OPTIMAL IMPLEMENTATION

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1. *Models versus reality*

If we dig down into microcosmos, we find that « reality » is something so tremendous in complexity and in number of aspects that a *complete* conception of it is impossible. Even the most resourceful team of research workers and the most fantastic electronic computers ever to be built cannot incorporate explicitly — together with other details — say, all the alternative combinations of the *dates* over the foreseeable future when Mr. N. N. will put a new blade into his razor; and the cross-classification of this with the dates when Mr. P. P. will do so and when Mr. Q. Q. and all the others will do so.

Therefore, we must remember that *all science* and *all scientific thinking* must proceed by way of abstractions and « models » of some kind or another. This is the only way. If we don't want to go it, nothing is left but to stop thinking.

Some models may, of course, be « more realistic » than others, as judged by the « practical » results we can get from them. But the measurement of the « degree of realism » and the « degree of practicalness » raises *new problems*, which in turn can only be discussed in terms of other models, perhaps stochastic ones.

This is a warning to all who believe they are doing something very respectable when they insist that they want to study « reality » and avoid « models ».

2. *Selection versus implementation.*

In economic planning, particularly in economic planning at the na-

tional or supranational level, it is important to distinguish between selection and implementation. Let me quote a passage from my paper, « Preface to the Oslo Channel Model » (pp. 266-267 in « Europe's Future in Figures », ed. R. C. Geary, Amsterdam, 1962).

> A selection model is primarily useful for the purpose of describing a constellation of the *volume figures* or the figures *in actual technical units* which has been realized or might be realized or one would like to see realized, provided one can find ways and means (institutional, administrative and financial) of bringing this constellation about.
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> In theory it would, of course, be possible to include also all these ways and means explicitly in the same programming analysis, but such a set-up would only be a formalistic one without much chance of leading to practical results at the present stage. It is more practical to separate the selection problem and the implementation problem. It is primarily in the selection problem that the biggest advantages of a precise quantitative analysis can be gained. In the implementation problem we must rely to a much larger extent on economic intuition and practical sense.
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> Another reason why it is a practical approach to separate the selection and the implementation problem is that the selection problem can be studied without stating a priori the kind of *economic institutions* (competitive markets or central controls etc.) one is prepared to accept.
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> After the selection problem has been solved, one will take up the implementation problem. If on scrutiny one should find that practical difficulties of implementation under an existing institutional, administrative and financial set-up make it impossible to reach the high goals — for instance a rapid rise in national product — which have emerged as feasible from the selection viewpoint, two ways are open: either to try to change the institutional, administrative and financial structure so as to make the high goals attainable, or to insist that this structure is not to be changed and that one will therefore have to acquiesce to the lower goals. In the latter case the computation of the difference between the two results will furnish a sound piece of information.

3. *A selection model is intended to have Rock Bottom (R.B.) character.*

A selection model contains two things: (1) The *unavoidable* relations and bounds which are technologically or behavioristically so *basic* that we have to accept them whether we like it or not. This constitutes the *structural* aspect of the selection model. (2) The *preferences* of the re-

sponsible political authority (whether it be a dictator, a junta of powerful men, or a democratic parliament).

The following are some excerpts from a paper which I published in the Proceedings of the International Operations Research Conference held in Oslo in the spring of 1963 :

A. *The Structure.*

Certain things that come into the picture when we discuss the selection problem are of such a sort that their elucidation depends solely on the work of *scientific experts*. These experts may be of very different sorts. Physical scientists, technological experts, medical experts, statisticians, economists and many others. The question that occupy these experts concern objective facts. They are of such a sort that any statement regarding these questions can in a scientific way be qualified as either « correct » or « not correct ». How much raw material is needed *if* we are to produce such and such a quantity of a given commodity? How many hospital beds and X-ray outfits and doctors and nurses etc. will be needed *if* we are to treat so many patients of a specific category? What is the traffic we can expect *if* we build a new railroad between such and such two points? How much of a given consumer good will we need *if* we are to statisfy the demand and *if* the prices of all the consumer goods (including that of this particular good) are such and such and *if* the purchasing power in the hands of the consumers is so large? And so on.

True enough there are cases when the scientific experts may not quite agree. Physicists may for instance disagree on the correct formulation of a physical or chemical property of a specific gas, technological experts may disagree on the breaking strength of some material, medical experts may disagree on the therapeutic value of some specific treatment. And so on.

But in all these cases the differences of opinion are of a purely scientific and objective character. As science progresses such differences will one by one disappear so that all competent experts in the field will reach a generally accepted view on what is *the « correct »* answer.

At the same time *new* differences may arise because scientific analysis is extended to *new* fields. But in time these new differences will disappear. The whole picture is essentially a dynamic *process* towards something which can be looked upon as a « correct » and objective characterization of the outer world. This characterization is something which man has to accept *whether he likes it or not*. He may

wish that the gravitational force be less than it is, but he can't do anything about it.

All these things which man can't do anything about is what I mean by the *structure* of the problem one has chosen to consider at a given point of time.

In particular I will speak of the structure of the national (or supranational) economy as the entity of all those things in the economy which man, whether he likes it or not, has to accept as *data* in his attempt at steering the economy. When describing the structure of the economy one must rid one's mind completely of the concepts « desirable » or « not desirable ». The question of whether we *want* to start a certain production must be eliminated when we study the inputs that *will have* to be made in this production *if we* decide to start it. Similarly the question of whether we *want* to build hospitals rather than to let the patients die must be eliminated when we study the structure.

B. *The Preferences.*

In sharp distinction to the structure of the economy are *the preferences* of the authority which is responsible for steering the economy. In this connection I am not concerned with the question which person or persons who constitute this responsible authority. It may be a single man (or woman), a small group of persons, an elected parliament or even the population as a whole, expressing its wishes through a plebiscite. What is essential to retain in this connection is that now we are *not* speaking of the structure of the economy — those things which man cannot change —, but we are speaking of precisely the opposite, namely of what man — represented by the responsible authority — would *like* to see realized, disregarding provisionally the question of whether it is structurally feasible or not.

When speaking of the structure of the economy we had to rid our minds completely of the concepts « desirable » or « not desirable ». Vice versa we now have to rid our minds completely of any considerations of what is structurally feasible, and concentrate only on what is desirable or not desirable if it could be obtained.

In the practical problems of everyday life within the smaller or larger field of vision to which we are accustomed, we are forced to reckon with the structural aspects of things. This is due to the fact that in our everyday life we are, each one of us, nearly every minute facing the need for *deciding* on some action, small or great. And in making these small or great decisions of everyday life we have, of

course, to take account of the structural elements of the decision. Sometimes it is done instinctively or subconsciously and sometimes by reasoning, but it is always there. It has become our second nature. Therefore, it takes quite a bit of mental energy to rid our minds of the structural possibilities when we now speak of the preferences at the national (or supranational) level. It looks as if it is very « unrealistic » to disregard these possibilities.

But when it comes to a *systematic* analysis of the complex alternatives of choice that are open to us in economic planning, the *truly realistic* approach is to segregate the preferences completely as a special point of view distinct from that of the structural elements of the problem. The national preference has to be formulated in the spirit « Would you like this thing or this other thing *if* you had the choice? ».

This is the basic idea of the construction of the *free* preference function (the « Santa Claus » function).

To retain at all times a clear cut distinction between the *structural* view point and the view point of *preferences* is a conditio sine qua non for clear thinking and for a rational classification of the problems in economic planning.

* * *

The most important of the *parameters of action* of central government should be brought out explicitly already in the selection model Investment startings in an underdeveloped country are examples in point. Likewise, politically imposed lower bounds on consumption, as well as a politically imposed upper bound on the peak load of foreign indebtedness, should appear explicitly. If these and others of the *most important* parameters of action are not introduced explicitly, the selectional model would not be of great interest from the *operational* viewpoint. The bounds mentioned are part of the expression for the preferences of the responsible political authority.

Relations derived by considering demand elasticities with respect to income or price do *not* belong in a Rock Bottom selection model, because these relations are institutionally contaminated to a high degree. Cf. section 4.

The actual decisions on what to do *in detail* in order to *steer* the economy in such a way that its dynamic constellation over time comes as close as possible to the constellation that has appeared as the selectionally optimal one, are not to be derived from the selection model itself.

Decisions on this will only appear after a *further* use of the selection model in the subsequent study of the whole implementation problem, as discussed in sections 4 - 7.

Thus, the selection model itself is essentially a device for studying the *number of degrees of freedom* — degrees of maneuverability — which we get when we only take account of the *basic* technological and behavioristic relations — those we have to accept whether we like it or not — and a device for studying how these degrees of maneuverability might be combined so that the whole constellation becomes *optimal* in relation to the top authority's purpose.

The existence of a politically determined preference function and politically determined bounds will *not* affect the number of degrees of freedom in the model.

A selection model may be something *very big* or something *very small* so far as the number of variables and number of degrees of freedom are concerned.

4. *The institutionally contaminated model.*

If we assume a given *institutional setup,* for instance a free-market economy (this is one of the things which we are *not* obliged to accept, whether we like it or not), a host of *additional relations* and *additional variables* — for instance, market prices — can be introduced in our model, provided we can get the necessary data.

But, even if we have reliable information, these relations are not of the R. B. character which the relations in the selection model possess. The coefficients of such an institutionally contaminated model will change as institutions change.

If we adopt a rather free attitude to the question of the economic institutions that may best serve our purpose in a developing country, we will, therefore, not derive much help from any specific institutionally contaminated model.

We may be more or less drastic in our inclusion of institutionally contaminated relations. The farther we go in this direction the more *doubtful and uncertain* will be the relations we include. For instance, if we assume that the economic institutional setup is such that purchasing power to consumers is in the main distributed through the activity of the consumers as primary input delivering factors, and if we assume that consumers are reasonably free to spend their income on various types of consumer goods, or to save it, we can with a fair degree of accuracy derive

demand elasticities with respect to income and with respect to price. Such data are among the best of the institutionally contaminated data at our disposal.

But if we go further, and want to derive similar elasticities for the propensities of financial investors to acquire assets of higher or lower degrees of liquidity, higher or lower risk, higher or lower chances of making « a big kill » in the stock market, then we are in a much more uncertain field and a model built on these elasticities will be of doubtful use. Such elasticities will not only be difficult to obtain, but will also be *extremely sensitive to institutional changes,* and even to changes that are, perhaps, more likely to occur than changes in the basic system which consists in determining the purchasing power of the consumer through his taking part in the production process.

Incidentally, even this latter aspect of the current institutional system has recently been challenged. A group of people, among whom we find such world-famous names as Professor Gunnar Myrdal of Stockholm, and Linus Pauling, the physicist, has made a declaration that says, among other things :

> The continuance of the income-through-jobs link as the only major mechanism for distributing effective demand — for granting the right to consume — now acts as the main brake on the almost unlimited capacity of a cybernated productive system.

The declaration asks for

> « an unqualified commitment » by society to provide, through its appropriate legal and governmental institutions, « every individual and every family with an adequate income as a matter of right ». It declared :
>
> « This undertaking we consider to be essential to the emerging economic, social and political order in this country ».
>
> « We regard it as the only policy by which the quarter of the nation now dispossessed and soon-to-be dispossessed by lack of employment can be brought within the abundant society ». (*The New York Times*, 23 March 1964).

An institutionally contaminated model may be big or small, just as an R. B. selection model may be big or small.

5. *The compartmentalization of decisions.*

If we look further into the actual system through which economic

planning has to be implemented, we find a number of specific *organizational groups*: ministries, industrial and other sectoral organizations, local governments, etc., that all have their own perferences and also, to a varying degree, have power to *make their own decisions*. Frequently, these groups are in competition with each other. At least we have no guarantee that all of them are able to, or desirous to, adopt that particular action which would be most effective in realizing the economic planning goals formulated by the central authority.

This constitutes the essence of the implementation problem in national and supranational planning.

If we had precise and complete information of how all these groups will behave under a given set of centrally imposed rules and conditions, and if we were mathematically powerful enough to work out the precise way in which this behavior under a given set of rules and conditions *determines* the whole constellation of the economy, including the variables contained in the R. B. selection model, and if we had a sufficiently large electronic computer (it would have to be enormous), — then the determination of the optimal set of rules and conditions in the administrative system would only be a matter of mathematical programming (of a strongly non-linear sort and with a very large number of variables).

In the present state of affairs, such an approach would seem utopian. Even the *mathematical* difficulties of including in a central planning function an explicit expression for the way in which the results of numerous suboptimalizations depend on the centrally fixed rules and conditions for these suboptimalizations would be extremely great, to say the least.

A further difficulty arises because we are not certain what the behavioral patterns of the various subdecisional organs actually are or might be. We would, therefore, want to experiment on this by putting live persons in simulated decision-making situations.

On this point administration games may be of considerable help.

But I would want to add that the logical principles of and the actual organization of, the games need to be worked out in considerable detail. I would also like to see the *aspiration level* of such games raised to a point where we might hope to get some information on what will most likely be an *optimal choice* of the implementation system. Otherwise, there is a danger that we will be wandering around from one series of games to another, without really being able to make full practical use of the results.

In the previously quoted paper in « Europe's Future in Figures », I suggested such multidecisional games in the following words (p. 278) :

> By the pyramidation problem I mean the general problem of the extent to which it is possible to decentralize the decisions, i. e. the problem of the optimum number of levels on which the decision making machinery is to be organized. This pyramidation viewpoint may be applied to any of the aspects that are brought out explicitly (types of commodities, types of investments, geographical regions etc.)...
>
> At the present stage and for some time to come it will, I think, be impossible to include the pyramidation problem explicitly in the model — which, of course, would have been the ideal solution —. We have at present to approach the problem by some kind of *simulation* technique. For instance in such a way that a number of competent (mixed economic and mathematical) groups are organized, each group representing a specific decision making unit in the general game. Each group would have to be allocated sufficient machine time on a good sized computer. An over-all central group would formulate rules and criteria which each special group would have to abide by. Within the confines of these specified rules and criteria each group would act in a perfectly selfish way. The global constellation which emerged from such a game would be studied in its desirable and undesirable features, and a new attempt would be made at reformulating the rules and criteria for the special game-groups in such a way that the resulting *global* constellation of the economy could conform better to that constellation which has emerged as the optimal one from the selection viewpoint in the economy as a whole. The pyramidation problem would then appear only as one aspect of the implementation problem.

In the sequel I shall try to be a little more explicit regarding this suggestion.

6. *The multidecisional administrative model.*

The rules and regulations of the gaming must be built on a clearcut and precise model which can define not only what the game *is going to be* (much in the same way as we design a flow diagram for an algorithm on an electronic computer), but also what the *purpose* of the gaming is, and the way in which it *ties in* with our R. B. selection model. The multidecisional administration model will contain much more detail than a

selection model or an institutionally contaminated model. In particular, the multidecisional element is an important new aspect.

There are three principles which I think should be followed when we construct an administration gaming model.

First principle : Specification of the administrative structure.

There are so many different administrative structures conceivable that I believe it would serve no useful purpose to try to classify them by some quantitative attributes. Each structure will have to be described as an individual. We should make a *list* of possible administrative structures to be considered, much as we make a *list* of investment projects to be considered for decision, or as we make *list* of patterns of geographical centers in regional analysis. In making out such a list, we should let the imagination « run wild », with the understanding that the inclusion of any item in the list does not imply any obligation to accept it. It is only a way to circumscribe the universe within which we will be reasoning.

A specific administrative structure will be denoted by the letter S.

Second principle : Specification of the ruling parameters.

Any given structure S will have a number of *ruling parameters* R^S_1, R^S_2, R^S_3 ... attached to it. These are numbers fixed by the central authority and which the subdecisional groups have to abide by. In a free-market economy, they may be tax rates, interest rates, maximum working hours, or the like. In a centrally planned economy, they may, for instance, be percentages expressing how much the directors of production establishments may retain and use at their own discretion for improving the capital stock or the working conditions within their establishments.

To any given set of ruling parameters, the various decisional groups will adopt a specific behavior, and this will in turn determine *the final outcome* of the game. We can therefore conceive of this outcome (all the values of the various variables in the administration model) as *functions* of R^S_1, R^S_2, R^S_3 ... The game itself can simply be looked upon as a way of calculating what the outcome will be when the values of the ruling parameters are given.

The ruling parameters associated with one specific administrative structure may be entirely different from those associated with another structure.

Third principle : Interpretability in terms of the selection model.

The administration model must be built in such a way that from the outcome of a certain game run we are able to compute what each and all of the variables in the selection model will be. The administration model will in general contain many more things than these variables, but by a suitable aggregation it must be possible to pass from the administration model to the selection model.

In particular, if the selection model contains a number of regions with specification of at least some key variables in each region (for instance, income generated there and the weight of this income in the overall preference function), it must be possible to compute the value of these key variables when the outcome of a game run is known.

The value of the selection model preference function computed from the outcome of a game can never be greater than the selectionally optimal value of the preference function computed in the selection model itself. If a higher value should emerge from our computations, this would either show that we have not been successful in establishing the selection model as a real R. B. model, or that there is something wrong with the administration model or its interpretability in terms of the selection model.

If we compare the outcome of a game run with the value of the preference function in an *institutionally contaminated* model that has been worked out under the assumption of an institutional structure that is different from the one underlying the game run, we may, of course, in principle find a game which is such that it « improves » upon the optimal solution of the non-game model. (Such a result may, if we like, be interpreted as an « improvement » of some of the coefficients of the non-game model). Different models may, of course, have different values of the preference function.

Such comparisons are, in my opinion, of little interest. The important conclusion to be derived from a game is to see whether a *feasible* solution of the selection model can be administratively produced and, if it can, to see how big is the *gap* between the selectionally optimal and the administratively attainable value of the selection preference function.

7. *Feasibility and optimality of a series of game runs under a given administrative structure.*

When an administrative structure S is given, two important questions arise : (1) Will it be possible to indicate a set of values of the ruling

parameters R^S_1, R^S_2, R^S_3 ... such that their values produce a game outcome which is *feasible* from the viewpoint of the selection model (where politically determined bounds were also present)? and (2) How close up to the selectionally optimal value of the preference function will it be possible to reach by a manipulation of the ruling parameters R^S_1, R^S_2, R^S_3...?

Both questions can *in principle* be answered by applying some of the subroutines contained in my nonplex method, for non connex programming. But it should be clearly understood that these computations in the case of a complicated subdecisional setup may be very heavy.

In the first place we note that we have no *analytical expression* for the way in which the outcome of a game run depends on the ruling parameter. Therefore, we can only use those parts of the nonplex algorithm which are applicable, even in the case where the value of the search function in any given point is only available in the form of an empirically given number. Fortunately, there are important parts of the nonplex algorithm that have this character.

In the second place, the computation of the value of the administratively defined search function corresponding to a given set of values R^S_1, R^S_2, R^S_3 ... is based on a game run and therefore must take considerably more time than the computation of the value of the search function in a regular application of the nonplex method when the variables involved in the search function are analytic functions of the basis variables.

With these reservations, let us see how the nonplex method may be applied to the problem at hand. For simplicity we drop the superscript S on R_1, R_2, R_3 ... These variables are now our basis variables. All the variables x_i ($i = 1, 2 \ldots m+n$) of the selection model are to be considered as functions of R_1, R_2, R_3 ..., the latter being the basis variables now. (It will greatly simplify matters if the investment startings of the selection model can be thought of as determined directly from the selection model, so that we need not incorporate them in the list R_1, R_2, R_3 ...).

We first use the method of partial moves in rotation. From a point R^0_k ($k = 1, 2, 3 \ldots$) we decide to change one specific of these variables, R_α by some conventional « quantum » δ_α, while leaving the rest of the basis variables unchanged. This gives values $x\delta_i$ of all the selection variables, and hence of the search function $M\delta_\alpha$. We also move from R^0_k to *another point* by changing R_α by the conventional « quantum » Δ_α ($\neq \delta_\alpha$). Now we have three points on a straight line with search function values M^0_α, $M\delta_\alpha$, and $M\Delta_\alpha$. Fitting a second degree parabola to these points we can employ an ω-optimal consideration similar to that on which the

nonplex method is based. In the case where it turns out that we ought to go to the maximum point on the fitted curve this gives:

$$(7.1) \qquad \Omega^{opt} = \frac{1}{2} \frac{M^0{}_\alpha (\Delta^2{}_\alpha - \delta^2{}_\alpha) - \Delta^2{}_\alpha M\delta_\alpha + \delta^2{}_\alpha M\Delta_\alpha}{M^0{}_\alpha (\Delta_\alpha - \delta_\alpha) - \Delta_\alpha M\delta_\alpha + \delta^2{}_\alpha M\Delta_\alpha}$$

which is to be inserted into

$$(7.2) \qquad R_\alpha = R^0{}_\alpha + \Omega_\alpha \qquad \text{with } R_k = R^0{}_k \text{ when } k \neq \alpha$$

We decide to move to that one of the points considered which gives the *highest* value of M, and from this point we make a partial move in the direction of the *next* variable, say, number ß, in the rotation order of the R_k.

We maintain a logbook and make a straight extrapolation move as in the nonplex method. Also we handle cycling in the standard nonplex way.

In this way, the administrative structures listed may be analyzed one by one.

8. *Types of selection models to be used as a basis for interpretation of administration games.*

As I have stated in previous sections, I believe that administration games may be highly useful and of great scientific relevance, *provided* they are associated with a selection model, in terms of which the outcome of the games can be *interpreted*, cf. the third principle of section 6. This selection model must be of the « rock bottom » sort, i. e., we must try to avoid making it « institutionally contaminated ». This means, inter alia, that the relations in it must be built as much as possible on volume concepts, or still better on genuinely physical data, i. e., on *engineering data*.

There can, of course, be no question of going through in detail here all possible variants of selection models that have R.B. character and therefore may be taken as a basis for interpretation of administration games. But it may be useful to indicate two broad types: (1) models of the *sector type*, and (2) models of the *process and goods type*. Both are selection models that aim at building the analysis as much as possible on *physical* concepts. But the process and goods models go farther in this direction than the sector models.

Since the models of the process and goods type go deeper down into the technical structure at the regional, national or supranational level,

they offer an opportunity of studying other, and is a sense *deeper,* aspects of national planning (product mix, joint products, technical processes in the production, industrial complexes) than the sector type models can be concerned with. But it must be clearly understood that there is a price to be paid for the introduction of these detailed technological considerations. The process-goods models become *larger,* and demand *even greater care* in working out the theoretical relations than the sector models do. It takes much more *scientific effort* to build a global process-goods model.

I think that *more,* and *more exact,* data exist for a technologically refined process-goods model than for a sector model. So far as the *existence* of data is concerned, the process-goods model is in a *better* position than the sector model. One reason for this is that it is so much easier concretely to define a « process » than to define a « sector ». But to *collect* these technical data and *systematize them* in a global decisional model is an enormous task.

For these reasons one may have to acquiesce with a model of the sector type extended however by including investment channels, as indicated in tab. 1.

Even though this model can properly be termed a « sector » model — or perhaps more appropriately a « sector and channel » model, it represents something way beyond the classical input-output table. An input-output table is incorporated in tab. 1, but it forms only a small corner of the table.

A process-goods model, cf. tab. 2, will in some respects resemble a sector model, but there are considerable differences. The sector concept is now dropped and replaced by the *process concept.* This process concept is much more technical in nature than the sector concept, and, from a formal viewpoint, there *is no process-to-process table* in the decision model itself. (A process-to-process table may, if wanted, be put together after the decision model has been solved). The flows are now much more complicated than in the sector analysis. They cannot be studied by a process-to-process concept, but can only be defined through the intervention of the concept of individual *goods.* Inputs to processes are now in the form of a goods spectrum. And outputs from processes are in the form of a goods spectrum. We are therefore able to take full account of the product mix. Within the limits set by the technology, the product mix may be varied. *This variation of the product mix constitutes a new and important dimension* in the decision process.

The goods-balances (the « material balances » in the terminology of

INTERFLOW TABLE FOR THE YEAR t
Millions of . . . at . . . prices

TABLE 1

		Inputs into receiving production sectors	Utilisation of Goods and Services outside of the Production Sectors						TOTAL (domestic production, etc.)
			Private consumption on current account	Government consumption on current account	Non-decisional sinking	DECISIONAL SINKING		Net increase in Stocks	Net Exports	
						In capacity channels	In infra channels			
			Consumer groups	Categories of gov't. use of goods & services		g 1,2...	f ... 1001, 1002...			
		$k = 1, 2...$	$j = 1, 2, 3$	$\gamma = 0, 1, 2...7$		$\sigma = 1,2...$	$\sigma = 1,2...$			
Delivering production sectors	$b = 1$ 2 . . .	X^t_{bk}	C^t_{bj}	$G^t_{b\gamma}$	$J^t_{b)}$	$J'^{t-\sigma}_{bg} H^\sigma_g$	$J'^{t-\sigma}_{bf} H^\sigma_f$	K^t_b	A^t_b	X^t_b
Primary input groups ("Labor")	$i = 1$ 2 3	L^t_{ik}	L^t_{ij}	$G^t_{i\gamma}$	$J^t_{i)}$	$J'^{t-\sigma}_{ig} H^\sigma_g$	$J'^{t-\sigma}_{if} H^\sigma_f$	K^t_i	A^t_i	L^t_i
Complementary imports	B	B^t_k	B^t_j	B^t_γ	$J^t_{B)}$	$J'^{t-\sigma}_{Bg} H^\sigma_g$	$J'^{t-\sigma}_{Bf} H^\sigma_f$	K^t_B	$-B^t$	0
Surplus ("Savings & Taxes")	S	S^t_k	S^t_j	S^t_0 (*otherwise* $S^t_\gamma = 0$)	0	0	0	0	0	S^t
TOTAL		X^t_k	L^t_j	G^t_γ	J^t			K^t	$E^t - E^{t-1}$	

INTERFLOW TABLE FOR THE YEAR t

Technical units. Horizontal summation possible.

TABLE 2

		Receiving process	Private consumption	Government consumption	Non-decisional sinking	DECISIONAL SINKING		Net increase in stocks	Net exports	Total Domestic Production
						In capacity channels	In infra channels			
		q = 1, 2 ... M	j = 1, 2 ...	γ = 0, 1 ...		g = 1, 2 ... T	f = 1001, 1002 ...			
						σ = 1, 2 ... T	σ = 1, 2 ... T			
Input of goods and services	v = 1 2 . . . N	X^t_{vq}	X^t_{vj}	$X^t_{v\gamma}$	$J^t_{v)}$	$J^{t\ \sigma}_{vg}\ H^\sigma_g$	$J^{t\ \sigma}_{vf}\ H^\sigma_f$	K^t_v	A^t_v	X^t_v
Input of labor	i = 1 2 3	L^t_{iq}	L^t_{ij}	$G^t_{i\ \gamma}$	$J^t_{i)}$	$J^{t-\sigma}_{ig}\ H^\sigma_g$	$J^{t-\sigma}_{if}\ H^\sigma_f$	K^t_i	A^t_i	L^t_i
Complementary imports	B	B^t_q	B^t_j	B^t_λ	$J^t_{B)}$	$J^{t-\sigma}_{Bg}\ H^\sigma_g$	$J^{t-\sigma}_{Bf}\ H^\sigma_f$	K^t_B	$-B^t$	0
Output of goods and services	v = 1 2 . . . N	*Delivering process* p = 1, 2 ... M X^t_{pv}			J^t			K^t	$E^t - E^{t-1}$	
Total Domestic Production		X^t_v								

the Eastern economists) will follow automatically in this setup because there will be one « goods-to-process » table and one « process-to-goods » table.

We use the notation

p = delivering process
q = receiving process
v = good

Nothing much is to be gained by introducing the « good-to-good » aspect explicitly.

In the process-goods type of model, the sector affixes no longer takes up the « best place » for affixes, namely, down and to the right. This place is now free. We take advantage of this and will use this space for the subscripts

vq = from goods to process (inputs)
pv = from process to goods (product mix output)

The letter t will, as before, be used as indication for time, say, year.

The tab. 2 type of model can be thought of as being worked out for a country as a whole, or as being incorporated into several or all of the regional parts of one all-inclusive national table.