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CHAPTER 9

PREFACE TO THE OSLO CHANNEL MODEL
A SURVEY OF TYPES OF ECONOMIC FORECASTING AND PROGRAMMING

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FIRST STAGE: THE ON-LOOKER APPROACH

The most primitive approach to medium and long term forecasting is a mechanical *trend extrapolation* for some specific variable which one may be interested in, or a mechanical trend extrapolation made separately for each of a number of variables.

Such a rough procedure may be of some use in very simple problems where accuracy is not essential and where the growth process of the phenomena in question is conspicuously stable. An example in point might be the extrapolation of the total population figure in an area where no immigration or emigration is likely to take place and the age composition and birth and death forces have been very stable in the past and are likely to remain so. Another example is an extrapolation of the 'development' of the national product in a stagnant economy where no economic reforms or other economic initiative are likely to occur.

In most cases, however, a more refined approach is needed. One will attempt to extrapolate *simultaneously* several demographic or economic variables, tying them together in their mutual dependency through a more or less elaborate dynamic model. Cases in point are population forecasts built on the classical interrelations between birth and death forces, immigration and emigration rates etc., or the extrapolation of the configuration of an economy built on some growth model of an aggregated or of a more sectorized sort.

The essential point in forecasts of these sorts is that the future course of any specific one of the variables – or constants – considered will throw light on the course of the others. All of the variables and constants should therefore be considered simultaneously. The essential point in this connec-

tion is not whether a magnitude is assumed to be variable or constant in the future, but whether it is deemed necessary to include it in the model or not. A magnitude has to be included in the model either because we like to forecast it for its own sake or because we need to do it since it is structurally connected with one or more variables which we like to forecast for their own sake.

Forecasts of this sort have methodologically something in common with the simultaneous smoothing of geodetic observations which are by definition connected by the geometric properties of the triangulation network used. Stochastic theory tells us that such a simultaneous smoothing will give higher precision than that obtained by smoothing each element separately. And for many practical purposes it is, of course, essential that the smoothed magnitudes correctly satisfy the definitional relations, and perhaps certain other relations.

In one respect the problem of demographic and economic forecasts differs, however, from that of the simultaneous smoothing of geodetic observations. In the forecasts many of the elements, which mutually influence each other, are not actually observed, but must be guessed at.

This leads logically to an analysis which does not yield one definite forecast but rather yields a number of *alternative* forecasts, each of them being derived from a specific combination of assumptions regarding the future course of some of the elements that are structurally tied together.

To make such alternative forecasts in the most effective way one must begin by analyzing carefully the number of degrees of freedom in the dynamic model used. And this being done, one must transform the relations of the model in such a way that the future course of all the unknowns is expressed in terms of the evolution – or levels – of the elements in a *basis set*. This basis set must contain some (inside the model independent) elements – variables or constants – equal in number to the number of degrees of freedom in the model. And this set should be chosen in a particular way, namely such that alternative values of these basis variables can generate the variety of forecasting alternatives (probable or improbable) in which we are most interested. All this can be thrown into more precise stochastic statements which it is, however, unnecessary to go into details about here.

The most plausible forecasting alternatives will be those which correspond to *alternative guesses* at the basis elements in this set up. Through the basis elements the alternative forecasts are systematized.

The simplest example of this sort of analysis is an economic forecast

based on a growth model which is so aggregated that the time evolution of its variables is determined by a few elements such as the capital-to-output ratio and the savings rate, and on a guess at the (perhaps constant) magnitudes of these few elements. And, lest we forget, on a scrutiny of the existing initial conditions.

One feature which is common to all these analyses that aim at systematizing alternative guesses, is that the attitude of the analyst is simply that of an on-looker. He simply tries to guess at what will happen without making any systematic attempt at finding out what somebody – the Government or a private organization or a coalition of private organizations – *ought to* do if they want to influence the course of affairs [1]).

In forecasts of the on-looker type a very serious problem is produced by what may be called the *publication effect*. In a free – or semi-free – market *economy* where private institutions and individuals are in the main left free to act according to what they think are their own interests, it will frequently happen that the very publication of an on-looker forecast will start a chain of reactions that *work counter* to the realization of the published forecast. This is an additional factor which often considerably increases the difficulty in a free or semi-free economy of making on-looker forecasts that do not deviate too much from subsequent observations.

Whether such deviations are nationally desirable or not is another question. In a semi-free economy where Government publishes economic targets – a situation which already goes beyond the on-looker attitude, cf. the second and the third stage below – the authorities may try to anticipate the reactions of the free part of the economy and not publish the goals they actually want to reach but something different which they calculate will enduce – through the publication effect – the free part of the economy to act in such a way as to lead the economy towards the goals which the authorities desire. When the free sector discovers this policy, it may take account of it and try to figure out what the true goals of the authorities actually are. The authorities may in turn try to take account even of this aspect of the behaviour of the free part of the

[1]) The on-looker may, of course, take a decision model of the kind discussed under the third stage below and use it, not for a policy oriented comparison of alternative choices of values of instrument variables, nor for a full-fledged programming, but simply for his on-looker purpose. He would then insert for the instrument variables his *guess* about the probable actions of Government. This procedure, however, will not change his on-looker attitude.

economy. And so on. This may start an endless game and supergame between the authorities and the free part of the economy, which raises a far-reaching question of public honesty in a semi-free economy. This aspect will not be discussed here.

In a more authoritatively steered economy the publication effect operates in a different setting. It may here be used to stimulate the efforts of institutions and individuals for fulfilment of national targets. This stimulation should not be carried so far as to cause an overfulfilment which entails a non-optimal allocation of national resources. But, of course, any overfulfilment which is made possible solely through a more effective utilization of allocated resources or an intensification of labour input or of the alertness and co-operative attitude of management will be only to the good.

SECOND STAGE: THE AD HOC INSTRUMENT APPROACH

Once the emphasis is shifted from the on-looker viewpoint to that of influencing the course of affairs, the analytical picture changes. Certain elements – variables or constants – will now attract particular interest, namely those which can be fixed in a rather direct way at will, at least within certain limits. They may be termed the parameters of action or the instruments or the *decisional elements*. In the most primitive version of this influencing-the-course-of-affairs approach one may perhaps not get so far as to discuss a complete model with a well defined number of degrees of freedom. One may not get further than simply picking ad hoc one or a few relations which connect some variables one would like to see evolve in a certain way, with some other variables which appear to be susceptible of direct control, at least to some extent.

The inadequacy of such an approach is obvious: In this analysis one is not even able to indicate which combinations of instrument fixations are in fact *feasible* from the viewpoint of the totality of all the realistically relevant relations that prevail in the economy. Before an analytical tool for describing this feasibility is available, no practically useful results can be produced. An ad hoc and haphazard fixation now of one instrument now of another – each time with some specific target in view – may indeed lead to quite unexpected, even chaotic, results, producing extreme tensions and contradictions in the economic structure. An ad hoc instrument approach to forecasting and programming is, therefore, warranted only

as a very first and tentative preparation for a *further analysis* that does lead to a precise dynamic model with a well defined number of degrees of freedom.

THIRD STAGE: THE FEASIBLE INSTRUMENT APPROACH

In the ad hoc instrument approach there was no coherent model. In the most refined form of the on-looker approach a coherent model was used, and in the feasible instrument approach a coherent model is also used but there is a difference.

When the emphasis is shifted towards the viewpoint of influencing the course of affairs and it is wanted to carry the analysis through in a logical way with emphasis on the feasibility of instrument selections, the principle for the selection of basis variables will be different from what it is in the on-looker approach. One will now try to select as basis elements as many as possible of the elements that have the character of instruments.

As a rule one will not be able to find a sufficient number of instruments to cover all the degrees of freedom in the model. This means that one or more *exogenous* elements will have to be left in the basis set. And the time evolution of these exogenous elements will have to be guessed at. For each such guess one may consider several alternative fixations of the decisional elements, each such fixation leading to a well defined forecast for the evolution of all the variables considered. In this way one will be able to systematize the possible alternative projections which it is worthwhile to consider in a study of ways and means to influence the course of affairs.

If the problem is coded for an automatic computer in such a way that the assumptions about the exogenous and the decisional elements can easily be changed, it will be a simple matter to run a whole series of alternative projections. One will be able to play national economic simulation games much in the same way as the military strategists play battles and even wars on the electronic computer.

During the last generation the shift from the on-looker viewpoint to the decision viewpoint has become more and more prevalent in economic thinking all over the world as witnessed, for instance, by the world-wide United Nations survey 'Evaluation of long-term economic projects', E 3379, 1960, produced in response to resolutions by the Social and Economic Council and the General Assembly. In most countries this shift in

viewpoint is, however, based on a sort of half-logic which I have never been able to understand and which, I think, will never be able to yield fundamental solutions. On one hand one still retains the on-looker viewpoint, and tries to make projections on this basis (growth models of the current types). And on the other hand one will *afterwards* try to use such projections as a basis for decisions. How can it be possible to make a projection without knowing the decisions that will basically influence the course of affairs? It is as if the policy maker would say to the economic expert: 'Now you expert try to guess what I am going to do, and make your estimate accordingly. On the basis of the factual information I thus receive I will then decide what to do.' The shift from the on-looker viewpoint to the decision viewpoint must be founded on a much more coherent form of logic. It must be based on a *decision model*, i.e. a model where the possible decisions are built in *explicitly* as essential variables. A more recent United Nations publication, 'Industrialization and Productivity', Bulletin 4, April 1961, moves away from this sort of half-logic and begins to approch the programming problem on a more rational basis.

One basically important aspect of this more rational approach to a study of feasible instrument choices is the need for *continuous co-operation* between the responsible authorities – whether Government or private – and the analytical experts. Only through such a co-operation with demonstration of alternatives will it be possible to map out to the authorities the feasible alternatives and to help them understand which one – or which ones – amongst the feasible alternatives are the most desirable from their own viewpoint. To develop a technique of discussing feasible policy alternatives in such a scientific way is one of the most burning needs in economic policy making to-day. This applies no matter whether the country has a democratic political structure of the Western type or an authoritarian structure or one of the mixed forms which are typical, for instance, for the developing countries of Asia and Africa that have only recently gained their independence.

Such a scientific approach to economic policy making is not least needed in connection with the activities which are now being organized in practically all the materially developed countries of the West in order to aid the materially underdeveloped countries towards an economic take off. The interest in the West in this great cause and the enthusiasm in the developing countries themselves are some of the really comforting aspects of the world of to-day. But this activity has its real dangers. It

may lead to a headlong plunge without a sufficiently careful study of how the development – and the aid to development – should be undertaken. Such a plunge may do more harm than good. Crucial questions which must be seriously considered in this connection are: What *sorts of investments* will be the most profitable ones from the national viewpoint? The manner and directions of capital utilization is probably an even more important problem than the lack of capital in the developing countries. And not to be forgotten is the parallel problem of what sorts of economic policies for the *current account* activities are most conducive to the goals one has in mind. A sensible answer to these questions depends not only on a careful study of the special conditions and traditions of the developing countries themselves and on a genuine co-operation with the national leaders in the developing countries – an aspect of the problem which is so obvious that hardly anybody can overlook it – but it also depends on a really effective technique of medium and long range economic forecasting and programming, and this aspect of the problem is far from being generally recognized. Here is where enlightenment on a scientific technique for discussing feasible alternatives in national economic policy making is indispensable. Techniques of the on-looker or ad hoc instrument kinds are inadequate.

Even a scientific technique for discussions on feasible alternatives is only a preliminary step towards rationalization of economic policy making. But it is at the present stage indispensable as a move in the right direction. It will pave the way and educate the public and the authorities towards an understanding of what is really needed: the optimalization approach to economic policy making.

FOURTH STAGE: THE OPTIMALIZATION APPROACH

When the effort to map out a spectrum of feasible alternatives has gone on for a while, the conclusion will inevitably force itself upon the public and the authorities that the number of feasible alternatives is so great that it is impossible to keep track of them simply by listing them and looking at them.

It will then be understood that one needs an analytical technique for picking that one – or those – alternatives that are in some sense of the word the *optimum* ones.

This leads directly to the problem of *mathematical programming* applied

to economics. Not only to economic programming in individual enterprises, but to economic programming regarding measures to be taken in the economic system at large. We need mathematical optimalization at the national – or even international level.

The Preference Function

Any such programming technique must be based on a definition of the nature of the preference function to be applied in national – or even international – economic policy.

This admittedly is a difficult problem that needs to be studied in all seriousness. During the last years I have worked so much both on the theoretical and on the practical aspects of this that I have been able to reach a rather definite opinion. I am convinced that the preference function problem for an economy at large can be solved when it is approached in an intelligent and cautious way.[1a]) Some details of an interview technique to be applied in this problem is discussed in several Oslo memoranda, in particular in [2] and [24].

The preference function cannot be formulated in one stroke. It can only be done through a series of attempts based on continuous co-operation between the responsible authorities and the analytical experts. A series of tentative solutions with different alternative formulations of the preference function (and of some of the bounds in the problem and other side conditions) is needed. In a sense we are thus back again to a study of alternatives, but they are now alternatives on a higher level in the hierarchy of analytical techniques.

This aspect of the problem assumes a specific significance in the democratic societies of the Western kind. Experience has shown that the *long term* rate of growth in the Western economies lags considerably behind that of the Soviet Union. In the cyclical ups and downs in the West there may be certain – perhaps relatively short – phases where the *actual* rate of growth comes at least near to that in the East (Western Germany, France and Italy in recent years). But such rates cannot be sustained as a long term tendency. We know too well that at times there is a frightening decline nearly to stagnation (the United States and Great Britain). A high long term growth rate can only be sustained through some form of national planning. Still better, of course, would be rational planning on a many-country level, for instance on an all Western Europe level, if an agree-

[1a]) This opinion is based not least on extensive interviews with leading policy makers.

ment about genuine economic planning at this level could be realized.

In an ideal Western form of planning – nationally or internationally – the way in which the preference function is fixed plays an absolutely fundamental role. In the Western kind of democracy the preference function cannot and should not be formulated *by dictate from above.* All layers of the population should participate in shaping its final form. How can this be done?

The solution must be that different political parties and different organizations which are engaged in economic questions at a national level formulate – in co-operation with analytical experts – the particular preference functions they want to suggest (and the particular extra conditions they want to impose on the problem). The optimal solution for economic policy measures which emerges from such a setting of the problem should be computed by the experts. And it should be published and *made the object of a public debate.*

It is impossible for the general public to see and understand all the direct and indirect repercussions of a specific economic policy measure or a set of such measures. Therefore when the public debate gravitates around such a specific measure or set of measures – as has practically always been the case in the West – the debate must be confused and superficial and ineffective. But the results of an optimal solution – arrived at by a conscientious and sound analysis by the experts – can be described in understandable terms. In particular the long term rate of growth which emerges from such a solution will be a fundamental feature easily understood.

Such solutions will have a power of persuasion enormously superior to lengthy verbal arguments. *It is therefore alternative optimal solutions and not alternative specific measures that should form the object of public debate.* On the basis of such discussions the responsible democratic authority should make the final choice of the preference function (and the particular extra conditions to be imposed).

From the viewpoint of public understanding the situation under such a system would be much the same as when there is a public debate, say, on the organization of a new social security scheme. In many such questions heavy actuarial computations are involved of which the general public understands nothing. But this does not prevent the enlightened layman from forming a very definite opinion on what the *results* of the actuarial computations *mean* from the viewpoint of social welfare and social justice in general.

To carry such techniques of analysis and debate over into the field of optimum choice of economic policy making at the national level is the crucial problem of the Western democracies to-day.

This is what I would like to call *liberty-planning*. It is planning under liberty, and at the same time it is planning *for* liberty because the safeguarding of a long term substantial rate of economic growth is a sine qua non for the continued existence of the Western democracies in a world of economic competition from the East.

Three Alternatives for the Future

The development of these techniques of analysis has such an enormous perspective for the Western democracies and such a basic meaning for the role which we as social scientists and social technicians are called upon to play in the history of the human race, that I must add a few words.

Seen in wide perspective there are three alternatives for the future.

The first alternative is what was at least until recently the Chinese thesis, namely that an atomic war might after all be an efficient way to get rid of the West. Of the enormous Chinese population at least one or two hundred millions would probably be left.

The second alternative is the Soviet thesis that atomic war must by all means be avoided not only because of its unbelievable horrors but also because it is entirely unnecessary. It suffices to let the West continue in its stubborn planlessness. It will then rapidly be lagging behind economically and will in due time fall from the tree like an overripe pear. This is their basic philosophy of peaceful coexistence.

The third alternative is that the West will at the last moment wake up and understand the situation in the world to-day and see the need for rational economic planning.

On the whole and as a general tendency the thinking and efforts of the West to-day are actually directed towards things which, as I see it, only constitute wrinkles in the surface of the problem. This even applies to its military alliances and it certainly applies to its efforts in the economic field. In this respect there is not a great deal of difference between countries with conservative Governments and countries with so-called socialist Governments.

This general tendency is clearly exemplified in the agreements and activities of the six common market countries. The general philosophy here is to revive what might be called the unenlightened financialism.

That is to say a laissez faire regime with free markets, free opportunities for big business, the private profit motive as the guiding criterion for what are the best investments and so on. This unenlightened financialism was the system that some centuries ago, at the end of the age of enlightenment, replaced absolute monarchy, e.i. replaced what might be called enlightened despotism. The unenlightened financialism which superceded the enlightened despotism was supposed to work so automatically that it did not need any enlightenment. To-day the tendency in the West is to revive this unenlightened financialism and only make it applicable within larger and larger geographical areas.

There are perhaps some tiny signs that point in another direction. In the recent declaration of Punta del Este in Uruguay signed by all the states in the organization of American states (the Council of American states) – including the United States – some reference is made to economic planning, and in the coming negotiations between Great Britain and the six countries of the Common market there is perhaps a slight possibility that some very mild sort of economic planning may be considered. I have indeed been told that in Brussels the bureaucrats of the Commission of the European Economic Community are not as convinced of the wisdom of unenlightened financialism as most of the politicians are. But these various manifestations of a tendency towards some sort of Western planning are as yet only very tiny beams of light. The massive background of the economic efforts of the West to-day is still a stubborn insistence on unenlightened financialism.

It is my deepest conviction that if this situation continues, the West will be hopelessly lost in its competition with the East. The outcome will be the end of the Western kind of democracy.

If it is to be at all possible to save the Western democracies, we must find a way to safeguard the freedom and ethical and moral dignity of the individual in the true spirit of the age of enlightenment and at the same time achieve full and effective use of all resources, natural resources as well as human know-how. This goal can never be reached through unenlightened financialism, but it has become technically possible since the advent of the electronic computer and the econometric methods. These aids to analysis have removed at least what was previously a technical obstacle to the third alternative. The question is whether the West will use these technical tools intensively and in a truly rational form of liberty-planning with emphasis not only on pecuniary measures of output, national income etc., but even more on *social goals*. This implies inter alia

breaking the power of private finance to exploit humans – in materially underdeveloped and highly developed countries alike – .

This is the perspective in which we should view our role as social scientists and social technicians. And this is the spirit in which my own labours and those of the Oslo Economic Institute have in all humility been guided in the last 10 or 20 years.

These various aspects of the optimalization approach I must consider in some detail.

A Few Historical Remarks

As far as I know the first elaborate attempt at discussing economic programming for the economy at large along the lines of the feasible instrument approach and the optimalization approach was the one contained in my memorandum 'Price-Wage-Tax-Subsidy Policies as Instruments in Maintaining Optimal Employment' (UN Document E|Cn. 1|Sub. 2|13, 18 April 1949) to the United Nations Sub-Commission on Employment and Economic Stability.

It is pathetic to think of the way this attempt was received at that time. But it is encouraging to think how much the atmosphere has changed since then, even though it has not yet changed as much as is needed for the survival of the democracies of the Western type.

My attempt at that time was built on what I called the Submodel. Its mathematical structure was prepared in advance in the Oslo University Institute of Economics and it was quantified by the use of Norwegian data. The data were far from being as good as I had wanted, but they could at least serve as an illustration of the type of analysis which I then thought – and still think – is absolutely necessary for reaching real solutions of problems for the economy at large, national and international. The mathematical model had 14 degrees of freedom. It was later published in Metroeconomica, cf. [18].

Naturally I did not dare to reveal this mathematical background to the Commission at that time, but I had brought along extensive mimeographed tables to display what sorts of feasible policy alternatives were available, and also to show examples of optimal solutions.

I tried, of course, to present the idea as simply and briefly as possible, but even so it was quite obvious that the members of the Commission got more and more into a state of panic in the face of such terrible waste of the Commission's precious time. And they felt great relief when I had finished my exposé.

This reception of the first attempt did not stop the efforts of the Oslo University Institute of Economics towards developing practical forms of programming techniques for the economy at large. Through generous grants from the Rockefeller Foundation and Norwegian sources – which I gratefully acknowledge also in the present connection – we have been able to continue the work.

As the years went by I not only found it necessary to concentrate on effective forms of the economic models and on the processing of the actual data needed and on a technique for co-operation with responsible authorities – one Norwegian Government member gave a considerable amount of his time for experiments along this line [2]) – but I also was led to consider further developments of computational techniques particularly adapted to the kind of mathematical programming problems encountered in our research.

The outcome was the multiplex method which has by now stood the acid test of solving big linear programming problems on an electronic computer at very low cost. In this connection I feel it a pleasant duty to give credit to the highly competent and constructive contribution made by Mr. Ole-Johan Dahl, mathematician and coding expert connected with the electronic computer of the Norwegian Defence Organization.

In a series of problems each involving approximately ten thousand coefficients (the big number of coefficients was due to the necessity of considering the dynamic repercussions over a number of years) the actual computing cost by the multiplex method on our Ferranti Mercury Computer turned out to be approximately one tenth of the cost estimate received from a big European computing office using the revised simplex on IBM 704. For other types of problems the comparison may, of course, give a different result.

The principles of the multiplex method were first published in Sankhyā [3]. Subsequently other authors – obviously without knowledge of the multiplex method – have worked along similar lines. In a new and fertile field it would indeed be strange if valuable ideas should not occur to several researchers nearly simultaneously. Without attempting anything like a complete bibliography I may mention the works of J. B. Rosen and G. Zoutendijk [4].

Prior to the elaboration of the multiplex method I had published

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[2]) He has now consented to the disclosing of his name. It was Mr. Trygve Bratteli, then Minister of Finance, now Minister of Transportation.

what I called the double gradient form of the logarithmic potential method. I used this method successfully on small problems. I abandoned, however, this method because I felt that it could not easily be mechanized for automatic computation. Recently the logarithmic potential idea has been taken up again and incorporated in a modified form in the method of Georges R. Parisot [5]. I have not tested Mr. Parisot's method numerically and will, therefore, reserve final opinion on the practical usefulness of his method. In this connection mention should also be made of the works of Tomasz Pietrzykowski [6].

The multiplex method has also been worked out for the case of linear bounds and a mixed linear and quadratic preference function with negative definite quadratic form. This was reported on at the Tokyo Meeting of the International Statistical Institute [7], and elaborated upon in a subsequent memorandum [8]. This version only involves very little additional cost as compared to the strictly linear case. A rather non-technical description of this version was given in the volume in honour of Professor Johan Åkerman [9]. We have also in an experimental way used the multiplex method on much more general cases. This is reported on in [10].

The real difficulties in the programming problem that emerges from a use of the Oslo Channel Model reside in the fact that we must be prepared to handle extremely general situations: mixed linear and quadratic preference function (which is mathematically equivalent to a singular quadratic part in the preference function), the non-concavity of the preference function and the non-convexity of the admissible region. Without these complications the problem is reasonably simple and may be attacked in a number of different ways. But when these complications are introduced, it seems next to impossible to develop a complete and explicit mathematical theory. At best it may take perhaps a generation before we get such a solution. But time is pressing for a solution because the problem has such immediate and important practical implications. At the present stage we must therefore make a dash forward, relying to a considerable extent on a heuristic, intuitive and empirical approach ('empirical mathematics'), but trying of course to push the exact mathematical criteria as far as possible in every direction where exact criteria seem practically and computationally feasible. This is the spirit in which we have tried to use the multiplex method on the most difficult cases.

To a considerable extent our work has been inspired by a desire to aid the underdeveloped countries, cf. [11] and [12]. I shall not go into detail regarding the extensive literature on economic development which

has appeared during the last years. Many references can be found, for instance, in the book by Gerald M. Meier and Robert E. Baldwin [13]. I would also like to mention the interesting brief survey given by Dr. Ibrahim H. Abdel Rahman, Director General of the Institute of National Planning, Cairo, [14].

I shall now proceed to summarizing what I think are the main types of problems we encounter when we face the tremendous task of optimal economic programming in all its complexity.

The Deterministic vs. the Stochastic Approach

The *probabilistic* or, if you wish, *stochastic* aspect of programming is important. I am convinced of the ultimate usefulness of this approach, but I have a feeling that at the present stage the minimum factor for programming at the national – or even international – level is a comprehensive analytical scheme where there is a *great number* of aspects included. We therefore have to economize on forces by provisionally neglecting the probabilistic refinement and reason as if we have certain structural equations to work with. In other words we assume that these structure equations themselves are constant. We must do the best we can and in the first approximation aim at a deterministic, i.e. a non-stochastic model. This is the viewpoint adapted in the sequel. Subsequently this deterministic model can offer a grip to probabilistic analyses.

Decision Models vs. Growth Models. Common Sense

The distinction between what is essentially a *growth* model and a *decision* model is important.

When I speak of a growth model I am not referring particularly to its dynamic character, because a useful decision model is also essentially dynamic, but I think of the rather too passive attitude to economic growth which is displayed in the use of the Western type of growth model approach, characterized by such simple notions as the general savings rate, capital to output ratios, marginal productivity of capital etc. without explicit introduction of the *decisional parameters* that will basically influence growth. The explicit introduction of these parameters in an operational way is what characterizes a decision model. We have to consider *a great number* of such decisional parameters, for instance those char-

acterizing many different types of investment and their relations to the current account activity of many different domestic sectors. These remarks will apply in all countries, less developed and more developed.

The need for introducing explicitly the decisional parameters in the analysis is most acute in a short range (say annual) or a medium range (say five or seven year) plan. In very long range (say twenty or fifty years forecasting) the future looses itself so much in the haze that we have to rely to a large degree on guesses of a growth kind. For instance what can we say to-day of the possibility of bringing to the blue-print stage certain bold ideas that linger in the heads of some prominent physicists? What can we say about the possibility of arrival of some ideas that are not yet in these heads? Here we can only guess about growth rates. But for the bulk of day to day planning work which is concerned with decisions between tangible and precisely formulated technical and economic alternatives the decision model viewpoint is absolutely fundamental .

A few remarks about the common sense of decision models might be appropriate.

A realistically constructed decision model is nothing more than systematized common sense. No sensible decision model builder believes that he can embrace everything and in an exact way. But he does know that it is possible through aggregations and approximations and simplifying assumptions to say something useful about a lot of things that are *relevant* and *too numerous* and related in *too complicated* ways to be grasped by simple talking. Through the models he will be able to build a useful plan-frame. Or several plan-frames one on each aspiration level in the hierarchy of problems.

To a large extent effective programming is an *art*, not a science. Creative intuition and practical sense wil always be needed. What scientific programming does is not to do away with these mental powers but to push forward by leaps and bounds the *frontier of demarcation* where we *have to* start using our intuition and practical sense. This is really what scientific programming does.

But are the data available and are they accurate enough? If they are not, can this fact be overcome by disregarding the consistent and quantitative reasoning by means of a model and using instead loose (and at times conveniently inconsistent) talking and strong convictions? Sometimes one gets the impression that some people think so. By running a number of solutions with randomly disturbed coefficients in a decision model we can get some idea of the stability significance of the solution.

The List of Economic Aspects

Compartmentalization According to Aspects

Economics in general and economic planning in particular is concerned with many different aspects of the economy. The following is a list of examples of what I mean by aspects:

1. *Goods* (material goods and services). Classified in a number of categories, agricultural products, industrial products etc.
2. *Power*, classified according to source such as coal, heavy oil, electricity (hydro or thermal), atomic energy etc.
3. *Transportation.* Classified in categories such as rail, road, shipping, air etc.
4. *Training, research and innovations.*
5. *Investment projects* as distinct from current account activity.
6. *Subsidiary vs. basic activity.* In investment or on current account. 'Induced' investment or 'induced' current account activity.
7. *Time range*, i.e. distinction between short, middle and long term.
8. *Primary and secondary importance.* From the viewpoint of what goal?
9. *The regional aspect* of interflows and planning. The territorial viewpoint. All national vs. local government decisions. Location problems for industry.
10. *Price problems.*
11. *Private finance and banking.*
12. *Public finance and money.*

Possibly some more examples could be added, but those given are the most important ones for discussing the type of administrative and practical planning difficulties I want to emphasize. (They are even illustrative of the organization of pure research work).

Since it is obviously impossible for any single human brain or any single team of administrators or policy makers to embrace all the above 12 aspects simultaneously, some sort of division of labour has to be made. This raises the organizationally and administratively very troublesome

problem of compartmentalization. All the 12 aspects are, of course, in reality interrelated, and decisions regarding any of them will have important repercussions on the setting of the problem for the others.

At every turn of the road a co-ordination problem thus arises which cannot be solved simply by asking the people to 'get together' and make their actions consistent. Nor can it be solved simply by appointing some new minister or committee of co-ordination. However honest and co-operative and enthusiastic the individual compartment leaders and the co-ordinators are, they will find themselves in a bureaucratic maze when they simply 'get together'. And so everybody is by the very nature of things driven into a compartmental way of action which is very detrimental to the purpose of the nation as a whole. In hard reality the solidarity of the whole does not permit splitting it.

Extremely enlightening examples of this are to be found in the history of planning in Soviet Russia. In the period before 1957 compartmentalization was done according to types of (material) goods, i.e. according to the above aspect No. 1. There was one all-Union central ministry for each group of goods. Because of frequent uncertainties of supply from other ministries each minister was tempted to set up his own ministerial factory for the component parts he needed. This led, of course, to inefficiencies of various sorts. There were also bureaucratic delays in settling questions with enterprises scattered over the whole nation. This motivated the abolition in 1957 of the central ministerial system and the introduction of a territorial system, i.e. a system compartmentalized according to the aspect No. 9 above.

But this reform only replaced certain kinds of difficulties by others. Professor Vasiutin, head of Gosplan's division for medium and long term planning, who gave a lecture in the Oslo University Institute of Economics in September 1961, told for instance with a good sense of humour how enterprise directors may have concealed their real productive capacity and material reserves in order not to get caught when they had to fulfil the region's productive targets.

The difficulties encountered in the regional compartmentalization have recently, I understand, released a desire to revert at least partly to principles of a more all-Union character.

Both in the goods-ministerial system and in the territorial system there were, of course, big difficulties in making the material balances really *balance*. Even several rounds of iterations with conferences and

'co-ordinations' frequently led to serious inconsistencies (for the solution of which big stocks had to be maintained).

So in whatever way one tries to solve the compartmentalization problem one runs into trouble.

These examples from the Soviet Union are not quoted in a critical spirit. They only depict certain general types of difficulties which are inherent in any human economic system. They are also present (but more concealed, which is even worse) in a free market economy. The growth rates achieved in the East and in the West over the last generation prove beyond doubt, I think, that at least from the viewpoint of economic growth the Eastern system with all its obvious difficulties is by far superior to the Western.

The answer to the troubles – whether in the Soviet Union or in any other country that wants to introduce an element of planning in its economy – is to base the planning work on a well designed decision model with built-in regional distribution and automatic consistency making of the material balances. Or rather to build the work on several such models on different aspiration levels. And subsequently to issue instructions in the form of a co-ordinated system of success indicators (which must go beyond the fixation of transaction prices, cf. the section on Prices on p. 267.

Even when the problem is approached through one or more decision models, it is, of course, necessary to compress it into a manageable form. But this compression does not proceed by compartmentalization according to the above 12 aspects, but in another way.

There may be alternative ways to compress the problem. The following is the way I suggest. It leads to the Oslo Channel Model [3]) for which the present paper may serve as a preface.

In the first place I distinguish between the *selection problem* and the *implementation problem*. To the latter sphere belong the aspects 10–12. They are approached only after an optimal solution of the selection problem is available. All the other aspects are covered simultaneously in the selection decision model.

Selection and Implementation

A selection model is primarily useful for the purpose of describing a

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[3]) The term channel model was suggested by Dr. Ibrahim Abdel Rahman. It was originally used for a simpler model applied to the Egyptian economy: The Cairo Channel Model.

constellation of the *volume figures* or the figures *in actual technical units* which has been realized or might be realized or one would like to see realized, provided one can find ways and means (institutional, administrative and financial) of bringing this constellation about.

In theory it would, of course, be possible to include also all these ways and means explicitly in the same programming analysis, but such a set-up would only be a formalistic one without much chance of leading to practical results at the present stage. It is more practical to separate the selection problem and the implementation problem [4]). It is primarily in the selection problem that the biggest advantages of a precise quantitative analysis can be gained. In the implementation problem we must rely to a much larger extent on economic intuition and practical sense.

Another reason why it is a practical approach to separate the selection and the implementation problem is that the selection problem can be studied without stating a priori the kind of *economic institutions* (competitive markets or central controls etc.) one is prepared to accept.

After the selection problem has been solved, one will take up the implementation problem. If on scrutiny one should find that practical difficulties of implementation under an existing institutional, administrative and financial set-up make it impossible to reach the high goals – for instance a rapid rise in national product – which have emerged as feasible from the selection viewpoint, two ways are open: either to try to change the institutional, administrative and financial structure so as to make the high goals attainable, or to insist that this structure is not to be changed and that one will therefore have to acquiesce to the lower goals. In the latter case the computation of the difference between the two results will furnish a sound piece of information.

Prices as One of the Implementation Instruments

In a free market economy the price system is the pivot around which the 'balancing' of the economy turns.

There is one strong point in favour of this sort of balancing: In

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[4]) There is perhaps a chance of proceeding *part* of the way towards the programming solution of the implementation problem by considering the interplay between *real* flows and *financial* flows. A research project in this direction, the refi-project (re = real, fi = financial) is going on at the Oslo University Institute of Economics, but I shall not be concerned with this here

its pure form it does not give the statesman or the economist any headaches. They do not need to think. The ship is steered *automatically* whither the wind blows. If there is a shortage, the prices rise and if there is an excess supply, the prices fall. And that is that.

But the moment the statesman – with the help of the economist – starts formulating *preferences* regarding the goals to be obtained – the course which the ship ought to follow – then the headaches begin. How can the 'bad effects' of price changes be corrected? Will a certain pattern of price changes or price stabilities be conducive to the goals one would like to see attained? If so, is it possible to influence the prices, say by direct controls, in the desired direction? Or is it just as well to give up all goals – such as for instance a high rate of economic groth – and only try to secure the stability of all prices except one, and throw all burden of variation on to this one – the interest rate – ?

The enormous literature on prices in the Western economies, from abstract philosophizing about justum pretium to learned books on 'how to avoid inflation', and the endless discussions amongst Soviet economists on how the prices 'ought' to be fixed, cf. [15], indicate the complexity of this matter.

From my long-time study I have reached three rather definite conclusions:

First, in any economy where one formulates preferences about the goals to be attained – and I think there is no modern society which does not have aspirations in this direction – it is *impossible* to leave the prices alone. They have to be 'tampered with'.

Second, even the strictest direct control over prices in all parts of the economy is *not sufficient* to steer the economy in a desired direction. Other types of controls or success indicators have also to be applied.

Third, the system of actual transaction prices to be used in order to steer the economy in a desired direction cannot successfully be fixed by ad hoc considerations. This will inevitably lead to all sorts of irrationalities and inconsistencies. They can only be fixed after a system of shadow prices has been found through the optimum solution of a selection decision model which is formulated in *technical units only*, without the use of the price concept. Cf. pp. 272–78.

The actual practice in the Soviet Union has not yet passed beyond the ad hoc approach. But such economists as *Nemchinov*, *Kantorovich*, *Sobolev* and *Novozhilov* are pressing for a really rational solution through

mathematical programming [5]). I am convinced that in this connection linear programming is inadequate. The programming technique must mathematically be of a much more general kind. Cf. pp. 272–78 and pp. 259–62. I am also convinced that the actual transaction prices cannot simply be put equal to the shadow prices obtained from an optimal solution. They must be worked out by taking account of the shadow prices but they must be of a much more differentiated nature where many sorts of detailed implementation considerations will have to be taken account of so as to make the actual transaction prices useful for micro-planners.

The Investment Projects

In a decision model it is absolutely inadequate to consider 'investment' as some sort of aggregated figure (perhaps to be compared with some other aggregated figure such as 'saving'). To work with such aggregated concepts would be evading the real problem of economic policy discussions. One of the most crucial aspects in a truly decisional analysis of the national economy is precisely to find out what *sorts* of investments to make. Practical planners are every day feeling the embarassing problem of picking amongst a great number of investment projects. A comparison between *different categories* of investment must, therefore, stand in the center of the analysis.

An investment project is defined through a description worked out according to a scheme with thorough specification of the data which are required to find out what the repercussions on the economy will be *if and when* it is decided to start the project.

The data contained in the project description can only be concerned with the repercussions that are *directly visible* and therefore can be given by the technicians connected with the elaboration of the project. The infinite variety of indirect repercussion will only appear when the project – with explicit formulation of the two alternatives: acceptance or rejection – has been incorporated in a decision model that has been solved by the overall planning experts.

[5]) Cf. p. 209 of [15]. Quite recently Mr. Tom Kronsjö, a Swedish guest associate in the Oslo University Institute of Economics, has translated from Russian into English the curriculum for economics students in Leningrad University as well as that for economics students in Moscow University. This material gives an overwhelming impression of the amount of mathematics in general and mathematical programming in particular which is now required in economics in these universities. This material has been issued as two memoranda from the Oslo Institute. Cf. [22] and [23]. Also [25].

There are four types of data to be included as direct repercussions in a project description.

First, a parameter that indicates the size of the project and its phasing, i.e. the year when it might be *started.*

Second, a set of coefficients describing the *carry-on-activity*, i.e. the investment inputs that have to be made into the project in the course of the construction period.

Third, a set of coefficients describing the effects which the project will have on the *capacities of production* when the fruits of the projects – if and when it is started – begin to emerge. It is essential to take account of the time shape of this capacity effect.

Fourth, a set of coefficients describing the *infra*-effect of investments, i.e. the effect which an investment project may have on the coefficients of the model.

Details about the current account inputs and outputs that will be connected with projects when it has reached the *state of operation* will depend on many things that cannot be finally decided upon until after a complete and overall programming solution has been obtained (unless one is prepared to make many simplifying assumptions). Final details about current account operations can therefore not be given in the project description but special types of information about this should be given.

All these aspects of the investment problem are discussed in detail in [16].

Rational planning at a national level can, of course, not consist simply in applying mathematical programming techniques to a decision model built on *any* given list of investment projects. For the possibility of obtaining a really high value of the preference function it is essential that investment projects of the right kind are *available.*

To increase the chance of having valuable projects included in the list of projects which is taken as a datum in the mathematical programming, the planning authorities should issue ahead of time what may be called a *project guidance* [6]). But this being said, I want to add that it is dangerous to go too far in this direction. We must rely on the creative imagination of all layers of the whole population. And for best results the project-makers should not be put in a Procrustean bed. Everybody should be encouraged to bring forth new ideas however wild they may look. But a collection of ideas is certainly not *enough.* One will need a systematic

[6]) Built on what project properties have proved useful in previous complete solutions or in solutions of pilot investigations.

method for distinguishing between what is valuable and what is useless from the viewpoint of the preferences of the policy makers. Only scientific programming can provide such a criterion.

For several reasons it is wanted to consider not only a detailed list of specific projects but also groups of projects, i.e. aggregation of projects into *channels*, each channel being defined by certain average project characteristics, cf. [16] and footnote 3 on p. 266.

Regional Aspects Described through a Pattern of Centres

In a concrete and realistic form of national planning it is impossible to disregard the regional aspect. The aims of national planning will indeed always to a smaller or larger degree be concerned with the development of certain underdeveloped parts of the country. And we have to consider the fundamental problem of an optimal development of the *transportation network* and the *location of industry*. And this in turn is connected with all sorts of micro-regional problems involving such questions as social layers, housing problems, shopping and entertainment centres, commuting lines between living place and working place etc.

In national planning all these problems must be seen in their connection with the goals set for the development of the nation as a whole. The way in which the macroaspects of these various questions can be co-ordinated is discussed in the next section. In the present context I shall only consider the question of how a 'region' can be defined and in what way the various regions should be included in the model.

I have no faith in a planning system where each region – defined in a more or less conventional way – is left free to submit, according to its own ideas, a suggestion for a plan – investment plan and|or plan for current account operations – within its border, and a subsequent attempt at 'co-ordinating' these regional plans at the top level by trial and error or rounds of 'iterations' by consultations between the top level and the regional authorities. For effective planning one must *start* by a rather definite *frame* to be prescribed for the subsequent detailed regional – or even enterprise – plans to be prepared at the lower levels. And this top level (necessarily aggregated) plan frame must be worked out by a simultaneous and explicit programming technique. This requires a specific definition of the concept of a region.

Abstractly any point within the border of a country can, of course, be defined in an exhaustive and non-overlapping way by indicating its

latitude and longitude. And in abstracto one may think of a sort of a super model where each such point is included with all the economic variables which one may think of as pertaining to this point.

But this is sheer formalism. To approach the regional problem in a practical way we must start by considering a certain *pattern of centres*. The centres should not be defined by conventional administrative geographical borders but be built on a consideration of the *economic significance* of each centre.

The centres may be large or small depending on the level of aggregation to which the decisional model in question pertains. In a model with high aspiration there may even be a pattern including a variety of both large and small centres. But in all cases the model must build on a *list of centres* that are so to speak authorized for inclusion in the model in question. This pattern may in very ambitious regional development planning call for a consideration of *alternative* patterns of centres. We face here a sort of a superlist of alternatives somewhat similar to the list of alternative investment projects among which a choice has to be made. But each optimal solution will have to be made on the basis of a *given* list of well defined centres.

The interflows between these standardized centres can, I think, best be characterized by four types of variables: Variables describing the flow of *goods* (material goods and services) inside the centres and between the centres, the *power* distribution within the centre and between the centres, the *local transportation* (rails, roads etc.) within the centre and the *inter-transportation* between the centres.

The *capacities* of production of each goods category within the centre as well as the capacities for power distribution and goods transportation within the centre and between the centres at the *beginning* of the planning period as well as the technical coefficients that describe the possibilities of increasing these capacities through the adoption of one or more of the investment projects in the available list of projects, and the various other dynamic repercussions of such an adoption, will be data for the analysis, cf. pp. 269–71.

These various aspects are all included in a systematic way in the structure of the regional interflow table (Table 1) given below.

Optimal Selection and Material Balances under a Given Pattern of Centres and a Given Aspiration Level

The precise structure of the regional interflow table which I advocate, is

TABLE 1 – Regional interflow table in technical units.

Figures on any given row are measured in a given technical unit			Receiving centres and sectors																Final deliveries				Grand total (Actual production at the centre)
			0				1				2				3								
			Goods	Power	Local transportation	Intertransportation	Goods	Power	Local transportation	Intertransportation	Goods	Power	Local transportation	Intertransportation	Goods	Power	Local transportation	Intertransportation	Private consumption on current account $i = \ldots$	Government consumption on current account $j = \ldots$	Gross investment in fixed real capital	Net increase in stocks at the centre	
Delivering centres and sectors	0	Goods																					
		Power																					
		Local transportation																					
		Intertransportation																					
	1	Goods																					
		Power																					
		Local transportation																					
		Intertransportation																					
	2	Goods																					
		Power																					
		Local transportation																					
		Intertransportation																					
	3	Goods																					
		Power																					
		Local transportation																					
		Intertransportation																					

given in Table 1. This table assumes that all the flows are defined in *technical units* only, with no explicit introduction of prices. Possibly some sort of conventional 'prices' may have to be introduced in order to define volume figures in an aggregate or average technical unit to be applied on any given row in the table. But such 'prices' intervene in a very *weak* form which will not influence the result of the solution in a basic way.

The upper left part of the table – its main part – consists of a square matrix with the same nomenclature vertically and horizontally. But so far as the contents of the cells are concerned, there is a difference. It is assumed that the items on a given row are measured in the same technical unit, but this unit may be completely different as we go from one row to the next. This means that a horizontal summation on any given row in the table is possible. But a vertical summation in a column has no meaning.

The nomenclature – the same vertically and horizontally – has as its *primary* principle of classification the list of geographical centres. The 'centre' No. 0 represents the Rest of the World, i.e. this 'centre' is only a balancing account giving a summary of the flows to and from the overall national economy (comprising all the concrete centres Nos. 1, 2...) and the outside world.

The *secondary* principle of classification in the nomenclature is a classification representing the economic nature of the flows. It is divided in four main categories.

The first category is what I have for brevity termed goods, with the understanding that it means goods produced in the centre in question. The breakdowns in this category will have to be made according to whatever kind of 'goods' it is deemed relevant and efficient to include in the concrete situation to which the model is to be applied. This breakdown will include material goods and whatever services it is deemed relevant to include (apart from power, i.e. energy production and transmission and transporting of material goods).

The second category is concerned with power. It is deemed necessary to consider this explicitly as a separate category since it is so fundamental from the viewpoint of the geographical distribution of the centres. This applies both to the production of power and to its transmission. Conceptionally this category includes both actual power production at the centre and whatever energy is 'imported' (as 'competitive imports', cf. below) into the centre from other centres. The energy distribution within the

centre of all the energy which the centre has at its disposal is also included in the power category. The distribution of this total energy to the various activities within the centre is recorded separately for each activity as is obvious from the structure of the table. If one wants a concrete interpretation of the 'import' of energy into a centre, one may think of the power production plant in a given centre as having a 'branch office' in each of the other centres, this 'branch office' delivering the energy to the centre where the 'branch office' is situated. The 'imports' into a centre will be expressed by a negative number and the 'exports' by a positive number on the row of this centre. Since there is a breakdown also for receiving centres, the table will give a complete picture of the energy flow from any given centre to any other centre. This interpretation of positive, negative or zero numbers under the power category will also apply on any row under the above first main category: goods. The breakdown within the power category will reveal the various forms of energy. Here electric energy will, of course, be the main form which can be carried from one centre to another.

An exactly analogous interpretation – with positive, negative or zero figures – applies to the third and fourth category comprising the transportation of material goods. Since here local and intercentre transportations are concretely of so different forms (trunk roads as distinct from side roads etc.) it was deemed useful to separate them as two main categories: local transportation and intercentre transportation. The breakdown within each of these main categories will concretely be roads, rails, transportation by barges on rivers and canals etc. For each such special transportation breakdown the conceptual handling of the items on a given specialized row will be precisely the same as everywhere else in the table. I use the terms 'local' and 'inter' because they are phonetically easier to distinguish than 'intra' and 'inter'.

As explained above there will exist capacity bounds for all the current account productions and all the power and transportation flows between centres. And the changes in these bounds that will be produced by investments are explicitly taken account of.

On any given row – whether representing a main category or a special breakdown – there are a number of items to the right of the square matrix (that has equal vertical and horizontal nomenclature). This right hand part of the table represents *final deliveries* from the activity which is represented by the row in question.

These final deliveries comprise the following four main categories.

First, private consumption broken down in whatever household groups ($i = 1, 2 \ldots$) which it is deemed relevant to consider.

Second, government consumption broken down into whatever government activities ($j = 1, 2 \ldots$) one wants to consider.

Third, the use of goods and services as inputs for gross investment purposes. If the table is used to report statistically the flows that take place in a given period, such as a given year or a given month, the items under this category will depict the gross investments *actually* made according to already decided upon, i.e. already-committed-to, projects. But if the table is used to depict the logical relations between repercussions of *possible* investments that enter into a comprehensive model for decisional analysis, the composition of the items here will need a breakdown for each one of the investment projects in the list of projects to be considered, see pp. 269–271 above. And each of these items will have te be expressed by the carry-on-structure for the project in question. Details about this are given in [16].

Fourth, the net increase (positive, negative or zero) of stocks of material goods within the centre.

In distinction to the final deliveries described under these four categories, all other deliveries may be termed *cross deliveries.*

Since the delivery items on any given row in the table may be positive, negative or zero as explained above, the grand total sum on this row will give the *actual production* within the centre (of goods, power, local transportation or intertransportation as the case may be).

The *total net exports* (positive, negative or zero) of a given kind (a kind of goods, a kind of power etc.) from a centre to activities in other centres is the sum of all the items on this row which are inside the big square matrix, except the items belonging to the vertical part of the table which represents the centre itself as a receiving centre.

If it is wanted to, we may for any particular kind of activity, say electricity production and transmission, extract from the big table a smaller (square) summary table showing for this special kind the delivery from any given centre to any other. A positive figure in a cell in this summary table would indicate that there is a net delivery *from* the centre whose name indicates the row and *to* the centre whose name indicates the column. A negative figure in the cell would indicate a net flow in the opposite direction.

The *preference function* for a decisional analysis based on the above set up would have to be expressed in terms of the various activities ex-

pressed in *physical units*. This is the most concrete and most relevant formulation of the preference function in a selection problem.

It will in point of principle depend on the size of the variables in each point of time within the horizon one wants to consider. A discount rate may or may not be used. If it is used, this rate would have nothing to do with an interest rate that emerges in a credit market, but would only reflect the time preference of the policy maker whoever he may be. Cf. the remarks on liberty-planning on p. 257.

The big table here described includes – if the table is taken as the basis for a decision analysis – in *explicit* form and *simultaneously* all the aspects 1–9 listed above on p. 264.

Indeed, the aspects 1, 2 and 3 are obviously included, and the aspect 4 is included provided it is taken as a breakdown under the first main category. Further the aspect 5 is obviously included.

The aspect 6 emerges automatically in the decisional analysis since all the activities are considered simultaneously in their mutual relationships. It seems difficult to give a precise meaning to the words 'subsidary' and 'basic' except through this decisional analysis. If it is wanted one may, of course, use a numbering that distinguishes between what one might be tempted to call basic and subsidiary. But this would be completely unnecessary and would only create headaches in classification because the distinction may be difficult in border cases and furthermore there may be activities that are 'subsidiary' in relation to some activity but 'basic' in relation to some others.

The aspect 7 is also automatically taken account of since the decisional analysis would be completely dynamic, distinguishing for any variable its size in the different points of time.

The aspect 8 is included automatically. It is simply depicted by the size of the variable as it emerges from the optimal solution.

The aspect 9 is obviously included.

The aspects 10–12 will not be further discussed here since they belong to the implementation problem, not to the selection problem which forms the object of the decisional analysis built on the above big table.

In practice the number of centres and other breakdowns will, of course, be much larger than what can be contained in a simple sheet table like Table 1, but for a discussion of the principles this single sheet table is useful.

In a separate chapter below some introductory remarks are made on the precise way in which the decisional analysis is built up. For simplicity

this discussion is confined to the case of a single centre. This may then be looked upon as a nation (or any lower order geographical region) which is in relation with the rest of the world.

The Pyramidation Problem in the Decisional Structure

By the pyramidation problem I mean the general problem of the extent to which it is possible to decentralize the decisions, i.e. the problem of the optimum number of levels on which the decision making machinery is to be organized. This pyramidation viewpoint may be applied to any of the aspects that are brought out explicitly in the big table above (types of commodities, types of investments, geographical regions etc.). It is therefore a viewpoint different from that of the 'aspects' discussed pp. 264–66.

At the present stage and for some time to come it will, I think, be impossible to include the pyramidation problem explicitly in the model – which, of course, would have been the ideal solution – . We have at present to approach the problem by some kind of *simulation* technique. For instance in such a way that a number of competent (mixed economic and mathematical) groups are organized, each group representing a specific decision making unit in the general game. Each group would have to be allocated sufficient machine time on a good sized computer. An over-all central group would formulate rules and criteria which each special group would have to abide by. Within the confines of these specified rules and criteria each group would act in a perfectly selfish way. The global constellation which emerged from such a game would be studied in its desirable and undesirable features, and a new attempt would be made at reformulating the rules and criteria for the special game-groups in such a way that the resulting *global* constellation of the economy could conform better to that constellation which has emerged as the optimal one from the selection viewpoint in the economy as a whole. The pyramidation problem would then appear only as one aspect of the implementation problem.

Moving Planning

In my work in underdeveloped countries like India and Egypt as well as in separate publications I have for several years advocated what may be termed moving (or sequential) planning, cf. for instance [17].

This simply means that each year we work out a new dynamic optimum

decision analysis for the planning horizon (say five or seven years) which is adopted, taking account of whatever fresh information has become available. This means inter alia that in the plan which is worked out in any given year, we have to include in the set of *non decisional* elements (i.e. in the set of already-committed-to elements) those things that were *decided upon* in the analysis made preceding year.

Professor Vasiutin in his lecture in the Oslo Institute in September 1961 told us that this idea of moving planning has been introduced in the actual planning work in the Soviet Union since last year.

THE OSLO CHANNEL MODEL FOR A SINGLE REGION

Simplifications are Possible

The analytical model based on Table 1 is extremely general and in this general form it gives a clear picture of the manysidedness of the problem as it exists in the real world. The logic of the interrelations is displayed by the table.

In a situation where it is not needed to consider all the relations in their full complexity or it is not possible to do so because of lack of data, there is no difficulty in using the general set up in Table 1 as a *machinery to generate simpler models.* Table 1 will then show in what *direction* from the realistic complexity the simplification is made.

It is even possible to carry the simplifications through so drastically that we are left with only a small macromodel with a few variables, but then, of course, also with a drastic loss of details.

As time goes on, one may want to and be able to extend and complete such an extremely simplified analysis. If so, it is essential to keep the general Table 1 in mind. The extension of the analysis can then proceed in an orderly and co-ordinated fashion so that the simple analyses *converge* towards a really satisfactory solution of the complete problem. For this purpose it is desirable to use a notation in the simplified problems which conforms as much as possible with the general notation.

To describe briefly the system of notation I have found it useful to standardize, it will here be sufficient to consider the case of a single region.

The Single Region Interflow Table

Table 2 indicates the notation to be used in the case of a single region.

This interflow table applies for any given year t. The flows in this table are expressed in value figures so that also *vertical* summations are possible. This permits us to introduce certain balancing principles in the table. But the change in interpretation which is necessary to come back to figures measured in the technical units used in Table 1 is obvious. We then simply have to drop the row Residual input in Table 2 as well as the idea of vertical summations.

The upper left $n \times n$ matrix of Table 2 represents the cross deliveries on current account. The element $X_{hk}{}^t$ represents the deliveries from sector h to sector k on current account (h = sec, k = sec, *sec* = 'sector'). These elements form a square matrix. If we only consider the deliveries of each sector to other sectors and disregard what the sector may deliver to itself, we will have

$$X_{kk}{}^t = 0 \qquad \text{for } k = sec \tag{1}$$

But this property will vanish if we aggregate sectors of bigger interflow table. In order to assure generality we will, therefore, in general not make the assumption (1).

The other rows in Table 2 – outside of its square part – record the values of the following technical inputs.

Primary input (labour etc.) is represented by other affixes h (h = prim, prim = 'primary'). To these affixes does not correspond any receiving sector. These affixes $\underline{h}$ may be used to designate special types of labour (for instance skilled, semi-skilled, unskilled etc.), or productive input from salaried personnel etc. If only one category of these affixes $\underline{h}$ is considered, it may be denoted $h = W$ (W = 'Wages').

A final group of technical input affixes h represents complementary imports (h = comp, comp = 'complementary') broken down, perhaps, according to country of origin if the case may be. If only one category of these affixes h is considered, it may be denoted $h = B$ (the letter B standing for complementary imports). Complementary (as distinct from competitive) imports mean imports that in practice cannot be produced domestically, at least not in the planning period.

The elements on the row *residual input* ε in Table 2 absorb all 'inputs' needed to make the column sum for a receiving production sector equal to its row sum. Concretely speaking this corresponds to (2), and this breakdown applies also to ε under other receiving categories.

TABLE 2 – Single Region Interflow Table of Value Figures

		Cross deliveries	Final deliveries $X_{h*}{}^t$					GRAND TOTAL
		Receiving sectors current account	Private consumption on current account (Commercialized deliveries on current account)	Government consumption on current account (Non-commercialized deliveries on current account)	Domestic gross investment in fixed real capital	Product balance for h-goods $\Delta_h{}^t$		
						Net increase in stocks of h-goods	Net exports of h-goods	
		$k = 1, 2, 3 \ldots$ (sec) $\ldots n$	$i = \ldots$(hous)	$j = \ldots$(gov)	J	L	A	
Technical inputs	Delivering sectors Current account $h = 1$ 2 3 . . . (sec) . . . n	$X_{hk}{}^t$	$C_{hi}{}^t$	$G_{hj}{}^t$	$J_h{}^t$	$\dot{L}_h{}^t$	$A_h{}^t$	(For production sectors: Actual domestic product at domestic market price) $X_h{}^t$
	Primary input (labour etc.) $h = n + 1$ $n + 2$ ... (prim) ... (W) ...							(For primary factors: Total input from = total income of the factor) $X_h{}^t$
	Complementary imports $h = \ldots$ ... (comp) ... (B) ...							0
Residual input	ε	$\varepsilon_k{}^t$	$\varepsilon_i{}^t$	$\varepsilon_j{}^t$	$\varepsilon_J{}^t$	$\varepsilon_L{}^t$	$\varepsilon_A{}^t$	$J^t + \dot{L}^t + E^t$
Grand total		$X_k{}^t$	$R_i{}^t$	0	J^t	$\dot{L}^t$	Improvement in national creditor position E^t	

Net taxes, direct and indirect (subsidies reckoned negatively). Import and export duties. $= T$

Net unilateral transfers (interests, dividends, gifts etc.). Positive for the sector of origin. $= \tau$

Depreciation on the fixed real capital used in the production sectors or by private consumers or by Government or used directly in the investment channel activities etc. Also depreciation on commodity stocks used for the same purposes. $= D$ (2)

Non distributed profits: Net column savings, after all taxes, unilateral transfers and depreciation on fixed real capital and on stocks. $= e$

Total residual input $= \varepsilon = T + \tau + D + e$

There are columns for delivery on current account to private consumer groups (i = hous, hous = 'households'). They represent *commercialized* deliveries on current account.

There are also columns for deliveries on current account to Government groups (j = gov, gov = 'Government'). They represent *non-commercialized* deliveries on current account.

Finally there are columns for inputs for *investment* purposes, also inputs into stocks of goods – the stocks being denoted L and their changes $\dot{L}$ – and deliveries to exports. All these columns are in obvious conformity with the columns of Table 1 except that the column for export in Table 2 represents a *summation* in Table 1 over all the columns pertaining to the 'centre' No. 0.

The export figures in Table 2 – denoted A – are, in conformity with Table 1, reckoned *net*, that is to say as gross exports of h-goods minus competitive imports of goods belonging to the h–category. The introduction of this net figure defined as a difference conceals in Table 2 – interpreted as a table in value figures – a statistical problem which is a bit troublesome because of the difference between f.o.b. and c.i.f. prices and the way in which import and export duties are reckoned etc. These questions are discussed in some detail in [19], but are not further considered in the present paper.

As the analysis is extended in the direction of a regional analysis, we must build up separate columns for net exports for a number of different regions, cf. the remarks on p. 276 on total net exports and on the (square) summary matrix.

From such data we may derive a *multicompensatory trade matrix* with the various types of programming and optimalization problems which I have discussed earlier, particularly in 1947, cf. [20].

These earlier analyses were a desperate attempt to persuade those responsible for the organization of international trade to consider not only the obviously beneficial specialization effect of international trade, but also the equally – nay more – important *payment* effect. The latter will periodically, and even chronically, produce a strangulation effect through an international scramble for liquidity if the problem is not faced square in its matrix form and with the inescapable programming problem that emerges from the matrix form.

It is tragic to see how the conservative or labour Governments persist in neglecting this matrix aspect of the payment effect, and concentrate on the specialization effect which they believe can be solved by such a naively simple measure as to lower import duties and introduce other forms of liberalization. In this connection I would like to quote a note added in the spring of 1958 to the 1959 edition of the 1947 paper: 'The developments of the last years show, I believe, that a study of the trade relations as a matrix is more needed than ever before. And action on this basis is vitally important. We are facing a downward spiral and an international scramble for liquidity that cannot be eliminated by conventional monetary weapons. International co-ordinated action of a new type is immediately needed'.

In this connection the calculations of Tibor Scitovsky (based on data by P. J. Verdoorn) are extremely enlightening. His conclusions in [21] are that the gains through specialization are 'ridiculously small'. Even though the data here included may not be too good and the calculations are audacious, these results tend, I believe, to illustrate the underlying fundamental fact which I tried to bring out in the 1947 paper.

The total of all deliveries which are not deliveries to receiving sectors, is called *final* delivery and denoted $X_{h*}{}^t$. This symbol is not used in the inner of Table 2.

For each delivering production sector the grand total in the row, i.e.

$$X_h{}^t = \sum_{k=\text{sec}} X_{hk}{}^t + \sum_{i=\text{hous}} C_{hi}{}^t + \sum_{j=\text{gov}} G_{hj}{}^t + J_h{}^t + \dot{L}_h{}^t + A_h{}^t \quad (h = \text{sec}) \tag{3}$$

will express the actual domestic (homeproduced) product in sector h.

The sum of the two last elements in (3), namely

$$\Delta_h{}^t = L_h{}^t + A_h{}^t \qquad (h = \text{sec}) \tag{4}$$

is the *product balance* for sector h. From the selection viewpoint it is a simplification to consider this as a single figure without using the breakdown (4).

The primary inputs delivered to the various receiving categories k, i, j, J, L and A express the *actual* deliveries to these categories. For instance, the services of private domestic servants in households will appear in one of the $C_{hi}{}^t$ items. The grand total row sum for a primary factor, namely

$$X_h{}^t = \sum_{k=\text{sec}} X_{hk}{}^t + \sum_{i=\text{hous}} C_{hi}{}^t + \\ + \sum_{j=\text{gov}} G_{hj}{}^t + J_h{}^t + \dot{L}_h{}^t + A_h{}^t \qquad (h = \text{prim}) \tag{5}$$

will, therefore, represent the *total income received* by the primary factor h, (not to be confounded with the disposable income, which is income after direct taxes and direct subsidies).

The primary factors delivering categories (h = prim) need not be in a one-to-one correspondence with the household groups (i = hous). But the total income of the primary factors will by definition correspond to the total income of the households. That is, we have by definition

$$\sum_{h=\text{prim}} X_h{}^t = \sum_{i=\text{hous}} R_i{}^t \tag{6}$$

The definition of gross national product is as follows

Private consumption on current account	$= \sum_{i=\text{hous}} (R_i{}^t - \varepsilon_i{}^t)$	
Government consumption on current account	$= \sum_{j=\text{gov}} (-\varepsilon_j{}^t)$	(7)
Gross domestic investment	$= J^t + \dot{L}^t$	
Net increase in the countries' net creditor position (including net gains on foreign transactions) = the sum in the A column	$= E^t$	
Total = GNP^t	= gross national product	

This sum is identical with the sum of the elements in the heavy hexagonal frame in Table 2.

By introducing in (6) the expression of $J^t + \dot{L}^t + E^t$ as the row sum on the row for the residual input – cf. Table 2 – it is easy to see that the

gross national product can also be expressed as a total of gross incomes [7]).

The corresponding *net* concepts are obtained by subtracting *depreciation D* on the capital goods (fixed capital and stock of goods) which are used in the various activities. In the interflow Table 2 these depreciation items are included as input items, namely in the residual input, cf. (2), and hence are included in the value figures which express the sector products $X_h{}^t$ [8]).

The *domestic* product concept (gross or net) as distinct from the *national* product concept is obtained by not including gifts and net current account profit elements on financial transactions with the rest of the world. These elements are in Table 2 included in the residual inputs, namely as a part of the item τ_A. Cf. (2). These items would have to be deducted from E^t in (6) if we wanted the domestic concept instead of the national concept. The problem of the actual statistical measurement of and the manner of handling the T_A and τ_A items is discussed in some detail in [19].

In the A column the elements on the rows for primary factor input will contain such items as direct exports of labour, if any.

On the row for complementary imports – possibly broken down in subcategories – the row sum for the elements to the left of A column is entered negatively in the A column, so that the complete row sum for complementary imports becomes zero.

The sum of the grand total column sums in the table must, of course, be equal to the sum of the grand total row sums, i.e. – cf. (6) –

$$\sum_{k=\text{sec}} X_k{}^t + \sum_{i=\text{hous}} R_i{}^t + J^t + \dot{L}^t + E^t = \tag{8}$$
$$= \sum_{h=\text{sec}} X_h{}^t + \sum_{h=\text{prim}} X_h{}^t + J^t + \dot{L}^t + E^t$$

For forecasting and programming methods of the more elaborate type, and, indeed, for all model building for the economy as a whole, it is absolutely necessary to co-ordinate the *absolute* figures (in value units or technical units) in an interflow table of the kind as Table 2 or the more general kind (Table 1). But one should not try mechanically to transform any of these tables into a table of *coefficients* simply by expressing the items in each column as a fraction of the *total column sum*

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[7]) We get: $\text{GNP}^t = \sum_{i=\text{hous}} R_i{}^t + \sum_{k=\text{sec}} \varepsilon_k{}^t + \varepsilon_J{}^t + \varepsilon_L{}^t + \varepsilon_A{}^t$.

[8]) The increase L_h of the stock of h-goods is *net* in the sense that the actual use of h-goods from stock for delivery to the various receiving categories are included as negative items in L_h, but *gross* in the sense that depreciation on the stock of goods is not subtracted. Cf. the remarks on depreciation in the text above and in (2).

in question and considering these coefficients as an expression for some sort of technical structure of the economy. This is an impossible approach in a decisional analysis. It evades an important aspect of the problem: the *substitution effect*. In the big Table 1 which considers also the distinction between centres, this is particularly striking. A very important aspect of the decisional analysis is precisely from what particular centre it is most economical to haul the material goods needed as inputs in a given activity. Similarly for substitution between different kinds of energy, etc.

The substitution effect is in the Oslo Channel model taken account of by considering what is called *ring structures*. This is discussed in detail in [16].

In the appended numerical Table 2 are given the actual flows in value figures for Norway 1956. These data are processed mainly by Mr. Kåre Edvardsen, working under the supervision of Mr. Tore Johansen, both research associates in the Oslo Institute of Economics. Part of this work is based on data processed by Mr. Arne Dag Johansen, also a research associate in the Institute.

In the numerical work a number of people in the Norwegian Central Bureau of Statistics have offered valuable advice which is here gratefully acknowledged. They are in particular Robert von Hirsch, Erik Homb, Anne Margrethe Martens, Arne Syversen, Reidar Øines and Thorleif Øines.

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Numerical TABLE 2 – Interflow Table – Norway 1956 – Millions of 1956 Kroner

		Nos.	Agriculture 501	Forestry and hunting 502	Fishing 503	Whaling 504	Mining 505	Products of fish and canning 506	Slaughtering, dairies and grain-mills 507	Other food manuf. industries 508	Beverage industries 509	Tobacco manufactures 510	Textiles 511	Made-up skin and leather goods 512	Made-up textile goods 513	Wood and cork 514	Pulp, paper and paper products 515	Printing and publishing 516	Leather, rubber and leather products 517	Products of oils and fats 518	Calcium and silicium carbid, fertilizers 519	Other chemical products 520
Technical inputs	Delivering sectors. Current account	Agriculture	001	14.5	42.0				23.9	1626.9	37.3	13.5		73.4	1.0	6.6	2.4			21.0	0.8		6.5
		Forestry and hunting	002					0.2	1.0	0.2	1.4			1.6	4.9	0.2	342.8	487.7	0.1	0.2			1.6
		Fishing	003	4.9		1.2			360.0	1.6	0.8			1.6	4.9	0.2	1.4			3.7	257.3		1.2
		Whaling	004																		84.2		1.0
		Mining	005	3.5		2.6		3.4	0.4	2.4	1.7	0.4		0.9			0.5	9.9	0.1		0.3	11.8	3.5
		Products of fish and canning	006	1.4		7.8			4.3	1.9	0.3										6.9		0.2
		Slaughtering, dairies and grain-mills	007	267.8		0.1			5.1	42.5	121.8	0.1									0.1		5.5
		Other food manufacturing industries	008						4.9	1.6	45.4	6.1		4.7				0.1			0.7		1.8
		Beverage industries	009	1.2							1.4	46.7					0.5	0.1					4.9
		Tobacco manufactures	010										1.6										
		Textiles	011	1.0		2.2				0.4				386.7	10.4	311.7	23.0	16.7	0.7	18.2	0.4	2.5	1.8
		Made-up skin and leather goods	012												3.2								
		Made-up textile goods	013	0.2						0.6	0.2			5.4	1.3	19.0		1.3		0.8	0.6	1.0	1.1
		Wood and cork	014	3.9				0.1	25.1	3.0	2.0	0.8		0.5	2.0	0.4	414.4	16.5	0.1	0.8	2.1		9.0
		Pulp, paper and paper products	015	7.6				0.2	13.9	15.3	25.2	0.4	8.5	5.9	3.6	2.7	5.3	541.3	94.7	3.3	3.5	38.0	45.1
		Printing and publishing	016						0.6	0.2	4.1	0.3	0.7	0.2	0.2	0.2		0.5	210.7				
		Leather, rubber and leather products	017						0.2					0.2	89.0	3.8	0.8	0.1	1.7	11.2			0.1
		Products of oils and fats	018	141.1		0.5			14.4	0.9	165.5			1.5				5.0		1.1	204.9	0.9	33.3
		Calcium and silicium carbid, fertilizers	019	97.5				0.6		0.9	0.3							1.5			0.1	48.8	10.4
		Other chemical products	020	23.6	1.2	40.0	3.2	10.2	3.0	7.8	11.6	7.9	2.2	35.1	1.2	5.4	18.6	84.6	9.0	6.2	20.7	14.9	128.6
		Non-metallic mineral products	021	6.0				0.1		5.3	5.2	6.4					3.3	4.5		0.1		0.7	5.9
		Basic metal industries	022						15.6					0.1			2.5	12.9		0.1	0.2	2.1	11.5
		Metal products except machinery and transport equipment	023	3.4	0.7	1.2	0.5	8.4	26.1	3.3	10.9	6.2	0.7	1.9	1.3	4.7	27.1	25.6	3.0	4.4	14.8	4.1	25.8
		Non-electrical machinery	024	0.1															0.1				
		Electrical machinery, apparatus, etc.	025														0.1	0.1					0.2
		Shipbuilding	026																				
		Transport equipment except ships	027	1.2	1.2												0.3						
		Other manufacturing	028						0.2	0.2	0.3			1.6		7.7	2.0	10.9		2.8	1.0	1.2	2.3
		House-building activity	029																				
		Construction activity	030																				
		Electricity, gas and water supplies	031	15.2				11.5	4.5	7.0	8.7	1.4	0.2	5.0	1.1	3.4	10.3	38.1	3.2	1.8	5.8	35.5	7.7
		Wholesale and retail trade	032	42.0	4.0	19.0	1.0	5.0	51.0	62.0	28.0	20.0	24.0	36.0	16.0	53.0	85.0	110.0	10.0	19.0	58.0	23.0	39.0
		Banking and insurance	033	4.0			1.0																
		Dwellings	034																				
		Business buildings	035																				
		Water transport	036				13.0	4.0															
		Land- and airtransport	037																				
		Communication	038	5.0																			
		Public service	039	8.0																			
		Private business service	040	12.0															27.0				
		Hotels and restaurants, recreation services	041																				
		Unspecified commodities	042	4.3	6.1	9.5	1.7	2.6	8.6	11.2	15.5	6.9	6.9	8.7	6.0	6.0	9.4	11.1	12.1	3.5	4.3	4.3	11.2
		Unspecified services	043	11.0	8.0	7.0	7.0	2.0	4.0	17.0	8.0	4.0	2.0	13.0	11.0	3.0	17.0	19.0	26.0	3.0	7.0	12.0	12.0
		Own investment services	044																				
		Services from assets owned by general government	045																				
		Wages	051	167.6	436.2	88.9	109.9	142.2	119.1	107.3	168.3	42.3	22.9	183.9	67.9	197.0	330.44	304.4	224.8	64.52	73.3	95.1	131.3
		Complementary imports	061	60.2	1.6	1.6	36.0	7.3	19.7	13.4	62.5	7.6	45.5	31.8	0.1	3.3	2.0	23.5	0.9	33.0	123.1	24.5	50.8
Residual inputs		Net indirect taxes	071	−128.6	−10.4	−2.0		−3.8	−15.8	−457.7	117.2	131.9	227.4	3.9	9.0	7.0	15.83	4.7	8.0	8.93	−8.2	10.4	−3.1
		Depreciation	072	412.2	18.9	214.8	113.9	29.8	26.6	42.3	24.6	15.8	3.0	50.1	10.0	9.0	41.55	120.6	31.0	12.91	31.4	75.2	29.5
		Net residual input	073	1343.7	456.0	385.7	−3.0	184.5	97.5	156.5	76.8	50.2	21.0	99.3	33.0	199.1	186.99	228.1	64.9	36.73	89.5	53.3	110.4
		Grand Total		2535.5	965.5	780.1	284.2	408.3	813.9	1674.0	945.0	368.9	366.6	951.4	272.2	843.2	1543.51	2078.8	728.1	257.29	982.8	459.3	690.1

Cross deliveries

Receiving sectors. Current account.

Non-metallic mineral products	Basic metal industries	Metal products except machinery and transport equipment	Non-electrical machinery	Electrical machinery, appar., etc.	Shipbuilding	Transport equipment except ships	Other manufacturing	House-building activity	Construction activity	Electricity, gas and water supplies	Wholesale and retail trade	Banking and insurance	Dwellings	Business buildings	Water transport	Land- and airtransport	Communication	Public service	Private business service	Hotels and restaurants, recreation services	Unspecified commodities	Unspecified services	Own investment services	Services from assets owned by general government	Private consumption on current account	Government consumption on current account			
																										Central	Local	Agriculture	Forestry
521	522	523	524	525	526	527	528	529	530	531	532	533	534	535	536	537	538	539	540	541	542	543	544	545	601	611	612	701	702
		0.1					4.2									4.3									906.7			71.0	
0.4	0.2	0.2	0.1		0.1	0.2	0.5	25.2	3.8						9.0			0.8							115.9 64.7	1.7	5.6		
															7.0													
27.6	134.2	2.9	0.5	0.2	1.4	0.5		22.9	2.6	15.2	0.6				8.7	28.6		0.9			19.8				25.2	2.5	1.3		
											9.1														158.9				
																									1169.3				
											0.2														841.3				
							0.1											1.0	1.0						334.5				
																									363.1				
1.0		5.8	1.0	3.6	3.1	2.8	1.4	23.9	0.6		7.3										4.5				605.9	3.1	2.0	2.0	
							0.4				0.6														264.4				
							0.4				1.7														820.6	3.1	6.4		
6.4	0.1	6.7	3.3	10.2	27.8	4.2	6.0	520.9	40.1		14.1														274.7	5.5	22.9	2.0	1.0
12.5		7.3	0.4	4.1		0.4	3.9	62.9	2.1	0.1	94.5								2.9		47.7				22.8	3.1	16.6		
								5.0	1.0		2.0										216.1				241.6	16.2	34.2		
		3.8	1.3	2.7	0.1	6.4	0.7	1.7	0.3		1.5										14.6				108.4	2.6		6.0	1.0
0.8		1.3	0.3	0.7	1.1			9.0			0.4										2.0				7.9				
																										1.0			
0.7	0.1	0.3		0.4	0.5		1.5																						
30.3	29.8	17.7	6.8	12.5	14.0	8.2	30.1	133.1	11.5	9.2	48.6				127.8	63.7		23.1	3.3		71.6				251.9	37.2	10.2		
50.2	4.8	5.5	1.3	8.2	3.3	1.2	2.2	269.3	45.2		2.0										1.0				72.0		0.3	1.0	1.0
4.5	183.7	291.4	64.1	40.9	138.1	17.3	6.7	142.5	41.5																				
5.5	20.7	233.1	40.9	48.3	75.1	29.8	19.8	301.2	102.8		9.4										1.8				201.6	30.5	5.0	22.0	2.0
	1.0	1.1	53.7	2.7	17.3	2.0		42.2	23.8																68.9	15.3	21.5	102.0	13.0
0.1		6.1	13.5	78.8	13.9	2.7	0.6	137.5	78.4												5.9				189.8	4.9	10.2	5.0	
	10.8	12.6	2.2	0.1	141.6	0.6		8.0																	7.5	11.5			
	0.1	0.5	1.4	0.4	0.1	92.0		4.2	0.8		3.0					13.1					4.0				89.1	13.2	18.2	15.5	
1.4	0.8	1.1	0.7	2.8	0.1		9.0	29.7	8.3	0.3	5.8							0.6			9.8				196.0	7.3	1.1		
					8.0																							332.0	
																										166.0	6.0		6.0
10.8	98.1	15.9	3.9	3.1	6.4	4.8	1.8			2.2						11.0		4.0			58.7				306.5	4.2	25.9		
35.0	60.0	77.0	29.0	29.0	32.0	40.0	12.0	296.8	61.2	7.0	113.0	3.0	1.0	1.0	48.0	119.0	2.0	16.0	6.0	13.0					3538.6	50.1	38.7	49.0	7.0
								1.0			29.0		14.0	6.0	51.0	11.0						112.0			377.7	2.1	1.1		
											127.0	12.0			11.0	22.0	1.0	11.0	28.0	43.0					1031.8	16.0	54.2		
					4.0						275.0				325.0		8.0					23.0			11.6 58.4	6.3 33.3	15.1 15.3		
											696.0				2.0	24.0	15.0			6.0	90.0				509.1	33.5	10.7		
												12.0	2.0								301.0				91.7	17.7	6.4		
													44.0	8.0							100.0				496.0	931.7	479.2		
								17.2	0.8				8.0	1.0							161.8				526.7	46.9	190.1		
											15.0									3.0	36.0				376.6				
5.2	6.8	11.3	5.2	8.5	7.7	4.4	1.7			4.3	79.1	22.3	8.6	3.4	20.6	26.6	14.5	29.0	24.8	25.5									
9.0	9.0	24.0	14.0	16.0	9.0	11.0	3.0	14.8	6.2	21.0	99.9	58.0	10.0	7.0	58.0	54.0	19.0	52.0	12.0	28.0	37.0					153.9	1.0		
																													54.0
																										146.0	211.0		
147.7	209.37	357.2	157.96	226.69	153.1	340.56	64.18	672.3	714.4	150.3	1314.5	284.3	7.0	2.0	1002.0	620.1	257.3	1872.9	650.0	215.3			202.0						
10.7	493.5	3.5	5.5	47.0	1.8	21.9	1.9	21.6	3.4	0.1	18.0	8.0			1625.0	68.5	21.0	4.9	2.8	8.6	1.9	136.0			1124.6	76.4	19.9	2.5	1.2
4.9	0.96	13.2	25.02	−8.05	316.8	−261.65	13.04	69.4	73.9	−2.8	2178.2	−11.3	−111.0	9.0	−9.0	21.8		−28.5	−7.0	81.1									
23.5	49.72	47.1	20.60	23.55	17.6	36.31	7.02	44.3	47.1	251.0	105.0		579.2	125.8	1558.9	418.8	66.8	34.6	10.0	22.0				357.0					
76.2	277.26	147.2	75.55	14.72	56.9	170.77	55.15	224.3	238.3	147.5	1249.5	231.7	539.2	124.8	1438.0	46.5	45.4	97.7	292.0	60.1					−3153.48	−1842.8	−1230.1		
464.4	1591.01	1293.9	528.23	577.11	1050.9	536.39	247.29	3100.9	1508.1	605.4	6500.0	620.0	1102.0	288.0	6283.0	1553.0	450.0	2120.0	1025.8	505.6	459.4	996.8	202.0	357.0	12698.52	0	0	610.0	86.2

Final deliveries																										
Domestic gross investment in fixed real capital																								Net exports		
Production sectors																					Government					
Fishing	Whaling	Mining	Food manufacturing, beverage and tobacco industries	Textile and clothing industries	Pulp, paper and paper products	Chemical products	Non-metallic mineral products	Basic metal industries	Ship building	Other metal products	Other manufacturing	Building and construction	Electricity supply	Wholesale and retail trade	Dwellings	Business buildings	Water transport	Land- and airtransport	Communication	Other industries	Central	Local	Net increase in stocks	Gross exports	Competitive imports (recorded negatively)	GRAND TOTAL
703	704	705	706	707	708	709	710	711	712	713	714	715	716	717	718	719	720	721	722	723	731	732	741	751	752	
																							114.4	107.0	−542.0	2535.5
																							29.3	40.7	−101.1	965.5
15.0																								64.9	−5.6	780.1
																								195.0	−3.0	284.2
																							−6.1	223.5	−146.1	408.3
																							12.0	616.1	−5.0	813.9
																							8.0	76.7	−23.0	1674.0
																							24.5	51.0	−37.3	945.0
																							1.0	5.5	−29.0	368.9
																							−1.0	5.9	−3.0	366.6
70.0	2.0		0.30	0.30	0.50	0.50	0.10	0.40	0.20	0.40	0.30	1.00		2.0			5.0	3.0		4.0			22.0	43.0	−647.3	951.4
																							2.0	4.6	−3.0	272.2
			0.02	0.02	0.27	0.27		0.26		0.14	0.02			1.0			4.0			5.0			10.0	14.0	−55.5	843.2
1.0			0.81	0.36	2.14	3.05	0.84	2.38		1.02	0.41			9.0			2.0		1.0	1.9			95.0	75.0	−79.0	1543.51
																							−24.0	1081.0	−74.0	2078.8
																							2.0	5.3	−13.0	728.1
		4.0	2.18	0.92	4.96	5.62	1.54	5.27	0.59	3.03	1.88	3.0		8.0				14.0		5.5			4.0	23.5	−88.9	257.29
																							38.6	435.1	−83.5	982.8
																							−11.1	314.4	−8.6	459.3
																							16.2	176.8	−878.5	690.1
			0.42	0.34	1.51	1.18	0.22	0.98	0.06	0.78	0.23									6.6			9.0	22.0	−84.9	464.42
		2.0	0.25		0.55	0.71	0.27	0.73		0.25	0.25						1.0						97.6	1177.0	−665.3	1591.01
10.0	6.0	5.0	4.49	18.42	12.37	10.54	7.48	10.36	3.80	5.39	12.15	16.0	2.0	19.0			42.0	2.8		1.7			54.1	172.0	−445.3	1293.9
1.0	6.0	8.8	39.35	24.89	76.15	70.63	9.73	42.24	19.99	43.39	32.66	53.8	27.0	37.0			9.0	10.0		3.6			22.0	83.0	−456.7	528.23
4.0	2.0	3.0	8.64	16.39	0.98	0.93	6.23	17.28	6.30	8.89	12.37	1.0	93.0	14.0			12.0	14.0	39.0	4.8			12.0	39.0	−286.5	577.11
144.0	32.0	1.0	4.59	2.79	8.47	11.04	5.18	10.10		3.61	2.22	2.0		3.0			2256.0	3.0		1.0			5.0	75.0	−1714.0	1050.9
1.0	1.8	4.6	4.52	4.88	16.01	17.62	1.63	14.16	2.35	10.08	4.34	15.2	1.0	62.8				188.4	1.4	13.1			−8.0	3.7	−82.5	536.39
1.0	1.0		1.53	1.32	4.72	5.69	0.91	4.46	0.49	2.74	1.13	1.0					5.0	1.0		18.0			6.0	14.6	−128.3	247.29
		19.0	37.10	37.00	38.40	59.10	11.30	66.90	42.90	69.80	33.50				1400.0	340.0					196.8	417.1				3100.9
3.0													518.0				29.0	198.0	103.0	47.2	127.7	296.2				1508.1
																							32.9	5.4	−150.6	605.4
21.0	5.0	8.0	17.70	27.54	41.53	37.89	9.54	33.63	11.80	25.07	22.30	36.0	20.0	59.0			25.0	60.0	5.0	43.0			35.0	566.6		6500.0
																								10.1		620.0
																										1102.0
																										288.0
	−1.0																−451.0							6087.0	−111.0	6283.0
																		−10.0						194.7	−18.0	1553.0
																								14.2		450.0
																								53.1		2120.0
			2.20	2.20	2.30	3.60	0.70	4.10	2.60	4.30	2.00		1.0				1.0							8.3		1025.8
																								75.4	−0.4	505.6
																										459.4
																								77.0		996.8
		9.0	6.00	4.00	33.00	26.00	5.00	15.00	10.00	19.00	4.00		1.0					8.0	8.0							202.0
																										357.0
																										12698.52
	16.2	4.6	3.60	3.21	10.34	10.84	1.13	8.15	1.85	6.32	3.26	24.0		36.5			510.0	171.8	0.6	4.6			−2.4	11.9	−5105.1	0
																										2282.68
																										5159.06
																								173.0	−318.0	3107.59
271.0	71.0	69.0	133.70	144.58	254.20	265.21	61.80	236.40	102.93	204.21	133.02	153.0	663.0	251.3	1400.0	340.0	2450.0	664.0	158.0	160.0	324.5	713.3	600.0	12422.0	−12393.0	75162.80

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