## i Candidate instructions

## ECON4325 - Monetary Policy

This is some important information about the written exam in ECON4325. Please read this carefully before you start answering the exam.

Date of exam: Monday, May 27, 2019
Time for exam: 09.00-12.00 (3 hours)
The problem set: The problem set consists of 3 problems - with subquestions. They will count as indicated.

Sketches: You may use sketches on all problems. You are to use the sketching sheets handed to you. You can use more than one sketching sheet per problem. See instructions for filling out sketching sheets below. It is very important that you make sure to allocate time to fill in the headings (the code for each problem, candidate number, course code, date etc.) on the sheets that you will use to add to your answer. You will find the code for each problem under the problem text. You will NOT be given extra time to fill out the "general information" on the sketching.

Access: You will not have access to your exam right after submission. The reason is that the sketches with equations and graphs must be scanned in to your exam. You will get access to your exam within 2-3 days.

Resources allowed: No written or printed resources - or calculator - is allowed (except if you have been granted use of a dictionary from the Faculty of Social Sciences).
Grading: The grades given: A-F, with $A$ as the best and $E$ as the weakest passing grade. $F$ is fail. Grades are given: Monday, June 17.

## 1 Problem 1 (20\%)

This part contains two short problems. You need to answer both to get full score.

1. The policy rate of the central bank (the reserve rate) affects market rates via the banking sector through three channels: the bank lending channel, the bank capital channel, and the deposit channel. Explain the mechanisms through which these channels affect the market rates.
2. What is forward guidance and what is the "forward guidance puzzle"? Use the following model to explain the "forward guidance puzzle":
$\pi_{t}=\beta E_{t}\left\{\pi_{t+1}\right\}+\kappa y_{t}$
$y_{t}=E_{t}\left\{y_{t+1}\right\}-\frac{1}{\sigma}\left(i_{t}-E_{t}\left\{\pi_{t+1}\right\}\right)$.

Hint: Solve the model forward.

## 1.

Most economic agents do not face the policy rate directly, but are affected by market rates via the banking sector. The banks are monopolistic and the policy rate is an opportunity cost for them. The banks maximise profits which is the revenue they earn from lending and from their reserves, minus the costs of deposits.
This is subject to their balance sheet constraint: Equity + Deposits = Loans + Reserves, and their capital constraint: Equity must be larger or equal to a fraction of total loans. In the sketching paper I will show the solution of the bank problem, which is what determines how banks choose their lending rates and deposit rates.

Bank lending channel: Banks choose their lending rates as a markup multiplied by the policy rate plus the shadow value of their capital constraint. (see sketching paper). When the central bank lowers the policy rate, banks will also lower their lending rates.

Bank capital channel: This is the channel that affects banks' capital constraints. When the central bank lowers the policy rate, asset prices will increase and so the bank will have a slacker capital constraint. (see sketching paper). This affects the kappa*lambda part of the optimal lending rate, so the lending rate will decrease as a response to a reduced policy rate. The bank will also be able to give out more loans with a slacker constraint, which will stimulate the economy.

Bank deposit channel: Banks choose their deposit rates as a mark-down of the policy rate. Since the rate on deposits is a cost for the bank, they will never choose a deposit rate that is lower than the policy rate. When the central bank reduces the policy rate, banks will also reduce their deposit rates.
2. Forward guidance is a type of unconventional monetary policy, especially effective when the interest rate is at the zero lower bound. The central bank can stimulate the economy by promising a lower rate than the Taylor rule in the future. The effect is through managing expectations. When the central bank gives a promise, the agents in the economy will revise their expectations, and hence both output gaps and inflation today will be affected. This will depend on both credibility and ability to commit in the future. Agents will not revise their expectations if they do not believe that the central bank will stick to its promise.

In the sketching paper we see the solutions for output gap and inflation when solved forward. We see that the output gap depends on expectations of interest rates in the future, which means that when the central bank promises to increase the interest rate in period $t+k$, this affects all output gaps from $t+k$ to today, which again will affect inflation today. In addition, in the solution we see that a promise to change the interest rate even further into the future will affect the economy even more today, something which is called the forward guidance puzzle. Some proposed solutions to the puzzle is to add discounting to the model, so that the future is discounted more than the current periods, or to add wage stickiness or information frictions.

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1) Barks' optimal lending rate:

$$
i^{L}=\frac{\epsilon_{L}}{\epsilon_{L}-1}\left(i^{r}+k \lambda\right)
$$

mark-up poling rate
$\tau$
ch ending
channel

Shadow value of cup ital constraint $\uparrow$
Bank capital channel

$$
\begin{aligned}
& \text { 2) } \pi_{t}=\beta E_{t}\left[\pi \pi_{t+1}\right]+k y_{t} \\
& \pi_{t}=\beta E_{t}\left[\beta E_{t}\left[\left[\pi_{t+2}\right]+k y_{t+1}\right]+k y_{t}\right. \\
& \pi_{t}=\beta^{2} E_{t}\left[\beta E_{t}\left[\pi_{t+3}\right]+k y_{t+2}+k y_{t+1}\right]+k y_{t} \\
& \vdots \\
& \pi_{t}=k E_{t} \sum_{k=0}^{\infty} \beta^{k}\left[y_{t+k}\right] \\
& y_{t}=E_{t}\left[y_{t+1}\right]-\frac{1}{\sigma}\left(i_{t}-E_{t}\left[\pi_{t+1}\right]\right) \\
& y_{t}=E_{t}\left[y_{t+2}-\frac{1}{\sigma}\left(i_{t+1}-E_{t}\left[\left(\pi_{t+2}\right)\right]-\frac{1}{\sigma}\left(i_{t}-E_{t}\left[\left(\pi_{t+1}\right]\right)\right.\right.\right. \\
& y_{t}=E_{t}\left[y_{t+3}-\frac{1}{\sigma}\left(i_{t+2}-E_{t}\left[\pi_{t+3}\right]\right)-\frac{1}{\sigma}\left(i_{t+1}-E_{t}\left[\pi_{t+2}\right]\right)\right]-\frac{1}{\sigma}\left(i_{t}-E_{t}\left[\pi_{t+1}\right]\right) \\
& \vdots \\
& \vdots \\
& y_{t}=-\frac{1}{\sigma} E_{t} \sum_{k=0}^{\infty}\left(i_{t+k}-\pi_{t+k+1}\right)
\end{aligned}
$$

1) Banks optimal deposit rate: deposit channel

$$
i^{d}=\frac{\epsilon d}{\epsilon d+1}
$$

mark-down policy rate

2 Problem 2 ( 30 \%)
In this problem, we are going to solve the firm problem with sticky prices. The firm problem is:
$\max _{P_{t}^{*},\left\{N_{t+k}, Y_{t+k}\right\}_{k=0}^{\infty}} E_{t} \sum_{k=0}^{\infty}(\beta \theta)^{k}\left\{\frac{U_{c, t+k}}{U_{c, t}} \frac{P_{t}}{P_{t+k}}\left(P_{t}^{*}(i) Y_{t+k \mid t}(i)-W_{t+k} N_{t+k \mid t}(i)\right)\right\}$
subject to
$Y_{t+k}(i)=N_{t+k}(i)^{1-\alpha}$
$Y_{t+k \mid t}(i)=\left(\frac{P_{t}^{*}(i)}{P_{t+k}}\right)^{-\epsilon} Y_{t+k}$
where $\theta$ is the probability that the firm cannot change the price in a period, $\beta$ is the discount factor, $P_{t}^{*}$ is the optimal price in period $t, U_{c, t+k}$ is the marginal utility of consumption in period $t+k, W$ is the aggregate nominal wage, $N$ is hours worked, $Y$ is output, $P$ is the price, $\alpha$ governs the curvature of the production function, and the notation ${ }_{t+k \mid t}$ refers to the value of that variable in period $t+k$ for a firm that last reset its price in period $t$.

1. The current period version of the demand for goods is $Y_{t}(i)=\left(\frac{P_{t}(i)}{P_{t}}\right)^{-\epsilon} Y_{t} \quad$ (DEMAND) Interpret equation (DEMAND).
2. Log-linearize equation (DEMAND).
3. The Lagrangian for the firm problem is

$$
\begin{aligned}
& L_{t}=E_{t} \sum_{k=0}^{\infty}(\beta \theta)^{k} \frac{U_{c, t+k}}{U_{c, t}} \frac{P_{t}}{P_{t+k}}\left[P_{t}^{*}(i) Y_{t+k \mid t}(i)-W_{t+k} N_{t+k \mid t}(i)\right. \\
& -\zeta_{t+k \mid t}(i)\left(Y_{t+k \mid t}(i)-\left(\frac{P_{t}^{*}(i)}{P_{t+k}}\right)^{-\epsilon} Y_{t+k}\right) \\
& \left.-\Psi_{t+k \mid t}(i)\left(Y_{t+k \mid t}(i)-N_{t+k \mid t}(i)^{1-\alpha}\right)\right]
\end{aligned}
$$

Find the first order conditions with respect to hours $N_{t}(i)$ and output $Y_{t}(i)$ and solve them for $\Psi_{t}(i)$ and $\zeta_{t}(i)$, respectively. What is $\Psi_{t}(i)$ ? What is $\zeta_{t}(i)$ ?
4. One can combine the first order condition with respect to $P_{t}(i)$ with the two first order conditions we have found in (3) to get the following price setting equation:
$\sum_{k=0}^{\infty} \underbrace{\theta^{k}}_{i} E_{t}\{\underbrace{\beta^{k} \frac{U_{c, t+k}}{U_{c, t}} \frac{P_{t}}{P_{t+k}} Y_{t+k \mid t}(i)}_{i i} \underbrace{\left[P_{t}^{*}(i)-\frac{\epsilon}{\epsilon-1} \Psi_{t+k \mid t}(i)\right]}_{i i i}\}=0$
Interpret this price setting equation. The answer should explain the intuition for why each of the three terms (i, ii, and iii) are in the optimal price setting equation.
1)

The demand for good $i$ is determined by its individual price relative to the aggregate price in the economy to the power of how elastic substitution between good is, multiplied by the aggregate demand. So if the price differs substantially from the average, it will affect demand more, either positively or negatively. Due to price stickiness, firms should choose their optimal price to be as close to the aggregate as possible, it may not be able to change prices in a while.
2) See sketching paper
3) See sketching paper

## 4)

The price setting equation is the solution for how firms choose their optimal prices. It depends on (i) which is the probability of being stuck with a price in one period. (ii) is how much the firm discounts future marginal utility of consumption and future prices, both relative to today, multiplied by the output. (iii) is the future marginal profits, which is the difference between the optimal price and marginal costs, like what was found in part 3).
So in conclusion, the firm chooses its optimal price as a markup of the weighted average of current and future nominal marginal costs, where it needs to take into account the cost of being stuck with a price, and how much it discounts the future. Also, it puts a higher weight on periods with more demand.

Besvart.

divide by steady -state to get:

$$
\begin{aligned}
& e^{\tilde{Y}_{t}(i)}=\left(\frac{e^{\tilde{p}_{t}(i)}}{e^{\tilde{p}_{t}}}\right)^{-t} e^{\tilde{Y}_{t}} \quad \text { take logs: } \quad \text { l } \quad \text { 䜹 }(i)+\epsilon \tilde{P}_{t}+\tilde{Y}_{t}
\end{aligned}
$$

3) $\partial \mathcal{L}$

$$
\begin{aligned}
& \frac{\partial \mathcal{L}}{\partial N_{t}(i)}=-W_{t}+(1-\alpha) \Psi_{t}(i) N_{t}(i)^{-\alpha}=0 \\
& \psi_{t}(i)=\frac{W_{c}}{(1-\alpha) N_{t}(i)^{-\alpha}}=\frac{W_{t}}{M P N_{t}(i)}
\end{aligned}
$$

$\psi_{t}(i)$ is the nominal marginal costs

$$
\zeta_{t}(i)=P_{t}^{*}(i)-\psi_{t}(i)
$$

$\delta_{t}(i)$ is the nominal marginal profits
under an interest rate rule. We assume that the model is
$\pi_{t}=\beta E_{t}\left\{\pi_{t+1}\right\}+\kappa \tilde{y}_{t} \quad(\mathrm{PC})$
$\tilde{y}_{t}=E_{t}\left\{\tilde{y}_{t+1}\right\}-\frac{1}{\sigma}\left(i_{t}-E_{t}\left\{\pi_{t+1}\right\}\right)+z_{t}$
$i_{t}=\phi_{\pi} \pi_{t}+\phi_{y} \tilde{y}_{t} \quad(\mathrm{TR})$
$z_{t}=\rho_{z} z_{t-1}+u_{t}^{z}$
$u_{t}^{z} \sim N\left(0, \sigma_{z}\right)$
$\kappa=\left(\sigma+\frac{\phi+\alpha}{1-\alpha}\right)\left(\frac{1-\theta}{\theta}\right)(1-\beta \theta)\left(\frac{1-\alpha}{1-\alpha+\alpha \epsilon}\right)$

1. By inserting (TR) into (DIS), we get
$\tilde{y}_{t}=E_{t}\left\{\tilde{y}_{t+1}\right\}-\frac{1}{\sigma}\left(\phi_{\pi} \pi_{t}+\phi_{y} \tilde{y}_{t}-E_{t}\left\{\pi_{t+1}\right\}\right)+z_{t}$
(DIS-TR)

Guess that

$$
\begin{aligned}
& \tilde{y}_{t}=\psi_{y z} z_{t} \\
& \pi_{t}=\psi_{\pi z} z_{t} \\
& E_{t}\left\{\tilde{y}_{t+1}\right\}=\rho_{z} \psi_{y z} z_{t} \\
& E_{t}\left\{\pi_{t+1}\right\}=\rho_{z} \psi_{\pi z} z_{t}
\end{aligned}
$$

Insert the guesses into (PC) and (DIS-TR) and show that the solution to the equation system is

$$
\begin{aligned}
& \psi_{y z}=\left(1-\beta \rho_{z}\right) \Lambda_{z} \\
& \psi_{\pi z}=\kappa \Lambda_{z} \\
& \Lambda_{z}=\frac{\sigma}{\left(1-\beta \rho_{z}\right)\left(\sigma\left(1-\rho_{z}\right)+\phi_{y}\right)+\kappa\left(\phi_{\pi}-\rho_{z}\right)}
\end{aligned}
$$

2. The figure below shows the impulse responses of the output gap, inflation, the nominal interest rate, and the discount rate shock to an initial shock to $u_{0}^{z}$ of -0.5 . We have used the following calibration: $\beta=0.99, \sigma=1, \phi=5, \alpha=0.25, \epsilon=9, \theta=0.75, \phi_{\pi}=1.5, \phi_{y}=0$, and $\rho_{z}=0.5$. Provide the economic intuition for the results in the figure. What is going on?

3. We now change the calibration by reducing $\epsilon$ from 9 to 2 . All other parameters are the same as in problem 2. The figure below presents the impulse responses in the benchmark scenario and in the new calibration. Explain what $\epsilon$ is and why it affects all the impulse responses to the discount rate in the way presented in the figure below.

4. We now change the calibration by reducing $\phi$ from 5 to 1 . All other parameters are the same as in problem 2. The figure below presents the impulse responses in the benchmark scenario and in the new calibration. Explain what $\phi$ is and why it affects all the impulse responses to the discount rate in the way presented in the figure below.

5. We are now going to solve for the optimal simple rule of the form:
$\tilde{y}_{t}=\omega z_{t} \quad$ (SR)

Assume that the central bank minimizes the following loss function

$$
L=\frac{1}{2} \mathbb{E}_{t} \sum_{k=0}^{\infty} \beta^{k}\left(\lambda \tilde{y}_{t+k}^{2}+\pi_{t+k}^{2}\right) \quad \text { (LOSS) }
$$

where $\lambda>0$. Minimize equation (LOSS) subject to the Phillips curve (PC) under the simple rule (SR). What is the optimal simple rule? How can one implement this optimal simple rule with an interest rate rule of the form $i_{t}=\omega_{i} z_{t}$ ?

## Fill in your answer here and/or on sketching paper

1) See sketching paper

## 2)

The figure show the impulse responses to a discount rate shock of -0.5 . What happens is that on impact, the discount rate shock itself decreases by 0.5 . Because the discount rate follows an $\operatorname{AR}(1)$ model, with $|r h o(z)|<1$, the shock will with time be dampened and it goes gradually back to its initial value after 7-8 quarters. A negative discount rate shock is a type of negative demand shock. So, when demand is lowered, output gap will also decrease on impact. The output gap decreases by approximately 0.6 on impact, but like the discount rate, the effect wears out and after approx. 7 quarters it has gone back to its initial value. Inflation is affected through the output gap, and decreases even more on impact: by -0.8 . Also inflation will go gradually back to its initial value as the shock dampens out. The nominal interest rate will be decreased on impact to stimulate the economy after a negative discount rate shock. As the economy goes back gradually to normal, so will the nominal interest rate.
3)

Epsilon is the elasticity of substitution between goods and the parameter affects our system through kappa. A reduced epsilon is the same as more inelastic substitution between goods, which implies that demand is less affected by differences in prices. So, the firm can afford to change prices more, since they will lose less demand than before, i.e. its market power has increased. When the cost of changing prices is decreased, it means that inflation is more affected by changes in the output gap (kappa has increased). So we see that the effects of a discount rate shock of -0.5 will be different from before. First, the effect on the output gap will be lower than before, now the output gap decreases only by 0.4 . The reason is that the demand will be reduced because of a negative discount rate shock, but less than before since the firm has a higher market power. It can afford to change prices more, which is reflected in the impulse response for inflation. More firms can afford to change prices on impact, so inflation will decrease by much more than in part 2). Because inflation is more affected by the shock, the nominal interest rate needs to change more to counteract the effect of the shock. So, nominal interest rate must decrease more relative to part 2).

## 4)

Phi is the curvature of the function of disutility of labour, and measures its convexity. A lower phi means that the disutility of labour function becomes less convex, and the individual will not need as high wage as before in order to work more. This implies that the firm gets lower marginal costs, because it gets cheaper labour. Kappa decreases as phi decreases. As the discount rate shock happens, output gap will decrease more relative to part 2), since the firm has lower marginal costs. Because inflation depends less on output gap than before, it will vary less on impact relative to part 2). Because of this, the nominal interest rate does not need to be decreased by as much as before, since the trade-off between inflation and output gap is not as big.
5) See sketching paper.

If the simple rule is of the form $\mathrm{it}=$ omega*zt, then the central bank is following a Taylor rule with zero weight on inflation and full weight on stabilising the output gap. This can be the case in an economy where kappa=0, so the inflation does not depend on the output gap. To fully stabilise the output gap comes with no inflationary cost. The tradeoff we see in the sketching paper is non-existent because kappa=0.

$$
\begin{aligned}
& \text { Tegneomràde Drawing area } \\
& \text { 1) } \psi_{\pi z} z_{t}=\beta \rho_{z} \psi_{\pi z} z_{t}+k \psi_{y z} z_{t} \quad(P C) \\
& \left.\psi_{y z} z_{t}=p_{z} \psi_{y z} z_{t}-\frac{1}{\sigma}\left(\phi_{\pi} \psi_{\pi z} z_{t}+\phi_{y} \psi_{y z} z_{t}-p_{z} \psi_{\pi z} z_{t}\right)+z_{t}{ }^{(D i s-T}\right) \\
& \text { (pc): } \psi_{\pi z}\left(1-\beta p_{z}\right)={ }_{k} \psi_{y z} \\
& \psi_{\pi z}=\frac{k}{1-\beta \rho_{z}} \psi_{y z}
\end{aligned}
$$

Insert into (D SS-TR):

$$
\begin{aligned}
& \psi_{y z}=\rho_{z} \psi_{y z}-\frac{1}{\sigma}\left(\phi_{\pi} \frac{k}{1-\beta \rho_{z}} \psi_{y z}+\phi_{y} \psi_{y z}-\rho_{z} \frac{k}{1-\beta \rho_{z}} \psi_{y z}\right)+1 \\
& \left(1-\rho_{z}+\frac{\phi_{\pi} k}{\sigma\left(1-\beta \rho_{z}\right)}+\frac{\phi_{y}}{\sigma}-\frac{\rho_{z} k}{\sigma\left(1-\beta \rho_{z}\right.}\right) \psi_{y z}=1 \\
& \left(1-\beta \rho_{z}\right)\left(\sigma\left(1-\rho_{z}\right)+\phi_{y}\right)+k\left(\phi \pi-\rho_{z}\right) \\
& \sigma\left(1-\beta \rho_{z}\right) \\
& \psi_{y z}=1 \\
& \psi_{y z}=\frac{\sigma\left(1-\beta \rho_{z}\right)}{\left(1-\beta \rho_{z}\right)\left(\sigma\left(1-\rho_{z}\right)+\phi_{y}\right)+k\left(\phi_{\pi}-\rho_{z}\right)}=\left(1-\beta \rho_{z}\right) \Lambda_{z} \\
& \psi_{\pi z}=\frac{k}{\left(1-\beta \rho_{z}\right)}\left(1-\beta \rho_{z}\right) \Lambda_{z}=k \Lambda_{z}
\end{aligned}
$$



