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Investment starting vs. investment sinking[1]

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1. A VERBAL DEFINITION

Investment *starting* in any given year is the total outlay which it is estimated that the projects started that year will have entailed when they are finally completed – perhaps at some future date. Investment *sinking* in any given year is the value of goods and services that were actually used (that were "sunk") that particular year in order to carry towards completion projects which were started that year or some previous year.

The distinction between investment starting and investment sinking is essential in an investment analysis that is to be realistically dynamic.

2. THE PROJECT DESCRIPTION

The project description is a collection of all the descriptive details regarding a project, that can be given by the specialists (technical engineers, etc) who have detailed knowledge about this project, but do *not* have a systematic knowledge of all the broader *social, economic and political* considerations at the national level that one must take account of before one can reach a well founded decision as to whether this project is to be accepted or not.

A rational and coherent treatment of investment criteria can in my view only be given by considering all the investment projects – defined through the project descriptions – as intergrated parts of a *complete macro-economic decision model* with all its *structural* and (politically) *preferential* aspects. The project descriptions are building stones in the complete decision model.

The great variety of so called "investment criteria" that are frequently

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discussed as criteria that should be applicable to any given project *per se* just as the project is defined through its project description, can therefore, in my opinion *never* lead to a really satisfactory solution of the investment decision problem. The per se approach to investment criteria is only escapism. It may be tempting because of its simplicity, but it has no decisional foundation.

Having adopted this point of view we must stress the fundamental distinction between information that is available *before* the optimization of the decision model and information that only emerges *after* this optimization.

This distinction is the crux of the matter in planning at the national level in any country. In this optimization all the geographical, material, cultural and political peculiarities of the country come into the picture. This broad perspective can, of course, not be compressed into the format of a project description.

Therefore, the project descriptions belong definitely to the kind of information that is available *before* optimization. Such information is a *necessary* basis, but very far from being a sufficient basis for reaching well founded investment decisions.

3. THE SINKING YEAR AND THE STARTING YEAR

Consider a single investment project No. g.

(3.1) t = *the interflow year* (the calendar year) is the year (or quarter or month) to which the complete macroeconomic interflow table with all its balancing and accounting relations apply. In particular, if we are discussing the difference between investment starting and investment sinking, t will be *the sinking year*.

(3.2) σ = *the starting year* is the year when actual work on the execution of the project was begun. The *decision* year i.e., the year when it was decided whether to accept or reject the project No. g, is the year when the whole plan was adopted. This may be an earlier year than σ. As a rule the decision year (the planning year) will be denoted as the year 0. Research work in connection with the plan and in particular research work in connection with the project No. g may have taken place even earlier.

(3.3) $s=t-\sigma$ is the *sinking delay*. Roughly formulated the sinking delay is "the number of years that have elapsed since starting." More precisely formulated: $s=0$ refers to the sinking that will take place in the *same* year as the starting (if the project *is* accepted for starting in a given year). $s=1$ refers to the sinking that will take place in the year that follows immediately after the year when the starting took place. And so on for the higher values of s.

4. SINKING FLOWS

Table (4.1). Sinking Flows for the Project No. g

				Sinking delay s (after the starting year)					
				$s=0$	$s=1$	$s=2$	...	$s=(g)-1$	Row Sums
SINKING INPUTS	From delivering Domestic Sectors	h	1 2 . .	J_{hg}^{0}	J_{hg}^{1}	...	...	...	J_{hg}
	From Domestic Primary Input Factors	i	101 102 . .	J_{ig}^{0}	J_{ig}^{1}	...	...	...	J_{ig}
	From Complementary (non-competitive) Imports		B	J_{Bg}^{0}	J_{Bg}^{1}	...	...	...	J_{Bg}
	Column sums			J_{g}^{0}	J_{g}^{1}	...	...	...	H_{g}^{tot}

J=Gross investment (as distinct from I=net investment after depreciation). H='Hardware'.

Here (g) denotes the number of different years in which sinking inputs for the project No. g will occur (roughly expressed: the *construction* period for project No. g.)

The symbols given in table (4.1) are *general* symbols for the sinking flows and their totals with respect to project No. g. These magnitudes may, for instance, refer to flows that are determined already in the project description, and if so they are denoted J^{*s}_{kg} $(k=h, i, B)$. This happens for all k and s only in the case where no *sinking substitution possibilities* exists. The flows that emerge after the decision model optimization are denoted $\hat{J}^{s}_{kg}$ $(k=h, i, B)$. These latter flows always exist and are well defined (possibly with some degrees of freedom if there remain degrees of freedom in the optimum). In a more general context the symbols J^{s}_{kg} $(k=h, i, B)$ in table (4.1) may be used simply as indicating *variables* that enter into the decision model before optimization.

H^{tot}_{g} denotes the sum total of all the sinking flows in table (4.1).

5. SINKING COEFFICIENTS IN THE NON-SUBSTITUTION CASE FOR SINKING INPUTS

In the special case where no possibility of sinking substitution is assumed to exist, all the flows in table (4.1) are *fixed* and *well defined* already in the project description. Cf. the comments to table (4.1).

In this case we may compute the corresponding system of *sinking coefficients*. They are defined by

$$(5.1) \quad J'^{s}_{kg} = \frac{J^{*s}_{kg}}{H^{*tot}_{g}} \quad \left[H^{*tot}_{g} \begin{array}{l} \text{denotes the total sinking, now} \\ \text{assumed to be determined already} \\ \text{in the project description} \end{array} \right] \quad (k=h, i, B)$$

When the coefficients (5.1) are computed, any of the sinking flows in the project description can be expressed as

$$(5.2) \qquad J^{*s}_{kg} = J'^{s}_{kg} \, H^{*tot}_{g} \qquad (k=h, i, B)$$

where the coefficients J'^{s}_{kg} can be read off from the project description.

Inserting (5.2) for each element in table (4.1) and performing a summation over all the cells of the table we see that the coefficients J'^{s}_{kg} must satisfy the equation

$$(5.3) \quad \sum_{h=\text{sec}} \Sigma^{s} J'^{s}_{hg} + \sum_{i=\text{prim}} \Sigma^{s} J'^{s}_{ig} + \Sigma^{s} J'^{s}_{Bg} = 1 \quad \left[\begin{array}{l} \text{For any } g \text{ for which no} \\ \text{sinking substitution} \\ \text{possibilities exist} \end{array} \right]$$

(sec = sectors, prim = primary input factors, s = sinking delays. The symbol Σ^{s} denotes a summation over all sinking delays.)

6. SINKING COEFFICIENTS AND EQUIVALENCE COEFFICIENTS IN THE CASE OF SUBSTITUTION POSSIBILITIES FOR SINKING INPUTS

If we have the case where sinking substitution possibilities exist, the total outlay H_g^{tot} – the sum of all items in table (4.1) – does not exist as a magnitude that is defined *in the project description*. The sum of all items will then have a definite meaning only *after* optimization.

In this case the *size* of the project in its full dress must in the project description be characterized by some other feature, for instance, by a capacity addition that may be associated with the project (if it is accepted in its full dress) or by some other *conventional* measure for the size of the project in its full dress. Let this conventional measure of the full dress size of the project be H_g^{*con}, the asterisk * indicating that this is a magnitude that can be read off from the project description and *con* indicating that the magnitude is a conventional measure of the full dress size of the project.

Such a conventional measure may, of course, be introduced regardless of whether substitution possibilities exist or not, but in the substitution possibility case it is *necessary* to rely on such a conventional measure.

Even in the substitution possibility case there may be *some*, and perhaps *many*, but not all, of the interflows in table (4.1) that exist as well determined magnitudes already in the project description. And for *these particular* flows we can introduce a project description determined coefficient-concept by expressing each such flow as a fraction of H_g^{*con}, namely

$$(6.1) \qquad J'^{s}_{kg} = \frac{J^{*s}_{kg}}{H_g^{*con}} \quad \text{(For the sinking flows No. } k \text{ – either } h \text{ or } i \text{ or } B \text{ – that } \textit{are} \text{ determined already in the project description)}$$

The dimension of (the denomination of) each such coefficient (6.1) will depend on the nature of the input flow in question, and on the conventional measure that is chosen for H_g^{*con}.

For the cells of table (4.1) for which the flow is *not* determined already in the project description, we assume that we have *instead* information about *equivalence coefficients*.

For instance, if the input elements in the three cells formed by the intersection of the three rows α, β, γ, and the column s of table (4.1) form a sinking input *substitution ring*, the three flows $J^s_{\alpha g}$, $J^s_{\beta g}$, $J^s_{\gamma g}$ are not determined in the project description, but we have instead information about three equivalence coefficients $J^{*eq.s}_{\alpha.rg}$, $J^{*eq.s}_{\beta.rg}$, $J^{*eq.s}_{\gamma.rg}$ such that the three flows $J^{s}_{\alpha g}$, $J^{s}_{\beta g}$, $J^{s}_{\gamma g}$ must satisfy the equation

(6.2) $J^{*eq.s}_{\alpha.rg} J^{s}_{\alpha g} + J^{*eq.s}_{\beta.rg} J^{s}_{\beta g} + J^{*eq.s}_{\gamma.rg} J^{s}_{\gamma g} = H^{*con}_{g}$ (r = a substitution ring $r = (\alpha\beta\gamma)$ for sinking input into the the pıoject g)

(eq = "equivalence" s = sinking delay)

[*Note.* For convenience we may divide through by H^{*con}_{g} in (6.2) so as to obtain the eqivalence coefficients (they are project description determined) given as fractions of H^{*con}_{g}. Cf. also the comments to (6.4).]

Here H^{*con}_{g} is the conventional measure of the full dress size of the project No. g, and $J^{*eq.s}_{\alpha.rg}$, $J^{*eq.s}_{\beta.rg}$, $J^{*eq.s}_{\gamma.rg}$, are equivalence coefficients for each of the three sinking input elements α, β, γ that together from the substitution ring $r = (\alpha\beta\gamma)$ for sinking inputs into the project g in the sinking delay year s.

For instance: Digging work connected with the project g in the sinking delay year s might be performed alternatively in any of the following three ways:

(6.3)

[α = manual labour unaided by digging machines ("chinese communes")
β = use of small and simple digging machines
γ = use of big and technically advanced digging machines]

The meaning of (6.2) is that the amounts to be used of the three kinds (6.3), namely $J^{s}_{\alpha g}$, $J^{s}_{\beta g}$, and $J^{s}_{\gamma g}$ are *not determined by the project description* but may be chosen freely, subject to the condition that the left member of (6.2) always be equal to the conventional full dress measure of the projects, namely H^{*con}_{g}. In other words, *before* optimization of the decision model we leave open the possibility that the necessary sinking input from the ring r to the project g in the sinking delay year s may be achieved either through the input element α, or thorugh β, or through γ or through any *desired combination* of these three elements, which is such as to make the linear form in the left member of (6.2) equal to H^{*con}_{g}. In section 7 we shall consider a type of *restrictions* which it may be realistic to impose in addition to the equivalence equation, but for the moment we will only discuss the equivalence equation as such.

The concrete meaning of the coefficients in (6.2) may be visualized as follows. Take for instance the coefficient $J^{*eq.s}_{\alpha.rg}$. This is a coefficient such that if we choose to use *only* the input element α, and *nothing at all* of

β or γ in the sinking delay year s for the project g, the actual flow $J^s_{\alpha g}$ will have to be equal to

$$(6.4) \qquad J^s_{\alpha g} = \frac{1}{J^{*eq.s}_{\alpha g}} \cdot H^{*con}_g \qquad \text{(if only } \alpha \text{ is used)}$$

Comparing (6.4) with (5.2) we see that the *reciprocal* value of $J^{*eq.s}_{\alpha.rg}$ has the same meaning as the *sinking* coefficient $J'^s_{\alpha g}$ that *would* prevail if only α is used. It other words, the reciprocal values of the three equivalence coefficients in (6.2) represent in a sense *partial* sinking coefficients inside the ring $r=(\alpha\beta\gamma)$. Another interpretation is obtained by differentiating (6.2) with respect to, say, $J^s_{\alpha\gamma}$ keeping $J^s_{\gamma g}$ constant. This shows that differentially $\frac{1}{J^{*eq}_{\alpha.rg}}$ units of α may be substituted for $\frac{1}{J^{*eq}_{\beta.rg}}$ units of β.

In the sinking substitution case the individual sinking flows such as $J^s_{\alpha g}, J^s_{\beta g}, J^s_{\gamma g}$ are not determined by the project description. They are only *variables* to be introduced in the complete decision model before optimization. Therefore, in the sinking substitution case the complete decision model will have *more degrees of freedom before optimization,* than a similar model where no sinking substitution is permitted.

For instance, if the model contains a single equivalence ring $r=(\alpha\beta\gamma)$ as exhibited in (6.2), the model will contain the three variables $J^s_{\alpha g}, J^s_{\beta g}, J^s_{\gamma g}$ which are connected by the equation (6.2). That is, the ring contributes *two degrees* of freedom to the model. On the other hand, if there had been no substitution possibilities among the three elements α, β, γ, we would have had

$$(6.5) \qquad J^s_{\omega g} = J'^s_{\omega g}\, H^{*con}_g \qquad (\omega=\alpha, \beta, \gamma)$$

where the sinking coefficients $J'^s_{\omega g}$ are determined by the project description. Hence, in this non-substitution case the set of three sinking input elements α, β, γ in the sinking delay year s for project g, would have contributed *no* degree of freedom to the model.

While the existence of input equivalence rings increases the number of degrees of freedom in the decision model, it does not introduce any non-linearity. Indeed, the equation (6.2) is a *linear* equation.

7. THE COMPLEMENTARITY RESTRICTIONS THAT MAY BE ASSOCIATED WITH AN EQUIVALENCE RING

Consider again the example (6.3). In concrete reality even the most automatically advanced digging machine can, of course, not be let loose to perform the work *alone* without the aid of any manual labour. This fact may be taken account of in a number of more or less elaborate ways when we construct the complete decision model. But the *simplest* way to do it might be *still to use* the concept of equivalence equations as explained in section 6, but now to complete this point of view by adding a certain type of restrictions which we may term *complementary restrictions*.

The meaning of the complementary restrictions can best be explained by changing slightly the definition of the input element α in the example (6.3), letting now α simply stand for "manual labour", i.e. dropping the specification "unaided by digging machines".

Having changed our example in this way, we may add a restriction expressing the fact that *a part* of the variable $J^s_{\alpha g}$ has to be used as a *complement* to the variable $J^s_{\beta g}$ and *another part* of the variable $J^s_{\alpha g}$ has to be used as a *complement* to the variable $J^s_{\gamma g}$. If we want to *avoid* the complication which it would be to split the variable $J^s_{\alpha g}$ into *several* variables, we can express the essence of the complementarity situation considered simply by introducing a restriction of the form

(7.1) $J^s_{\alpha g} \geqq$ some coefficient times $J^s_{\beta g}$ plus some cofficient times $J^s_{\gamma g}$

We can formalize this idea by imposing a restriction of the form

$$J^{*com.s}_{\alpha.\varrho rg} J^s_{\alpha g} + J^{*com.s}_{\beta.\varrho rg} J^s_{\beta g} + J^{*com.s}_{\gamma.\varrho rg} J^s_{\gamma g} \geqq 0 \tag{7.2}$$

where

$$J^{*com.s}_{\alpha.\varrho rg},\ J^{*com.s}_{\beta.\varrho rg},\ J^{*com.s}_{\gamma.\varrho rg} \tag{7.3}$$

are three coefficients *that are determined in the project description.* ($\varrho =$ "restriction", or more explicitely: $\varrho =$ a restriction associated with *rg*. The superscript *com* indicates "complementarity").

There may be *several* restrictions ($\varrho = 1$, 2, 3 etc.) of the form (7.2) expressing, for instance, the fact that if we choose to use some big digging machines – input elements γ – we may need also some small and simple digging machines – input elements β – as a complement to γ.

The fact that the *coefficients* J^* in (7.2) are determined by the project description, does, of course, *not* mean the actual flows $J^s_{\alpha g}$, $J^s_{\beta g}$ and $J^s_{\gamma g}$ are also determined by the project description. They are in fact still variables. But *if* we choose to put one of the actual flows equal to a given magnitude, any complementarity restriction of the form (7.2) will reduce the *admissibility range* for the other actual flows.

The formal set up (7.2) which introduces a set of restrictions ϱ associated with rg is a very general one. It opens the possibility of expressing *a great variety of complementary* restrictions which we may find it necessary to introduce in order to make the complete decision model *realistic* enough to cover an actual situation. *The equivalence* equations express the fact that substitution possibilities exist, while the *complementary* restrictions express the limitations that exist to these substitution possibilities.

In each complementarity restriction of the form (7.2), – i.e. for any ϱ – we may, since the restriction is homogenous in the actual variables $J^s_{\alpha g}$, $J^s_{\beta g}$, $J^s_{\gamma g}$, *normalize* the coefficients $J^{*com.s}_{\alpha.\varrho rg}$, $J^{*com.s}_{\beta.\varrho rg}$, $J^{*com.s}_{\gamma.\varrho rg}$ in any way which we find convenient, for instance by putting one of the three coefficients equal to unity.

Since restrictions of the form (7.2) do not introduce any *new variable*, and since all such restrictions are of the *inequality* form, the introduction of the complementarity restrictions will neither change the number of degrees of freedom in the model, nor introduced non-linearities. The only exception occurs if we find it realistic *a priori* to replace in any of the complementarity restrictions the symbol $\geqq$ by $=$. In this special case we will, of course reduce the number of degrees of freedom in the complete decision model by as many units as the number of restrictions ϱ for which the symbol $\geqq$ is *a priori* replaced by $=$. Even in this case no non-linearities are introduced.

8. THE HYPOTHETICAL STARTING VARIABLES AND THEIR RÔLE IN THE DECISION MODEL

If we want to have a programming approach from which we require a solution which, for each investment project, will indicate exactly *either* rejection of the project *or* acceptance of the project for starting in its full dress in a specific year σ in the plan period, we would *in principle* have to use *integer* programming. While methods for integer programming are known, it would in practice be a next to impossible task to proceed in this way when *a great number* – perhaps several hundred – of possible investment projects and several possible starting years for each project are considered together with the many other features of a complete decision model.

We will circumvent this practical difficulty in the following way.

Let us introduce the hypothetical starting variables H_g^σ, where σ is any year in the plan period and g is any of the projects. The number of such variables is equal to the number of investment projects (or channels) *times* the number of possible starting years in the plan period. The *phasing* of the investment startings is thus introduced explicitely in the optimality analysis. This is important.

Each of the variables H_g^σ is assumed to be a *continous* variable between 0 and H_g^{*con}, i.e.

$$0 \leqq H_g^\sigma \leqq H_g^{*con} \qquad \text{(for any } \sigma \text{ and any } g\text{)} \tag{8.1}$$

where H^{*con} is the conventional measure of the individual project No. g in its full dress (or the stockpile of projects in the channel g, see below).

We also impose the bounds

$$\sum^\sigma H_g^\sigma \leqq H_g^{*con} \qquad \text{(for any } g\text{)} \tag{8.2}$$

where $\sum^\sigma$ denotes a summation over all the starting years in the plan period. It is not necessary to put on the lower bound zero in (8.2) because this is already assured by (8.1).

The bounds (8.1) and (8.2) will prevent the optimal solution from coming out with *negative* starting variables H_g^σ or with a *repetition* of startings, the sum of which go beyond the total H_g^{*con} for the plan period.

The bounds (8.1) and (8.2) apply equally to the case of *individual* projects g and to the case of investment startings in channels of similar investment projects, any such channel being denoted by g. In the latter case H_g^{*con} must be interpreted as the total stockpile of projects in the channel g. For a number of minor projects way may use the channel concept, and for some of the big and important projects we may consider individual projects.

The rôle to be played by the starting variables H_g^σ in the complete decision model, is brought out by the following rule.

If the optimal solution of the model comes out with the value $H_g^\sigma = 0$, we interpret this as expressing that the project is *not* to be started in the year σ, and if the optimal solution comes out with $H_g^\sigma = H_g^{*\sigma}$ we interpret this as expressing that the project *is* to be started in the year σ in its full dress.

In many cases – particularly in a model that is completely linear – it will be found that *most of the variables* H_g^σ will in the optimum have hit *either* the lower bound 0 or the upper bound H_g^{*con}. If for any project g or channel g one of these two alternatives emerges, the situation is clear.

For an individual project that is *marginal*, i.e. for a project for which the optimal solution comes out with a value of H_g^σ in the *interior* of the interval (8.1), we interpret the result *cum grano salis*. If the optimal value is close to the lower bound 0, we interpret it as rejection, and if it is close to the upper bound H_g^{*con} we interpret it as acceptance.

For a few *extremely important* marginal projects for which it is found that this rule is not sufficiently accurate, we may *after* the general and approximate solution has been obtained, perform a special and partial analysis, now by means of integer programming and assuming as *data* the great variety of *other* starting variables that emerged from the first solution where all H_g^σ were considered as continuous variables.[1]

Now for the question of *how* the hypothetical starting variables are to be imbedded in the model.

They are imbedded by expressing the *sinking flows* that would follow *in any given sinking year t* ($\geqq \sigma$), cf. (3.1), *if* the starting starting variables H_g^σ assumed such and such magnitudes in the various *starting* years σ, cf. (3.2). These expressions being formalized, a summation over σ for any constant t will express the hypothetical structure of the interflow table for the calendar year t in the plan period as a function of all the H_g^σ. These hypothetical interflow tables for each calendar year t in the plan period form the backbone of the dynamic decision model.

I shall indicate some of the crucial formulae that intervene in this connection.

The sinking flows $J_{kg}^{t\sigma}$ ($k=h, i, B$) that *occur* in the sinking year t and are *entailed* by H_g^σ, are determined on the same principles as those we discussed in sections 4–6, with the only difference that now we have added the extra superscript σ indicating the *possible* starting year or years σ ($\leqq t$). In other words, we now use to full extent the *two dimensional* system of time indications which was defined in Section 3.

Consider first the *proportionally* determined sinking goods. They are

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[1] The decision regarding the choice between *alternative sizes* of a certain big and important project g, may be handled without integer programming by entering the alternatives as so many different projects g', g'',... etc. In addition to (8.1) and (8.2) for each alernative we would then impose the *alternativity bound*

$$\text{(8.2b)} \quad \Sigma^\sigma H_{g'}^\sigma + \Sigma^\sigma H_{g''}^\sigma + \ldots \leqq H_{g\max}^{*con}$$

where the right member is the conventional size of the *biggest* alternative.

the sinking goods of category k ($k=h, i, B$) for which the sinking *coefficients* J'^{s}_{kg} can be read off from the project description. (Cf. 5.1).

These sinking flows are determined by

$$(8.3) \qquad J^{t\sigma}_{kg} = J'^{t-\sigma}_{kg} H^{\sigma}_{g} \qquad \text{(proportionality determined sinking flows)}$$

The flow (8.3) is a flow of goods of categories k ($k=h, i, B$) that are *sunk* in the calendar year t and are entailed by (are "due" to) the hypothetical *starting* in year σ in the project g and with the hypothetical starting size H^{σ}_{g} (from the decision model point of view possibly less than the full dress size H^{*tot}_{g} of the project). Similar interpretation if g is a channel.

We assume for simplicity that the sinking coefficients J'^{s}_{kg} that can be read off from the project description, cf. (6.1), are *stationary* in the sense that they do not depend on the particular year σ in which the project might be started. This is why the affix σ on the sinking coeffcient in the right member of (8.3) only appears through the difference $t-\sigma$ (i.e. through the sinking delay, cf. (3.3)), and *not* on t and σ taken *separately* as two independent time indications. A similar remark applies to the equivalence coefficients in (8.4) below and to the complementarity coefficients in (8.5). On the other hand, for the *actual flows* $J^{t\sigma}_{kg}$ (which are *variables* in the model) the *separation* of the two affixes t and σ as two *independent* time indications in the superscript, is absolutely essential.

Next considered the sinking input of the k goods ($k=h, i, B$) that are *not* proportionally determined. For these we have equivalence equations of the form

$$(8.4) \qquad J^{*eq.t-\sigma}_{\alpha.rg} J^{t\sigma}_{\alpha g} + J^{*eq.t-\sigma}_{\beta.rg} J^{t\sigma}_{\beta g} + J^{*eq.t-\sigma}_{\gamma.rg} J^{t\sigma}_{\gamma g} = H^{\sigma}_{g}$$

(For the sinking input elements α, β, γ that are substitutionally connected in the ring r in the sinking year t and are due to starting in the year σ in the project – or channel – g.)

To summarize: In the H^{σ}_{g} formulation the magnitudes which the actual sinking flows $J^{t\sigma}_{\alpha g}$, $J^{t\sigma}_{\beta g}$ and $J^{t\sigma}_{\gamma g}$ will assume are *not* determined as functions of the single variable H^{σ}_{g} (as in the case (8.3)), but when H^{σ}_{g} is given, the three sinking flows considered will have to satisfy the equivalence equation (8.4).

Several such sinking input rings as examplified in (8.4) may exist for an individual project g or a channel g.

The existence of *complementarity restrictions* in a model built on hypothetical starting variables H^{σ}_{g} is easily formalized.

Just as the left member of (8.4) was derived from the left member of (6.2) simply by replacing the superscript s by $t-\sigma$ on the *coefficients*, and by replacing the superscript s by the two *independent* time indications t and σ on the *actual flows*; we now get in analogy with (7.2).

$$J^{*com.t-\sigma}_{\alpha.\rho rg} J^{t\sigma}_{\alpha g} + J^{*com.t-\sigma}_{\beta.\rho rg} J^{t\sigma}_{\beta g} + J^{*com.t-\sigma}_{\gamma.\rho rg} J^{t\sigma}_{\gamma g} \geqq 0 \tag{8.5}$$

Several such complementarity restrictions as examplified in (8.5) may exist for each r and g where the *coefficients* $J^{*com.s}$ $(s=t-\sigma)$ are read off from the project description.