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A GENERALIZED FORM OF THE REFI INTERFLOW TABLE



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1. INTRODUCTION

IN THE theory of the refi model (*re* = real, *fi* = financial) of the Institute of Economics at the University of Oslo, we need an interflow table which is considerably more refined than the usual input-output tables, and even more elaborate than the forms discussed in the Oslo median model (memorandum of 10 October 1956), the interflow table with competitive imports (memorandum of 7 June 1959) and the interflow tables for investment planning (for instance tab. (16.1) in the memorandum of 15 July 1959).

The main problems in the refi-table are to take account of the following complications:

1) The distinction between establishments (plants) and enterprises (firms) in the production. This leads to the concept of *ownership sectors* as distinct from production sectors. The investment and operation decisions rest to a large extent with the ownership sectors.

2) The explicit introduction of *financial objects* (money, credit documents etc.) and the trading in such objects. These operations must be analytically coordinated with the operations in real objects so as to arrive at a coherent common system.

3) The explicit introduction of *financial sectors*, i.e. sectors whose main purpose is to produce and exchange financial objects.

4) A more detailed consideration of the various kinds of *transfers* (taxes, social security contributions and benefits, interests, dividends etc.).

All these desiderata must be weighed against each other so as to arrive at a presentation that is reasonably simple and can be represented in a central table of *two dimensions* (with whatever special appended tables that may be needed).

The main idea of the central table here presented is that it is *general* in the sense that it allows for a variety of *different conventions* on the concrete content of the individual figures. The various special cases are generated by putting some of the elements of the general table equal to zero or equal to some specific concretely determined figures. This general approach is a necessary basis for discussing the best form to be actually used in a specific work on numerical data.

TABLE

		Receiving production sectors (establishments). Current account		Consumer groups. Current account		
		Agriculture, Forestry ...	Accounting sector for foreign trade	Government consumption	Private consumption	
		<i>h</i>	ω	<i>G</i>	<i>j</i>	
Delivering production sectors (establishments). Current account.	Agriculture	<i>k</i>	X_{kh}	$X_{k\omega}$	C_{kG}	C_{kj}
	Forestry					
	...					
	Unspecified services					
	Accounting sector for foreign trade	ω	B_h	B_ω	B_G	B_j
	Non-competitive imports at domestic market prices.					
Distributed remuneration to consumer groups.	Government sales of goods and services	<i>o</i>	X_{oh}	$X_{o\omega}$	C_{oG}	C_{oj}
	Wage households	<i>i</i>	W_{ih}	$W_{i\omega}$	W_{iG}	W_{ij}
	Ownership households					
	...					
Categorized transfers recorded positively for the sector of origin. (Excluding transfers made on rows for consumer groups, ownership sectors and financial sectors).	Direct taxes (positive for the tax paying sector)	μ	$T_{\mu h}$	$T_{\mu\omega}$	$T_{\mu G}$	$T_{\mu j}$
	Indirect taxes (positive for the tax paying sector). Subsidies (negative for the subsidy receiving sector)					
	Social security contributions (positive for the contributing sector)					
	Gifts and similar unilateral transfers (positive for the sector of origin)					
	Interests (positive for the interest paying sector)					
	Dividends (positive for the dividend paying sector)					
	Accounting transfers for balancing the table (positive for the sector of origin)					
Ownership sectors (enterprises as distinct from establishments).	State enterprises	<i>e</i>	δ_{eh}	$\delta_{e\omega}$	δ_{eG}	δ_{ej}
	Local Government enterprises					
	Big private enterprises in industry and mining					
	...					
	Single-sector enterprises					
Financial sectors (main purpose: trading in financial objects).	State budget	<i>f</i>	δ_{fh}	$\delta_{f\omega}$	δ_{fG}	δ_{fj}
	Local Government budgets					
	...					
	Central Bank of issue					
	...					
GRAND TOTAL			$X_{(h)}$	$X_{(\omega)}$	$X_{(G)}$	$R_{(j)}$

Note. The table is general in the sense that it may be applied under a variety of special certain magnitudes zero by definition, or equal to other preassigned figures.

(2.1).

Domestic gross investment in fixed real capital				Net increase in inventories. By sector of origin	Rest of the world receiving		SUBTOTAL (Actual production in production sectors)	Categorized transfers recorded negatively for the sector of origin. (Same description as in rows μ , but with positive changed to negative and vice versa)	Minus increase in financial objects (see specified list)	GRAND TOTAL
In production sectors		In consumer groups			Exports at domestic market price	Competitive imports at domestic market price				
Agriculture ..	Accounting sector for foreign trade	Government administration	Private consumer groups							
g	ω	G	j	L	A	$-Z$	v	λ		
J_{kg}	$J_{k\omega}$	J_{kG}	J_{kj}	\dot{L}_k	A_k	$-Z_k$	X_k	S_{kv}	$-\dot{F}_{k\lambda}$	$X_{(k)} + J_{(k)}$
B_g^j	B_ω^j	B_G^j	B_j^j	B_L	$B_A + X_\omega -$ $-B$	$-B_Z$	X_ω	$S_{\omega v}$	$-\dot{F}_{\omega\lambda}$	$X_{(\omega)} + J_{(\omega)}$
J_{og}	$J_{o\omega}$	J_{oG}	J_{oj}	\dot{L}_o	A_o	$-Z_o$	X_o	S_{ov}	$-\dot{F}_{o\lambda}$	$X_{(o)} + J_{(o)}$
W_{ig}^j	$W_{i\omega}^j$	W_{iG}^j	W_{ij}^j	W_{iL}	W_{iA}	$-W_Z$	R_i	S_{iv}	$-\dot{F}_{i\lambda}$	$R_{(i)}$
$T_{\mu g}^j$	$T_{\mu \omega}^j$	$T_{\mu G}^j$	$T_{\mu j}^j$	$T_{\mu L}$	$T_{\mu A}$	$T_{\mu Z}$	T_μ	0	0	T_μ
δ_{eg}^j	$\delta_{e\omega}^j$	δ_{eG}^j	δ_{ej}^j	δ_{eL}	δ_{eA}	δ_{eZ}	...	S_{ev}	$-\dot{F}_{e\lambda}$	$J_{(e)}$
δ_{fg}^j	$\delta_{f\omega}^j$	δ_{fG}^j	δ_{fj}^j	δ_{fL}	δ_{fA}	δ_{fZ}	...	S_{fv}	$-\dot{F}_{f\lambda}$	$J_{(f)}$
$J_{(g)}$	$J_{(\omega)}$	$J_{(G)}$	$J_{(j)}$	$\dot{L}_{(l)}$	E	...	S_v	$-E$

conventions on the concrete content of the figures. The special cases are generated by putting

Work is now being done in a cooperative effort of several Norwegian Government Departments (Ministry of Finance, Ministry of Industry, Ministry of Commerce), the Norwegian Central Bank of issue, the Central Bureau of Statistics and the Institute of Economics at the University of Oslo with a view to building up a better analytical foundation for the national budget—including investment plans—and, in general, for objective discussions on economic policy.

The form of the interflow table which is presented in the sequel, is the outgrowth of many and long discussions that have taken place in the Institute of Economics at the University of Oslo. Valuable contribution to these discussions have in particular been made by the research associates Hans Heli (the role of ownership sectors), Tore Johansen (symbolism), Hans Jacob Kreyberg (general aspects), Jan Serck-Hanssen (competitive imports and the relation between domestic market prices and cif and fob prices) and Tore Thonstad (accounting principles and the typographical shape of the table). Subsequently the results have been reformulated and to a considerable extent modified by me. I am responsible for the table as now presented.

2. GENERAL REMARKS ON THE STRUCTURE OF THE TABLE

The table is built up in rows and columns as indicated in the accompanying tab. (2.1). The rows are grouped together in certain *horizontal parts* and the columns in certain *vertical parts*. The order of succession of these parts are chosen in such a way that the upper left region of the refi table resembles as much as possible the previously used interflow tables that were mentioned in Section 1.

The specification of the individual rows within a certain horizontal part in the refi table and the specification of the individual columns in a certain vertical part is to be decided in each concrete case. But the horizontal parts and the vertical parts are standard. Each part is indicated by a standardized affix, for instance k , ω , i etc. for the horizontal parts and h , ω , j etc. for the vertical parts as exhibited in tab. (2.1). Each of these affixes runs through a certain number of values indicating the individual rows or columns.

It will be found convenient to let the numbering of the individual rows be different from that of the columns. One may, for instance, let the individual row numbers be selected from among 1–499 and the column numbers from among 501–999, and in such a way that if there is a *correspondence* between a certain row and a certain column, the column number should be exactly 500 larger than the row number. If it is not found inconvenient to use four digits, one may instead let the column number be exactly 1000 larger than the corresponding row number. There is no necessity of letting the numbering run continuously. We may for instance number the rows in the first horizontal part 001, 002 etc. and those in the second horizontal part 051, 052 etc. And similarly for the columns.

As a general principle of balancing it may be convenient to require that whenever there is a correspondence between a row and a column, the row sum should be equal to the column sum, and for a row that has no corresponding column, or a column that has no corresponding row, the sum should be zero. This we may call the correspondence principle for balancing. In the general form of the refi table we will, however, not impose these conditions strictly, but allow certain exceptions as explained below. By so doing we may in certain cases let a row sum or a column sum record some essential figure which it seems more logical to have as a marginal total than as a (perhaps negative) balancing item in the interior of the table.

Whatever the exceptions to the correspondence principle are, it will, of course, always be true that the sum of the column sums is equal to the sum of the row sums. This gives a means of numerical checking and will also help to clear up the logic of the exceptions to the correspondence principle. Examples are given below.

The whole interflow table is assumed to apply to *a given period*, say a given year. For this period we assume in essence that the prices of real goods and services are given. They are observed or estimated as the average prices prevailing in the period in question.

Financial objects may be assumed to be traded at the prices actually prevailing. If we do not consider the *chaining* of the refi tables from one period to another in such a way that the value of the stocks of financial objects at any given point of time is logically connected with the interflow tables for each period, we do not need to consider (positive or negative) income elements arising from *changes* in the prices of stocks and bonds and other financial objects. Such elements need therefore not be considered in the table. If it is wanted to do so, it may be done through specifications in appropriate rows and columns in the table. Some special considerations on this aspect of the financial interflow are offered in Section 15 of the memorandum *General theory of the kernel model*, of 7 February 1958.

Depreciation on real capital (leading from the concept of gross investment to that of net investment, and correspondingly from the concept of gross national product to that of net national product) is not considered explicitly in the table, but it could be introduced by the inclusion of an extra row and column. Compare tab. (1.1) in the memorandum of 7 June 1959.

3. THE HORIZONTAL PARTS OF THE TABLE

I. The first horizontal part of the table represents *delivering production sectors*, current account. The classification over the individual rows is here made according to the production sector in which the establishments (plants) are classified. It is not made according to enterprises (firms). The standard affix denoting delivering production sector is *k*. As an additional sector in this group we con-

sider an accounting sector for foreign trade. Its affix is ω . In the actual numerical work now going on the number of sectors in this part will be between 40 and 50.

II. The next horizontal part represents *consumer groups*. On a given row in this part are recorded the distributed primary factor remunerations that go to the consumer group in question. The fact that these items represent *distributed* remuneration means that we here only consider the amounts that go to households. One or more of these consumer groups pertain, for instance, to ownership groups—and for these only the distributed remuneration is recorded here, while the rest is retained as a surplus in the production sectors (perhaps later to be transferred to an ownership sector as explained below).

On each row the distributed remunerations are broken down according to the sectors where the primary factor input was made (represented by columns, as discussed in Section 4).

On the first row in the horizontal consumer group part we record Government sales of goods and services. In other words we consider the amount of these sales as a remuneration of a primary factor. (This means that these sales will have to be included in the concept of gross national product). This special factor is denoted by the affix 0. We could, of course, have recorded Government sales of goods and services on a production sector's row, but it has been found convenient rather to take them as pertaining to a consumer group.

III. The third horizontal part of the table represents what may be called *categorized transfers*. These are transfers classified according to the special kind of the transfers, for instance direct and indirect taxes, social security contributions, interests, dividends etc. The rows of this horizontal part pertain to transfer items that are recorded as positive numbers for the sector that *makes the payment* (which can be interpreted as the sector that receives a corresponding productive service). The standardized affix for the rows in this part is μ . An example of the specification of these rows is given in tab. (2.1). The last row is a row for accounting transfers that may be needed for the purpose of balancing the table.

In many cases we have a choice of whether we want to let a transfer pass through one of the categories now considered or go directly from the paying to the receiving sector (which may, for instance, be an ownership sector or a financial sector). This is exemplified in Section 6 below.

The interflow table is not constructed so as to exhibit the three dimensional breakdown according to paying sector, receiving sector and the category of the transfer. We can only choose between two out of the three breakdowns. If a three dimensional breakdown is wanted, it must be made in a special table appended to the interflow table.

The horizontal part representing categorized transfers has its counterpart in a vertical part of such transfers as explained under V in Section 4.

IV. The fourth horizontal part represents *ownership sectors*, i.e. enterprises. They are denoted by the standardized affix e (e = enterprise).

A rough breakdown in this horizontal part of the table may, for instance, be:

1. State enterprises.
2. Local Government enterprises.
3. Big private enterprises whose main field of operation is in industry and mining.
4. Non-big enterprises whose main field of operation is in industry and mining.
5. Enterprises whose main field of operation is in shipping and whaling.
6. Other enterprises operating in more than one production sector.
7. Single-sector enterprises, i.e. enterprises each of which only operates in one of the classified production sectors.

V. The fifth and last horizontal part represents the *financial sectors*, i.e. sections whose main purpose is the operation in financial objects. They are denoted by the standardized affix f (f = financial). Some examples of sectors in this part are given in tab. (2.1). A last sector in this part is the Rest of the world. Its row is only used for balancing purposes, when needed.

The ownership sectors and the financial sectors have certain features in common, but concretely they are sufficiently distinct to warrant a separate treatment of them.

4. THE VERTICAL PARTS OF THE TABLE

I. The first vertical part represents receiving production sectors, current account. This part corresponds to the horizontal part I. The standardized affixes for the columns are h and ω .

In a given column in this vertical part are recorded inputs into the sector in question. Some of these inputs may be recorded under the horizontal parts representing delivering sectors or categorized transfers or ownership sectors (when the ownership sector takes over the surplus from the production sector) or financial sectors.

Since there is a choice of recording categorized transfers in a horizontal part or in a vertical part, as explained below under VI, the column sum of a given receiving sector need not be equal to the actual production in the sector. This is why it is denoted $X_{(h)}$ rather than X_h . And similarly for the receiving sector ω .

II. The second vertical part represents the *consumers as receiving units*. The first column (or set of columns) represents Government consumption on current account. It will contain big items. The standard affix is G . If Government consumption on current account is split into headings (for instance in conformity with the state budget or local Government budgets), G will run through a certain number of affixes.

The items W_{iG} are particularly important for i = Wage or salary households.

The rest of the columns in this vertical part, i.e. those with standard affix j , represent private consumer groups. They correspond to the rows i . That is to say, for each value of i there is a corresponding j and *vice versa*.

The balancing of the individual consumer groups may be done in different ways; either vertically or horizontally. Through the accounting transfer row or column any balance from an individual household's row may be transferred to its column, or *vice versa*. In whatever way this is done, it is convenient to impose the rule that the sum in the column of a private consumer group should be equal to the sum in its row. This is indicated in tab (2.1) where the same symbol is used for column sum and row sum, i.e. $R_{(j)} = R_{(i)}$ for $i = j$. This sum may be, say, zero or actual consumption in the consumer group in question.

It is also possible to arrange the balancing in such a way that δ_{ej} (for $e =$ single-sector enterprises) records the total saving (real and financial) in the private consumer group j .

Similar remarks apply to Government consumption. Here it will be convenient to impose the condition

$$X_{(\omega)} = \sum_G X_{(G)} \quad (4.1)$$

III. The third vertical part represents the inputs in various *investment directions*.

There are two main vertical sections here: investment in production sectors and investment in consumer groups.

In the former of these sections there is a subsection for investment in the regular production sectors—with standard affix g —and investment in the accounting sector for foreign trade—with standard affix ω . The latter of these subsections is relatively unimportant and nothing much would be lost by putting all elements in this column equal to zero by definition. Occasionally there may, however, occur a concrete item which would find its natural place here. The former of the two subsections is important. Its standard affix g is different from the standard affix h of receiving sectors on current account. There are several reasons for this distinction. One is that the investment directions for production sectors may be either more aggregated or less aggregated than the receiving sectors on current account¹. In the actual numerical work now being done, there will be between 20 and 25 investment directions in this subsection.

In the section for investment in consumer groups there are also two subsections, one for investment in Government administration—with standard affix G —and one for investment in private household groups—with standard affix j . The former of these subsections may be divided in different special directions, i.e. different values of the affix G . This breakdown may or may not be similar to that used for Government consumption on current account. For the latter of the sub-

¹ In Sections 15 and 16 of the memorandum of 15 July 1959 a great variety of investment directions are considered.

sections one may conveniently use a breakdown similar to that for consumer groups on current account, i.e. j may in both cases run through the same private consumer groups (but the individual column numbers will be different, compare the remarks in Section 2). If such a breakdown is found difficult for lack of data, an aggregation will have to be made, even down to a single column j in the private investment subsection.

In the investment column or columns for private consumer groups we may—if the data are available—record investment in semi-durable or durable consumer goods. An attempt should be made to include at least residential building and private automobiles.

In all the investment columns we allow in principle for the same whole variety of inputs which we considered in the regular production sectors, i.e. labour input and other types of primary factors remuneration (the rows i) and all sorts of taxes and other transfers (the rows μ) as well as surpluses (the rows e and f). In other words each investment direction may in principle be considered itself as a sort of a production sector. In the general form of the table it is very convenient to keep this possibility open. One only has to think of the need for expressing the effect of a diversified system of taxes on special investment directions. If it is not wanted to take advantage of the book-keeping possibilities that are thus kept open, one just has to put some of the items in the investment columns equal to zero.

The concrete meaning of the column sums $J_{(g)}$, $J_{(\omega)}$ etc. will depend on the particular conventions adopted for the recording of the individual items in these columns. For further comments on this point see Section 7.

IV. Net increase in *inventories* is recorded in a separate column (with standard affix L) immediately after the vertical part for investment in fixed real capital. The items in the inventory column are broken down according to the sector of origin (the rows) for the inventory items. The possibility is kept open of considering all sorts of inputs here in the same way we followed for the investments in fixed real capital.

V. The *Rest of the world* as a receiving sector is represented by two columns, one for exports and the other for competitive imports.

As a general rule the items in these columns are recorded in domestic market prices, the correctional term X_{ω} which is needed in order to correct the values in such a way as to bring the balance in conformity with what the Rest of the world is actually to be credited or debited for, is entered as one component of the export item on the ω row. This correctional term X_{ω} may be looked upon as the "total actual product" in the accounting sector for foreign trade. This question is discussed in greater detail in connection with (16.2) in the memorandum of 15 July 1959 (where f was used instead of the now standardized affix ω).

The item B_A represents reexport (at domestic market prices) of non-competitive import items, and B_Z represents competitive imports, if any, into the accounting sector for foreign trade. By the definition of the subtotal X_{ω} the symbol B ,

recorded as a component in the same cell as B_A , will be equal to the sum of all the B -items on row ω , i.e.

$$B = \Sigma_h B_h + B_\omega + \Sigma_G B_G + \Sigma_j B_j + \Sigma_g B_g^j + B_\omega^j + \Sigma_G B_G^j + \Sigma_j B_j^j + B_L + B_A - B_Z \quad (4.2)$$

There will always be some practical difficulty in defining concretely what is to be included in the concept of "non-competitive" imports, but the difficulties can be surmounted with a sufficient degree of approximation.

The principle will be to include imports of such *kinds* of goods and services which cannot be conceived of as produced domestically under existing technical conditions and the system of prices for which the interflow table is constructed. It should be noted that it is only a question of whether the *kinds* of goods can be conceived of as produced domestically or not. If a certain kind of good can actually be conceived of as produced at home, it should *not* be classified as non-competitive, even if the domestic *capacity of production* is so limited that it is likely that a smaller or larger quantity of the good will in any case have to be imported. Any such import that may take place because of limited domestic capacity of production should be taken as competitive import. An essential point in the subsequent programming formulation—where the competitive imports represent degrees of freedom—is precisely that the amount of these imports will be determined by taking account of the domestic capacity bounds.

By going through the list of commodities imported and deciding each case according to the above criterion (for Norway say tea, coffee, bananas, certain kinds of machinery etc.), it will be possible to reach sufficiently accurate figures for the non-competitive imports in a given year (actual work in Norway will be done on 700–800 import goods for the year 1955). From these figures one may in the usual way derive non-competitive import *coefficients* (expressed in relation to the actual production in the production sectors or in relation to disposable income in consumer groups etc.).

Categorized transfers for the Rest of the world are recorded either in the horizontal μ part or in the vertical ν part (with $f =$ Rest of the world) and with accounting transfer to the $T_{\mu A}$ or $T_{\mu Z}$ cells (with $\mu =$ accounting transfer row) so that the sum total E of the columns A and $-Z$ becomes equal to the actual net *export surplus* (positive or negative) for which the Rest of the world is to be debited. This amount E is to be compensated through the change in financial objects with the Rest of the world. Compare the remark below under VII on the column sum $-E$ for the aggregation of all the columns of minus increase in financial objects.

VI. The vertical part *categorized transfers* is broken down into individual columns that correspond exactly to those in the horizontal part μ . We thus have a great freedom of choice in deciding whether we want to record such a transfer in a row or in a column. A given item of transfer will either have to be recorded positively in a row or negatively in the corresponding column or *vice versa*.

Since every transfer that is made *from* one sector or group must be made *to* another sector or group, the grand total row sum T_μ for the transfer category μ , must be equal to the grand total column sum for the same category of transfers. That is we have

$$T_\mu = S_\mu \quad \text{for all } \mu \quad (4.3)$$

But we do not, of course, have any corresponding equality for individual sectors or groups. For instance

$$T_{\mu h} \text{ may be different from } S_{k\nu} \text{ even if } \mu = \nu \text{ and } h = k \quad (4.4)$$

VII. In the last vertical part are recorded changes in the holdings of *financial objects* (financial assets). An actual increase in such a holding is recorded as a negative number. That is, the items record minus the increase. The standard affix for the individual columns here is λ . The following is an example of how the breakdown in the increase in financial assets, i.e. the specification of the individual columns, may be made.

1. Net purchase of existing real capital (as distinct from the real investment of the period to which the interflow table pertains). Such a purchase has in this connection to be considered as "financial" even if the underlying object is real.
2. Net increase in cash and deposits in the Central bank of issue.
3. Net increase in the holdings of Government bills.
4. Net increase in the holdings of Government bonds.
5. Net increase in the holdings of non-Government bonds issued by others than the holder.
6. Net increase in the holdings of non-Government bonds issued by the holder.
7. Net increase in the holdings of stocks issued by others than the holder.
8. Net increase in the holdings of stocks issued by the holder.
9. Net increase in special forms of financial capital investment.
10. Net increase in the holdings of banking deposits (the net increase will be positive if the public has actually increased its banking deposits).
11. Net increase in bank loans.
12. Net increase in the holdings of other domestic financial assets.
13. Net increase in the holdings of foreign financial assets.

Since the items recorded in these columns pertain to changes in what any *domestic* sector holds of the various kinds of assets², the grand total of all the λ columns, must correspond to the opposite of the change in the financial holdings of the Rest of the World (in its relation to our country). In other words the grand total of all the λ columns in the interflow table must be equal to $-E$, where E is the net export surplus reckoned in the prices at which the rest of the world is to be debited. Compare V of Section 4.

² The rest of the world row under f is only a balancing row which may be used if it is wanted to produce zero sums in certain rows and columns.

5. AB SECTOR PRICES VS. PRICES TO BE PAID BY THE RECEIVING SECTORS

In principle the total input into a domestic sector should be reckoned in such a way that this total input includes domestic indirect taxes and similar items. In this sense the total input (to be balanced against the sectors output) is a market price concept, not a factor cost concept.

This, however, still leaves open the question of the particular way in which all the inputs, including indirect taxes and similar items, are distributed over the cells of the column for the receiving sector in question.

Special indirect taxes levied on the production in each separate sector can from the practical statistical viewpoint easily be charged directly to the sector in question (as is done in the Norwegian input-output work). But for the general sales tax such a distribution is not so easy to obtain when this tax is levied on the last stage of the circulation process. In this case it is statistically simpler to charge the sales tax to the special sector "Internal trade" and from this sector to pass it on as part of the inputs that go to the various receiving sectors from the sector "Internal trade". If this is done, we may say that the deliveries from one sector to another are reckoned in "ab sector" prices. (This is done in the Norwegian input-output work).

In point of principle it would be more satisfactory to distribute all indirect taxes over the various delivering sectors. This would be done by including also the general sales tax in the respective h columns for receiving sectors so that these taxes would be passed on as part of the delivery from each sector. This would mean a system of "prices to be paid by receiving sectors" instead of "ab sector prices".

From the programming viewpoint it does not matter very much which one of the two procedures is followed, provided the input to all sectors from the "Internal trade" sector is properly counted (and the input-output coefficients reckoned accordingly). In the "ab sector" system this means a higher input to all sectors from the "Internal trade" sector.

The distinction between domestic market price and *cif* and *fob* prices in relation to the rest of the world is discussed under V, in Section 4.

6. TRANSFERRING SURPLUS FROM PRODUCTION SECTORS

The transferring of surplus from a production sector may be done in a great number of different ways, and the particular way that is chosen will entail different interpretations of the grand total column sum $X_{(h)}$, $X_{(\omega)}$ etc. and the grand total row sums $X_{(k)} + J_{(k)}$, $X_{(\omega)} + J_{(\omega)}$ etc.

Before we discuss these various ways of transferring we will state an equation which in the refi interflow table replaces the rule that the grand total in the column for sector h is equal to the grand total in the row of this sector.

Let

$$Y_h = \sum_k X_{kh} + B_h + X_{oh} + \sum_i W_{ih} + \sum_\mu T_{\mu h} \quad (6.1)$$

be the subtotal in column h up to and including the T -items.

Then by the definition of the column sum—see tab. (2.1)—we have

$$X_{(h)} = Y_h + \sum_e \delta_{eh} + \sum_f \delta_{fh} \quad (6.2)$$

On the other hand we have by the definition of the grand total on row h

$$X_{(h)} = X_h + \sum_\nu S_{h\nu} - \sum_\lambda \dot{F}_{h\lambda} - J_{(h)} \quad (6.3)$$

From (6.2) and (6.3) follows

$$X_h - (Y_h - \sum_\nu S_{h\nu}) = \sum_\lambda \dot{F}_{h\lambda} + J_{(h)} + \sum_e \delta_{eh} + \sum_f \delta_{fh} \quad (6.4)$$

The left member of (6.4) is the surplus of sector h in the classical (median model) sense. Hence the right member of (6.4) also expresses this surplus. This right member can now be interpreted in a refi sense. We can break it down into the following three parts

$$\sum_\lambda \dot{F}_{h\lambda} = \text{the financial objects which the production sector } h \text{ acquires on its own account. This is the increase in what may in a large sense be looked upon as the "cash holdings" of the establishments in the sector.} \quad (6.5)$$

$$J_{(h)} = \text{the gross real investment which the production sector } h \text{ acquires on its own account, including in this real investment both investment in fixed real capital and in inventories.} \quad (6.6)$$

$$\sum_e \delta_{eh} + \sum_f \delta_{fh} = \text{the net surplus of the production sector } h \text{ over and above (6.5) and (6.6). This surplus is distributed to the } e \text{ and } f \text{ sectors.} \quad (6.7)$$

If we allow the sector to acquire nothing on its own account, the whole surplus will be in the form of (6.7), the sum of which corresponds to the median model surplus concept δ_h .

On the other hand if an accounting rule is adopted whereby (6.7) is put zero, the sector h must invest all its surplus in the form of either (6.5) or (6.6), or in the form of a mixture of these two elements such that their sum is equal to the surplus.

Further, if the sector is not allowed to acquire any financial objects, and (6.7) is by definition put equal to zero, the sector must invest all its surplus in the form of $J_{(h)}$.

Still another alternative is that we let all the surplus in sector h be transferred in the form (6.7) *except* that we put the elements $\delta_{eh} = 0$ for $e =$ single-sector enterprises, meaning by this that we do not record any δ -surplus for the *independent* establishments in the production sector h (i.e. those belonging to single-sector enterprises). In this case the sum of (6.5) and (6.6) would express the total surplus in these independent establishments. And we would also have an expression for

the forms in which this surplus is retained by the sector, namely in the financial form (6.5) or in the real form (6.6).

If we allow certain (6.7) items, i.e. surplusses to be transferred to the e and f sectors, these surplusses must, of course, in turn be disposed of in some way. The way in which it is done is expressed by the *horizontal balancing* of the e and f rows. The grand total on each such row, i.e. $J_{(e)}$ and $J_{(f)}$, respectively, can then be looked upon as the gross real investment (including investments in fixed real capital and in inventories) that is required by these sectors *as their property*. And the δ elements on the e and f rows can be looked upon as the surplusses which accrue to the e and f sectors through various activities including the investment directions considered as production activities.

We can throw further light on this way of balancing by considering the equality between the sum of the row totals and the sum of the column totals. Making use of (4.1) and (4.3), we get³.

$$\Sigma_g J_{(g)} + J_{(\omega)} + \Sigma_G J_{(G)} + \Sigma_j J_{(j)} + \dot{L}_0 = \Sigma_k J_{(k)} + J_{(\omega)} + J_{(o)} + \Sigma_e J_{(e)} + \Sigma_f J_{(f)} \quad (6.8)$$

This equation expresses the fact that the sum of everything that is invested in all investment directions (the left member of (6.8))—with the interpretation of the elements in the investment direction columns which we have chosen—is equal to the sum of all investments that are acquired as the property of any sector or group (the right member of (6.8)).

In the special case discussed above as an illustration of the horizontal balancing of the e and f rows, the elements $J_{(g)}$, $J_{(\omega)}$ etc. in the left member of (6.8) are equal to the *actual* gross investments. These we denote with a subscript without parenthesis, i.e.

$$\Sigma_g J_g + J_\omega + \Sigma_G J_G + \Sigma_j J_j + \dot{L} \quad (6.9)$$

This equality between the investment totals symbolized with and without a parenthesis around the subscript need not always hold. It all depends on how we want to perform the horizontal balancing in the e and f rows. We may perhaps decide to distribute the previous grand total $J_{(e)}$ over the various investment direction columns as negative items so as to have the grand total on each e row equal to zero. And similarly on each f row. If this is done, the left member of (6.8), i.e. the sum of the column totals for the investment directions, would only indicate that part of the gross investments which is acquired as the property of the sectors indicated by the affixes in the right member of (6.8) when the terms with $J_{(e)}$ drop out.

If this way of balancing the e and f rows is combined with the procedure of including also the surplus of the independent establishments in the sum (6.7), so that the sum of (6.5) and (6.6) is zero, we may still have $J_{(k)}$, $J_{(\omega)}$ and $J_{(o)}$ different from zero, but the gross investments that are acquired as property by the production sectors could now only be brought about through a decline in the financial hold-

³ Compare the remark before (4.1). For more precision we could introduce "Households" as an ownership sector, and add $J_{(i)}$ as a term beside $R_{(i)}$.

ings of these sectors. Hence the column totals written in the left member of (6.8) would only represent those investments that are acquired by the production sectors through depletion of the financial holdings of these sectors (plus the unimportant item $J_{(0)}$).

If in the latter case we do not allow the production sectors to make any change in their holdings of financial assets (for instance because we never reckon with any such holdings), all the items in both members of (6.8) would be zero (except $J_{(0)}$).

In all the above cases we may, if we like, include in the investment direction cells of rows e and f a component representing a surplus accruing to the e and f sectors through the investment directions considered as production activities—as we suggested above in the special case first considered when we discussed the horizontal balancing of the e and f rows. But if the gross investments acquired by the e and f sectors as their property is distributed back as negative elements over the e and f rows and in the investment direction columns, the *net* negative elements here would not represent the total investments acquired by the e and f sectors, but only the investments acquired by the e and f sectors through the surpluses in these sectors *exclusive* of the surpluses which these sectors have realized in the investment directions considered as production activities.

Even so, there is much to be said for this way of balancing horizontal the e and f sectors, because we would get a picture of how the surplus of a given e or f sector is *distributed* over gross real investments in the various production sectors (and over the acquisition of financial assets), while we only get an expression for the e and f sectors' *total* gross investment in real capital (and their acquisition of financial assets) if we perform the horizontal balancing in such a way that the gross investment in real capital appears as a grand total.

With these general rules in mind we may consider in more detail some examples of ways of recording transfers of surpluses.

7. A CLOSER EXAMINATION OF TRANSFERS AND THE BALANCING OF THE VARIOUS PARTS OF THE INTERFLOW TABLE

For more precision in the subsequent discussion we will make some additional explanations about the symbols in tab. (2.1). In so doing we will for completeness follow the indication in the footnote 1) to the text immediately before (6.8). That is, we assume that the grand total on row i is denoted $R_{(i)} + J_{(i)}$ instead of simply $R_{(i)}$. We may, if we like, also assume that households is one of the rows in the e part, but this is not necessary. As suggested in Section 3. V, we may also use a row for the Rest of the world, but that is not necessary either. Compare the remark below under (7.15).

We begin by stating explicitly an assumption that was already contained implicitly in the reasoning about (6.1)–(6.4), namely that

$$X_{(k)} = X_{(h)} \text{ when } h \text{ corresponds to } k \quad (7.1)$$

A similar statement must be made for the accounting sector for foreign trade. Following the principle of numbering of rows and columns explained in Section 2, we might for full typographical precision have used a different affix for the row and the column of the accounting sector, instead of the single affix ω . We don't need to do it, however, if we only remember the convention

$$\text{column sum } X_{(\text{col})}^{\omega} = X_{(\omega)} \quad (7.2)$$

equals the item $X_{(\text{row})}^{\omega} = X_{(\omega)}$ that occurs in the grand total of row ω

We also state explicitly the assumption that

$$R_{(i)} = R_{(j)} \text{ when } j \text{ corresponds to } i \quad (7.3)$$

Finally we recall (4.1).

When these assumptions are made, the equality of the sum of row sums and the sum of column sums in tab. (2.1) lead to the equation (6.8) where, however, the item $\Sigma_i J_{(i)}$ now has to be added in the right member so that the equation becomes

$$\Sigma_g J_{(g)} + J_{(\text{col})}^{\omega} + \Sigma_G J_{(G)} + \Sigma_j J_{(j)} + \dot{L}_{(i)} = \Sigma_k J_{(k)} + J_{(\text{row})}^{\omega} + J_{(\omega)} + \Sigma_e J_{(e)} + \Sigma_f J_{(f)} + \Sigma_i J_{(i)} \quad (7.4)$$

We will not make any assumption about a correspondence between the J elements in the left and right member of (7.4) similar to the correspondence assumptions (7.1)–(7.3) and (4.1).

To arrive at a precise interpretation of the J items in (7.4) we must consider in general the way in which the various parts of tab. (2.1) are balanced. In this connection there are three different parts of the table to consider.

I. *The correspondence balancing part* consisting of the rows k, ω, o, i and the corresponding current account columns h, ω, G, j .

II. *The vertical balancing part* consisting of the investment columns g, ω, G, j, L and the two Rest of the world columns A and $-Z$, as well as the ν and λ columns.

III. *The horizontal balancing part*, consisting of the rows μ, e and f .

Regarding the correspondence balancing part we note in the first place that by (6.4)—which follows from (7.1) together with (6.1)—we have the interpretation (6.6) of $J_{(h)}$. For complete typographical precision we might have written

$$J_{(k \text{ cor. } h)} \text{ instead of } J_{(h)} \quad (7.5)$$

The symbol to the left in (7.5) means the magnitude $J_{(k)}$ for that value of the row affix k which corresponds to the column affix h . Similarly for $\dot{F}_{h\lambda}$. The simpler notation used in (6.3)–(6.7) is, however, clear enough.

It should be noted that $J_{(h)}$ means the gross real investment which emerges as the *property* of production sector h , regardless of what is the nature of these investments. In principle the ownership distribution of the new investment items may be different from the distribution of these items according to the kind of investment or the production sector where the new capital goods are to be *used*.

In practice certain simplifying conventions in this respect may be made, but such conventions are only specializations, not included in the standard conventions on the symbols of the table.

By a reasoning similar to (6.1)–(6.7) we are by (7.2)–(7.3) and (4.1) led to the interpretations specified in tab. (7.6).

Through (6.5)–(6.7) and tab. (7.6) the balancing of the correspondence part of the table is specified. Even with these standard conventions a great variety of specialized conventions may be made according to what we decide to include in the transfer elements T and S and the surplus elements δ .

Regarding the vertical balancing part we first note that each investment column is considered more or less as a complete production sector that may receive inputs and deliver a total output. The same applies to the inventory column. The inputs in any of these columns may be of all sorts: deliveries from the regular production sectors, for instance J_{kg} , non-competitive imports B_g^j etc. Categorized transfers for the investment directions and the inventory columns can only be recorded on the μ row, but on these rows all categorized transfers can be specified. In principle, surplus items of the categories δ can also be recorded for each investment direction.

TABLE (7.6). *Ways of placing the surplus arising on current account operations in the following sectors.*

Compare also (6.5)–(6.7).

	Accounting sector for foreign trade ω	Sector for Government sales of goods and services, and Government consumption. Compare (4.1)	Consumer group No. i . Current account i
The financial objects which the sector acquires <i>on its own account</i> (i.e. as its property).	$\Sigma_{\lambda} \dot{F}_{\omega\lambda}$	$\Sigma_{\lambda} \dot{F}_{o\lambda}$	$\Sigma_{\lambda} \dot{F}_{i\lambda}$
The gross real investment goods (produced the same year) which the sector acquires <i>on its own account</i> (i.e. as its property), including both investment in fixed real capital and in inventories.	$J_{(\omega)}$ in the sense of $J_{(\text{row})}^{\omega}$	$J_{(o)}$	$J_{(i)}$
The surplus of the sector over and above the preceding two items. This surplus is passed on as a transfer to the e and f sectors.	$\Sigma_e \delta_{e\omega}$ $+\Sigma_f \delta_{f\omega}$	$\Sigma_e \Sigma_G \delta_{eG}$ $+\Sigma_f \Sigma_G \delta_{fG}$	$\Sigma_e \delta_{ej}$ $+\Sigma_f \delta_{fj}$ (j corresponding to i)

The total output in each investment direction is in tab. (2.1) expressed by the column sums $J_{(g)}$, $J_{(\omega)}$... We have for generality put a parenthesis around the subscript on these magnitudes, but in all cases where the δ -items are taken to denote actual surplusses to be transferred to ownership or financial sectors, the investment column sums will be identical with the actual gross investments. The same applies to the column sum \dot{L}_0 . Compare the comments to (6.9). In order to standardize the notation as much as possible, we will in the following make this convention. I.e. we assume

$$J_{(g)} = J_g, \quad J_{(\omega)} = J_\omega, \quad J_{(G)} = J_G, \quad J_{(j)} = J_j, \quad \dot{L}_0 = \dot{L} \quad (7.7)$$

where the symbols without parenthesis around the subscript are the actual real investments in the various directions. In accordance with (7.7) we let the δ -items in the fixed capital investment columns and the inventory column denote actual surplusses to be transferred to ownership or financial sectors.

We assume that the investment directions and the inventory column have no other form of surplus than those of the δ -form. That is, they cannot place their surplus in any of the categories corresponding to (6.5) and (6.6), i.e. to those in the first two horizontal parts of tab. (7.6).

The vertical balancing of the two Rest of the world columns is simple. The sum total of these two columns is equal to the export surplus E (positive, negative or zero) conceived of as that amount for which the Rest of the world is to be debited.

The vertical balancing of the ν columns simply consists in recording the column sums.

Finally, the sum of the column sums in the λ -columns must be equal to the negative of the export surplus, i.e. it must be equal to $-E$. This sum expresses the total (positive, negative or zero) of all financial objects which the domestic sectors have acquired as their property.

Now for the horizontal balancing part.

On each μ row the balancing consists simply in recording the row sums T_μ , the assumption being that the entries in this horizontal part T_μ and those in the vertical part S_ν are made in such a way that (4.3) is fulfilled. For more typographical precision we could write this convention

$$T_\mu = S_\nu \quad \text{when } \mu \text{ corresponds to } \nu \quad (7.8)$$

For instance, since all direct taxes are recorded on row $\mu =$ direct taxes, or in column $\nu =$ direct taxes⁴, the two balancing elements in the column $\nu =$ direct taxes on the row $f =$ state budget and Local Government budgets together will show the total of all direct taxes (and direct subsidies, if any)⁵.

⁴ On the row $\mu =$ direct taxes an actual tax would be recorded as a positive number and that for an actual subsidy would be recorded as a negative number, while in the column $\nu =$ direct taxes we will have the opposite rule.

⁵ From a formal viewpoint the horizontal μ -part and the vertical ν -part could be looked upon as belonging to the "correspondence balancing part" of the table.

Each of the e rows or f rows is considered only as an accounting sector that can receive payments (i.e. render services) only in the form of surplus transfers δ or in the form of categorized transfers S . If material production services are to be taken account of for any e or f sector, it must be done by a δ -transfer or an S -transfer from a production sector or a consumer group. For any e sector the total surplus that is available and must be placed, is consequently

$$\sum_h \delta_{eh} + \delta_{ew} + \sum_G \delta_{eG} + \sum_j \delta_{ej} + \sum_g \delta_{eg}^J + \delta_{ew}^J + \sum_G \delta_{eG}^J + \sum_j \delta_{ej}^J + \delta_{eL} + \delta_{eA} + \delta_{eZ} + \sum_v S_{ev} \tag{7.9}$$

By the definition of the total on row e in tab. (2.1), the sum (7.9) is equal to

$$\sum_\lambda \dot{F}_{e\lambda} + J_{(e)} \tag{7.10}$$

The sum (7.9) expresses the various *sources* of the surplus in sector e and (7.10) represents the two ways in which this surplus can be *placed*. These two ways are completely analogous to (6.5)–(6.6) and the first two horizontal parts of tab. (7.6).

For an e sector there is no concept corresponding to (6.7) or the lower part of tab. (7.6). If we should introduce such a concept for an e sector, it would have to be the *negative* of (7.9), and in this case the surplus of the e sector would by definition be zero. It is more natural to consider (7.9) as a specification of the ways in which the surplus in sector e is created, and (7.10) as a specification of the ways in which it is placed.

An exactly similar reasoning applies to the f sectors.

We can therefore formulate the interpretations given in tab. (7.11).

TABLE (7.11). *Ways of placing the surplus arising on current account operations in the following sectors.*

Compare also (6.5)–(6.7) and tab. (7.6).

	Ownership sector e	Financial sector f
The financial objects which the sector acquires <i>on its own account</i> (i.e. as its property). (Since we want to have the column sum of all the λ columns equal to $-E$, we do not record separately the Rest of the world's acquisition of financial objects. This appears as the various λ column sums).	$\sum_\lambda \dot{F}_{e\lambda}$	$\sum_\lambda \dot{F}_{f\lambda}$
The gross real investment goods (produced the same year) which the sector acquires <i>on its own account</i> (i.e. as its property) including both investment in fixed real capital and in inventories. (Since the J items only pertain to <i>domestic</i> investments, the Rest of the world has no such investments. Export of fixed capital goods is recorded in the A column and export of previously existing fixed capital goods is recorded in the column λ = existing real capital. Compare Section 4.VII.1).	$J_{(e)}$	$J_{(f)}$

We have now given a complete description of the *standard* meaning of the magnitudes entering into tab. (2.1).

Apart from the meaning of the symbols there is also the question of how the figures are to be entered in the various cells of the table. We will not discuss all possibilities, but only mention how the table will look if the $J_{(e)}$ item is distributed over the various investment columns and similarly for $J_{(f)}$. It is sufficient to indicate that part of the table that is changed. This is shown in tab. (7.12).

TABLE (7.12)

	Domestic gross investment in fixed real capital				Net increase in inventories L	Grand total
	g	ω	G	j		
e	δ_{eg}^J $-J_{(eg)}$	$\delta_{e\omega}^J$ $-J_{(e\omega)}$	δ_{eG}^J $-J_{(eG)}$	δ_{ej}^J $-J_{(ej)}$	δ_{eL} $-J_{(eL)}$	0
f	δ_{fg}^J $-J_{(fg)}$	$\delta_{f\omega}^J$ $-J_{(f\omega)}$	δ_{fG}^J $-J_{(fG)}$	δ_{fj}^J $-J_{(fj)}$	δ_{fL} $-J_{(fL)}$	0
Grand total	J_g $-\Sigma_e J_{(eg)}$ $-\Sigma_f J_{(fg)}$	J_ω $-\Sigma_e J_{(e\omega)}$ $-\Sigma_f J_{(f\omega)}$	J_G $-\Sigma_e J_{(eG)}$ $-\Sigma_f J_{(fG)}$	J_j $-\Sigma_e J_{(ej)}$ $-\Sigma_f J_{(fj)}$	\dot{L} $-\Sigma_e J_{(eL)}$ $-\Sigma_f J_{(fL)}$...

In this table $J_{(eg)}$, $J_{(fg)}$ etc. denote how much of the gross investment in the g direction that has been acquired as the property of the e and f sectors respectively. Similarly for $J_{(e\omega)}$, $J_{(f\omega)}$ etc.

The individual row sums e and f in tab. (7.12) will be zero and the sum of all the column sums in the investment directions and in the inventory column will now indicate those parts of the new real investment that was acquired by the production sectors, the accounting sector ω , the sector for Government sales of goods and services and the consumer groups as their own property. In other words we have

$$\begin{aligned} & \Sigma_g [J_g - \Sigma_e J_{(eg)} - \Sigma_f J_{(fg)}] + [J_\omega - \Sigma_e J_{(e\omega)} - \Sigma_f J_{(f\omega)}] + \Sigma_G [J_G - \Sigma_e J_{(eG)} - \Sigma_f J_{(fG)}] + \\ & + \Sigma_j [J_j - \Sigma_e J_{(ej)} - \Sigma_f J_{(fj)}] + [\dot{L} - \Sigma_e J_{(eL)} - \Sigma_f J_{(fL)}] = \Sigma_k J_{(k)} + J_{(\omega)} + J_{(G)} + \Sigma_i J_{(i)} \end{aligned} \quad (7.13)$$

This follows from the fact that the sum of column sums must be equal to the sum of the row sums also when tab. (2.1) is reshaped as indicated in tab. (7.12) (and the term $J_{(i)}$ added in the grand total on row i).

The various investments in Government administration will ordinarily not be included either in the left or in the right member of (7.12). In the left member their positive entries are cancelled by the subtraction in the form of $J_{(fG)}$ with f

= State budget and Local Government budgets, (except perhaps for small amounts included in $J_{(\omega)}$ or $J_{(o)}$).

Several of the investment items in (7.13) may ordinarily be put equal to zero by definition, for instance $J_{(\omega)}$. [If a small amount is included in $J_{(o)}$, this same amount will ordinarily be equal to the difference expressed by the third bracket in the left member of (7.13).

We will now give some examples of ways of recording categorized transfers and transfers of surpluses.

Take for instance a large gift from abroad accruing to the state budget. It may be recorded in anyone of a number of different ways.

(7.14) One way—and perhaps the simplest—is to enter the amount positively on the μ row gifts in the column A , and at the same time positively on the f row state budget and in the ν column gifts. In this way the relation (4.3) is fulfilled and the result will be that the export surplus—i.e. the total amount by which the Rest of the world is to be debited—is increased by the amount in question. And this amount appears also as an increase of the surplus creating items of the state budget (compare the last term of (7.9) when the equation is applied to the f row state budget). How the state balances this increase, depends on its various activities. It may do it in the \bar{F} or J form (whether the recording is made as in tab. (2.1) or in the reshaped (7.10) form), or it may do it by increasing Government consumption—in which case it would decrease δ_{fG} —or it may do it in other ways. In any case the State budget would be balanced.

(7.15) Another way of recording the foreign gift would be to use the Rest of the world row f , if such a row is included in the table. The gift could then be entered twice on the f row Rest of the world, namely positively in the A column and negatively in the ν column gifts. At the same time the amount would be entered positively on f row State budget in the ν column gifts. This would zero balance the f row Rest of the world, and it would also zero balance the ν column gifts, but E and the State budget surplus would be increased. The question of how the increased State surplus is used, would be the same as under (7.14). This recording via the Rest of the world row is more complicated than that by way of (7.14), so transactions of this sort are no argument in favour of introducing a Rest of the world row.

As another example consider the dividends collected by the State on its holdings of stocks in independent establishments that can be classified under the receiving production sectors h .

(7.16) One possibility is to enter the dividend positively under δ_{fh} (with f = State budget). If so, the effect on the horizontal balancing will appear immediately under $-\bar{F}_{f\lambda}$ or $J_{(f)}$, or be used by the State in some other way.

(7.17) Another possibility is to enter the dividend positively under δ_{eh} (with e = State enterprises). The horizontal balancing will then either appear in $-\bar{F}_{e\lambda}$ or

in $J_{(e)}$, or it may be passed on to the State budget by entering the total of the collected dividends negatively under S_{ev} (with e = State enterprises, and v = accounting transfers) and positively under S_{fv} (with f = State budget, and v = accounting transfers). In the latter case the dividends will finish up by effecting the State budget in the same way as under (7.16).

(7.18) A third possibility is to enter the State collected dividends positively under $T_{\mu h}$ (with μ = dividends) and positively under S_{fv} (with f = State budget and v = dividends). This will not disturb the equality (4.3), and will affect the State surplus in the same way as under (7.16)–(7.17).

(7.19) A fourth possibility is to enter the State collected dividends negatively under S_{kv} (with k = the delivering sector k corresponding to the receiving sector h and v = dividends), and at the same time enter the collected amount positively under S_{fv} (with f = State budget and v = dividends).

Second, consider the dividends collected by the State on its holdings of stocks in combined enterprises.

(7.20) One possibility to record these dividends is to enter them negatively under S_{ev} (with e = the various ownership sectors involved, and v = dividends) and at the same time positively under S_{fv} (with f = State budget and v = dividends). The effect on the State budget would be similar to that under (7.18)–(7.19).

(7.21) Another possibility is to record these dividends negatively under S_{ev} as in (7.20) (with e = the combined enterprises involved and v = dividends) and positively under S_{ev} (with e = State enterprises, and v = dividends). The ensuing surplus on the State enterprises may be handled as under (7.17).

Third, consider in a more general way the recording of any surplus of the establishments in a given receiving production sector h . The surplus is *defined* as the left member of (6.4). Some remarks on the placing of this surplus were made after (6.7), we will now add some further possibilities, also including the sectors mentioned in tab. (7.6).

(7.22) The logically most straightforward way to record the surplus of sector h is to distribute all of it over the δ_{eh} and δ_{fh} cells in the column h (one of the values of e corresponding to single-sector enterprises). In this case the sum of the δ_{eh} and δ_{fh} items becomes equal to the left member of (6.4), and hence

$$J_{(k)} = -\sum_{\lambda} F_{k\lambda} \quad k \text{ corresponding to } h \quad (7.22a)$$

The only gross real investment which the delivering production sector k can acquire on its own account, will in this case be what it purchases through depletion of its holdings of financial objects. Compare also the discussion after (6.7).

(7.23) In the case (7.22) it might be natural to record on the e and f rows not only the total surpluses realized in the h columns, but also those realized in the sectors mentioned in tab. (7.6). For the rows ω , 0 and i we would then get equations similar to (7.22a).

As to the surplusses in the investment directions we must remember that the items δ_{eg}^J , $\delta_{e\omega}^J$, δ_{eG}^J , δ_{ej}^J and δ_{eL} cannot be deduced from other individual entries in the table, but will have to be determined by some additional information or conventions and then entered directly on the e and f rows. That is to say, for the investment directions we assume no other form of surplus than the δ -kind. Compare the paragraph after (7.7).

In any case the horizontal balancing of the e and f rows can be made by entering the items $-\dot{F}_{e\lambda}$ and $J_{(e)}$, respectively $-\dot{F}_{f\lambda}$ and $J_{(f)}$, as in tab. (2.1), or by splitting the totals $J_{(e)}$ and $J_{(f)}$ in the way indicated in tab. (7.12).

(7.24) If we follow (7.23) except for the fact that the total surplusses of the *independent* establishments are not recorded as δ -elements, compare (6.4), or, to be more specific, if we put the sum $(\sum_e \delta_{eh} + \sum_f \delta_{fh})$ for the independent establishments in sector h equal to zero, the independent establishments in sector k (k corresponding to h) would increase the sum of their holdings of financial objects and their real investments by an amount equal to their total surplus.

Now let the symbol $J'_{(k)}$ indicate the real investments retained as the property of the independent establishments in sector k and the symbol $J''_{(k)}$ indicate the real investments that are acquired as the property of the other establishments in sector k so that $J_{(k)} = J'_{(k)} + J''_{(k)}$.

The investments $J''_{(k)}$ can only be brought about through a decline in the financial holdings of the sectors in question, compare (7.22a). That is, we may write

$$J''_{(k)} = -\sum_{\lambda} \dot{F}_{k\lambda}'' \quad (7.24a)$$

And similarly for the whole of $J_{(\omega)}$, $J_{(o)}$ and $J_{(i)}$. For all these sectors we will now use the right hand expression corresponding to (7.24a). For independent establishments, on the contrary, we retain the expression $J'_{(k)}$.

If further all $J_{(e)}$ and $J_{(f)}$ items are distributed as in tab. (7.12), the sum of all the column sums in the investment directions g , ω , G , j , L would be equal to

$$\sum_k J'_{(k)} - \sum_{\lambda} [\sum_k \dot{F}_{k\lambda}'' + \dot{F}_{\omega\lambda} + \dot{F}_{o\lambda} + \sum_i F_{i\lambda}] \quad (7.24b)$$

For instance, if by convention the sectors indicated in the bracket of (7.24b) are not allowed to have any change in their financial holdings, the sum of all the column sums in the investment directions mentioned would be equal to the first term in (7.24b).

The above reasoning will, of course, apply to *any* splitting of the production sectors in two groups, one of which has its total surplus transferred to the e and f rows, and another for which the sum $(\sum_e \delta_{eh} + \sum_f \delta_{fh})$ is zero.

(7.25) If we want to eliminate the $J_{(k)}$, $J_{(\omega)}$, $J_{(o)}$ and $J_{(i)}$ items altogether, we can, for instance, increase δ_{eh} —and/or δ_{fh} —(with h corresponding to k and with e = the various ownership sectors, including the single-sector enterprise ones and f = the various financial sectors) sufficiently to make up for the amount previously defined as $J_{(k)}$. And similarly for $J_{(\omega)}$, $J_{(o)}$ and $J_{(i)}$ (with households

as one of the e -rows). The ensuing items $J_{(e)}$ and $J_{(f)}$ could, if we want to, be distributed over the investment columns as in tab. (7.12). If so, the sum in each of the investment columns g , ω , G , j and L would be zero (if we assume that none of these column sums can be negative).

(7.26) If we proceed as in (7.25) except for the *independent* establishments and if we further use the (7.12) principle, the sum of the column sums in the investment directions would be equal to $\sum_k J'_{(k)}$. This is a more general reasoning than the one in the comment immediately after (7.24b).