OPTIMAL SKEWNESS IN INTERNATIONAL ECONOMIC COOPERATION

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INTRODUCTION. THE PROBLEM

The peculiar theoretical problem here to be considered, springs from a fundamental feature in world economics today which I can best describe by reproducing some passages from my paper "An Implementation System for Optimal National Economic Planning without Detailed Quantity Fixation from a Central Authority. Part I. Prolegomena: Selection."[1]

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Small or big communities?

In human society there are many forms of *groupings*.

Some of these groupings are of a *geographical* sort: The people living within a national border are knitted together by many economic and sentimental ties, similarly for those living in a main region within a country (perhaps in a state in a union), similarly there will be county groupings or even small locality or neighbourhood grouping and, smaller still, the family or household groupings.

But there are also groupings which are not based on geographical criteria. For instance: People belonging to a particular trade or a particular social class will in various ways be tied together through similarity of thoughts and feelings.

Many aspects of the grouping problem will be the same whether we think of a geographical grouping or some other kind of grouping. In the sequel I will by way of example only speak of the geographical groupings.

[1] Cf. the Proceedings of the 3rd International Conference on Operations Research, held in Oslo 1963.

In nearly all parts of the world there is today a spectacular tendency towards geographical integration of decisional power and institutions. Examples in point are: The Western European Common Market, the Comecon in Eastern Europe, attempt at economic integration in Latin America, efforts at realizing what is called the Arab Nation, further the Pan-African movement (which made signal progress at the 22–26 May 1963 Addis Abeba Conference), and so on.

The widespread interest in such integrations stems, of course, from the fact that unity of decision offers sizable *material advantages.*

But at the same time there is something priceless, something human that is lost in this geographical concentration of decisional powers. It must necessarily lead to suppression of the freedom and self-rule of the smaller geographical units.

There is also the difficulty of agreeing on how the material gains of integration is to be *shared* between the participants.

Any number of examples of such difficulties could be given. We only have to think of the heated discussions that have in recent years gone on amongst the countries of The Western European Common Market, both on agricultural and industrial policy, or the frictions among Arab countries in the Middle East.

These difficulties cannot be brushed aside simply by saying that those who oppose that particular form of geographical integration which is à la mode today, lack understanding of "what is needed in our time". The real problem is one of weighing against each other the comparative advantages of small and big communities and of finding *some sort of optimal arrangement* for cooperation which can preserve as much as possible of the good points in integration and in national independence respectively.

This is not only a question of separating the kinds of decision that ought to be integrated from those that ought to be left to the smaller geographical units, but it is also – and perhaps primarily – a question of *justice* in the sharing of burdens and advantages.

To work towards such optimal arrangements in supra-national cooperation is a basic problem of our times. It cannot be neglected when we discuss the technique of macroeconomic planning.

Material gains through geographically integrated decisions.

I will make no attempt at giving a systematic survey of the material gains that may be realized through geographically integrated decisions. A single pick of quotations will be enough to exemplify the kind of gains one might expect.

In a penetrating article Julius K. Nyerere, President of the Republic of Tanganika says inter alia:[1] "African unity is essential to the continent

[1] *The Review of International Affairs*, Belgrade, Yugoslavia, 20 April 1963.

as a whole and to every part of it . . . disagreements would lead to a waste of resources . . . The political arguments for African unity are certainly overwhelming. Yet the economic arguments are just as strong. . . We sell in competition with each other, often to the same major consumer. Thus we are all in a desperately weak bargaining position. . . Many developments would only be possible if Africa could be considered as a single economic unit. The multitude of small economic plans could be coordinated".

Some great human qualities will only thrive in small independent communities

But the medal has also a reverse. There are some great human qualities that will only thrive in small independent communities.

The small community is a breeding and testing ground for true democracy, for cultural values and for the priceless asset which consists in historical tradition and the feeling of partnership with one's fellow countrymen.

If the administrative distance from the individual to the decisional centre is long, the individual will not have any *real influence* on the course of affairs. All important matters will in fact be decided by a junta of bureaucrats. In such a situation the individual will have little chance of experiencing that dignity and selfrespect that develop when he has to assume a certain measure of responsibility for what is to happen to his community. The absence of this experience means the death of true democracy.[1]

I shall never forget the manifestation of living and vigorous democracy which I experienced in the summer of 1962 on a lecture and study tour in the small communities in the north-eastern part of Iceland.

Also when it comes to cultural developments in a more specific literary and artistic sense it is the small communities we have to relie upon. Let me just give one pertinent quotation. Dr. Edvard Beyer, professor of literature in the University of Oslo, says:[2] "The richest bloom in European cultural history has taken place in quite small communities: The Greek cities, the small Italian states, Iceland, London at the time of the renaissance, Weimar at the time of Goethe. Cultural life in the small German

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[1] There is, of course, no objection to the small community having many sorts of links *within special fields* (trade relations, membership in the United Nations etc.) with a higher geographical unit. The essential point is that there are *certain important things* which the small community can decide itself. This is the meaning of 'liberty' and 'liberation' etc. And since the community is small, the individual feels that he can play a significant role.

What I have said here applies at all levels: The local region in its relations to the country, the country in its relations to a supra national organization etc.

[2] In the weekly *Orientering*, Oslo, 12 December 1962.

states was much richer that it became later in "das Reich". Small communities stimulate the individual and create a diversified cultural activity, while the big communities create a tendency towards passiveness and repress the creative genius of the individual."

Therefore the problem is not so easy that it can be solved simply by hastening that particular form of geographical integration which is à la mode today. The problem is rather how to find ways and means of reaping the fruits of supra-national cooperation in material and economic affairs in our age, while at the same time salvaging the priceless assets of the small communities.

Equity and justice are prime prerequisites

Without equity and justice there can be no lasting peace in the world and no real solution of the problem of supra-national economic cooperation.

Through military and economic domination it might be possible to maintain "integration" *for a time*. But lasting stability can only be achieved if built on equity and justice.

What do these words mean?

Take as an example the colonial system as it prevailed in the 19th and the first half of the 20th century. The general economic and cultural situation in the colonies would probably have been even worse if the colonial system had not been in operation. (A fact which the defenders of the colonial system never failed to stress.) But even so all unbiased people will feel that there was something wrong in the way in which the system was practised. The advantages gained by this particular form of "integration" were not shared with equity and justice between the colonies and the colonial powers. From a moral standpoint we would require that a larger share should have gone to the colonies and a correspondingly smaller share to the industrially developed colonial powers.

THE DISTRIBUTION SKEWNESS AND ITS EFFECT ON THE TOTAL GAIN

But – and here we are approaching the essence of the problem – would not such a lowering of the share that goes to the industrially developed colonial powers have *reduced the incentive* for these powers to invest in the colonies and would not this have *reduced the total economic gain* which were available for distribution among the two parties? I think it is fair to say that such an effect would certainly have been produced.

We are thus facing a peculiar theoretical problem: There exists an opportunity of gain through cooperation between two parties. The *total gain* from this cooperation will be the larger the *skewer* the distribution of the proceeds between the two parties is. This in itself would indicate

that the skewness ought to be very great. But the size of the total gain is not the only thing to take into account. There is also the moral requirement that the proceeds should be shared in an equitable and just way, and this in itself would indicate that there ought to be *no skewness*, that is, it would indicate that the sharing of the proceeds ought to be made more or less on a fifty-fifty basis. Taking account of both requirements we are led to the conclusion that some sort of *compromise* will have to be made. What is the *optimum amount* of skewness that ought to be introduced?

This type of fundamental optimum problem is, I think, characteristic not only for what is in our mind when we speak of the colonial powers as having "exploited" the colonies, but also for a large range of other fundamental problems which we encounter in our modern world where full utilization of the modern techniques of production can only be achieved through cooperation between different parties.

But for the sake of simplicity in expounding the basic principles let us stick to considering the cooperation between countries.

Comparative utilities as a principle of justice

Is there any hope of finding a reasonable principle by which to approach this optimum problem? I believe there is, but it must build on a concept which both Western and Eastern economists – each in their particular way – have tried to discredit, namely the consideration of *comparative utilities*.

Many Western economists have indulged in learned proofs of the impossibility of comparing the utilities enjoyed by different individuals or different groups whereupon they embark on such studies of social institutions and income distribution which have no sense unless comparisons of utilities are possible. And many Eastern economists seem to reason about as follows: "I don't like bourgois economists. Since some of the bourgois economists use the concept of utility, this concept must be something bad."

Real scientific progress is impossible as long as we are spellbound by words or by misunderstanding of words. Utility is only a convenient term for something which in a certain sense is measurable and which can give us a clue to solving the optimal skewness problem I have outlined above.

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In the paper quoted I gave by way of examples some numerical computations to illustrate the principle of the coalition preference function and its use in determining the optimum distribution of skewness in economic

cooperation between two countries. Here I shall generalize the approach and briefly map out the theory as it will appear in the case of n countries.

1. MARGINAL UTILITY AND TOTAL UTILITY IN A GIVEN COUNTRY

Let Ω_i be the total utility of income enjoyed by a "typical" individual in country No. i. It will depend on his income measured in real (physical) terms. If this income is denoted x_i we may write

$$\Omega_i = \text{total utility of income as a function of } x_i = \Omega_i(x_i) \tag{1.1}$$

The corresponding marginal utility of income is defined as

$$\omega_i = \frac{d\Omega_i}{dx_i} \tag{1.2}$$

It is a well established fact that ω_i will be a *decreasing* function of x_i. We write this function

$$\omega_i = \text{marginal utility of income as a function of } x_i = \omega_i(x_i) \tag{1.3}$$

The way in which ω_i depends on x_i can be described by studying the *income flexibility* defined by

$$\tilde{\omega}_i = \frac{\text{dlog } \omega_i}{\text{dlog } x_i} = \frac{d\omega_i}{dx_i} \cdot \frac{x_i}{\omega_i} \tag{1.4}$$

By the principle of decreasing marginal utility the income flexibility will be a negative number.

There is evidence to indicate that the absolute value of the flexibility will *decrease* as income increases. In other words we may assume that we have

$$\frac{d\,|\tilde{\omega}_i|}{dx_i} < 0 \tag{1.5}$$

I think that the schedule according to which $|\tilde{\omega}_i|$ decreases – and decreases monotonically – is so pronounced and so typical that we may take $|\tilde{\omega}_i|$ as an *indicator* of the level of real income, or, if you like, the level of economic wellbeing.

If the magnitude of $|\tilde{\omega}_i|$ can be used to fix the point on the scale of

real income (economic wellbeing) – and I believe it can – we have here an indicator which is *independent* of technical units of measurement, and thus can be used in *international comparisons* without entering into the endless and insolvable questions of what different types of commodities may mean to individuals living under completely different circumstances and with access to completely different kinds of commodities. (Is a pound of seal-lard worth more or less to an escimo than a pound of rice is worth to an inhabitant of India?)[1]

The above considerations about $|\tilde{\omega}_i|$ as an indicator useful in international comparisons will have no meaning if $|\tilde{\omega}_i|$ cannot be concretely meaured. In fact I think it can be measured. It can be measured with a degree og approximation comparable to that with which we can measure the general run of ordinary demand elasticities. *Nothing more and nothing less.*[2]

In the present paper I shall assume that $|\tilde{\omega}_i|$ is actually measurable at least as an "average" or "typical" magnitude characterizing the population as a whole in the various countries that enter into the international comparisons made in the present paper.

Since we are only concerned with international comparisons and with alternatives that introduce "relatively small" changes in the real income of the various countries, it will not introduce too much of an extraneous assumption if we presuppose that *within any given country* No. i $|\tilde{\omega}_i|$ is constant. I.e.

$$\tilde{\omega}_i = \text{independent of } x_i \text{ (for any given } i) \tag{1.6}$$

It will be a high negative number, say -10 in a poor country and have a smaller negative value, say -2 in a richer country. Our rough approximation will only work if all the countries entering into the cooperation in question have a value of $|\tilde{\omega}_i|$ well above 1. Otherwise a more refined theoretical set up must be used.

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[1] The magnitude of $|\tilde{\omega}_i|$ is also connected with the problem of international or interregional comparisons of the cost of living. I have suggested that $|\tilde{\omega}_i|$ is at present the only theoretically sound parameter by which, in international or interregional comparisons, we can define income levels on which we can say that specific population groups are *equally well off*. Cf. for instance section 9 of my book *New Methods of Measuring Marginal Utility*, Tübingen 1932.

[2] Cf. for instance my own measurements at different income levels in the US (summarized p. 63 in the book quoted above). Also my July 1964 paper in *Econometrica* where I quoted estimates by Leif Johansen, I. F. Pearce, A. P. Barten and Arne Amundsen. The fact that I found lower absolute values than the authors mentioned here, is easily explained by the US having a higher real income level than the European countries.

Integrating the differential definition (1.4) on the assumption (1.6) we get

$$\omega_i = c_i x_i^{\tilde{\omega}_i} \qquad \text{(for all i)} \tag{1.7}$$

where c_i is a normalization constant which is introduced through the integrating process and which remains undetermined so long as no further assumptions are made. It must be positive in order to assure a positive marginal utility.

Integrating further the differential definition (1.2), now on the assumption (1.7), we get

$$\Omega_i = B_i + C_i x_i^{\tilde{\omega}_i + 1} \tag{1.8}$$

where B_i is a constant of integration, so far undetermined, and C_i is the constant

$$C_i = \frac{c_i}{\tilde{\omega}_i + 1} \tag{1.9}$$

c_i being the normalization constant occuring in (1.7).

In the sequel we will only consider *changes* around the income point x_{io} that existed *before* the international economic agreements in question were put into effect. I. e. we consider *surplus* in total utility

$$\Omega_i - \Omega_{io} = A_i + C_i (x_{io} + y_i)^{\tilde{\omega}_i + 1} \tag{1.10}$$

where

$$A_i = - C_i x_{io}^{\tilde{\omega}_i + 1} \tag{1.11}$$

and y_i is defined as the *increment* in the income of the "average" or "typical" individual in country No. i, which is produced through the international economic cooperation considered. That is, we have by definition

$$y_i = x_i - x_{io} \tag{1.12}$$

Assuming that in all the countries in question $\tilde{\omega}_i$ can be measured through econometric analyses (and is found to be well above unity in absolute value), the only unknown element in (1.10) is the positive normalization constant c_i occurring in (1.7).

It will, I think, be hard to find a principle which would permit us to say that a given level of marginal utility of (real) income in one coutry, should in our comparisons be counted, say, twice or three times as high as that in an other country. It therefore seems plausible to put all the c_i equal to unity, i. e

$$c_i = 1 \qquad \text{(for all i)} \tag{1.13}$$

With this convention the right number of (1.10) *does not contain any unknown* element.

The convention (1.13) seems plausible but is in fact not indispensable. The foundation for the rest of our argument is only that *in some way or other* we remove the n degrees of freedom that exist so long as the normalization constants c_i are not yet determined.

Now for the *goal* to be aimed at. It would seem natural in principle to put up as a goal for the international economic cooperation considered to realize

$$\Sigma_i N_i (\Omega_i - \Omega_{io}) = \max! \tag{1.14}$$

where N_i is the population in country No.i. More sophisticated formulations are, of course, possible. But as a close to hand example let us consider (1.14).

2. THE DISTRIBUTION SKEWNESS

Let us assume that we introduce skewness parameters λ_i, one for each country, measuring the degree of skewness with which the total profit (in terms of real income) that is created through the particular kind of economic cooperation considered, is distributed amongst the countries. To be precise: If Y is the total expected profit, the *part* falling to country No. i is by definition equal to

$$\left(\frac{N_i}{N_1+N_2+\ldots+N_n}+\lambda_i\right)Y \tag{2.1}$$

where the λ_i must satisfy

$$\lambda_1+\lambda_2+\ldots+\lambda_n=0 \tag{2.2}$$

The problem of how to reconcile equity and justice on the one hand with economic efficiency on the other, is to *decide* what particular magnitude one "ought to" attribute to the various λ_i.

The case

$$\lambda_i = 0 \qquad \text{(for all } i\text{)} \tag{2.3}$$

expresses a distribution scheme where the total profit is simply shared according to the population size of the countries in question. (The way in which any national part of the total profit is to be *utilized*, is an internal question to be decided independently by each country.)

Whatever the λ_i are – subject to (2.2) – and whatever the magnitude of Y, the definition (2.1) satisfies the consistiency condition

$$\Sigma_i\left(\frac{N_i}{N_1+N_2+\ldots+N_n}+\lambda_i\right)Y = Y \tag{2.4}$$

The principle (1.14) is one element in the answer to the question of how the λ_i "ought to" be fixed, but this principle in itself is not sufficient. We must further add an estimate of or an assumption about the way in which the *total* profit Y depends on the skewness parameters $\lambda_1, \lambda_2 \ldots \lambda_n$ We will denote this function by the symbol φ, i.e

$$Y = \varphi(\lambda_1, \lambda_2 \ldots \lambda_n) \tag{2.5}$$

Note that by (2.2) φ is in reality a function of only $(n-1)$ variables. For the sake of symmetry in the notation we write it nevertheless in the form (2.5).

The case where there is reason to believe that Y is independent of all the λ_i, is simply a *special case*, and should be handled as such. But we cannot stick to this simple case. We need a more general theory. Cf. the example of the colonial powers and colonies mentioned in the introduction.

When the nature of the function φ is estimated or assumed, the function to be maximized emerges in the form

$$W = \Sigma_i N_i C_i (x_{io} + y_i)^{\tilde{\omega}_i + 1} \tag{2.6}$$

The constant term $N_i A_i$ in the summation which emerges when (1.10) is introduced into (1.14), can be left out because it will not affect the position of the maximum point.

In (2.6) y_i is the increase in (real) income which is enjoyed by the "average" or "typical" individual in country No. i, and which is due to the international cooperation in question. In other words

$$N_i y_i \text{ is by definition equal to (2.1)} \tag{2.7}$$

This means that we have

$$y_i = \left(\frac{1}{N_1 + N_2 + \ldots N_n} + \frac{\lambda_i}{N_i}\right) \varphi(\lambda_1, \lambda_2 \ldots \lambda_n) \qquad \text{(for all } i\text{)} \tag{2.8}$$

The right member of (2.8) depends only on the unknowns $\lambda_1, \lambda_2 \ldots \lambda_n$ and on empirically knowns constants (including the parameters that determine the shape of the function φ). If the expression (2.8) for y_i as a function of $\lambda_1, \lambda_2 \ldots \lambda_n$ is introduced into (2.6), we see that the function W to be maximized is a function of the n variables $\lambda_1, \lambda_2 \ldots \lambda_n$. These variables are not free but are subject to the condition (2.2).

The solution of this maximization problem can be achieved by any of the known methods, for instance by the method of Lagrange. This

method will in the present case have only one single Lagrangean multiplier.

The approach by the Lagrange method is applicable if one is willing to accept any values of the λ_i which may emerge, even such values as may *not* satisfy

$$-\frac{N_i}{N_1+N_2+\ldots N_n} \gtreqless \lambda_i \qquad \text{(for all } i\text{)} \tag{2.9}$$

This last condition is simply the condition that none of the countries involved is actually to *loose* national income by the cooperation in question.

There are however both moral and other considerations which would indicate that the conditions (2.9) be adopted. If they are, the problem is not solvable by a straightforward application of the classical Lagrange method, but becomes a mathematical programming problem with a non linear preference function and with linear (and hence convex) bounds. This problem too is mathematically speaking fairly simple, and may be handled by any of a number of different methods, for instance by my multiplex method. As a preliminary step in such a programming procedure we may use the Lagrange method, provisionally disregarding (2.9).

If by chance the solution obtained in this preliminary step, or one of the solutions, satisfy (2.9), well and good. But if none of the solutions satisfy (2.9), we must proceed to the full programming analysis (and in this case the maximum value obtained for W will be smaller – or at least not larger – than that found in the Lagrangean solution).

What are the practical possibilities of proceeding along the theoretical lines suggested in this paper? Many details must, of course, be elaborated and prolonged negotiations will be needed to replace coercion through military and economic domination in the field of international economics by the principle of equity and justice as here suggested. But if there is a will, there is a way. The beginning might be difficult, but later this procedure would, let us hope, appear more and more as a solution which is able to combine the two seemingly contradictory requirements of (1) true equity in the distribution of advantages and (2) high technical efficiency from the view point economic integration.