_w^{sem} and π_b^{sem}. Instead of Table 3 we now get Table 4.

It should be understood that Tables 2, 3 and 4 — as well as a table similar to Table 2 with sem added as superscript on the π and ε — exist at the same time.

In the complete system now considered we again have $2n$ degrees of freedom which may be unfolded by, say, the X_k and the $\varepsilon_k'^{sem}$. In the case of 2 sectors, there will be 4 degrees of freedom. In the strictly physical system we also had 4 degrees of freedom, when the factor prices π_w and π_b as well as the 6 production coefficients in (8) were given.

If we lean on the interpretation in strictly physical quantities, it is easy to indicate what the semi-volume figures will be in terms of the volume figures X_k, X_{kh} etc.

TABLE 4

Interflow Table of Semi-Volume Figures Reckoned under the Prices that Prevail when Factor Prices are Constant and Residual Rates are Arbitrarily Given

		Receiving sector No.		Final deliveries	Total deliveries
		$h = 1$	2		
Delivering sector No.	$k = 1$	0	X_{12}^{sem}	X_{1*}^{sem}	X_1^{sem}
	2	X_{21}^{sem}	0	X_{2*}^{sem}	X_2^{sem}
Primary input	Labour	W_1^{sem}	W_2^{sem}	$-(W_1^{sem}+W_2^{sem})$	0
	Non competitive imports	B_1^{sem}	B_2^{sem}	$-(B_1^{sem}+B_2^{sem})$	0
Residual input		ε_1^{sem}	ε_2^{sem}	$-(\varepsilon_1^{sem}+\varepsilon_2^{sem})$	0
Grand total		X_1^{sem}	X_2^{sem}	0	

Indeed, adding the superscript sem for the price elements in (22) (except for the factor prices, which are the same) and using (31)—(33), we get

(35)
$$\pi_1^{sem}(1-\varepsilon_1^{\prime sem}) - \pi_2^{sem}x'_{21} = \pi_w w'_1 + \pi_b b'_1$$
$$-\pi_1^{sem}x'_{12} + \pi_2^{sem}(1-\varepsilon_2^{\prime sem}) = \pi_w w'_2 + \pi_b b'_2$$

where

(36)
$$\varepsilon_k^{\prime sem} = \frac{\varepsilon_k^{sem}}{X_k^{sem}}.$$

Similarly we get

(37)
$$\pi_1^{sem} - \pi_2^{sem}x'_{21} = \pi_w w'_1 + \pi_b b'_1 + \bar{\bar{\varepsilon}}_1^{sem}$$
$$-\pi_1^{sem}x'_{12} + \pi_2^{sem} = \pi_w w'_2 + \pi_b b'_2 + \bar{\bar{\varepsilon}}_2^{sem}$$

where

(38)
$$\bar{\bar{\varepsilon}}_k^{sem} = \frac{\varepsilon_k^{sem}}{x_k}$$

i.e.

$$(39) \qquad \varepsilon_k'^{\text{sem}} = \frac{\bar{\varepsilon}_k^{\text{sem}}}{\pi_k}.$$

We may look upon the two sets of prices π_k and π_k^{sem} simply as special values assumed by the product price functions for different values of the residual rates considered as arguments in these functions.

Through (13)—(15) and (31)—(34) we get, remembering that the strictly physical quantities x_k, x_{hk} etc. are independent of how residual rates are chosen

$$(40) \qquad \frac{X_k^{\text{sem}}}{X_k} = \frac{\pi_k^{\text{sem}}}{\pi_k} \qquad \frac{X_{kx}^{\text{sem}}}{X_{kx}} = \frac{\pi_k^{\text{sem}}}{\pi_k}$$

$$(41) \qquad \frac{X_{kh}^{\text{sem}}}{X_{kh}} = \frac{\pi_k^{\text{sem}}}{\pi_k}$$

$$(42) \qquad \frac{W_k^{\text{sem}}}{W_k} = 1 \qquad \frac{B_h^{\text{sem}}}{B_h} = 1.$$

That is to say, we have

$$(43) \qquad X_k^{\text{sem}} = p_k^{\text{sem}} X_k \quad X_{k*} = p_k^{\text{sem}} X_{k*}^{\text{sem}}$$

$$(44) \qquad X_{kh}^{\text{sem}} = p_k^{\text{sem}} X_{kh}$$

$$(45) \qquad W_k^{\text{sem}} = W_k \qquad B_h^{\text{sem}} = B_h$$

where

$$(46) \qquad p_k^{\text{sem}} = \frac{\pi_k^{\text{sem}}}{\pi_k}$$

the p_k^{sem} are *index numbers* of prices, with the base situation chosen as the situation in relation to which the volume figures are defined. These index numbers have a meaning even if no strictly physical quantities are defined. For $p_k^{\text{sem}} = 1$ the semi-volume figures are equal to the volume figures.

The number $2n$ of degrees of freedom is not changed by introducing the semi-volume figures through (31)—(34) or by introducing the price indices through (46). Indeed, to each new magnitude introduced corresponds a definitional equation.

To unfold the $2n$ degrees of freedom, we may use the volume figures X_k and the residual rates $\varepsilon_k'^{\text{sem}}$ defined by (36) (the ε_k' are fixed as mentioned above, see in particular the comments to (13)—(15)). Instead we may use as basis variables the X_k and the p_k^{sem}. Or the X_k^{sem} and the X_k. Or some other linearly independent set of $2n$ of the variables entering into the complete set up.

From (35) we get by (46), (18) and (19)

$$(47)\qquad \begin{aligned} p_1^{sem}(1-\varepsilon_1'^{sem}) - p_2^{sem}X_{21}' &= W_1' + B_1' \\ -p_1^{sem}X_{12}' + p_2^{sem}(1-\varepsilon_2'^{sem}) &= W_2' + B_2'. \end{aligned}$$

The coefficient X_{12}', X_{21}' etc. in (47) are determined by (16)—(17) applied in the base year situation. The p_k^{sem} are therefore well defined as functions of the $\varepsilon_k'^{sem}$. The generalization to n sectors is obvious.

We could also have considered the semi-volume residual rates in the form

$$(48)\qquad \bar{\varepsilon}_k^{sem} = \frac{\varepsilon_k^{sem}}{X_k}.$$

By (36) and (40) this is the same as

$$(49)\qquad \bar{\varepsilon}_k^{sem} = p_k^{sem}\,\varepsilon_k'^{sem}.$$

With the $\bar{\varepsilon}_k^{sem}$ given (47) takes the form

$$(50)\qquad \begin{aligned} p_1^{sem} - p_2^{sem}X_{21}' &= W_1' + B_1' + \bar{\varepsilon}_1^{sem} \\ -p_1^{sem}X_{12}' + p_2^{sem} &= W_2' + B_2' + \bar{\varepsilon}_2^{sem}. \end{aligned}$$

Note the analogy — and also the difference — between the equations (47) and (50) on one hand and on the other the equations (22) and (9), and also the equations (35) and (37).

For $\bar{\varepsilon}_k^{sem} = \varepsilon_k'$ we should by (46) get $p_1^{sem} = p_2^{sem} = 1$. That this is in fact so, is seen by inserting these values for the p_k^{sem} and comparing with (24).

The solution of (50) is

$$(51)\qquad \begin{aligned} p_1^{sem} &= \frac{W_1' + W_2'X_{21}' + (B_1' + B_2'X_{21}') + (\bar{\varepsilon}_1^{sem} + \bar{\varepsilon}_2^{sem}X_{21}')}{1 - X_{12}'X_{21}'} \\ p_2^{sem} &= \frac{(W_1'X_{12}' + W_2') + (B_1'X_{12}' + B_2') + (\bar{\varepsilon}_1^{sem}X_{12}' + \bar{\varepsilon}_2^{sem})}{1 - X_{12}'X_{21}'}. \end{aligned}$$

Again the generalization to n sectors is obvious. Instead of (51) we get

$$(52) \qquad \sum_{k=1}^{n} p_k^{\text{sem}}(\delta - X')_{kh} = W'_h + B'_h + \bar{\varepsilon}_h^{\text{sem}} \quad (h = 1, 2 \ldots n).$$

The solution of this is

$$(53) \qquad p_k^{\text{sem}} = \sum_{h=1}^{n} (W'_h + B'_h + \bar{\varepsilon}_h^{\text{sem}})(\delta - X')^{-1}_{hk} \quad (k = 1, 2 \ldots n).$$

The last formula suggests immediately the following three component parts of the price p_k^{sem}

$$W^{\cdot}_k = \sum_{h=1}^{n} W'_h(\delta - X')^{-1}_{hk} \text{ due to labour input anywhere in the system.}$$

$$(54) \qquad B^{\cdot}_k = \sum_{h=1}^{n} B'_h(\delta - X')^{-1}_{hk} \text{ due to imports anywhere in the system.}$$

$$\varepsilon_k^{\cdot\text{sem}} = \sum_{h=1}^{n} \bar{\varepsilon}_h^{\text{sem}}(\delta - X')^{-1}_{hk} \text{ due to residual input anywhere in the system.}$$

The last expression in (54) is *aggregate* residual substance in X_k reckoned per unit of X_k. It satisfies the equations

$$(55) \qquad \begin{aligned} \varepsilon_1^{\cdot\text{sem}} - \varepsilon_2^{\cdot\text{sem}} X'_{21} &= \bar{\varepsilon}_1^{\text{sem}} \\ -\varepsilon_1^{\cdot\text{sem}} X'^{\text{sem}}_{12} + \varepsilon_2^{\cdot\text{sem}} &= \bar{\varepsilon}_2^{\text{sem}} \end{aligned}$$

where as before $\bar{\varepsilon}_k^{\text{sem}}$ is direct residual input reckoned per unit of X_k. These equations are analogous to (26). In both cases the residual substance is reckoned per unit of the sector product measured in volume figures. In n sectors (55) is written

$$(56) \qquad \sum_{k=1}^{n} \varepsilon_k^{\cdot\text{sem}}(\delta - X')_{kh} = \bar{\varepsilon}_h^{\text{sem}} \quad (h = 1, 2 \ldots n).$$

G. *The Semi-Volume Coefficients and a Modified Definition of the Sector Products*

In analogy with (16)—(17) let semi-volume coefficients be defined by

$$(57) \qquad X'^{\text{sem}}_{kh} = \frac{X^{\text{sem}}_{kh}}{X^{\text{sem}}_h}$$

$$(58) \qquad W'^{\text{sem}}_h = \frac{W_h}{X^{\text{sem}}_h} \quad B'^{\text{sem}}_h = \frac{B_h}{X^{\text{sem}}_h}.$$

Note in this connection (42).

Inserting from (43)—(44) into (57)—(58), we get

(59) $$X'^{\text{sem}}_{kh} = \frac{p^{\text{sem}}_k}{p^{\text{sem}}_h} X'_{kh} \text{ i.e. } X'^{\text{sem}}_{kh} = \frac{p^{\text{sem}}_k X_{kh}}{p^{\text{sem}}_h X_h}$$

(60) $$W'^{\text{sem}}_h = \frac{1}{p^{\text{sem}}_h} W'_h \text{ and } B'^{\text{sem}}_h = \frac{1}{p^{\text{sem}}_h} B'_h.$$

This shows that if the volume coefficients X'_{kh} are constant, the semi-volume coefficients can*not* be constants under changes in residual rates, because, such changes will by (51) make the price indices p^{sem}_k change. There would, of course, be no logical inconsistency in assuming the *semi*-volume coefficients constant, but then the volume coefficients would change under the changes in residual rates.

The real question at issue is to know which is the most *realistic* assumption.

If we assume such a market organization and such a technological structure that an increase in the price of a product will cause an equally large relative decline in its use for cross delivery as well as for total delivery, then the semi-volume coefficient would be constant while the volume coefficient would change. This is seen from (59)—(60).[1])

But if we can assume fixed coefficients in the strictly physical structure, then the volume coefficients must be constant. In what follows I will assume constant volume coefficients.

Then a second question arises: Can we modify the definition of sector product in such a way as to compensate for the variability in semi-volume coefficients?

An obvious answer is that if the *volume* coefficients X'_{kh} are *known* and also the residual rates — either in the form $\varepsilon'^{\text{sem}}_k$ or in the form $\bar{\varepsilon}^{\text{sem}}_k$ — the price indices p^{sem}_k will follow by (47) or (50). Hence we can always by (43)—(44) compute "compensated" variables — namely the X_k and the X_{kh} — which are such that they will be connected by the constant volume coefficients. This procedure is however highly *non linear* and it does not seem

[1]) How realistic such a case would be, is another question.

very promising to proceed to a study of the semi-volume variables along such lines.[2])

Starting from the concepts of semi-volume figures it is, however, possible to introduce certain *modified* definitions of sector products and cross deliveries which are such that they are *approximately* related through the constant volume coefficients.

To arrive at such a formulation we will first rewrite the expressions (51) in the forms

$$(61)\quad \begin{aligned} p_1^{\text{sem}} &= 1 + \frac{(\bar{\varepsilon}_1^{\text{sem}} - \varepsilon_1') + (\bar{\varepsilon}_2^{\text{sem}} - \varepsilon_2')X_{21}'}{1 - X_{12}'X_{21}'} \\ p_2^{\text{sem}} &= 1 + \frac{(\bar{\varepsilon}_1^{\text{sem}} - \varepsilon_1')X_{12}' + (\bar{\varepsilon}_2^{\text{sem}} - \varepsilon_2')}{1 - X_{12}'X_{21}'}. \end{aligned}$$

The first of these equations follows by writing the numerator in the first equation of (51) in the form

$$(62)\quad (W_1'+B_1'+\varepsilon_1')+X_{21}'(W_2'+B_2'+\varepsilon_2') + (\bar{\varepsilon}_1^{\text{sem}}-\varepsilon_1')+X_{21}'(\bar{\varepsilon}_2^{\text{sem}}-\varepsilon_2').$$

The first and second parenthesis here are respectively $(1-X_{21}')$ and $(1 - X_{12}')$ by (24). This part of (62) therefore becomes $(1 - X_{21}') + X_{21}'(1 - X_{12}') = 1 - X_{12}'X_{21}'$. This establishes the first equation in (61). Similarly for the second equation in (61).

As a check on (61) we see that p_1^{sem} and p_2^{sem} reduce to 1 if $\bar{\varepsilon}_1^{\text{sem}} = \varepsilon_1'$ and $\bar{\varepsilon}_2^{\text{sem}} = \varepsilon_2'$.

In the case of n sectors, we have

$$(63)\quad p_k^{\text{sem}} = 1 + \sum_{h=1}^{n} (\bar{\varepsilon}_h^{\text{sem}} - \varepsilon_h')(\delta - X')_{hk}^{-1} \quad (k = 1, 2 \ldots n).$$

In the regular case the coefficient of $(\bar{\varepsilon}_1^{\text{sem}} - \varepsilon_1')$ in the first equation of (61), namely $1/1 - X_{12}'X_{21}'$ (in general: the diagonal element $(\delta - X')_{kk}^{-1}$) will be slightly above unity, while the coefficient of $(\bar{\varepsilon}_2^{\text{sem}} - \varepsilon_2')$ in the first equation of (61) will be small since it is multiplied by the coefficient X_{21}', and may therefore be neglected in a first approximation. Similarly in the second equation in (61). That is, we have

$$(64)\quad p_k^{\text{sem}} = 1+\bar{\varepsilon}_k^{\text{sem}}-\varepsilon_k' \quad \text{(approximately)} \quad (k = 1, 2 \ldots n).$$

[2]) The prices are by (50) linear in the $\bar{\varepsilon}_k^{\text{sem}}$, but by (47) non linear in the $\varepsilon_k'^{\text{sem}}$. In any case the deflation by the prices makes the set up non linear.

In other words, as a first approximation the price p_k^{sem} depends only on the residual rate $\bar{\varepsilon}_k$ and not on the other residual rates. And the relation is a simple addition. The "dependency" we speak of now is an (approximate) accounting dependency which hold good regardless of behaviouristic relations.[1])

Multiplying (68) by X_k, we get

(65) $X_k^{sem} - \varepsilon_k^{sem} = X_k(1-\varepsilon'_k)$ (approximately) $(k = 1, 2, \ldots n)$.

The input-output coefficient in semi volume figures, i.e. X'^{sem}_{kh} as defined by (57) is not constant. There is, as is seen from the left hand expression in (59) a correction to be applied in the numerator as well as in the denominator in order to reach something that is constant. The correction in the *denominator* can be done with the approximation (65) simply by using the left hand expression in (65) to measure the sector product instead of X_k^{sem}.

We will first consider the case where we make this denominator correction without making the numerator correction. Is this a sound procedure?

By analogy consider the difference $(x_1 - x_2)$ between two stochastic variables. The variance of this difference will be equal to $\text{var.}\, x_1 + \text{var.}\, x_2 - 2r\sqrt{\text{var.}\, x_1 \cdot \text{var.}\, x_2}$ where r is the correlation coefficient. This expression is larger than var. x_1 if, and only if $\sqrt{\text{var.}\, x_2/\text{var.}\, x_1} > 2r$. Therefore, if we know that var. x_2 is *appreciably* larger than var. x_1, it will pay to correct x_2 — that is making it non stochastic — even if we do not correct x_1. And this will apply regardless of the nature of the correlation, whether positive or negative.

In our case the question is if we shall correct for p_h^{sem} in the denominator of the expression to the right in (59) even if we do not correct for p_k^{sem} in the numerator. We know that a change in p_h^{sem} will produce a change in p_k^{sem} in the *same* direction (positive correlation), but the change in p_k^{sem} will be proportionally much smaller if there are many highly intertwined sectors. Hence we

[1]) It is the equation itself, i.e. (64) — or more exactly (61) — which has accounting character. This, of course, does not prevent one or more of the variables from entering into some other relations that are behaviouristic. The expressions "accounting" vs. "behaviouristic" can be used about a relation, not about a variable.

ought to get a more correct result by correcting for p_h^{sem} even if we do not do it for p_k^{sem}.

The above argument is particularly adapted to the case where there is a change in the residual rate in a single sector. To some extent a similar reasoning can be applied in succession to any of the sectors. In each step the correction contemplated will be better than nothing. But it is quite clear that if all residuals change simultaneously, there may occur cases where it would have been better to make no corrections at all in the variables.

For instance if all $\varepsilon_k'^{\text{sem}}$ are equal — i.e. $\varepsilon_k'^{\text{sem}}$ independent of k — we see from (72) that we obtain a better approximation by not making any corrections on the variables, because in this case X_{kh}^{sem} is a constant times X_h^{sem}. The constant is equal to $(1 - \varepsilon_k'/1 - \varepsilon_h')X_{kh}'$.

On the other hand if $(\bar{\varepsilon}_k^{\text{sem}} - \varepsilon_k')$ is independent of k, and hence by (64), p_k^{sem} independent of k, we see from (57)—(59) that X_{kh}^{sem} is again a constant times X_h, but now the constant is simply X_{kh}'.

These cases where the semi volumes figures themselves are connected by constant coefficients are, however, very special. They resemble the case where the residual rates are constant and we get the volume figures.

If we want an approximation that holds — at least roughly — for *any* changes in residual rates — in particular for changes with a small covariance between the individual residual rates — the correction of the denominator to the right in (59) — which leads to (68) — seems to be a workable formula.

The correction for p_h^{sem} can be achieved simply by starting from the exact relation

$$(66) \qquad X_{kh}^{\text{sem}} = p_k^{\text{sem}} X_{kh}' X_h$$

and introducing here the expression for X_h taken from (65). This gives

$$(67) \qquad X_{kh}^{\text{sem}} = p_k^{\text{sem}} \left[\frac{X_{kh}'}{1 - \varepsilon_h'}\right] (X_h^{\text{sem}} - \varepsilon_h^{\text{sem}}) \quad \text{(approximately).}$$

Dropping at this stage the correction p_k^{sem}, we can write

(68) $$X_{kh}^{\text{sem}} = \left[\frac{X'_{kh}}{1 - \varepsilon'_h}\right] (X_h^{\text{sem}} - \varepsilon_h^{\text{sem}}) \quad \text{(approximately).}$$

The expression in bracket is a constant and can be determined from the data in the base year. (If the sector product has been defined as $(X_h - \varepsilon_h)$ already in the base year where the coefficients were determined, the value of the bracket will emerge directly.)

This procedure, while rough has the great advantage that it *keeps the model linear*, and it will as a rule — compare the discussion above — at least be better than simply to put

(69) $$X_{kh}^{\text{sem}} = X'_{kh} X_h^{\text{sem}} \quad \text{(incorrect)}$$

in a case where the residual rates do *not* remain constant.

It is possible to make the first order correction also for the factor p_k in the numerator to the right in (59) but then the model does not remain linear. Indeed we have

(70) $$p_k^{\text{sem}} = \frac{X_k^{\text{sem}}}{X_k} \quad \text{(exactly).}$$

Introducing here for X_k from (65), we get

(71) $$p_k^{\text{sem}} = \frac{X_k^{\text{sem}}}{X_k^{\text{sem}} - \varepsilon_k^{\text{sem}}} (1 - \varepsilon'_k) \quad \text{(approximately).}$$

And inserting this in (67), we get [1])

(72) $$X_{kh}^{\text{sem}} = \frac{X_k^{\text{sem}}}{X_k^{\text{sem}} - \varepsilon_k^{\text{sem}}} \left[\frac{(1 - \varepsilon'_k) X'_{kh}}{1 - \varepsilon'_h}\right] (X_h^{\text{sem}} - \varepsilon_h^{\text{sem}}) \quad \text{(approximately).}$$

H. *The Formulation in Non-Residual Cost*

Let us introduce *the aggregate non-residual part* of the price of the output from sector k. This is that part of p_k^{sem} which is due to the input of primary factors (in Table 4) labour and imports). This part of p_k^{sem} is given as the first two terms in (54) which — in the case $n = 2$ — are given by the terms in

[1]) Dividing by the first fraction in (72), the left member becomes $X_{kh}^{\text{sem}}(1 - \varepsilon_k'^{\text{sem}})$ If this is taken as definition of a *corrected* cross delivery, we get a relation with constant coefficient. But this relation is not linear in X_{kh}^{sem}, X_k^{sem} and $\varepsilon_k^{\text{sem}}$.

(51) that do not depend on $\bar{\varepsilon}_1^{\text{sem}}$ and $\bar{\varepsilon}_2^{\text{sem}}$.

We denote this part

$$(73) \quad p_k^{\text{nori}} = \sum_{k=1}^{n} (W_h' + B_h')(\delta - X')^{-1}_{hk} \quad (k = 1, 2 \ldots n).$$

As long as the production coefficients reckoned in volume figures are constant, the prices (73) are constant. The corresponding non residual parts of the sector products, cross deliveries and final deliveries are

$$(74) \quad X_k^{\text{nori}} = p_k^{\text{nori}} X_k \quad X_{k*}^{\text{nori}} = p_k^{\text{nori}} X_{k*}$$

$$(75) \quad X_{kh}^{\text{nori}} = p_k^{\text{nori}} X_{kh}$$

$$(76) \quad W_h^{\text{nori}} = W_h \qquad B_h^{\text{nori}} = B_h.$$

Since these non residual parts of the volume figures are simply *proportional* to the volume figures, nothing is gained by working with these variables instead of the volume figures. Both sets of variables will exactly satisfy relations with constant coefficients, provided the volume structure coefficients are constant. No further attention will therefore be paid to the structure in non residual parts.

I. *Formulation in Factor Costs*

Finally we will consider a breakdown of $\varepsilon_k^{\text{sem}}$ in the two parts

$$(77) \quad \varepsilon_k^{\text{sem}} = \delta_k^{\text{sem}} + T_k^{\text{sem}} \qquad (k = 1, 2. \ldots n)$$

where δ_k^{sem} stands for profits and T_k^{sem} for taxes. More specifically we may interpret δ_k^{sem} as profits before the deduction of *direct* taxes, so that T_k^{sem} will stand for *indirect* taxes.

We have now n more degrees of freedom, i.e. $3n$ degrees altogether. They may be represented, say, by the X_k, and the rates $\bar{\varepsilon}_k^{\text{sem}}$ and $\bar{T}_k^{\text{sem}}$, where $\bar{\varepsilon}_k^{\text{sem}}$ is defined by (48) and

$$(78) \quad \bar{\delta}_k^{\text{sem}} = \frac{\delta_k^{\text{sem}}}{X_k} \qquad \bar{T}_k^{\text{sem}} = \frac{T_k^{\text{sem}}}{X_k}$$

so that

$$\bar{\varepsilon}_k^{\text{sem}} = \bar{\delta}_k^{\text{sem}} + \bar{T}_k^{\text{sem}}.$$

Assuming that the coefficients in the volume structure are constant, we know that if the $\bar{\varepsilon}_k^{\text{sem}}$ are changed, the p_k^{sem} must follow in the way previously discussed. This, however, does not say anything about the way in which the prices will change if the T_k^{sem} change. Conceivably any change in the T_k^{sem} may be compensated by opposite changes in the δ_k^{sem} so that the ε_k^{sem} remain constant and hence the p_k^{sem} constant. Or a smaller or larger part of the change in T_k^{sem} may be absorbed in the prices.

In a market of a more or less conventional sort it is perhaps plausible to assume, as a very simple case, that the T_k^{sem} will affect the prices *directly and fully* in the sense that we have

$$(79) \quad p_k^{sem} = 1 + T_k^{sem} - T_k' \quad \text{(approximately)} \quad (k = 1, 2 \ldots n)$$

where

$$(80) \quad T_k' = \frac{T_k}{X_k} = \text{indirect tax rate in the base year.}$$

If this is so, we get by a reasoning analogous to that connected with (64)—(65).

$$(81) \quad X_k^{sem} - T_k^{sem} = X_k(1 - T_k') \quad \text{(approximately)} \quad k = 1, 2 \ldots n).$$

so that in analogy with (71)—(72) we may put

$$(82) \quad X_{kh}^{sem} = \left[\frac{X_{kh}'}{1 - T_h'}\right] (X_h^{sem} - T_h^{sem}) \quad \text{(approximately)}.$$

The expression in brackets is a constant to which we may attach comments similar to those connected with (68).

The set up (81) has been used in the Oslo median model.[1])

J. *Formulation in current values*

If we consider also the factor prices as variables, we are led to the concepts $X_k^{cur} = p_k X_k$, $X_{kh}^{cur} = p_k X_{kh}$, etc. Note that p_k^{sem} stands for the price concepts that emerge when the factor prices, i.e. the price concepts for W_k and B_k, are *constant*, while p_k stand for the corresponding concepts when the wage rate and the import prices may change. I.e. that emerge when the factor price concept is expressed by a general wage index q (with

[1]) The Oslo median model contained several specifications that go beyond those considered here, but in essence we can say that what is here denoted $(X_h^{sem} - T_h^{sem})$ and X_{kh}^{sem} respectively, was there denoted X_h and X_{kh}.

$q = 1$ in the base situation) and the import prices are arbitrarily given. Correspondingly the X_k^{cur} and the X_{kh}^{cur} are current values as they emerge when applying the general price indices p_k.

When the current values are *deflated*, we get back to the volume figures. For any individual sector product this deflation is simply

$$\text{(83)} \qquad \text{defl. } X_k^{\text{cur}} = \frac{X_k^{\text{cur}}}{\text{Pr. ind. } (X_k^{\text{cur}})} = \frac{X_k^{\text{cur}}}{p_k} = X_k.$$

For the global output as a whole we have

$$\text{(84)} \qquad \text{defl. } (X_1^{\text{cur}} + X_2^{\text{cur}}) = \frac{X_1^{\text{cur}} + X_2^{\text{cur}}}{\text{Pr. ind. } (X_1^{\text{cur}} + X_2^{\text{cur}})}.$$

If we use a Laspeyre price index, (84) is further reduced to

$$\text{(85)} \qquad \text{defl. } (X_1^{\text{cur}} + X_2^{\text{cur}}) = \frac{X_1^{\text{cur}} + X_2^{\text{cur}}}{\dfrac{p_1 X_1 + p_2 X_2}{X_1 + X_2}} = X_1 + X_2.$$

A similar reduction takes place if we deflate the semi volume figures. That is to say, in order to arrive at a measurement of the global output that is independent of such price effects as may be produced by residual input elements, or more specifically that part of these elements which is represented by indirect taxes, it is not necessary to use measurements as $(X_k^{\text{cur}} - \varepsilon_k^{\text{cur}})$ or $(X_k^{\text{cur}} - T_k^{\text{cur}})$ for the sector products (compare by analogy (65) and (81)). We can use the total value X_k^{sem} and afterwards deflate.

Another aspect of this question is that such differences as $(X_k^{\text{cur}} - \varepsilon_k^{\text{cur}})$ or $(X_k^{\text{cur}} - T_k^{\text{cur}})$ only represent first order corrections. The complete correction is obtained by computing the prices p_1, p_2 by formulae analogous to (51) or (53) — or observing them — and then using these prices for the deflation process.