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Rational price fixing in a socialistic society[1]

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CONTENTS

PART 1

1. THE SETTING OF THE PROBLEM

One characteristic feature of a socialistic economy is that prices are fixed officially by a central authority or by one of the subordinate organs. This price fixation may be more or less rigorous, allowing perhaps for some deviations in special cases, and it may extend to a smaller or larger

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[1] This paper is based on an address given on 18 March 1966 in the Economic Institute of the Academy of Sciences of the Soviet Union. The meeting was chaired by Professor D. A. Allachverdjan, Deputy Director of the Institute, and by Professor Hachaturov. A very active participant in the ensuing discussion was Professor Istislavsky. My thanks to Professor Allachverdjan are also due because of an elaborate personal memorandum which he wrote me in the fall of 1963 on the rules for price fixation in the Soviet Union.

list of goods, leaving perhaps the prices of some minor agriculture products to the free play of the market forces. But the *main* principle is that prices are not only controlled, but actually *fixed* officially.

When prices are fixed, it is inevitable that here and there in the economy a *pressure* may arise against the barrier which is constituted by the fixed prices. A *tendency* towards the creation of grey markets or black markets may thus arise. To some extent such pressures are inevitable in an economic organization of society that aims at *steering* the economic ship instead of letting the ship drift whither the wind blows. An entirely pressure free price system can only be realized by falling back into a completely free market economy with all the evils, which we know only too well. See Section 4 below.

An essential aspect of the problem of price fixing in a socialistic economy is therefore not to find a price system that is *entirely pressure free*, but to find a system where the ensuing pressures can be expected to remain within bounds that are deemed *tolerable*. This aspect of the problem can be worked into an optimal programming analysis by means of certain parameters, which may be termed pressure coefficients, see [1].

The problem of *how* to fix a system of prices in a socialistic economy – and indeed in any economy where prices are *not* left completely free – is both important and difficult. In the Soviet Union for instance, there has over the last decades probably been no single problem in economics about which one has talked so much in theory and done so little in practice to solve efficiently.

The present paper is a modest attempt at contributing to ground-clearing in this field.

The question of how prices "ought to" be fixed, has no meaning unless one specifies explicitely what *purpose* one has in mind by the fixing of the prices: Facilitating the keeping of accounts amongst individuals or organizations? Controlling and checking quantitative planning directives? Steering the use of rational resources? Etc.

In the earlier phases of the socialistic regime in the Soviet Union and other Eastern European countries the main purpose of fixing a price system was – implicitely and explicitely – taken to be the facilitation of account-keeping and control under an economic steering system which was in all essentials based on detailed *quantity fixation*. Under such a steering system there is not much room for considering the price system as an important and directly active vehicle for steering the economic activity. Even as late as in the early 1960-ies an English translation (printed in the Soviet Union) of a 1959 book, designated as a "Popular

Course", [2] p. 275 says explicitely: "The law of value is not the regulator of production and of the distribution of the means of production and of labour among the branches of the national economy. This is all done by the state planning organs...". In this context the law of value means the labour theory of value which states that in a capitalistic society the exchange of commodities is effected in accordance with the amount of socially necessary labour expended on their production. This law was originally formulated forcefully by Adam Smith (1733–1790) and subsequently used by Karl Marx (1818–1883) in his penetrating analysis of the capitalistic system. Cf. Section 2 below.

Even as late as at the 22nd party congress of the communist party – held in 1961 – it is this law of value (which, according to the quotation above, is *not* the regulator of production and distribution in a socialistic economy) that was accepted as the *foundation* for price fixation. The resolution adopted at this congress says indeed:[1] "The prices ought to reflect the socially necessary labour's consumption and assure the compensation of the costs of production and distribution, and also assure some gain in each normally functioning enterprise". Only small *deviations* from the main principle may have had some steering effect.

Subsequently the picture has, as we know, changed considerably. The system of direct quantity planning is to a large extent to be replaced by an indirect system based on *incentives* of various sorts. The realization of this indirect system has not yet got very far in practice in the Soviet Union – apart from its introduction in a number of experimental enterprises – but the intention is clear and has been officially adopted in a unanimous resolution passed on 27 September 1965 in the Central Committee of the Socialist Party.

This resolution was heavily influenced by a penetrating criticism of the old system by a number of leading Soviet economists. A prominent figure in this constructive criticism was Soviet Economists' grand old man V. S. Nemchinov (1894–1964).[2]

In this new situation it was only natural that the *price system* as an instrument for steering the economy – and in particular for steering the use of scarce resources – should come into the foreground. Professor Leif Johansen in [3] p. 84 and pp. 92–96 has given an excellent survey of this discussion.

The most radical reforms have been advocated by L. V. Kantorovich (the originator of the method of linear programming which was later, and

[1] Quoted here from Professor Allachverdjan's memorandum to me.

[2] Obituary by Paul Medow in *Economics of Planning*, No. 1–2, 1965.

independently, developed by the American George B. Dantzig), and also advocated by V. V. Novozhilov and by the dynamic director of the Central Economic Mathematical Institute of the Academy of Sciences of the USSR, Nikolay Fedorenko. This group of people advocates that prices should be fixed on the basis of what might be called *the optimal prices* – or shadow prices – as derived from the solution of a mathematical programming problem for the economy as a whole.

This latter approach is very much in line with my own way of thinking, with this important *proviso*, however, that a price system based on the concept of optimal prices is only a *necessary*, but far from a *sufficient* means of implementation for steering the economy in a direction which conforms with the intentions of the responsible political authority. Cf. Section 4 below.

In discussing the role of prices as part of such an implementation system I shall leave completely aside such much talked about principles as the principle that prices "ought to" be proportional to marginal costs, or the principle that they "ought to" be just high enough to clear the market, i.e. creating balance between demand and supply. These principles are entirely inadequate for a discussion of the price fixing problem in a socialistic society. They can only serve to throw the discussion into a barren procrustean bed.

2. THREE MAIN IDEAS INVOLVED IN THE CLASSICAL MARXIAN THEORY OF VALUE FOR A CAPITALISTIC SOCIETY

It is impossible to proceed to a discussion of a rational theory of price fixing under modern conditions in a socialistic society without stating briefly the essential points of the Marxian theory of value for a capitalistic society. These points can be indicated by the three catch-words: Labour theory of value, the iron law of wages and the theory of the surplus value (Mehrwert).

The labour theory of value

The labour theory of value originated in clear form with Adam Smith whose whole approach to the study of value centers around labour as the guiding and dominating principle. Already in the Introduction to his famous treatise "The Wealth of Nations" (1776) he states his point of view unambiguously: "The annual labour of every nation is the fund which supplies it with all the necessaries and conveniences of life which it annually consumes". And, perhaps in an even more pointed form, directing his polemical argument against the mercantilists: "It is not with gold and silver but with labour that all wealth in the world has originally been acquired".

Having adopted this basic labour point of view, it was only natural that Adam Smith considered the amounts of labour that were incorporated in the various commodities as the "cause" of their different *values in exchange*. When in a primitive hunting society, he says, one beaver is exchanged for two deers, it is because it takes twice as many hours to kill a beaver as to kill a deer. Already at this point in the argument we see clearly the contours of the theory which was later propagated with such force by Karl Marx. Cf. the quotations in Section 1 above regarding the theory of labour cost ("the law of value") and the practical rules for price fixing adopted in the Soviet Union at the 22nd Party Congress.

But the similarity of theoretical thought between Adam Smith and Karl Marx goes even much deeper. In a more advanced society, Adam Smith says, when capital and land *have been made the subject of private ownership*, the price of a commodity will consist of three component parts: wages, profits (on capital) and rent (on land). Here is a theory of "surplus value" (Mehrwert) over and above labour cost. It is linked with the advent of *private ownership*, and as a theoretical proposition it is formulated quite clearly.

It is puzzling to ponder over the fact that two great men with such similarities in their *theoretical* thinking, should reach so contradictory conclusions with regard to the most desirable *practical* organization of society: The former became the advocate of economic liberalism and the free play of the market forces, while the latter became the father of modern socialism with a regulated and planned economic life as its goal.

Also David Ricardo (1772–1823) propounded the theory that commodities exchange in the ratio of their respective costs in terms of labour.

The iron law of wages

This is a theoretical element which is *quite distinct from* the theory of labour cost, although there are, of course, many complex forms of reasoning where both elements may be used simultaneously.

The idea which later came to be known under the name "iron law of wages", goes back at least as far as to Anne Robert Jacques Turgot (1727–1781). In his "Réflexions sur la Formation et la Distribution des Richesses" (printed 1769–1770) he says explicitely that *competition will force* *artisans' wages down to subsistence level.* This is the doctrine of necessary wages. Here quoted after Vol. III. p. 594 of [4].

The same basic viewpoint we also find in David Ricardo. He formulates the doctrine of the standard-of-comfort theory of wages. This is basically the same idea as we find in Turgot, with this modification, however, that in Ricardo less emphasis is put on the bare *physical* minimum of existence, and more emphasis on the *conventional* minimum, which may change with the general economic standard in society. The difference with Turgot is only one of degree.

Also Robert Mathus (1776–1834) must be mentioned in this connection. The "iron law" is implicit in his theory, which states that as soon as new economic resources are made available, for instance in a new country, the human race will – after having for a while lived comfortably because of the new resources – *multiply* in number up to a point where a further increase is checked because the new population pressure against resources has *reestablished* the "normal" situation where the great masses of the population live in poverty.

The term "iron law of wages", ein ehernes Gesetz, was coined in 1862 –1863 by Ferdinand Lassalle (1825–1864), "Denker und Kämpfer" as he is called in the inscription on his tomb. In his "Working Mens' Programme" of 1862 he insisted that "by an iron and inexorable law... under the domination of supply and demand, the average wages of labour remain always reduced to the bare subsistence which, according to the standard of living of a nation, is necessary for maintenance and reproduction". The same thought he again expressed in 1863 in an Open Letter written on the occasion of the convocation of a general congress of the working men of Germany.

The theory of the surplus value (Mehrwert)

When the labour theory of value and the iron law of wages are combined, one reaches quite naturally the theory of the surplus value (Mehrwert) so basic in Karl Marx's diagnosis of the capitalistic system. I shall give Marx's argument in the words of Friedrich Engels (1820–1895): The labour power exists in the form of the living worker who needs a certain quantity of means of subsistence for himself and his family. The value of the labour power is determined by the number of hours necessary for the production of the means of subsistence.

"Suppose", writes Engels, "that these means of subsistence represent a working time of six hours per day... The fact that only six hours a day are needed to keep the worker alive for 24 hours a day, does not prevent him from working 12 out of these 24 hours. *The value of the labour power and the value which the labour power creates in the production process are two different magnitudes* (Italics by R. F.)... The difference is appropriated by the money owner... six hours non-paid surplus work... six hours non-paid surplus product in which six hours work is incorporated. The trick is completed. Surplus value has emerged. The money has been transformed into capital". P. 250–251 of "Anti-Dühring", here quoted after p. 251–252 of [5].

It is quite clear that Marx's theory of value, which culminated in the doctrine of the surplus value, was developed in order to diagnose the *capitalistic* system. It was not developed as a basis for the theory of value in a socialistic society. Marx states this explicitely, as appears, for instance from the quotations given on p. 14 of [6].

When it comes to the economic organization of a *socialistic* society, Marx and Engels have only discussed generalities without going into concrete details. And in the subsequent history of the Soviet Union and other Eastern countries we have witnessed many changing phases.

The basic fact expressed in that part of the quotation from Engels which I have rendered in italics, will, however, *persist in any society*, also in a socialistic society. Even here the surplus value (Mehrwert) will exist, as a logical category. The only difference is that its *size* may be different from what it will be under the regime of the iron law of wages. The whole situation now assumes *one degree of freedom*, depending on how much it is decided to let the workers retain in the form of wages.

All this will become perfectly clear when the various categories pertaining to value are expressed in the formal mathematical apparatus of a macro-economic programming model as the one I have used in Section 3.

Through the analysis of Section 3 will also appear that in reality there is no contradiction between a utility theory of value and a cost theory of value. Too frequently the discussion has proceeded on the assumption that if one of the two theories is true, the other must be false. The concept of utility – in the form of value in use – was present also in many classical economists, including Marx, Cf. for instance p. 11 of [6].

The mathematical programming approach will bring the whole issue into focus, and show the logic of the interrelations between the various concepts.

3. THE MACRO-OPTIMAL PRICES

I shall first give a table summarizing the formulae I need for my discussion of macro-economic programming and its impact on price-fixing in a regulated economy. Subsequently these formulae will be explained and commented upon.

Tab. (3.1) *Main principles of macro-economic programming*

With special emphasis on the concepts of value and price.

I. THE AUTHORIZED LIST OF VARIABLES

$$x_1, x_2 \ldots x_N$$

II. THE SYSTEM OF EQUATIONS

A. Primary equations

$$S_f(x_1, x_2 \ldots x_N) = 0 \qquad f = \text{eq},\ m \text{ in number (assumed mutually independent)}$$

B. Reduced equations

The analytically free variables: x_h (h=free) $n = N-m$ in number

The dependent variables: x_j (j=dep) m in number

$n = N - m$ = number of equational degrees of freedom

The dependence functions: $x_j = r_j(x_h)$ $\begin{pmatrix} j = \text{dep} \\ h = \text{free} \end{pmatrix}$

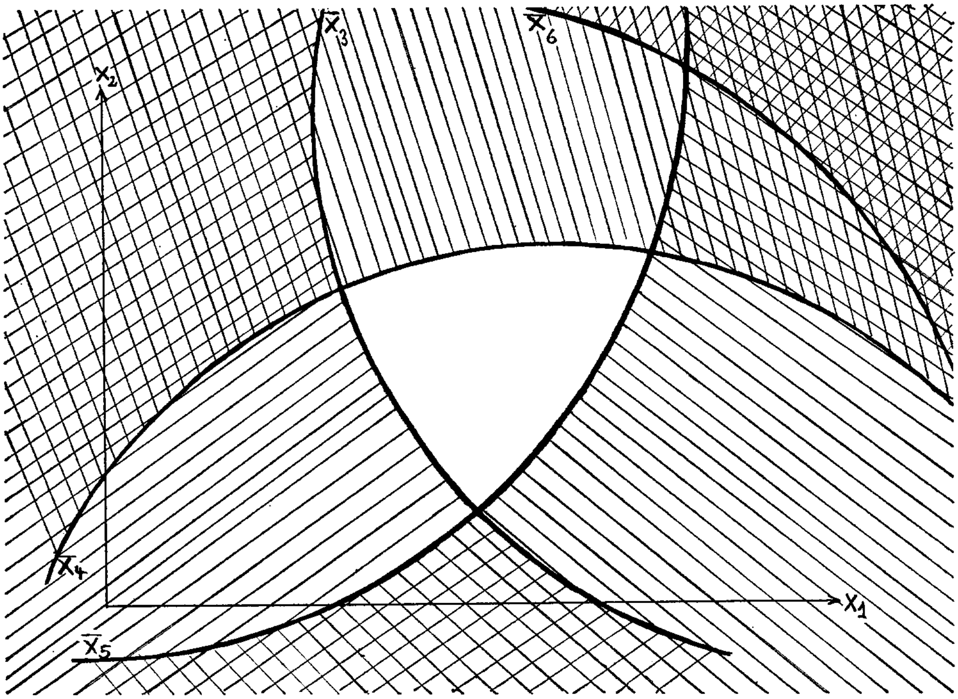
(The reduced form of the equations, r indicating "reduced")

$$\text{If } k = \text{free: } r_k(x_h) = x_k$$

III. THE BOUNDS

$$\underline{x}_i \gtreqless x_i \gtreqless \bar{x}_i \qquad (i = \text{all}) \quad \underline{x}_i \gtreqless \bar{x}_i \text{ given}$$

The admissible region:



IV. THE PREFERENCE FUNCTION

Gross form $\qquad P(x_i,\ i=\text{all})$

Net form $\qquad p(x_h,\ h=\text{free}) = P(r_i(x_h))$

$$\frac{\partial p}{\partial x_h} = \sum_{i=\text{all}} \left(\frac{\partial P}{\partial x_i}\right) \cdot \left(\frac{\partial r_i}{\partial x_h}\right) \qquad (h=\text{any free}) \tag{1}$$

In the linear case this reduces to

$$p_h = \sum_{i=\text{all}} P_i r_{ih} = P_h - \sum_{j=\text{dep}} (-P_j) r_{jh} \qquad (h=\text{any free}) \tag{2}$$

$p_h = \dfrac{\partial p}{\partial x_h}$, $P_i = \dfrac{\partial P}{\partial x_i}$ and $r_{ih} = \dfrac{\partial x_i}{\partial x_h}$ are constants in the linear case.

V. THE OPTIMAL SOLUTION

$$\hat{x}_i \qquad (i=\text{all})$$

The number of *optimal degrees of freedom* may be anything from *o* to *n*, *n* being the number of equational degrees of freedom.

VI. DEFINITION OF OPTIMAL PRICES

$$\hat{p}_i = \lim_{\delta_i \to +0} \frac{p^{\text{opt}}(\delta_i) - p^{\text{opt}}(0)}{\delta_i} \qquad \begin{array}{l}(i = \text{any variable for which a} \\ \text{bound is increased, that is } \delta_i > 0)\end{array}$$

= marginal efficiency of increasing a bound on x_i (increasing the lower bound or increasing the upper bound on x_i). Because of possibly existing discontinuities, we have here specified the sign of δ_i. In any case the optimal price on a lower bound will be non positive and that on an upper bound non negative. It is therefore unnecessary to introduce a special notation to distinguish the optimal price of a lower bound from that of an upper bound. The optimal price of a non-boundattained variable is 0.

In the definitional formula nothing would have been changed if in the right member we had used the gross form P instead of the net form p, because the *ordinate values* of the two forms are equal.

VII. COMPUTATION OF OPTIMAL PRICES

$$\text{(1)} \qquad \textit{In general:} \left(\frac{\partial p}{\partial x_h}\right)^{\text{opt}} = \sum_{i=\text{all}} \hat{p}_i \left(\frac{\partial r_i}{\partial x_h}\right)^{\text{opt}} \qquad (h = \text{any free})$$

Fundamental formula stating that in the optimum the preference vector must belong to the multiplicity that is unfolded by the gradients on the boundattained variables. The optimal prices are the multipliers by which the preference vector is expressed as a linear combination of the gradients on the boundattained variables. The preference vector is here taken in the net form.

Examples: If *no* bound is attained in the optimum, the preference vector must be zero, by (1). (The traditional first order necessary condition for the maximum of a function when no bounds are imposed on the variables.)

If *one* bound is attained in the optimum, the preference vector components must be proportional to the gradient components for the variable that is boundattained. (The traditional condition for a substitumal point in production theory.) The proportionality factor is the optimal price of the variable that is bound attained. If *two* bounds are attained in the optimum, the preference vector must belong to the two dimensional multiplicity that is unfolded by the gradients on the two boundattained variables. The two optimal prices are the multipliers by which the preference vector is expressed in terms of the two gradients.

(2) *Linear*: $p_h = \sum_{i=\text{all}} \hat{p}_i r_{ih}$ $\quad (h = \text{any free})$

In the linear case $p_h = \dfrac{\partial p}{\partial x_h}$ and $r_{ih} = \dfrac{\partial r_i}{\partial x_h}$ are independent of where the optimum happens to be. Hence, at least n variables must be boundattained in the optimum (if the optimum is situated at a finite distance from origin).

VIII. INTERPRETATION OF OPTIMAL PRICES

(0) *Marginal preferency in the optimum* ("marginal utility", "marginal productivity", "marginal efficiency"):

By comparison of (VII.1) with (IV.1): $\hat{p}_i = \left(\dfrac{\partial P}{\partial x_i}\right)^{\text{re}}_{\text{opt}}$ $\quad (i = \text{all})$

(1) *In general*: $\hat{p}_h = \left(\dfrac{\partial p}{\partial x_h}\right)^{\text{opt}} + \sum_{j=\text{dep}} (-\hat{p}_j)\left(\dfrac{\partial r_j}{\partial x_h}\right)^{\text{opt}}$ $\quad (h = \text{any free})$

(2) *Linear*: $\hat{p}_h = p_h + \sum_{j=\text{dep}} (-\hat{p}_j)\, r_{jh}$ $\quad (h = \text{any free})$

(3) *Labour theory of value in Mehrwert form*: $P_h = p_h + (-P_1)\, r_{1h}$ $\quad (h = \text{any free})$

(4) *Friedrich v. Wieser marginal utility theory of value in Mehrwert form*: $P_h^{\text{opt}} = p_h^{\text{opt}} + \sum_{j=\text{dep}} (-P_j)^{\text{opt}}\, r_{jh}^{\text{opt}}$ $\quad (h = \text{any free})$

(3) is the special case of (4) where there is only one j and where it is unnecessary to use the superscript opt.

Note the difference between $P_h = \dfrac{\partial P\ (\text{all})}{\partial x_h}$ and $p_h = \dfrac{\partial p\ (\text{free})}{\partial x_h}$.

*

I now proceed to commenting upon the concepts and formulae that are summarized in tab. (3.1).

The main idea of my approach to rational price fixing is that it is futile to discuss this problem so to speak in a vaccuum, i.e. by considering trading prices as something that can be discussed *per se*. The problem of rational price fixing can only be handled in a really rational and fruitful way by considering it as a *part* of the general problem of decision making for the economy as a whole. This is why a considerable part of the subsequent discussion is orientated towards general principles for the building of macro-economic decision models.

3.I ad: THE AUTHORIZED LIST OF VARIABLES

I shall adopt an extremely general point of view when I speak of "the variables that are to be included" in an analysis aiming at building up a computable macro-economic model. These variables may indeed be anything under the sun, all depending on the purpose and nature of the analysis one has in mind. They may for instance express static or dynamic concepts, exogenously estimated variables or endogenous variables, parameters of action of government or variables that are not of this sort, and so on.

3.Ia *The authorization of the list*

Whatever the concrete nature of a certain variable that is introduced in the analysis, it is vitally important that the analyst is concious of the fact that he *has* introduced it in his analysis, and that he takes full account of all the logical implications that follow from the fact that he has done so.

Too frequently we witness that a verbal economist (and sometimes even a so called mathematical economist) when he comes to a difficult point in his argument, stretches out his hand and picks some *new* variable that seems to come in handy at this point. Too frequently this is done without the analyst being aware of the fact that by so doing he might have disturbed the logical balance which determines the all important concept of the *degrees of freedom* in his model. Cf. section 3.II below. Also cf. section 4.5 of [7].

In any respectable analysis the list of variables should therefore so to speak be *officially authorized*. And in the argument it ought to be forbidden to mention any non-included variable, except by either openly revising the authorized list or by introducing a careful distinction between the model as such and *accessoric* "on the side" reasonings of the sort I have described in my preface to [8].

3.Ib *A simple example*

One very simple example of the specification of variables to be included in the authorized list, is the one where all the variables are *quantities* (or volume indices) which fall into two categories: *products* emerging from various production processes, and *resources* (say different types of labour, real capital and land) that are used in these production processes. In the sequel I shall occasionally use this simple example to illustrate the meaning of the concept of optimal prices.

3.Ic *Decision models as distinct from growth models*

The distinction between what is essentially a *growth* model and a *decision* model is important.

When I speak of a growth model I am not referring particularly to its dynamic character, because a useful decision model is also essentially dynamic, but I think of the rather too passive attitude to economic growth which is often displayed in the use of the Western type of the growth model approach, characterized by such simple notions as the general savings rate, capital to output ratios, marginal productivity of capital etc. without explicit introduction of the *decisional parameters* that will basically influence the rate of growth. The explicit introduction of these parameters in an operational way is what characterizes a decision model. We have to consider *a great number* of such decisional parameters, for instance those characterizing many different types of investments and their relations to the current account activity of many different domestic sectors. Cf. sections 3.Ig and 3.Ih belows. These remarks will apply in all countries, less developed and more developed.

The need for introducing explicitly the decisional parameters in the analysis is most acute in a short range (say annual) or a medium range (say five or seven years) plan. In the very long run (say twenty or fifty years forecasting) the future loses itself so much in the haze that we have to rely to a large degree on guesses of a growth kind. For instance: what can we say to-day of the possibility of bringing to the blue-print stage certain bold ideas that linger in the heads of some prominent physicists? What can we say about the possibility of arrival of some ideas that are not yet in these heads? Here we can only guess about growth rates. But for the bulk of the day to day planning work which is concerned with decisions between tangible and precisely formulated technical and economic alternatives, the decision model viewpoint is absolutely fundamental. Cf. also [9].

3.Id *The dynamic model with temporal splitting of variables*

In most cases the model will be much more complex than the simple one mentioned in 3.Ib. In particular the model will usually have to be dynamic.

When it is a question of building a computable decision model with a great number of variables, I believe that the most fruitful and most practical approach to dynamification is through temporal splitting of variables (the simultaneous multistage method).

This means that, say, consumer use of a certain commodity, or the output of production in a certain production sector, or the net foreign creditor position of the country etc. is not introduced as a *single* variable but as so many variables as there are years (or quarters, or months etc.) in the dynamic analysis. In all practical analyses this will be a finite number. Problems with an infinite horizon and the abstract proofs of "convergence" etc. that go with them, have little relevance to economic reality.

A model with temporal splitting can in an excellent way exhibit how one situation grows out of the foregoing ones. This is the essence of a dynamic analysis.

My reasons for thinking that the P. Massé – A. Wald – R. Bellman type of *recurrent* programming is not the most useful in macroeconomic planning work, are given in [10]. I think it is unfortunate that one sometimes speaks of "dynamic programming" as synonomous only with this recurrent type. Recurrent programming is only one special sort of dynamic programming.

3.Ie *Selection vs. implementation*

A selection model is a model which is primarily useful for the purpose of describing a constellation of the *volume figures* in the economy or figures *in actual technical units* which might be realized or one would like to see realized, *if* one could find ways and means (administrative, institutional and financial) of bringing this constellation about, i.e. of *implementing* this constellation.

In theory it would, of course, be possible to include also *all* these ways and means explicitly in the same programming analysis. But models of this type might be very complicated and run the risk of becoming only a formal exercise without much practical significance. It is primarily in the selection problem that the biggest advantages of an explicite quantitative model with optimal solution can be gained. In the complete implementation problem we must rely to a larger extent on *simulation* techniques and on economic intuition and practical sense. Cf. [12].

There is, however, a possibility of proceeding *part* of the way towards a formal programming solution of the implementation problem by considering the interplay between *real* flows and *financial* flows, leading to what may be called a *refi* model (*re* = real, *fi* = financial). Cf. [11]. I have been informed that the Oslo refi model has attracted a certain amount of interest in the Moscow Central Economic Mathematical Institute.

Another reason why it is a practical approach to separate the selection and the implementation problem, is that the selection problem can be studied without stating a priori the kind of *economic institutions* (competitive markets or central controls or a mixed system) one is prepared to accept. Such a separation and the study of a rock bottom selection as distinct from an institutionally contaminated model, is more or less a necessity if the purpose of the analysis is to compare different kinds of regimes. Cf. [12]. Cf. also the end of section 3.II c.

These considerations have great consequences for the kind of variables one decides to introduce in the authorized list. A selection model will, for instance, not contain *trading prices*, but it will nevertheless – after the optimal solution – lead up to the concept of *optimal prices*, and these optimal prices might be taken as the basis for a further analysis of a rational method of fixing trading prices – these trading prices now being part of the means of implemation. A study of such a procedure is precisely the aim of the present paper.

If it *is* desired to work with an institutionally contaminated model where trading prices do occur explicitly, and bounds are prescribed for the trading prices, one will encounter such concepts as "the optimal price on a trading price".

3.If *The ring structure*

This is a very general concept by which one can, for instance, replace the uncomfortable assumption of fixed input coefficients in an interindustry table or in a process-to-goods table, by a linear scheme which expresses *substitution* possibilities.

The idea of ring structure is applicable also in a great number of other fields in macro-economic planning, for instance in the study of consumer demand and in the study of export distribution over geographical areas. Another important general use is in the study of *compartmentalization,* i.e. the study of how to coordinate central planning with planning in subordinate organs (compartmentalization according to regions or according to spheres of economic activity, for instance according to a system of "trusts" in the terminology of socialist planning).[1]

The theory of substitution rings has been given for instance in [13].

The use of substitution rings does not introduce non-linearities if the coefficients in the substitution rings are constants, but it increases the number of variables to be considered. It is therefore an important point to take account of in the elaboration of the authorized list of variables.

3.Ig *Investment starting vs. investment sinking*

One must distinguish between investment starting and investment sinking. Investment *starting* in any given year is the total outlay which it is estimated that the projects started that year will have entailed when they are finally completed – perhaps at some future date. Investment *sinking* in any given year is the value of goods and services that were actually used (that were "sunk") that particular year in order to carry towards completion projects which were started that year or some previous year. The distinction between investment starting and investment sinking is essential in an analysis that is to be truly dynamic.

Variables that express investment startings are extremely important in a *decisional* analysis that aims at *choosing* between various investment projects. The simplest approach is, for each investment starting variable, say H (H for "hardware"), to assume $0 \gtreqless H \gtreqless \bar{H}$ where $\bar{H}$ is the "full dress" size of the project according to the project description. An optimal value zero for such a variable H means that the project is rejected or postponed. And an optimal value equal to the "full dress size" will indicate acceptance. Most frequently one of the bounds will be attained – or nearly attained – in the optimum. If we want to assure in an *exact* way that the starting variable H assumes one of some given discrete values $H^1, H^2, H^3 \ldots$ in the optimum (for instance 0 or 1), we impose in the programming work the condition $(H-H^1)(H-H^2)(H-H^3)\ldots=0$. But in most practical cases we will get a sufficiently clear result without introducing such a condition.

The optimal price on a starting variable, cf. sections 3.VI – 3.VIII below, will measure the *importance* of non-accepting, respectively accepting the project. Often starting variables will make up a great part of the authorized list.

[1] Compartmentalization use of the idea of substitution rings have been emphasized in private communications to me by professor Paul Medow of Ruthger's University of New York.

A theory built on investment starting variables is developed in section 5 of [13] and more fully, for instance, in my Oslo lectures (in English), which, however, have not yet been prepared for publication.

The introduction of investment starting variables may – if the set up is sensibly handled – reduce rather than increase the number of variables in an investment analysis that aims at being a *decision* analysis and not simply an aggregated growth analysis of the on-looker kind. This reduction is obtained by utilizing to the fullest extent the available *engineering data*. Such data are more lavishly available and as a rule more reliable than statistical data of the time series kind or the cross section kind.

Any investment starting variable, like other variables, may be temporally split.

3.Ih *Capacity effects and infra effects*

Through the use of investment starting variables we can study the effect which any project – if accepted for starting in a given year (or quarter or month) – will have on existing *production capacities* in each of the subsequent years. This is an important element in investment decisions. Since the investment starting variables are non-determined before the optimal solution, they offer a means of taking all the possible capacity effects simultaneously into consideration in our search for an optimal decision on investment starting.

A similar argument applies to any *infra effect* which the investments may have. The infra effect is the effect which an investment starting may have on current account input *coefficients* in the subsequent years (for instance an investment in labour saving machinery, or in machinery for changing from coal to dieseloil etc.), or, more generally, the effect on *any coefficient* in the model as it existed before the infra effect producing investments were introduced (the "postinfra" as distinct from the "preinfra" model).

The introduction of infra effects is a powerful means of using an optimal decison technique for choosing between different directions of technical progress, weighing material and labour costs of the investments against the advantages which may be obtained over the subsequent years.

This is another reason why investment starting variables are important items to consider when we work out the authorized list of variables.

The introduction of infra effects will not *per se* increase the number of variables, but it will make the model conspicuously non linear.

The *combination* of temporally split variables, ring structures, the distinction between investment starting and investment sinking, the capacity effects and the infra effects, will make a decision model dynamically rich and decisionally much more *realistic* than models without these features. But the introduction and quantification of these features will, of course, make the analysis more difficult than the study of growth models of the traditional type. Nature *is* difficult whether we like it or not. And if we are not willing to face analytical difficulties in optimal decision problems, we have to acquiesce with highly unrealistic models.

3.Ii *Moving planning. Decisional magnitudes and already-committed-to magnitudes*

For many years I have in lectures and in writing advocated the use of moving planning. This means that a plan with a given horizon, say a five year plan, or a seven year plan etc. should be elaborated *each year*, taking account of whatever new developments that may have occurred since the five year plan or the seven year plan was last elaborated.

These new developments may pertain not only to new technical, historical and statistical data, but also to possible changes in the policy maker's evaluation of the things that he desires.

The *planning* year and the *happening* years thus become two distinct time indications. The time scale becomes two dimensional, and this has an important consequence for the composition of the list of magnitudes that are to be considered as *variables* – as distinct from constants – in a decision model.

Indeed, the most powerful means of formalizing the interplay between the two time indications mentioned, is a systematic distinction in the model between magnitudes that are *already-committed-to* and therefore enter into the decisional analysis as constants, and those that are still variables in the *decisional* sense. A special system of notation for this distinction has been introduced, and a rather detailed explanation on how to compute the already-committed-to magnitudes on a moving basis (which is not a very simple matter) and how to distinguish them from the decisional variables, has been given, for instance on pp. 31–33 and pp. 126–140 of [14]. Also pertinent in this connection is [15] which relates the problem to a very general conception of rules of strategy under repeated planning with full use, respectively partial use of new information. Cf. also the highly interesting paper [16] by Jaroslav Habr. In view of the remarks, and references given above, his statement: "So far, however, this planning method (sliding, i.e. moving plans) has not been explored theoretically", appears as slightly incorrect.

3.II ad: THE SYSTEM OF EQUATIONS

3.IIa *Models vs. reality*

All science and all scientific thinking *must* proceed by way of abstraction and "models" of some kind or another. This is the only way. If we don't want to go it, nothing is left but to stop thinking.

Some models may, of course, be "more realistic" than others, as judged by the "practical" results we can get from them. But the measurement of the "degree of realism" and the "degree of practicalness" raises *new problems*, which in turn can only be discussed in terms of other models, perhaps stochastic ones.

This is a warning to all who believe they are doing something very respectable when they insist that they want to study "reality" and avoid "models".

3.IIb *Structural vs. administrative aspects of planning*

By structural aspects I mean all those "patterns" or "regularites" in the economy which can *not* be changed directly by a Parliament decision, or more generally, by a human action. A structural aspect is something we have to accept whether we like it or not, whether we are of the bluest sort of conservative politicians or of the reddest sort of radical politicians.

All *other* aspects are what I mean by administrative aspects of planning, taking now administrative in the widest sense of the word.

The structure may be expressed by *equations* or lower and upper *bounds* on the variables. The equations or bounds may be deterministic or stochastic. Through various forms of certainty equivalence theorems the stochastic approach may often be reduced to a deterministic approach. In what follows I shall therefore confine myself to considering deterministic equations and bounds.

The structural equations and bounds may be of the definitional sort or of the technological sort, taking now also the word technological in its widest sense.

Definitional equations (accounting relations) and definitional bounds for the variables are equations or bounds that follow from the very definition of the variables. For instance: a delivery from a production sector is by definition the sum of two magnitudes, namely that part which is used as *cross* delivery, i.e. delivery to other production sectors, and that part which constitutes *final* delivery. And the latter magnitude is again by definition the sum of the six magnitudes: private consumption on current account, government consumption on current account, private use of goods and services for investment sinking, government use of goods and services for this purpose, net increase in stocks and net exports.

An example of a definitional bound: The output from a production sector has by definition the lower bound zero.

Technologically structural conditions are for instance all equations and all bounds that follow from such technological "patterns" or "regularities" which we can not hope to change within the perspective under which our planning takes place: the gravitational forces, the number of tons of bauxit needed in order to produce one ton of aluminum, the way in which the human need for and craving for food decrease as the supply of food increases. And so on.

3.IIc *Imposing conditions means restricting manoeuvrability*

The structural aspects of the economy constitute a set of conditions which *must* be imposed on the solution of a macro-economic programming problem, whether we like it or not.

The acceptance of a specific system of administrative rules and institutions constitutes an *additonal* set of conditions.

As long as we only impose the structural conditions we have a fairly high degree of manoeuvrability, but it is manoeuvrability with mathematical items only. After having imposed a *specific* system of administrative rules and institutions our degree of manoeuvrability is much lower. Therefore, we will not be able to reach as good a result as (or more precisely: any better result than) we would have been able to reach if we had *not* prescribed a *specific* system of administrative rules and institutions, but had – temporarily – left the administrative question open.

This will amplify what I mean by distinguishing between selection and implementation, cf. 3.Ie. In the selection analysis we only impose the structural conditions, not any specific system of administrative conditions. The result obtained from the optimal solution of a selection problem is therefore always better than (or at least as good as) the optimal solution obtained by adopting any specific administrative system. And the *shortcoming* of the latter result as compared with the former, is a good measure of how good the administrative system in question is.

This test – the comparison with the selectionally optimal result – ought to be applied to any set of administrative rules and institutions. Hence the great importance of studying the selectionally optimal solution. It will give us a means of testing the goodness of a concrete administrative system. Cf. [12].

3.IId *A summary of the selection model*

A general discussion on bounds is given in section 3.III, but it was necessary to anticipate part of that discussion already in the present section devoted to equations. If we further anticipate the concept of a preference function, cf. section 3.IV, and that of an admissible region, cf. section 3.III, the nature of a selection model may be summarized as follows:

<table>
<tr><td colspan="2">Tab. (3.IId.1)</td><td>Equations</td><td>Bounds</td><td></td></tr>
<tr><td rowspan="2">Structural aspects</td><td>Definitional</td><td>Definitional equations</td><td>Definitional bounds</td><td rowspan="2">If to this we add a preference function (cf.3.IV) we get a selection model</td></tr>
<tr><td>Technological</td><td>Technological equations</td><td>Technological bounds</td></tr>
<tr><td></td><td></td><td colspan="2">This gives rise to the selectionally admissible region (Cf.3.III)</td><td></td></tr>
</table>

Thus, the only kinds of equations and bounds that occur in a selection model are the definitional and technological ones.

The question may arise if there really *exists* a selectionally admissible region, i.e. if the definitional and technological equations and bounds are compatible. The answer is *yes*. Provided, of course, that the model is not so unrealistic that it must be disregarded. The proof is simply that from experience we know that "societies exist".

We can formulate this in the following:

Proposition (3.IId.2): A realistic selection model has always a non-empty admissible region.

All the above formulations are only an amplification of the definition in section 3.IIb which states that the structural aspects are the "patterns" or "regulartities" which we *have* to accept whether we like it or not.

3.IIe *Preference equations added to a selection model*

So much for a selection model. Let us now proceed to a discussion of certain new types of equations that may occur if we decide to work with models that are more or less implementationally (or, if you like, institutionally) contaminated.

A selection model – as summarized in tab. (3.IId.1) – contains an important political element, but this element occurs only in the form of a preference *function*, i.e. a function whose maximization defines what is meant by "optimization". The addition of such a function to any model has no consequence for the existence of equations and bounds in the model, and hence it has no consequence for the form of the admissible region, and hence – in the case of a selection model – it does not contradict proposition (3.IId.2).

The situation is fundamentally changed if the preferences of the policy maker are also expressed in terms of some *equations*. The policy maker may for instance desire that private consumption should move over time as a certain *proportional part* of gross national product. Private consumption is one of the variables in the selection model – when private consumption is expressed as a volume index (i.e. as a "constant-price measure" of consumption).

A policy maker's requirement of the sort mentioned constitutes a political *equation*, as distinct from a politically defined preference function. Such an equation has definitely the character of a "pattern" or "regulartity" which we are *not* forced to accept whether we like it or not. Hence it belongs to what I have called the administrative aspects.

Another example is the imposition of conditions to the effect that the technical production has to go on according to certain politically decided *proportions* between factors of production, or between factors and products. In Norway there have been "Buttermix" rules stating that one *had to* add a certain quantity of natural butter to margarine (to get rid of a surplus of natural butter). In Egypt there have been rules to the effect that certain production sectors *had to* use such and such numbers of workers (to counteract unemployment).

Political equations of the various types here mentioned, constitute in principle only a *first* step into the administrative – or if you like implementational or institutional – field, because they only impose relations between variables that *could have been* incorporated in a (more or less disaggregated) selection model. Institutionally contaminated models that emerge in this way are therefore among those that are *most akin* to a selection model.

The distinction between the two types of models is, however, quite clear, not only because of the criterion "whether we like it or not", but also because proposition (3.IId.2) is not any longer *necessarily true* in a model containing one or more political equations. Indeed, an *equation* – as distinct from a preference function – may influence the shape of the admissible region, and in particular it may make this region empty, i.e. it may introduce non-compatibility in the set of conditions.

3.IIf. *Political equations containing refi concepts or other implementational concepts*

If we introduce variables that by their very nature are *not* of the structural sort, for instance introduce variables based on monetary and financial concepts, market-price and profit concepts, tax rules, categories of subsidies, or other refi concepts, then we have moved still further into the implementational field. Now the list of variables in itself is enough to characterize the model as institutionally contaminated, even though we have not yet imposed any equations which these administrative variables ought to satisfy, i.e. even if we have not yet *fixed* any prices or interest rates or tax rules etc. In Section 3.IIe the implementational aspect came in only through the introduction of one or more political equations.

The mere introduction of implementational concept in addition to a selectional structure, without saying anything about the magnitudes that these new concept ought to have, will *not* influence the admissible region as it emerged from the selectional structure. But if we proceed further and introduce political *equations* and/or political *bounds* which some of or all the structural or implementational variables, e.g. the refi concepts, ought to conform to, we have *opened the barrage* for a flood of possibilities for changing the shape of the admissible region, and thereby, perhaps, creating flagrant inconsistencies.

In big systems of variables and equations these inconsistencies may not be of the conspicuous sort that attracts the attention of the casual observer. They may be of a rather *hidden* sort that will only be brought to light by an advanced form of mathematical analysis.

Therefore, if implementational requirements and fixations are introduced by the political authority – perhaps more or less on the spur of the moment – there is a great danger of producing inconsistencies, i.e. producing an empty admissible region. Such inconsistencies life itself will correct by making some of the implementational rules and regulations – here or there in the system – *give way*, so that we get back to a situation similar to the one described by proposition (3.IId.2). "Societies do exist", even though political decisions are not always consistent.

3.IIg. *Target setting, a special case of political equations*

By target setting I mean the decision by a political authority that the aim of the economic policy should be to realize such and such a *magnitude* for each of the economic variables in a certain specified list of variables. Such a procedure is obviously a special case of the imposition of political equations.

Target setting as thus defined is, as we know, a very popular procedure in economic planning. But most often it is used in a form which is little short of wandering in the fog. This applies even if the political target setting has been made on the advice of so called experts.

The *ultimate* goal of an advanced analysis, that begin with the study of a selection model, is to arrive at a set of quantity targets for the development of the economy over the years to come. Such targets are the ultimate manifestation of planning. But before reaching this ultimate stage there is a long way to go.

If we *start* economic planning on a hunch that we ought to build one sort of factory here and another sort of factory there and perhaps try to increase wheat production next year etc. then we would start wandering in the fog. For one thing we would not even know if all these quantity targets are *feasible*, i.e. *consistent* among themselves. And even if they were, we would have no guarantee that they really represent the *best* – the optimum – use of the resources at our disposal. I.e. that they represent that particular combination of quantity targets that come nearest to achieving what the policy maker *really would want to see achieved* – if it could be done.

In the beginning of the development of an underdeveloped country there may be *some* projects – some big and conspicuous projects – which are of such a nature that we would probably not make a big mistake by accepting them more or less on a hunch. But not all projects are of this sort. And as time goes on, and more and more – small or big – aspects of the economy come into the picture, the situation becomes so complex that we will make a big and serious mistake if we also now start by the target setting approach, and subsequently attempt by trial and error to make the complex of our targets as consistent as possible. Even several rounds of trial and error attempts may not produce effective consistency – iteration methods may be very unrealisable things, as every mathematician knows. And even if consistency were attained, optimality would not be assured.

One example of this mistaken and naive approach is the one which – more for reasons of simplicity than for reasons of realism – is so popular in many Western countries (including the Common Market as represented by its Commission in Brussels, cf. [17]) – namely the procedure of starting by guessing at a *growth rate* of the gross national product that might be attainable, and subsequently from this guess trying to deduce, by input-output analyses, national accounts etc., the consequences for different sectors of the economy, and put these figures up as targets. There are indeed many *different* alternative developments – many *different* sets of quantity targets – that will give the *same* growth rate for gross national product (or some other invented statistical concept). Which one of these alternatives is "the best"? And why was the growth rate put at the figure used?

Much unclear thinking on planning methodology stems precisely from the crude target-setting way of thinking. In particular much nuclear thinking about the usefulness or the futility of a precise formulation of the overall national preferences stems from the target-setting way of thinking. Some of the arguments against the possibility of a precise preference formulation at the overall national level, is based on the erroneous conception that such a formulation ought to pertain to a complex of quantity targets. If it did pertain to quantity targets, the criticism would be well founded. But in fact the situation is quite different. In a rational planning system the precise formulation of the national preferences does *not* pertain to a complex of quantity targets but to something quite different, as explained in section 3.IV below.

In a rational macro-economic planning system we must start by ridding our minds completely of the target-setting approach, and proceed through the successive steps that begin with a study of a powerful selection model which contains an overall preference function. Cf. tab. (3.IId.1).

3.IIf *Primary equations, the equational degrees of freedom and the reduced equations*

The primary equations may be any equations which for abstract or concrete reasons are imposed on some of or all the variables in the authorized list. These equations may be linear or non-linear. We write them

$$S_f(x_1, x_2, \ldots x_N) = 0 \qquad (f = eq) \tag{3.IIf.1}$$

where S indicates the forms of the functions, and the affix f runs through the numbering of all the equations (definitional, technological or administrative) that have been imposed. The variables $x_1, x_2, \ldots x_N$ (or shorter x_i, $i=$all) indicate all the variables in the authorized list. In practice most of the equations (3.IIf.1) will contain only a limited number of the x_i.

A fundamental question is how many degrees of freedom is left in the x_i (i=all) after the imposition of the equations (3.IIf.1). This number is the equational degrees of freedom. This number depends on the *unfolding capacity*, or *rank* of the system (3.IIf.1). This rank can never be larger than the number of equations, but it may be smaller.

If all the equations (3.IIf.1) – let m be their number – are independent within the complete model (if they are "intramodelly independent"), the rank of the system is m. This means that the m equations have really imposed m different conditions, and hence

(3.IIf.2) n = number of degrees of freedom = N-m
(when the equations (3.IIf.1) are intramodelly independent)

More generally: If the rank is ϱ the number of degrees of freedom is

(3.IIf.3) n = number of degrees of freedom = N-ϱ
(ϱ = rank of the system (3.IIf.1))

In the case of a great number of variables, perhaps several thousands, it is not a trivial problem to find out for certain what the rank of the system (3.IIf.1) is, and hence what the number of degrees of freedom is.

To illustrate my point let me briefly summarize some classical facts from linear algebra. Consider any system of linear equations:

$$a_{fo} + \sum_{i=\text{all}} a_{fi}\, x_i = 0 \qquad (f=\text{eq}) \tag{3.IIf.4}$$

The number of equations may be smaller than, equal to or larger than the number of variables. The a_{fo} and a_{fi} are given constants.

Necessary and sufficient for the existence of a set of values of the x_i which will satisfy all the equations (3.IIf.4), is that the rank of the matrix a_{fi} is equal to the rank of the *augmented* matrix, i.e. the matrix obtained from a_{fi} by adding the column a_{fo}.

The rank of any matrix is defined as the row number of the highest rowed, square and non zero determinant that can be formed by picking some rows and some columns from the matrix.

From the classical fact quoted follows that if there are exactly as many equations as variables, and if the determinant value of the matrix a_{fi} is different from zero, there exists one and only one set of values of the x_i

that satisfies the equations. (This is the usual case which is – explicitely or tacitly – *assumed* when presenting the classical methods of "solving" linear equations, by elimination or iteration).

If the rank of the system is less than the number of variables (obviously it can't be larger than this number) we may have either no solution set or an infinity of solution sets, i.e. a certain number of degrees of freedom in the solution.

In the case where the functions S_f are non linear, similar considerations apply in the *vicinity of any given point* in space, the a_{fi} being now replaced by the derivatives $\frac{\partial S_f}{\partial x_i}$ taken at the point considered. Therefore the number of degrees of freedom will in this case be a function of the point in space.

It will easily be seen that in the case of a great number of variables, it might – even in the linear case – be next to impossible *in practice* to find out the number of degrees of freedom by following the *theoretical* rule given in connection with (3.IIf.4). And it would be even more impossible in the non-linear case.

In practice we therefore have to proceed in a different way. Without entering into details I can suggest my way of thinking as follows.

If we assume that in the set up (3.IIf.1) the rank is ϱ in the vicinity of a given point, it is possible to pick a certain set of the variables N-ϱ in number, – in what follows to be called the analytically free variables, and denoted x_h (h=free) – with the following two properties:

(3.IIf.5) The variables x_h (h=free) are independent inside the model (and in the vicinity of the point considered). This means that inside the model there does *not* exist any relation which contains some of (or all) these variables x_h (h=free) and *none of the other variables in the model.*

(3.IIf.6) It is possible to express all the other variables in the model in terms of the picked set x_h (h=free).

The functions that express how these other variables depend on the x_h (h=free), may be termed the *dependence* functions and written

$$x_j = r_j(x_h,\ h=\text{free}) \qquad (j=\text{dep}) \tag{3.IIf.7}$$

The equations (3.IIf.7) constitute the *reduced* form of the primary equations (3.IIf.1), (r for "reduced").

In the most general case it may, of course, happen that the dependence functions (3.IIf.7) are *not* singlevalued, even though the primary functions S_f are singlevalued. Therefore, taking the reduced form (3.IIf.7) as our starting point and assuming the dependence functions r_j to be singlevalued, is really considering only a special case. In what follows I shall consider this special case.

A practical way of testing the number of degrees of freedom will then be to *start* from a reduced system of the form (3.IIf.7) which we have been able to deduce *in some way or another*, and then test that, when the dependence functions r_j thus postulated are inserted into (3.IIf.1), then all the equations (3.IIf.1) reduce to identities $0=0$, for *any* set of values we may have inserted for the x_h (h=free). In this case the number of degrees of freedom is at least equal to the number of variable in the set x_h (x_h=free).

In the last sentence we have to say "at least" and not "exactly". For instance, if the complete dependence functions actually contain three free variables, the test mentioned would be met also if we put the third of these variables equal to some arbitrarily chosen fixed number. But this would mean testing a set of dependence functions of only two free variables.

We ought therefore always to try to make the set of believed free variables *as large as possible*. There is no risk in doing so, because if we overdid it, the abovementioned test would stop us. The primary equations would then not reduce to identities, but to equations by which one or more of the believed free variables could be expressed in terms of the others.

These are useful practical rules both for checking the logic and for checking numerical computations in the case of a large number of variables.

(To be continued)